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Treatment of neutrino oscillations in a thermal environment

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We consider the mixing of neutrinos being scattered randomly by a background medium. We point out that quantum damping effects will tend to inhibit neutrino oscillations and provide some implications for the early Universe. If the off-diagonal terms in the neutrino mass matrix are small compared to an "effective collision rate" then oscillations will be blocked.

The treatment of particle mixing, as among neutral K mesons or among neutrino types, is well understood when we deal with the propagation of a beam of particles, whether it be in a vacuum or in a medium. There is another class of problems, however, where we are concerned with the evolution of a statistical ensemble simultaneously mixing and scattering in a medium: for example, neutrinos during stellar collapse or in the early Universe. Such problems cannot be easily understood in terms of the propagation of a beam. There is no beam in the first place and in any event we do not expect coherent oscillations of the wave function under conditions of interaction with a stochastic background medium. In particular, there is the somewhat subtle question of the effect of interactions with the medium on the coherent superpositions of states which are involved. If one has, say, $\alpha \mid \nu_e \rangle + \beta \mid \nu_\mu \rangle$, and a collision occurs, do we have, after collision, a linear combination again or an incoherent mixture with probabilities $\mid \alpha \mid^2$ and $\mid \beta \mid^2$? It will be seen shortly that the answer to this kind of question is physically significant and actually affects the way in which the mixing proceeds.

A method has been developed for treating such problems.[1] One works with the density matrix in the variable which "mixes," and studies the time evolution of this matrix under the simultaneous influence of mixing, as it would occur in vacuum alone, and the influence of repeated random interactions with the medium. For a two-component system, as with two neutrino types (which we shall refer to as ν_e and ν_μ) we have a 2-by-2 density matrix which may be written in terms of the Pauli matrices $\boldsymbol{\sigma}$ and a "polarization vector" $\mathbf{P}$:

$$\tfrac{1}{2}(1+\mathbf{P}\cdot\boldsymbol{\sigma}) , \tag{1}$$

where "spin up" signifies a ν_e and "spin down" a ν_μ. Thus P_z gives the excess of ν_e over ν_μ, such that $P_z = +1$ implies that all neutrinos are ν_e and $P_z = 0$ implies that half are ν_e and half are ν_μ. P_x and P_y, which we call the "transverse" components $\mathbf{P}_T$, contain information on the phase relations between the components. The length of $\mathbf{P}$ gives the degree of coherence: length 1 corresponds to a pure state, shorter P's to some degree of incoherence, and length zero is the completely mixed or incoherent state.

The time evolution is described through the time dependence of the vector $\mathbf{P}$. In a vacuum, given a certain $\mathbf{P} \neq 0$ at $t=0$, the polarization vector simply precesses without loss of length according to the equation

$$\frac{d\mathbf{P}}{dt} = \mathbf{V}^{\text{vac}} \times \mathbf{P} , \tag{2}$$

where the vector $\mathbf{V}^{\text{vac}}$ contains the information concerning the mass-mixing matrix. This information is as follows: The angle which $\mathbf{V}^{\text{vac}}$ makes with the vertical (z) axis (Fig. 1) is twice the usual mixing angle for the two neutrino types, and the length of $\mathbf{V}^{\text{vac}}$ is determined by the mass difference of the neutrino mass eigenstates 1 and 2:

$$\mid \mathbf{V}^{\text{vac}} \mid = E_2 - E_1 \simeq \frac{{m_2}^2 - {m_1}^2}{2K} \tag{3}$$

for relativistic neutrinos of momentum K. In a spin visualization $\mathbf{V}$ represents a magnetic field around which the "polarization" $\mathbf{P}$ precesses. Note that the transverse components of $\mathbf{V}$, $\mathbf{V}_T$ are given by the off-diagonal elements of the mass matrix.

