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Fritzsch mass matrix with the fourth generation and the renormalization-group equations

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We study the effect of the fourth-generation quarks on the lighter-generation quarks. We take the Fritzsch form for the Yukawa couplings with the simple phases conjectured by Shin at the grand unified scale, and investigate three cases: the one-Higgs-doublet model, the two-Higgs-doublet model, and the supersymmetric model for the Higgs sector. We calculate the evolution of mixing among the second-generation quark, the third one, and the fourth one by use of the renormalization-group equations, and then obtain rather large mixings at the electroweak scale. Furthermore, we study the effect of these mixings on the B_s^0-$\bar{B}_s^0$ system.

I. INTRODUCTION

Recently, a great deal of attention has been focused on the question whether or not there is the fourth generation of quarks and leptons.[1–4] The number of generations is very important to build up the unified theory beyond the standard model. From the experimental point of view,[1,5] the fourth generation of quarks and leptons is allowed for the present. The important questions concerning the fourth-generation fermions are their masses and mixings with the three lighter generations. Some authors have already studied the masses of the fourth-generation fermions by taking into account experimental[5] or theoretical constraints.[2,3,6]

In the standard grand-unified theories, the origin of the fermion masses lies in the Yukawa couplings between fermions and Higgs bosons at the grand-unified-theory (GUT) scale M_X. The mass matrices at low energies (e.g., the electroweak scale M_W) are connected with the Yukawa couplings at M_X by the renormalization-group equations (RGE's) if the perturbative unification and the desert are assumed.[7] As is well known, the Yukawa couplings converge to the infrared fixed points controlled by RGE's.[2,3] If the Yukawa couplings have possibly large values in the framework of perturbative unification, they approach the infrared fixed point at the physical low-energy limit M_W. Since the fourth-generation quarks are expected to have large Yukawa couplings at M_X, their masses are near the value which is given by the fixed point. On the other hand, their mixings with the lighter generation quarks depend on the structure of the Yukawa couplings in the generation space at M_X. Once the matrices of the Yukawa couplings in the generation space are given at M_X, the generation mixings are obtained through the RGE's at the low energy M_W. Of course, one has not yet found the final answer for the Yukawa coupling matrices, but we know very attractive matrices, that is the Fritzsch form[8] of the Yukawa couplings, whose phase structure has been studied by some authors.[9,10]

In this paper, we give *a priori* the Fritzsch form for the Yukawa couplings, in which the simple phases conjectured by Shin[11,9] are used in the framework of four generations, and then study the effect of the heavy fourth-generation quarks on the lighter generation ones. One of the authors has already done numerical studies of the mixing between the fourth-generation quarks and the third ones using the Fritzsch matrix.[12] Here we calculate mixing among second, third, and fourth generations. Then we have found that the top quark should exist in the restricted mass regions. Because the fourth-generation quark mass is controlled by the fixed point, our results are different from other analyses[11] taking arbitrary fourth-generation quark masses even if the Fritzsch mass matrices are used.

We deal with the case of the two-Higgs-doublet model[2] and the supersymmetric model[3] for the Higgs sector in addition to the one-Higgs-doublet model. In the former two models, we have found that the evolution of the mixing between the fourth-generation quarks and the second (third) ones is clearly contrasted to that of the one-Higgs-doublet model.

We represent the general framework of RGE's in the standard $SU(3)_c \times SU(2) \times U(1)$ gauge theory and that of the Fritzsch matrix in Sec. II. The numerical results in each case of the one-Higgs-doublet model, the two-Higgs-doublet model, and the supersymmetric model are given in Secs. III, IV, and V, respectively. The magnitude of the B_s^0-$\bar{B}_s^0$ mixing is estimated for the three models in Sec. VI. Section VII is devoted to conclusions.

