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Signatures of vector-boson compositeness in $e^+e^- \rightarrow \mu^+\mu^-$ for the strongly coupled standard model

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We study the cross section and the forward-backward asymmetry of $e^+e^- \rightarrow \mu^+\mu^-$ in the strongly coupled standard model, at energies appropriate for the colliders TRISTAN at KEK, the Stanford Linear Collider, and LEP at CERN (60–180 GeV). Constraints from low-energy measurements are used to calculate the expected deviations from the standard-model results, arising as contributions of excited W and isoscalar vector bosons. The experimental observability, especially at the Z pole, of these deviations is discussed.

In this note we examine the possible deviations from the standard-model predictions in $e^+e^- \rightarrow \mu^+\mu^-$ at energies appropriate for the colliders TRISTAN at KEK, the Stanford Linear Collider, and LEP at CERN (but especially at the Z pole) as a consequence of the possible compositeness of the intermediate vector bosons in the manner of the Abbott-Farhi model. The Abbott-Farhi composite model, hereafter referred to as the strongly coupled standard model (SCSM), has been studied in Refs. 1–3, where it is shown that, provided certain conditions are satisfied, it reproduces the low-energy phenomenology of the standard Glashow-Weinberg-Salam (GWS) model. Here we turn to higher energies and examine the magnitude of the predicted deviations between the SCSM and GWS models, using the constraints imposed on the parameters obtained from comparison of the low-energy effective Lagrangians with the corresponding experimental results.[3]

We want to emphasize that our results, especially on the Z pole, are different from the results of Refs. 4 and 5, where similar issues were discussed. There it was assumed that the effects of compositeness would show up only as interference between the standard-model interactions and contact interactions because of compositeness. But the presence of additional four-fermion interactions modifies the relation between the W-$f\bar{f}$ coupling and the Fermi constant (which is measured very precisely). Hence the Z couplings will be different from the standard-model predictions based on the measured G_F and, as our calculations show, the deviation in the cross section at the Z pole could be quite large.

We concentrate on two contributions not present in the GWS model and present in the SCSM: the excited partners of the standard vector bosons and the vector-boson isosinglet dileptons which appear in this model.[6] In the context of general composite models, several properties of the excited W bosons (here denoted by W') have been studied previously by various authors.[7] In the SCSM their effect on the low-energy effective Lagrangian was analyzed in Ref. 3 where, contrary to the usual approach,[7] no use was made of sum-rule results, as it has been shown[8] that their use in the SCSM cannot give any information about the mass and couplings of excited vector mesons.

It is important to note that, regarding radiative corrections, the leading-order contribution from the GWS model can also be used in the SCSM (Ref. 3). This is due to the fact that, at energies below the mass of the W, the two models have the same particle spectrum and the effect of the small deviations between these models can be neglected in calculating these effects. Thus we expect that, even if a detailed inclusion of the radiative corrections changes the following results in the GWS model, the SCSM values will follow closely and the discrepancy will not be significantly altered.

The physical parameters of the W' bosons determining its effect on the fermion-fermion interaction are its mass M'_W, coupling g'_W to the isovector current, and the mixing strength of the neutral component with the photon, λ'_W. Actually it is convenient to introduce the ratios of these parameters to the ones corresponding to the W bosons: $\mu \equiv M_W/M'_W$, $r \equiv g'_W/g_W$, $k \equiv \lambda'_W/\lambda_W$. Using low-energy $(E \ll M_W)$ measurements and the known value of M_Z/M_W, one can find certain constraints on the parameters of the W'. As is shown in Ref. 3, the best one can do is to specify an allowed region in the μ-r-k space, as depicted in Fig. 4 of Ref. 3. Such constraints shall be used below.

Consider first the inclusion of the W' in the process $e^+e^- \rightarrow \mu^+\mu^-$. It is straightforward to calculate the differential cross section following, for example, a simple variant of the method used in Ref. 9. We also assume vector-meson dominance in the isovector channel with the contributions of the W and W' pole only, this gives the well-known relation $g_W\lambda_W + g'_W\lambda'_W = |e|$. It must be kept in mind that the mixing with the photon will modify the axial-vector and vector couplings, as well as the mass, of the neutral component of W'. The differential cross sections for two different center-of-mass energies (M_Z and 180 GeV) are shown in Fig. 1, where