Now let a medium be present. We ask not merely how a beam propagates but also for the evolution of the whole ensemble, including those particles scattered out of the "beam." Two types of effects will be induced. One will be an index of refraction for the neutrinos which, in effect, gives a medium-induced contribution to $\mathbf{V}$—changing the axis and speed of precession but not the length of $\mathbf{P}$. If it is true, as is the case for the ordinary neutrinos, that to a good approximation the scattering on the medium conserves neutrino flavor, then this contribution to $\mathbf{V}^{\text{med}}$ must be along the z axis. Otherwise $\mathbf{V}^{\text{med}}$ would rotate $\mathbf{P}$ in a flavor-changing direction. When this $\mathbf{V}^{\text{med}}$ is added to the vacuum $\mathbf{V}^{\text{vac}}$, the tendency will be to produce a new vector making an even smaller angle with the z axis, and hence a smaller mixing. This is the point noted by Wolfenstein[2] that, in general, the index of refraction will tend to suppress mixing. An exception, however, can arise if the vacuum $\mathbf{V}$ and the medium-induced $\mathbf{V}$ point in opposite directions and are about the same in size. Then the angle and therefore the mixing can be increased. This is the circumstance exploited by Michaeev and Smirnoff[3] as a possible way of getting enhanced $\nu_e \rightarrow \nu_\mu$ conversion in the Sun. The second effect induced by the medium is a shrinkage of the vector **P**. This is due to collisions which destroy the coherence of the evolution. This incoherence is the new element not present for a beam of neutrinos. When this shrinkage is strong it can influence the qualitative behavior of the mixing process. Once again the assumption that flavor is conserved in the scattering on the medium means that this shrinkage must be perpendicular to the z (flavor) axis. One thus arrives at the generalization of Eq. (2) to the case of the medium

$$\frac{d\mathbf{P}}{dt}=\mathbf{V}\times\mathbf{P}-D\mathbf{P}_T \ , \tag{4}$$

where $\mathbf{V}=\mathbf{V}^{\text{vac}}+\mathbf{V}^{\text{med}}$ and D is a damping parameter giving the rate of loss of coherence of the ensemble. $\mathbf{P}_T$ is the "transverse" part of **P**.

The two parameters **V** and D are the real and imaginary parts of a certain expression[1] which for scattering on a species i of the medium is given by the scattering amplitudes of ν_e and ν_μ on i:

$$\mathbf{V}_z^{\text{med}}=Nv\frac{\pi}{K_{\text{c.m.}}{}^2}\text{Re}\,i\langle i \mid 1-S^\dagger_{\nu_e}S_{\nu_\mu} \mid i\rangle \ , \tag{5a}$$

$$D=Nv\frac{\pi}{K_{\text{c.m.}}{}^2}\text{Im}\,i\langle i \mid 1-S^\dagger_{\nu_e}S_{\nu_\mu} \mid i\rangle \ . \tag{5b}$$

A sum must be taken over the various momenta and species, which in the present problem will be a sum over thermal distributions. The quantity D may be thought of as a rate parameter measuring the effectiveness of the collisions in interupting the mixing of the two states, i.e., the frequency at which collisions stop the coherent development of the wave function. In the case where only one neutrino type interacts and the other one does not, Eq. (5b) gives simply one-half the collision rate of the interacting neutrino or, expressed as a distance, one-half the inverse mean free path for the interacting neutrino. In the opposite extreme where both neutrinos scatter identically, $D=0$ since $S^\dagger S=1$. There is no damping—the medium has not "measured" which neutrino is present.

One anticipates regimes of different types of behavior for **P** according to the relationship between the time between (effective) collisions and the times involved in the vacuum oscillations. Weak damping changes the sinusoidal oscillations of **P** in the vacuum situation to damped oscillations, and increasing in strength, eventually gives overdamped behavior in the strong damping limit. The strong damping limit is particularly interesting since it can qualitatively change the nature of the mixing process. When D becomes very large, the **P** vector changes very slowly, becoming fixed to the z axis. This is because with strong damping the transverse components of P are quickly shrunk away, which tends to align **P** strongly along the z axis. Note that in the absence of a transverse component of **V** a constant **P** along the z axis would be a solution of the equations. Thus, in this limit the time dependence is essentially due to the P_T generated by V_T, the transverse component of V.