II. GENERAL FRAMEWORK

We consider the standard theory based on the group $SU(3)_c \times SU(2) \times U(1)$ with the fourth generation. In the following calculations, we shall work with the three heavy generations: the second, third, and fourth generations. Since the first-generation fermions have extremely small Yukawa couplings, they have practically no effect on our results. The six heavy doublets are expressed as

$$\begin{bmatrix}\nu_\mu \\ \mu\end{bmatrix}, \quad \begin{bmatrix}c \\ s\end{bmatrix}, \quad \begin{bmatrix}\nu_\tau \\ \tau\end{bmatrix}, \quad \begin{bmatrix}t \\ b\end{bmatrix}, \quad \begin{bmatrix}\nu_L \\ L\end{bmatrix}, \quad \begin{bmatrix}a \\ v\end{bmatrix} . \tag{2.1}$$

The fourth generation consists of a pair of very heavy quarks (a,v) and a charge heavy lepton (L) with its neutrino (ν_L).[1] In the following, all neutrino masses are neglected. The 3×3 matrices of the Yukawa couplings in the generation space for the up quarks, the down quarks, and the charged leptons are denoted by U, D, and E, respectively. Then, the RGE's are given in the one-loop approximation as[13]

$$\begin{aligned}\frac{1}{U}\frac{dU}{dt} &= G_U - \mathrm{Tr}(3U^\dagger U + 3aD^\dagger D + aE^\dagger E) \\ &\quad -\tfrac{3}{2}(bU^\dagger U - cD^\dagger D) , \\ \frac{1}{D}\frac{dD}{dt} &= G_D - \mathrm{Tr}(3aU^\dagger U + 3D^\dagger D + E^\dagger E) \\ &\quad +\tfrac{3}{2}(cU^\dagger U - bD^\dagger D) , \end{aligned} \tag{2.2}$$

$$\frac{1}{E}\frac{dE}{dt} = G_E - \mathrm{Tr}(3aU^\dagger U + 3D^\dagger D + E^\dagger E) - \tfrac{3}{2}bE^\dagger E ,$$

with

$$t = \frac{1}{16\pi^2}\ln\left|\frac{M_X}{M}\right| , \tag{2.3}$$

where M denotes the running energy scale. The coefficients (a,b,c) are $(1,1,1)$, $(0,1,-\frac{1}{3})$, and $(0,2,-\frac{2}{3})$ for the one-Higgs-doublet model, the two-Higgs-doublet model and the supersymmetric model, respectively. The values of G_U, G_D, and G_E are given by the linear combinations of the squares of the gauge coupling constants $g_1{}^2$, $g_2{}^2$, and $g_3{}^2$.

Before solving the RGE's, we should determine the low-energy parameters at M_W. By setting $\alpha_{\rm em}=1/128$,[14] $\sin^2\theta_W=0.226$,[15] and $\Lambda_{\overline{\rm MS}}=0.1$ GeV at M_W (where $\overline{\rm MS}$ denotes the modified minimal-subtraction scheme), we get the values of $g_3(M_W)$, $g_2(M_W)$, and $g_1(M_W)$, which are the gauge coupling constants at M_W for $SU(3)_c$, SU(2), and U(1), respectively. In our scheme, known masses are of the b quark, c quark, s quark, τ lepton, and muon. The values of quark masses are given in Ref. 16 as

$$\begin{aligned} m_b(1\ \mathrm{GeV}) &= 5.3\pm 0.1\ \mathrm{GeV} , \\ m_c(1\ \mathrm{GeV}) &= 1.35\pm 0.05\ \mathrm{GeV} , \\ m_s(1\ \mathrm{GeV}) &= 175\pm 55\ \mathrm{MeV} . \end{aligned} \tag{2.4}$$

Since these values are the running masses at 1 GeV, we calculate the running masses at the M_W scale by use of the QCD formula by Georgi and Politzer.[17] With the help of $\Lambda_{\overline{\rm MS}}=0.1$ GeV and the relevant number of flavors, we get the quark masses at M_W, which is denoted by m_q^W, as

$$\begin{aligned} &m_b^W=3.5-3.7\ \mathrm{GeV}, \quad m_c^W=0.76-0.83\ \mathrm{GeV} , \\ &m_s^W=70-130\ \mathrm{MeV} . \end{aligned} \tag{2.5}$$

The τ-lepton and muon masses are, respectively, $m_\tau^W=1.78$ GeV and $m_\mu^W=106$ MeV by neglecting the QED correction. The top-quark mass is taken as a free parameter in this paper. Furthermore, the observed Kobayashi-Maskawa (KM) mixing matrix element V_{cb} is taken as $|V_{cb}|=0.035-0.049$ (Ref. 18) at the M_W scale.