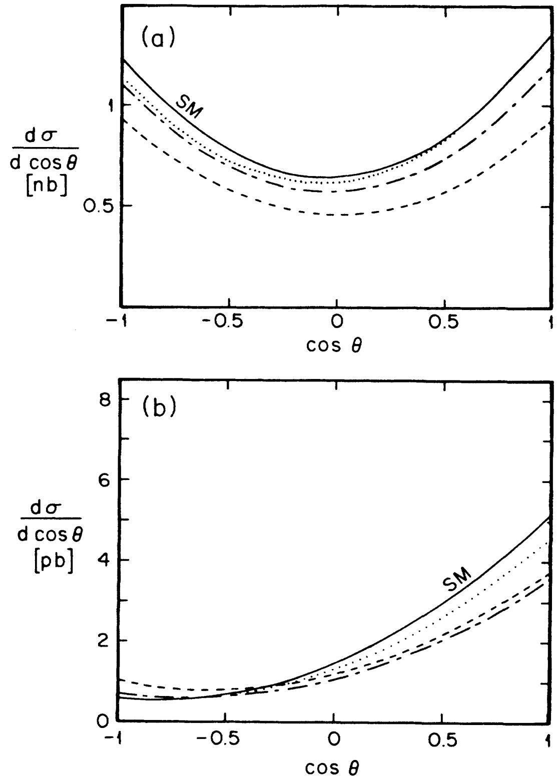

FIG. 1. The cross section $d\sigma/d\cos\theta$ for $e^+e^- \to \mu^+\mu^-$ with the contribution of excited W bosons included, for (a) center-of-mass energy $\sqrt{s}=M_Z=93$ GeV, and (b) center-of-mass energy $\sqrt{s}=180$ GeV. Solid line: standard-model result, forward-backward asymmetry: (a) $A=4.0\%$, (b) $A=60\%$; dashed line: $g'_W=g_W$, $\lambda'_W=0.05\lambda_W$, $m'_W=210$ GeV, (a) $A=-0.06\%$, (b) $A=43\%$; dot-dashed line: $g'_W=0.7g_W$, $\lambda'_W=0.05\lambda_W$, $m'_W=500$ GeV, (a) $A=3.3\%$, (b) $A=51\%$; dotted line: $g'_W=g_W$, $\lambda'_W=0.05\lambda_W$, $m'_W=500$ GeV, (a) $A=6.6\%$, (b) $A=56\%$.

the corresponding values of the forward-backward asymmetry A are also included. The results are compared with the standard model ones which were calculated using $M_Z=93$ GeV, which value was used to calculate $\sin^2\theta_W \equiv 1-(M_Z/M_W)^2$, with $O(\alpha)$ radiative corrections included,[10] to determine the coupling of the Z. The SCSM values were obtained by adjusting M_W to yield the same value for M_Z, and the electromagnetic constant e was scaled from zero to the energy considered using the results for the GWS model.[11]

For the center-of-mass energy in the range of TRISTAN (around 60 GeV) the effect of the W' is very small, even if it is very light and if its coupling is close to g_W. However at the Z pole [Fig. 1(a)], where the statistics will be best, the deviation in the cross section can be substantial, especially if M'_W is in the 200–300-GeV range and g'_W is comparable to g_W. We note that the W'-photon mixing cannot be very large as otherwise the M_Z/M_W ratio deviates too much from its experimental value.[3] It should be pointed out that A, the asymmetry, can differ significantly from its GWS-model value even for large M'_W. This is because, even in the $M'_W \to \infty$ limit, the vector-meson-dominance equation $g_W\lambda_W + g'_W\lambda'_W = |e|$ forces the Z coupling away from its GWS value. The contribution of the W' bosons goes to zero only if at least two out of three ratios μ, r, k vanish.

In the SCSM two fundamental left-handed fermions can form spin-one bound states which are weak-isospin singlets and also mediate current-current interactions (for discussion in general composite models see Ref. 12). The fundamental fermions carry a "flavor" index (which includes color) and which, for three generations, runs from 1 to 12. If we neglect the breaking of this SU(12) symmetry by $SU(3)_{\text{color}}$, $U(1)_{\text{em}}$, and Yukawa couplings,[1,2] then in this approximation all these isoscalar vector bosons $V_a^{\mu b}$ $(a,b=1,\ldots,12)$, have the same mass and coupling constant. The coupling of these vector bosons to the fermions is through the isoscalar current

$$\mathcal{L}' = -\frac{g_s}{\sqrt{2}} \sum_{a,b} V_a^{\mu b} J_{Lb}^{\mu a}, \quad J_{Lb}^{\mu a} \equiv \bar{\psi}_{Lb}\gamma^\mu \psi_L^a \ , \tag{1}$$

where ψ_L are the physical left-handed fermion fields and $g_s/\sqrt{2}$ is the coupling constant.