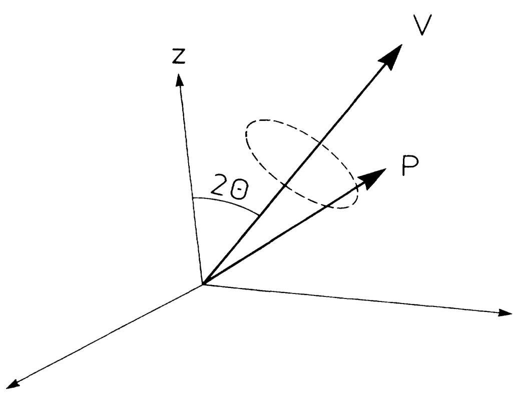

FIG. 1. Relationship of **P** and **V** in the absence of damping. The z or flavor axis is vertical so that **P** "up" represents a ν_e and **P** "down" a ν_μ. **P**, characterizing the density matrix, precesses around **V** which represents the influence of the mass mixing matrix and the index of refraction of the medium.

We can make these remarks more quantitative by examining the general solution[4] with constant parameters for Eq. (4). This can be obtained by assuming exponential time behavior for **P** and finding the eigenvectors and eigenvalues of the right-hand side (RHS) of Eq. (4). First, we find that with $\mathbf{V}_T=0$ there is one eigenvalue zero, corresponding to **P** along the z axis and constant. When a small $\mathbf{V}_T$ is switched on this eigenvalue becomes $\mathbf{V}_T{}^2D/(D^2+V_z{}^2)$. The other eigenvalues are approximately equal to D and correspond to **P** approximately perpendicular to the z axis. We thus conclude that the strong damping limit, where **P** is pinned to the z axis, is given by

$$D \gg V_T \tag{6a}$$

and that in this limit the relaxation time of **P** is

$$\mathbf{V}_T{}^2\frac{D}{D^2+\mathbf{V}_z{}^2} \ . \tag{6b}$$

At finite temperatures, larger than the energy splitting of the two neutrinos, the true equilibrium state is, of course, always an equal mixture of both particle types, corresponding to $P=0$; and in a sufficiently long time, all solutions of Eq. (4) do indeed relax to $\mathbf{P}=0$. However, in an actual problem the "freezing in" of **P** via the strong damping may be so effective on the scale of other relevant times that, in effect, equilibrium never exists.

To understand when this may or may not happen, it is necessary to estimate the values of **V** and D, to which we now turn. We begin with $\mathbf{V}^{\text{med}}$, as given by Eq. (5a). Since we are dealing with weak interactions, we expand to first order in G and find by writing $S=1+iT\sim 1+iG$ that, to order G, $\mathbf{V}^{\text{med}}$ involves the difference in the forward elastic scattering amplitudes for ν_e and ν_μ. Since the forward-scattering amplitude f can be related to the refractive index via $n=1+(2\pi/K^2)Nf$, N the number density,

$$\mathbf{V}^{\text{med}}=\mathbf{V}_z^{\text{med}}=K(n_{\nu_e}-n_{\nu_\mu}) \ . \tag{7}$$

We therefore need Kn for ν_e and ν_μ, as calculated from the forward, no-spin-flip, elastic scattering amplitude. For standard-model interactions at low energy, where we can neglect the nonlocal effects of the intermediate-boson propagators, the effective Hamiltonian governing neutrino scattering can be written as a charged-current plus a neutral-current part. For the elastic scattering amplitude, as needed to calculate the refractive index, the Fierz transformation invariance property of the $V-A$ interaction allows us to rewrite the charged-current part in charge-retention order, giving an effective neutral-current interaction. The effective weak current of the target will then be constructed of fields such as $\bar{e}e$, $\bar{\nu}_\mu \nu_\mu$, etc. Since all the fields are classified as isosinglets or isodoublets in the standard model, the effective current of the medium can then be represented as a sum of the third component of an isovector and an isoscalar, so that the index of refraction can be expressed as a combination of the electric charge and weak-isospin densities. For ν_e, for example, we thus arrive at

$$K(n_{\nu_e}-1) = -G\sqrt{2}(I_3^L - 2Q\sin^2\theta_W) - G\sqrt{2}(N_e^L + \tfrac{1}{2}N_{\nu_e}^L) \,, \tag{8}$$