The fermion masses at M_W are written as

$$m_U = U(M_W)v_U, \quad m_D = D(M_W)v_D, \quad m_E = E(M_W)v_E , \tag{2.6}$$

where v_U, v_D, and v_E are the vacuum expectation values of the neutral components of the relevant Higgs doublets. For the one-Higgs-doublet model, $v_U=v_D=v_E=175$ GeV and for both the two-Higgs-doublet model and the supersymmetric model, $v_D=v_E$ and $v_U{}^2+v_D{}^2=(175\ \mathrm{GeV})^2$.

The KM quark mixing matrix V (Ref. 19) is defined by

$$V = R_U^\dagger R_D = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uv} \\ V_{cd} & V_{cs} & V_{cb} & V_{cv} \\ V_{td} & V_{ts} & V_{tb} & V_{tv} \\ V_{ad} & V_{as} & V_{ab} & V_{av} \end{bmatrix} , \tag{2.7}$$

where R_U and R_D are the unitary matrices diagonalizing $m_U^\dagger m_U$ and $m_D^\dagger m_D$, respectively.

In the following calculations, we use the GUT condition of the Yukawa couplings[7]

$$D(M_X) = E(M_X) , \tag{2.8}$$

which is a condition of the minimal model, where only the **5** scalar of SU(5) [or **10** of SO(10)] can be coupled to fermions, but this condition may be too tight. For example, under this condition, the m_s/m_μ ratio cannot be predicted correctly although the m_b/m_τ ratio is successfully done.[20] Since our following result depends mainly on the quark sectors $U(M_X)$ and $D(M_X)$, we use the condition Eq. (2.8) as a conventional one and then abandon it attempting to predict the muon mass correctly in this paper.

The Yukawa coupling matrices in the generation space have not been credibly determined at M_X by any theory, but a very attractive matrix is known, the Fritzsch matrix.[8] We shall show the Fritzsch form of the Yukawa coupling with four generations in the following. After a suitable redefinition of the phases of the quark fields, the Yukawa couplings between up quarks and the Higgs scalar ϕ are written as

$$(\bar u_0, \bar c_0, \bar t_0, \bar a_0)_L \begin{bmatrix} 0 & \delta & 0 & 0 \\ \delta & 0 & \gamma & 0 \\ 0 & \gamma & 0 & \beta \\ 0 & 0 & \beta & \alpha \end{bmatrix} \phi \begin{bmatrix} u_0 \\ c_0 \\ t_0 \\ a_0 \end{bmatrix}_R + \mathrm{H.c.} , \tag{2.9}$$

where α, β, γ, and δ are real, u_0, c_0, t_0, and a_0 are weak eigenstates, and L and R denote left handed and right handed, respectively. A similar form is assumed for the down quarks in which the elements α, β, γ, and δ are replaced by $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$, respectively. These symmetric matrices U and D can be diagonalized by a suitable rotation in the quark space described by an orthogonal matrix R_U and R_D, respectively. Then the KM matrix V (Ref. 19) at the M_X scale is given by

$$V=[R_U]^T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\sigma} & 0 & 0 \\ 0 & 0 & e^{i\tau} & 0 \\ 0 & 0 & 0 & e^{i\eta} \end{bmatrix} R_D \ , \tag{2.10}$$

where σ, τ, and η are phases which are not absorbed into quark fields. On the basis of phenomenological analyses, $\sigma=\tau=\pi/2$ $(-\pi/2)$ has been conjectured by Shin.[9] In the following calculations, we take $\sigma=\tau=\eta=\pi/2(-\pi/2)$ for the four-generation scheme as used in Ref. 11. Furthermore, we neglect the first-generation fermions because this effect on the RGE's is very small. Then, the imaginary parts of the KM matrix elements vanish and CP violation disappears. It is emphasized that CP violation is caused by the first generation if we take $\sigma=\tau=\eta$ in this scheme. After solving the RGE's, the matrix form of Eq. (2.9) is no longer maintained at the M_W scale, but the phase structure of the KM matrix in Eq. (2.10) is not changed between the M_X scale and the M_W scale. We will find that the mixings between the fourth generation and the lighter ones are drastically changed because the large Yukawa couplings of the fourth-generation quarks have a significant effect on the renormalization of the mass matrices.