Consider first the contribution to the reaction $e^+e^- \to \mu^+\mu^-$ coming from the flavor-nondiagonal terms in the interaction (1). In this case we need only to take into account the terms containing $V_a^{\mu b}$, where a stands for electron and b for muon or vice versa. After a Fierz transformation (which introduces a minus sign), we obtain the amplitude

$$\mathcal{M} = -\frac{i}{t-M_s^2}\frac{g_s^2}{8}\bar{e}\gamma_\alpha(1-\gamma_5)e\,\bar{\mu}\gamma^\alpha(1-\gamma_5)\mu \ , \tag{2}$$

where $t=(p_e-p_\mu)^2$, e represents the electron spinor, etc.

Let us now turn to the flavor-diagonal terms. The corresponding contribution vanishes identically provided we ignore possible mixings with the photon. This is physically obvious if we choose to work in the basis where the flavor is diagonal, then in the absence of mixing with the photon V_e^e cannot go to V_μ^μ. To consider the general case we note first that the linear combination

$$V_1^\mu \equiv \frac{1}{\sqrt{\bar{y}}} \sum_{a=1}^{12} y_a V_a^{\mu a} \tag{3}$$

of flavor-diagonal isoscalar bosons represents the only state that can mix with the photon. In (3) y_a is the hypercharge and $\bar{y}=\sum_a y_a{}^2$. Therefore it is convenient for V_1 to be treated separately. To this effect we introduce an orthogonal transformation

$$V_a^\mu = \sum_{b=1}^{12} R_{ab} V_b^{\mu b}, \quad J_a^\mu = \sum_{b=1}^{12} R_{ab} J_{Lb}^{\mu b} \ , \tag{4}$$

where $R \cdot R^T = 1$, and $R_{1b}=y_b/\sqrt{\bar{y}}$. We will not need to specify R further, see below. The exchange of bosons V_a, $a \neq 1$ gives an effective four-fermion interaction

$$\mathcal{L}_{\text{eff}}^{(1)} = \frac{1}{2}\frac{g_s{}^2/2}{s-M_s{}^2} \sum_{a,b=1}^{12} [\delta_{ab} J_{La}^a \cdot J_{Lb}^b - (R_{1a}J_{La}^a)\cdot(R_{1b}J_{Lb}^b)] \ , \tag{5}$$

with $s=(p_e+p_{\bar{e}})^2$. Only the second term contributes to $e^+e^- \to \mu^+\mu^-$ and yields an amplitude

$$\mathcal{M}^{(1)} = -\frac{i}{32}\frac{g_s^2}{s-M_s^2}\bar{e}\gamma_\alpha(1-\gamma_5)e\,\bar{\mu}\gamma^\alpha(1-\gamma_5)\mu \ . \quad (6)$$

On the other hand, a straightforward calculation for the contribution from V_1 exchange yields an expression identical to (5) but with opposite sign and the mass of V_1 replacing M_s. If there is no photon mixing the two terms cancel; however in general the mixing strength λ_s is nonzero and this produces a mass shift so that the cancellation is only partial. The value of the mass shift due to the mixing is easily evaluated following the method of Ref. 9, the final result is

$$M_{V_1}{}^2 = M_s{}^2 \frac{1-\lambda_W{}^2}{1-\lambda_W{}^2-\lambda_s{}^2}\left[1-\left[\frac{\lambda_s\lambda_W M_W}{(1-\lambda_W{}^2)M_s}\right]^2\right] + O((M_W/M_s)^4) \ . \quad (7)$$

The constant λ_s can be expressed in terms of other parameters if we assume vector-meson dominance. Indeed, considering only the contribution from the lowest-mass pole, we immediately obtain $\lambda_s g_s = \sqrt{2}\,|\,e\,|$. Concerning the constraints on these parameters coming from experiment, we find that the strongest experimental bound on g_s comes from the requirement that there be no tachyons in the theory; this implies[3] $g_s \gtrsim 0.5$; moreover from low-energy phenomenology it is known that[3] $M_s/g_s \gtrsim 550$ GeV; therefore we must have $M_s \gtrsim 250$ GeV. Using the above formulas, and taking into account the above constraints on the parameters, we have calculated the cross section and the forward-backward asymmetry for the reaction $e^+e^- \to \mu^+\mu^-$; the final results are presented in Figs. 2 and 3.