I_3^L and Q are the densities per cm^3 of weak isospin, and charge. (I_3^L, N_e^L, and N_ν^L are left-handed densities. For example, N^L is equal to the number density of particles minus antiparticles for slow particles and equal to two or zero times the number density for relativistic left- or right-handed particles, and reversed for antiparticles. For the refractive indices of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ reverse the sign of the expression.) When the neutrino scatters on a lepton of its own flavor there is an extra contribution to the forward-scattering amplitude due to charged-current or identical particle effects. This extra contribution has been written in the second set of parentheses on the RHS in Eq. (8), so that the first set of parentheses is common for ν_e and ν_μ. For the index of refraction of ν_μ, then, change the label in the second set of parentheses. A consequence of Eq. (8) is that n does not deviate from one if a particle and its CP-conjugate state are present in equal amounts, and so at sufficiently low temperatures the only contributions to $n-1$ arise from the matter-antimatter excess of the medium. We stress, however, that this is only a consequence of the low-energy approximation and does not follow from general symmetry arguments. In fact, Fukugita, Nötzold, Raffelt, and Silk[5] point out that the small effects coming from the energy dependence of the intermediate-boson propagators at low energy are important in calculating the refractive index since they lead to terms in n involving the total matter density, instead of just the matter-antimatter excess. Therefore deviations from the low-energy approximation Eq. (8) may set in rather rapidly, around an MeV in the early Universe.

We now turn to the evaluation of the damping parameter D. D may be viewed as a kind of "unitarity defect." Writing $S=1+iT$, we have that D is essentially the imaginary part of

$$T_{\nu_e} + T_{\nu_\mu} - 2iT_{\nu_e}^\dagger T_{\nu_\mu} \,, \tag{9}$$

an expression that would be zero by the optical theorem if both T's were the same scattering amplitude. Indeed, using the optical theorem for the linear part of the expression to express it as a sum of cross sections for each final channel F we can write $D=\sum_F D_F$:

$$D_F \sim \sigma_{F,\nu_e} + \sigma_{F,\nu_\mu} - 2\,\mathrm{Re}T_{F,\nu_e}^\dagger T_{F,\nu_\mu} \,. \tag{10}$$

This expression is simple to evaluate in two extreme limits. If $T_{F,\nu_e}=T_{F,\nu_\mu}$ it is zero, and the channel F makes no contribution to D. At the other extreme, where one T is zero, the contribution is given by the cross section for the other neutrino, the one which does interact. In the general case the contribution may be expected to involve some intermediate value which might be typically similar to $\sigma_{F,\nu_e}+\sigma_{F,\nu_\mu}-2(\sigma_{F,\nu_e}\sigma_{F,\nu_\mu})^{1/2}$ (equal cross sections do not necessarily mean $D=0$). We may thus estimate the value of D by considering the various reaction channels for the two neutrinos. For those where both scatter identically there is no contribution to D. The medium does not "measure" the neutrinos. For those where only one type scatters there is a "full" contribution to D which, according to Eq. (5b) (Ref. 1), is one-half the interaction rate, or one-half the inverse mean free path for the reaction in question. In the intermediate case we anticipate some smaller fraction of the interaction rate of the reactions involved. D is then the sum of these "full" or "partial" rates.

To illustrate these remarks let us consider the contributions to D in ν_e-ν_μ mixing from reactions involving various quarks, leptons, and nucleons, as might be relevant in the early Universe or in stellar collapse. In Table I we have listed various target species i to be considered (horizontally). We begin with a discussion of elastic scattering reactions where the incoming neutrino is also present in the final state. Assuming standard-model interactions with ν_e-ν_μ universality, there will be no contribution to D from processes which proceed via Z exchange without identical particle effects: elastic scattering on quarks and antiquarks. We are thus left with the elastic scattering on leptons. Here there are no cases where ν_e and ν_μ have the same scattering amplitude. Consider first elastic scattering on the charged antileptons. We have the reaction $\nu+e^+\to\nu+e^+$, where the scattering amplitudes for both types exist but are not exactly the same, leading to a partial damping. For the elastic scattering on antineutrinos, $\nu+\bar{\nu}_e\to\nu+\bar{\nu}_e$, $\nu+\bar{\nu}_\mu\to\nu+\bar{\nu}_\mu$, the amplitudes are also not identical for the two types, giving partial damping. Similarly, for $\nu\nu_e, \nu\nu_\mu, \nu e^-$, the target can "partially" distinguish the neutrino.