Here, we comment on the Fritzsch form of the Yukawa couplings at M_X because its origin is less clear. A few authors[21] have proposed the derivation of the Fritzsch-type matrix at M_X introducing extra symmetries in addition to the standard symmetries, for example, new U(1)'s. If these extra symmetries are broken spontaneously at M_X, and extra symmetries do not remain at low energy, our analyses for the Fritzsch matrix may be justified. Although the derivation of the Fritzsch-type matrix seems to depend on the symmetry at M_X, we analyze using the standard RGE's without considering the specific extra symmetries. We emphasize that our study will clarify the effect of the fourth generation on the lighter ones such as the mixings at M_W.

III. ONE-HIGGS-DOUBLET MODEL

In this section, the minimal model with one Higgs doublet is studied. The RGE's are given in Eq. (2.2) with $(a,b,c)=(1,1,1)$. The values of G_U, G_D, and G_E are written as[2,13]

$$\begin{aligned} G_U &= 8{g_3}^2+\tfrac{9}{4}{g_2}^2+\tfrac{17}{12}{g_1}^2 \ , \\ G_D &= 8{g_3}^2+\tfrac{9}{4}{g_2}^2+\tfrac{5}{12}{g_1}^2 \ , \\ G_E &= \tfrac{9}{4}{g_2}^2+\tfrac{15}{4}{g_1}^2 \ , \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} (g_3^0/g_3)^2 &= 1+2(\tfrac{4}{3}N_g-11)g_3^{02}t \ , \\ (g_2^0/g_2)^2 &= 1+2(\tfrac{4}{3}N_g+\tfrac{1}{6}N_H-\tfrac{22}{3})g_2^{02}t \ , \\ (g_1^0/g_1)^2 &= 1+2(\tfrac{20}{9}N_g+\tfrac{1}{6}N_H)g_1^{02}t \ , \end{aligned} \tag{3.2}$$

here N_g and N_H denote the number of generations and the Higgs doublets, respectively, and g_1^0, g_2^0, and g_3^0 are the initial values of the gauge couplings at M_X. We take $M_X=10^{15}$ GeV, $N_g=4$, and $N_H=1$, then the GUT condition $g_3^0 \approx g_2^0 \approx (\frac{5}{3})^{1/2} g_1^0 \approx 0.58$ is obtained within a few percent error using RGE's together with low-energy parameters given in Sec. II.

We start with the following Fritzsch form[8] of the 3×3 matrices U, D, and E at M_X:

$$U(M_X)=\begin{pmatrix} 0 & \gamma & 0 \\ \gamma & 0 & \beta \\ 0 & \beta & \alpha \end{pmatrix} \begin{matrix} c \\ t \\ a \end{matrix}\ , \quad (\text{columns } c,\ t,\ a)$$

$$D(M_X)=\begin{pmatrix} 0 & \tilde{\gamma} & 0 \\ \tilde{\gamma} & 0 & \tilde{\beta} \\ 0 & \tilde{\beta} & \tilde{\alpha} \end{pmatrix} \begin{matrix} s \\ b \\ v \end{matrix} = E(M_X) \ . \quad (\text{columns } s,\ b,\ v) \tag{3.3}$$

The six parameters α, β, γ, $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ are unknown ones, but some of them are to be determined so as to correctly predict the quark masses m_b^W, m_c^W, m_s^W, and the observed KM mixing element $|V_{cb}|$ (Ref. 18) (0.035–0.049). The matrices U, D, and E develop by RGE's as the energy scale M goes down from M_X to M_W, and then these matrices give the fermion masses after the spontaneous symmetry breakdown at the M_W scale as given in Eq. (2.6), where $v_U=v_D=v_E=175$ GeV for the one-Higgs-doublet model. In our calculations, we have determined some unknown parameters to fit the observed masses m_b^W, m_c^W, m_s^W, and V_{cb} and then two parameters are left practically undetermined. We take a possible large Yukawa coupling $U_{33}(M_X)=3.0$ because we are interested in the case of the large Yukawa couplings as discussed in Sec. II. The larger value than $U_{33}(M_X)=3.0$ is dangerous in the perturbative GUT because of the constraint $U_{33}(M_X)^2/4\pi<1$. On the other hand, $D_{33}(M_X)$ is varied in the region below 3.0, and then some predictions are given for the one-Higgs-doublet model.