Given the above results we now turn to the question as to whether the calculated effects will be observable in present and future colliders. First, regarding TRISTAN, we found that at the corresponding energies and luminosities there is no hope of observing the effects of either of the new particles discussed above. In the case of SLC we use the following estimated luminosities in $\text{cm}^{-2}\,\text{sec}^{-1}$ (Ref. 13):

July 1987–February 1988: 6×10^{27} ,

February 1988–June 1988: 10^{29} ,

June 1988–June 1989: 6×10^{29} ,

June 1989–June 1990: 6×10^{30} .

With the standard-model cross section for the reaction $e^+e^- \to \mu^+\mu^-$ (at the Z pole) of ~ 1.5 nb, this will yield about 5000 $\mu\bar{\mu}$ pairs by the end of 1988. Since at this energy the GWS model A will be $\sim 4\%$, we conclude that statistics alone will allow observation of deviations of the order of 1.4% or greater. By the end of 1989 the number of $\mu\bar{\mu}$ events will be about 50 000 with the corresponding statistical error of about 0.4%. Here we mention that one has to assure good beam energy stability,

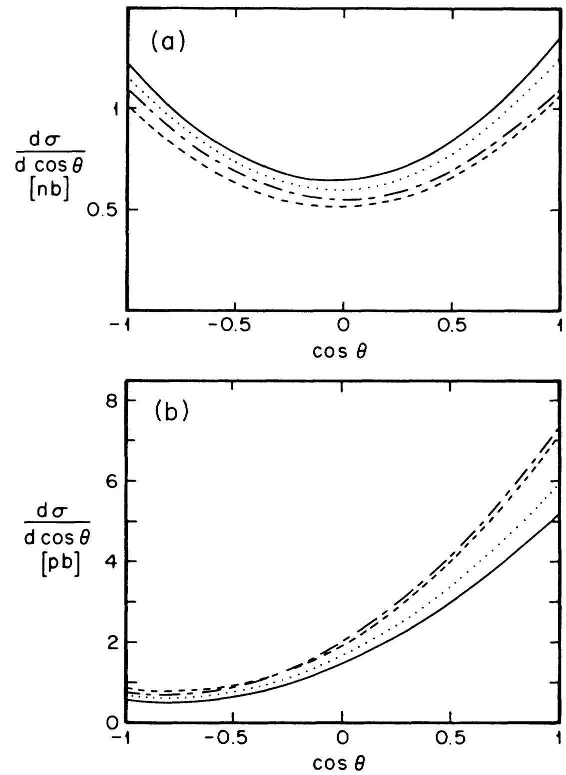

FIG. 2. The cross section $d\sigma/d\cos\theta$ for $e^+e^- \to \mu^+\mu^-$ with the contribution of isoscalar vector bosons included, for (a) center-of-mass energy $\sqrt{s}=M_Z=93$ GeV, (b) center-of-mass energy $\sqrt{s}=180$ GeV. Solid line: standard-model result, forward-backward asymmetry: (a) $A=4.0\%$, (b) $A=60\%$; dashed line: $g_s=0.6$, $M_s=330$ GeV, (a) $A=2.5\%$, (b) $A=58\%$; dot-dashed line: $g_s=1$, $M_s=500$ GeV, (a) $A=0.8\%$, (b) $A=60\%$; dotted line: $g_s=0.8$, $M_s=700$ GeV, (a) $A=2.6\%$, (b) $A=59\%$.