This completes the discussion of the contributions to damping due to elastic scattering. We now examine the inelastic reactions. In the context in which Eqs. (5) were originally derived, processes where the mixing particle is created or annihilated were not anticipated. However, in the present problem we can exploit the fact that we expect to have approximate thermodynamic equilibrium at any given time so that the disappearance of neutrinos through a given reaction is balanced through their creation in the inverse reaction. This may be viewed as a multistep elastic process, one which, however, may be expected to wipe out any possible traces of coherence of the linear combination of ν_e and ν_μ. For example,

TABLE I. Contributions to the damping parameter D for the ν_e-ν_μ system. Zero means no contribution; "full" means a contribution equal to one-half the rate (inverse-mean-free path) for the reaction in question. "Partial" means some intermediate value. Each entry for an inelastic reaction gives a "full" contribution, when energetically allowed (see text). For the $\bar{\nu}_e$-$\bar{\nu}_\mu$ system reverse the role of particles and antiparticles.

Target species i	e^-	e^+	ν_e	$\bar{\nu}_e$	ν_μ	$\bar{\nu}_\mu$	u	d	$\bar{u}$	$\bar{d}$	Proton	Neutron
Elastic reactions $\nu+i\to\nu+i$	Partial	Partial	Partial	Partial	Partial	Partial	0	0	0	0	0	0
Inelastic reactions $\nu+i\to X$	$\nu_e\mu^-$	$\nu_\mu\mu^+$	0	e^+e^- $\mu^+\mu^-$ $\nu_\mu\bar{\nu}_\mu$ μ^-e^+ $\nu_\tau\bar{\nu}_\tau$	0	e^+e^- $\mu^+\mu^-$ $\nu_e\bar{\nu}_e$ $e^-\mu^+$ $\nu_\tau\bar{\nu}_\tau$	0	e^-u μ^-u	$e^-\bar{d}$ $\mu^-\bar{d}$	0	0	e^-P μ^-P

$\nu+d\to u+e^-$ provides a strong "measurement" of whether ν was ν_e or ν_μ. We therefore count such reactions with a full contribution to the damping factor. The entries for the inelastic reactions are shown in the bottom half of the table, each one giving a "full" contribution. The table has been set up for the ν_e-ν_μ system; for the antiparticle ($\bar{\nu}_e$-$\bar{\nu}_\mu$) system the role of particles and antiparticles in the medium should be exchanged.

The net effect of these considerations on D may be viewed as follows. The naive expectation that every collision completely interrupts the coherent evolution of the mixing would lead to a D equal to the interaction rate or, expressed as a distance, the inverse neutrino mean free path. However, certain collisions do not distinguish, or only distinguish partially between the two neutrino types, reducing the effective integration rate or increasing the effective mean free path for damping.

Let us take the early Universe with the temperature above an MeV, but substantially less than either the μ mass, or the temperature for a possible QCD phase transition, which would lead to a high quark-antiquark density. The reaction rate for a single channel is roughly

$$D_F \simeq \frac{1}{2}\frac{G^2}{\pi}T^5 \simeq (1.0\times 10^{12}\ \mathrm{cm})^{-1}\left[\frac{T}{\mathrm{MeV}}\right]^5 . \qquad (11)$$

We note that this is very strongly temperature dependent. From the table we count six "partial" channels and six "full" channels accessible, so we expect D to be approximately $(10^{+11}\ \mathrm{cm})^{-1}$ around 1 MeV.

In the actual problems of stellar collapse or the early Universe the solutions of Eq. (4) cannot be found explicitly since the parameters $\mathbf{V}$ and D are varying in time, and possibly in space, and must be found numerically, in general. With slow variation of the parameters or in various limiting cases it should, however, be possible to see the behavior of the system qualitatively.