Let us show the numerical result. The evolutions of $|V_{ab}|$ and $|V_{as}|$ are shown in Fig. 1, where $U_{33}(M_X)=D_{33}(M_X)=3.0$, $m_t(\text{physical})=m_t^P=50$ GeV and the relevant m_s^W are taken, and they are compared with the results of other models in IV and V, which are also shown in Fig. 1. The values of parameters at M_X and the masses at M_W are summarized in Table I. As the energy scale M decreases from M_X to M_W, both magnitudes of the mixings V_{ab} and V_{as} are reduced to about 60% at M_W. The behavior of the mixing V_{ab} has been already shown in our previous paper.[12] The structure of the matrix at M_W is considerably different from that at M_X. The matrices at M_X and M_W for the case

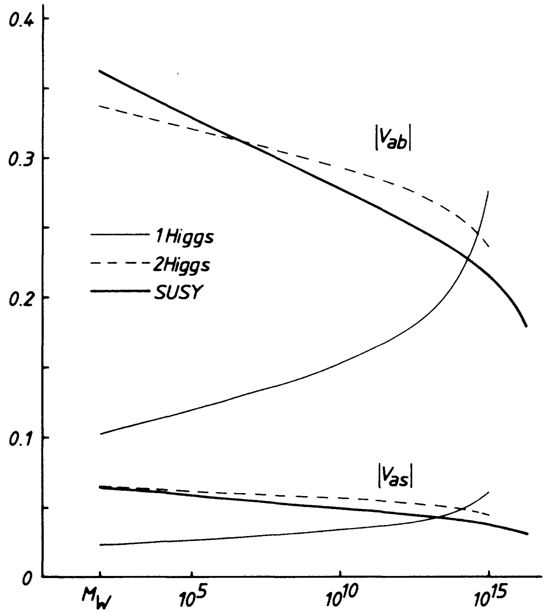

FIG. 1. Evolutions of $|V_{ab}|$ and $|V_{as}|$ for the three models. Here $m_t^P=50$ GeV is taken and other parameters are shown in Table I.

shown in Fig. 1 are

$$U(M_X)=\begin{bmatrix} 0 & 0.114 & 0 \\ 0.114 & 0 & 1.69 \\ 0 & 1.69 & 3.00 \end{bmatrix},$$

$$D(M_X)=\begin{bmatrix} 0 & 0.011 & 0 \\ 0.011 & 0 & 0.421 \\ 0 & 0.421 & 3.00 \end{bmatrix},$$

$$U(M_W)=0.306\times\begin{bmatrix} 0.000 & 0.116 & -0.011 \\ 0.115 & -0.162 & 1.783 \\ -0.015 & 1.435 & 3.000 \end{bmatrix},$$

$$D(M_W)=0.347\times\begin{bmatrix} -0.000 & 0.011 & 0.001 \\ 0.012 & 0.040 & 0.415 \\ 0.015 & 0.712 & 3.000 \end{bmatrix}. \tag{3.4}$$

TABLE I. Values of parameters at M_X and the mass values at M_W, corresponding to each model shown in Fig. 1.

	One Higgs	Two Higgs	SUSY
α	3.000	3.000	3.000
β	1.690	1.275	0.867
γ	0.114	0.066	0.032
$\tilde{\alpha}$	3.000	3.000	3.000
$\tilde{\beta}$	0.421	0.334	0.243
$\tilde{\gamma}$	0.011	0.006	0.003
v_D/v_U		1.000	1.000
m_a^W (GeV)	199	156	142
m_v^W (GeV)	191	146	136
m_t^W (GeV)	45.5	45.8	45.5
m_b^W (GeV)	3.61	3.62	3.60
m_c^W (GeV)	0.800	0.800	0.801
m_s^W (GeV)	0.110	0.099	0.097

Thus, the clear deviation from the Fritzsch matrix is obtained at the M_W scale for both up-quark and down-quark matrices.