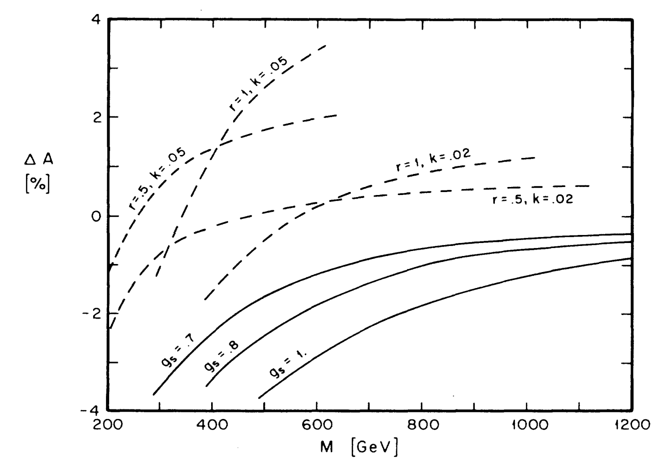

FIG. 3. The shift $\Delta A \equiv A_{\text{SCSM}} - A_{\text{GWS}}$ in the forward-backward asymmetry for center-of-mass energy $\sqrt{s}=M_Z=93$ GeV. Solid lines correspond to the contribution of isoscalar vector bosons (M denotes M_s in this case), while dashed lines (with indicated ratios $r=g'_W/g_W$, $k=\lambda'_W/\lambda_W$) describe the effect of excited W bosons, M being the mass of the W' boson in this case.

since at the Z pole $\Delta A/\Delta E \approx 1\%/100$ MeV. Also, the shift of the maximum of the cross section with respect to the Z mass in the SCSM is not significantly different from that of the standard model (after one imposes the constraints on the masses and coupling constants of the exotic sector, coming from low-energy measurements). Comparing the above estimates with the calculated expected effects of compositeness we conclude that the effects could be observable in the later stages of operation of this collider. On the other hand, the deviations in the total cross section at the Z pole can be 10–20 %, as compared to 5–8 % expected experimental accuracy of the absolute cross-section measurement. For LEP the situation is even better, because of the projected luminosity of 10^{31} cm^{-2}sec^{-1}. In this case the statistics will allow precision of up to $\sim 0.3\%$ which will be sufficient for the effects of these particles to be easily distinguishable. As one can see, the expected statistics at the Z pole are very good, but the expected deviations from the standard model are not always significant, and those which are (e.g., the absolute value of the cross section), are difficult to measure. Still, the deviations are not completely negligible, contrary to some previous claims.[4,5] On the other hand, at 180 GeV the deviations can be quite large, but the cross section is now very small, giving poor statistics. We also expect that future experiments will impose more stringent restrictions on the parameters of the theory and this could decrease the magnitude of the effects. It should be clear that the above discussion is not intended to be exhaustive but that we only try to give a guideline as to the observability of the calculated effects.

Finally we would like to emphasize that to be able to confirm the presence of new particles a good determination of the parameters of the GWS model is essential, for only then one can determine the contributions from the radiative corrections and thus pinpoint possible new effects.

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[1]L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981); Nucl. Phys. **B189**, 547 (1981).

[2]M. Claudson, E. Farhi, and R. L. Jaffe, Phys. Rev. D **34**, 873 (1986).

[3]C. Korpa and Z. Ryzak, Phys. Rev. D **34**, 2139 (1986).

[4]I. Bars, J. F. Gunion, and M. Kwan, Phys. Rev. D **34**, 122 (1986).

[5]L. J. Hall and S. F. King, Nucl. Phys. **B287**, 551 (1987).

[6]J. Wudka, Phys. Lett. **167B**, 337 (1986).

[7]M. Kuroda *et al.*, Nucl. Phys. **B261**, 432 (1985); M. Kuroda and D. Schildknecht, Phys. Lett. **121B**, 173 (1983); P. Chen and J. J. Sakurai, *ibid.* **110B**, 481 (1982); R. Barbieri and R. N. Mohapatra, *ibid.* **120B**, 198 (1983); S. Narison, *ibid.* **122B**, 171 (1983); G. Gounaris *et al.*, *ibid.* **133B**, 118 (1983); A. B. Kaidalov and A. V. Nogteva, Yad. Fiz. **42**, 233 (1985) [Sov. J. Nucl. Phys. **42**, 147 (1985)].

[8]S. Devyanin and R. L. Jaffe, Phys. Rev. D **33**, 2615 (1986).

[9]R. Kögerler and D. Schildknecht, CERN Report No. TH.3231-CERN, 1982 (unpublished).

[10]M. Consoli and A. Sirlin, in *Physics at LEP*, LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. Peccei (CERN Report No. 86-02, Geneva, 1986); Lynn *et al.*, *ibid.*, Vol. 1, p. 63.

[11]L. Hall, Nucl. Phys. **B178**, 75 (1981).

[12]B. Schrempp and F. Schrempp, DESY Report No. DESY 84-055, 1984 (unpublished).

[13]These are the figures given by B. Richter in November 1986, with some dates slightly changed due to difficulties encountered in the meantime. Some recent estimates for expected luminosity are lower, in which case it would take longer to achieve the same statistical accuracy.