In the Michaeev-Smirnoff mechanism, for example, one can see that the total $\mathbf{V}$ begins by pointing approximately up, swings through a horizontal position and ends up pointing down, assuming the ν_μ-like particle is heavier than the ν_e-like particle. This is due to the density-induced variation in $\mathbf{V}$. If this variation is sufficiently slow, $\mathbf{V}$ will carry $\mathbf{P}$ along with it just as a spin follows a slowly moving magnetic field, and the direction of $\mathbf{P}$ will be inverted. This adiabatic inversion will turn ν_e into ν_μ and vice versa, and is suggested as an explanation for the apparent deficit of solar ν_e in the ^{37}Cl experiment. The variation of $\mathbf{V}$ is due to the changing density seen by the neutrino passing through the Sun, but it is also possible to imagine such a neutrino "swap" in the early Universe due to the change of density in the expansion. In a recent paper[6] the possibility of such an exchange around 1 MeV and its effect on nucleosynthesis was studied. Since the matter-antimatter excess around 1 MeV corresponds to the density in the Sun, the Michaeev-Smirnoff explanation of the solar-neutrino problem might also imply interesting effects around this time, including possible adjustments in the primordial abundances of light elements. Although the authors of Ref. 6 conclude that observable effects would be small and there are further complications (see below) it is interesting to examine illustratively the types of possibilities for a "neutrino swap" according to the methods we have outlined.

We must first assume, naturally, that through some mechanism a nonzero value for $\mathbf{P}$ exists. There would then appear to be three regimes in the case of a potential neutrino swap. First the damping may be very strong in the sense that Eq. (6a) applies and that the relaxation time, Eq. (6b), is longer than the time during which $\mathbf{V}$ swings around (passes through "resonance"). Then it seems clear that the swap will be inhibited; $\mathbf{P}$ will remain roughly in its original position. As the expansion continues, the damping will weaken as the neutrino mean free path increases but the moment of "resonance" has been missed, so $\mathbf{P}$ will either retain some finite value in the original direction or relax to zero, depending on how rapidly the damping dies off with the expansion.

Second, there would appear to be a more subtle case of strong damping. Although the damping may not be strong enough to prevent $\mathbf{V}$ from entraining $\mathbf{P}$ along with it, the damping time may still be shorter than the time it takes to pass through "resonance." In this case $\mathbf{P}$ can be shrunk toward zero, so we anticipate a rather complicated behavior where the swap is inhibited and **P** is relaxed toward zero.

Finally, the damping time can be longer than both ${V_T}^{-1}$ and the time for **V** to swing around, in which case the swap proceeds and P is inverted. If the damping then disappears rapidly the new asymmetry is preserved.

For the early Universe around MeV energies it is not immediately clear which situation prevails. We found above that D is approximately $(10^{11}\ \mathrm{cm})^{-1}$ around an MeV. Coincidentally, this is not far from the time scale for the expansion of the Universe at this time which, through its effect on the density, would be the time scale for **V** to swing around. It is also not far from the solar radius, the other parameter involved in Michaeev-Smirnoff inversion, so it appears that around MeV's the damping is neither distinctly fast nor slow on any of the relevant time scales and a detailed study is necessary. In addition, it should be noted that the important contributions to the index of refraction, pointed out in Ref. 5 and not included in the low-energy approximation, mean that it is not possible to identify the index of refraction in the Sun with that in the early Universe at the same matter-antimatter excess. Although these effects cannot change V_T, they can change V^{med} and so lead to a shift in the temperature for "resonance." Since damping is so temperature sensitive, this can have a great effect on the possible behavior. Furthermore, there is a point noted by Seckel[7] that the "swap" in changing the number of ν_e also changes the refractive index which reacts back on the swap, giving in effect a nonlinear problem which must be looked at in detail.

It is also important to note that we have not discussed the dark matter. If it should turn out that the dark matter interacts significantly with neutrinos, then its effects must be taken into account for **V** and D.

It seems there will be a large number of interesting effects, some of them nonlinear, which can be studied through Eq. (4) and its generalizations to more than two neutrinos, to heavy neutrinos, and to spin, as in magnetic fields.

I would like to thank the authors of Ref. 5 for communication of their results before publication and G. Raffelt and D. Nötzold for discussion of them. I would also like to acknowledge discussion with S. T. Petcov and D. Seckel.

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