Since the calculated value of V_{cb} depends fairly on the top-quark mass, the allowed region of the top-quark mass are obtained by use of the experimental data of $|V_{cb}|$ (0.035–0.049).[18] In Fig. 2, we show the allowed region of the physical top-quark mass (m_t^P) versus the running s-quark mass at M_W, m_s^W in the case of $U_{33}(M_X)=D_{33}(M_X)=3.0$. Two allowed regions are obtained as

$$23\text{ GeV}\lesssim m_t^P\lesssim 30\text{ GeV},\quad 38\text{ GeV}\lesssim m_t^P\lesssim 95\text{ GeV}\ , \tag{3.5}$$

because the s-quark mass is allowed in a wide region due to the experimental large error. We have checked that this result is almost independent of the value of $U_{33}(M_X)$ and $D_{33}(M_X)$ in $2\le U_{33}(M_X)\le 4$ and $0.5\le D_{33}(M_X)\le 3$, which means that the fourth-generation Yukawa couplings reach near the fixed point if we use a large initial value of $U_{33}(M_X)$ and $D_{33}(M_X)$. The calculated value of V_{cb} decreases about 15% as the energy scale goes down from M_X to M_W. The numerical values of $|V_{cb}|$ at M_X and M_W are shown together with the result of the two-Higgs-doublet model and the supersymmetric one in Table II. These results suggest the importance of the renormalization effect in the four-generation scheme. Shin and Fritzsch have already given the allowed region for m_t^P such as 30–80 GeV in the case of $\Lambda_{\overline{\rm MS}}=0.1$ GeV for the three-generation scheme.[9,22] If the fourth-generation quarks exist, the allowed region of the top-quark mass is somewhat larger, but the region 30 GeV$\lesssim m_t^P\lesssim$38 GeV is not allowed, which is contrasted with the case of the three-generation scheme. In Fig. 3, we show $|V_{ab}|$ and $|V_{as}|$ at M_W vs $D_{33}(M_X)$ in the case of $m_t^P=50$ GeV. The lower limit of $D_{33}(M_X)$(≈ 1.1) is obtained by using $m_L>41$ GeV given by the UA1 Collaboration.[23] The obtained mixings are

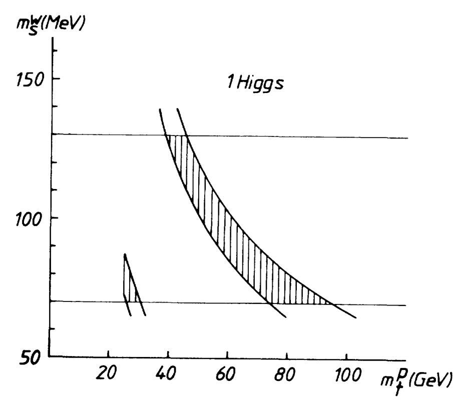

FIG. 2. The allowed region of m_t^P vs m_s^W for the one-Higgs-doublet model, where $U_{33}(M_X)=D_{33}(M_X)=3$ is taken. Two shaded regions are allowed setting $|V_{cb}|=0.035$–0.049.

TABLE II. The values of $|V_{cb}|$, $|V_{as}|$, and $|V_{ab}|$ at M_X and M_W for the typical parameters in the three models.

	v_D/v_U	m_a^W (GeV)	m_v^W (GeV)	m_t^p (GeV)	m_s^W (MeV)	$\|V_{cb}\|$ M_X	$\|V_{cb}\|$ M_W	$\|V_{as}\|$ M_X	$\|V_{as}\|$ M_W	$\|V_{ab}\|$ M_X	$\|V_{ab}\|$ M_W
One Higgs		200	192	40	137	0.050	0.043	0.056	0.021	0.236	0.088
		199	191	50	110	0.052	0.043	0.061	0.023	0.276	0.103
Two Higgs	0.42	205	80	40	131	0.041	0.043	0.026	0.038	0.122	0.178
		205	80	50	107	0.041	0.044	0.031	0.045	0.157	0.228
	1	157	146	40	126	0.037	0.042	0.041	0.059	0.195	0.281
		156	146	50	99	0.037	0.043	0.045	0.065	0.236	0.339
SUSY	0.42	186	74	40	133	0.040	0.042	0.018	0.038	0.088	0.185
		185	74	50	106	0.039	0.043	0.021	0.035	0.118	0.245
	1	142	136	40	125	0.037	0.043	0.028	0.059	0.145	0.298
		142	136	50	97	0.034	0.043	0.031	0.065	0.178	0.363

$$0.076 \lesssim |V_{ab}| \lesssim 0.103\,, \qquad 0.015 \lesssim |V_{as}| \lesssim 0.023\,, \tag{3.6}$$

which depend on the top-quark mass. In the case of $m_t^p=70(40)$ GeV, the allowed regions of $|V_{ab}|$ and $|V_{as}|$ are as follows:

$$0.105(0.060) \lesssim |V_{ab}| \lesssim 0.126(0.088)\,, \qquad 0.019(0.013) \lesssim |V_{as}| \lesssim 0.027(0.021)\,. \tag{3.7}$$

Furthermore, these mixings also depend on the $U_{33}(M_X)$. In the region of $U_{33}(M_X)=2$–4, these values increase or decrease within 20%. In any case, we predict rather large values of the mixings V_{ab} and V_{as}.

IV. TWO-HIGGS-DOUBLET MODEL

In this section we investigate the two-Higgs-doublet model, in which a scalar doublet couples to leptons and down quarks, and another doublet couples to up quarks. The RGE's are given as Eq. (2.2) with $(a,b,c)=(0,1,-1/3)$ (Ref. 2), and G_U, G_D, and G_E are written in the same form as in Eqs. (3.1) and (3.2) where $N_H=2$ is taken, and the fermion masses are given in Eq. (2.6) where $v_D=v_E$ and $v_U{}^2+v_D{}^2=(175\ \text{GeV})^2$. We also take $g_3^0 \approx g_2^0=(\frac{5}{3})^{1/2}g_1^0 \approx 0.58$ at $M_X=10^{15}$ GeV in this model. The matrices of the Yukawa couplings U, D, and E at M_X are taken in the same form as in Eq. (3.3).

For the two-Higgs-doublet model, one unknown parameter is introduced in addition to the parameters in Sec. III, which is the ratio v_D/v_U. Then, the prediction of the two-Higgs-doublet model is less definitive than that in the case of the one-Higgs-doublet model. Therefore, we tentatively choose the specific but interesting condition $U_{33}(M_X)=D_{33}(M_X)$.

The evolutions of $|V_{ab}|$ and $|V_{as}|$ are shown in Fig. 1, where we have taken that $U_{33}(M_X)=D_{33}(M_X)=3.0$, $m_t^p=50$ GeV, and $v_D/v_U=1.0$. The values of parameters at M_X and the masses at M_W are summarized in Table I. As the energy scale M decreases from M_X to M_W, both magnitudes of the mixings V_{ab} and V_{as} increase to about 145% at M_W. This tendency, which has already been pointed out by Komatsu,[7] is in remarkable contrast to the one-Higgs-doublet model.

In Fig. 4, we show the allowed region of the physical top-quark mass versus the running s-quark mass at M_W in the case of $v_D/v_U=1.0$. Two regions of m_t^p are obtained as well as in the case of the one-Higgs-doublet model as follows:

$$23\ \text{GeV} \lesssim m_t^p \lesssim 28\ \text{GeV}, \quad 36\ \text{GeV} \lesssim m_t^p \lesssim 73\ \text{GeV}\,. \tag{4.1}$$

These regions are somewhat dependent of the value v_D/v_U. If we take smaller value, $v_D/v_U=0.42$, the allowed ranges are rather wide as

$$23\ \text{GeV} \lesssim m_t^p \lesssim 30\ \text{GeV}, \quad 37\ \text{GeV} \lesssim m_t^p \lesssim 84\ \text{GeV}\,. \tag{4.2}$$

In Table II, the calculated values of $|V_{cb}|$, $|V_{as}|$, and $|V_{ab}|$ at M_X and M_W are summarized in the case of the typical parameters. The value of $|V_{cb}|$ increases as the energy scale slightly goes down from M_X to M_W as well as $|V_{ab}|$ and $|V_{as}|$. As seen in Eqs. (3.5) and (4.1), we

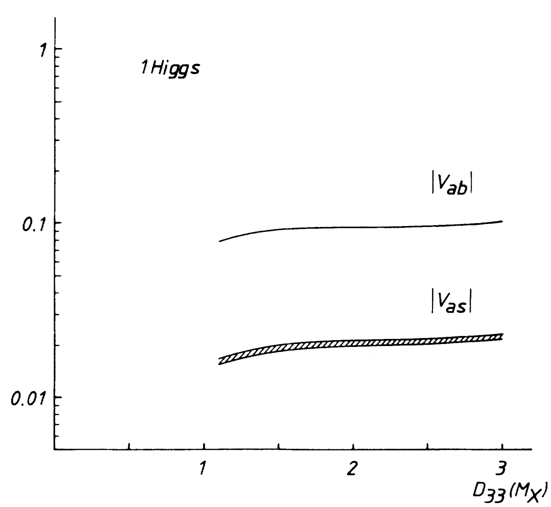

FIG. 3. The value of $|V_{ab}|$ and $|V_{as}|$ vs $D_{33}(M_X)$ for the one-Higgs-doublet model, where $U_{33}(M_X)=3.0$ and $m_t^p=50$ GeV are taken. The shaded region of $|V_{as}|$ is due to the large experimental error of m_s^W.

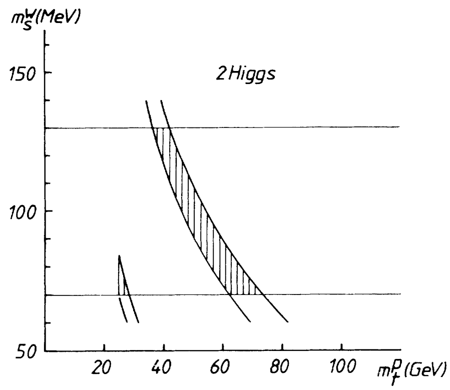

FIG. 4. The allowed region of m_t^p vs m_s^W for the two-Higgs-doublet model, where $U_{33}(M_X)=D_{33}(M_X)=3$ and $v_D/v_U=1$ are taken.

get different allowed regions of m_t^p for each different Higgs-doublet model. Even if we take the same value of $|V_{cb}|$ at M_X in both Higgs-doublet models, we obtain significantly different values of $|V_{cb}|$ at M_W or each model due to the contrast evolutions (the increasing behavior and the decreasing one). This fact means that the renormalization of the mass matrices are very important if the large Yukawa couplings exist.

In Fig. 5, $|V_{ab}|$ and $|V_{as}|$ at M_W are shown vs v_D/v_U in the case of $U_{33}(M_X)=D_{33}(M_X)=3.0$ and $m_t^p=50$ GeV. The lower limit of $v_D/v_U=0.37$ is obtained by using $m_L>41$ GeV given by the UA1 Collaboration.[24] We have cut off v_D/v_U around 1.2, above which the value of m_v^W becomes considerably larger than m_a^W. The obtained mixings are for $m_t^p=40$ (50,70) GeV,

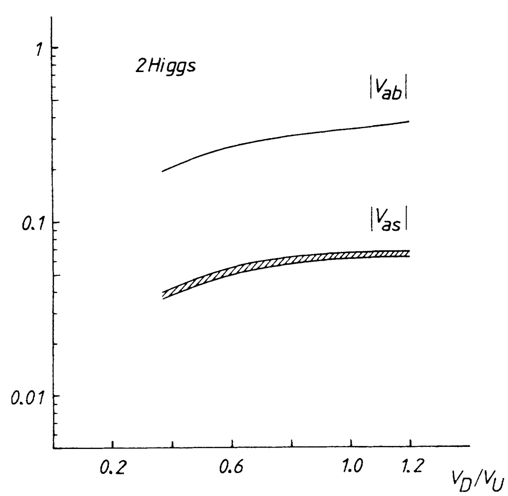

FIG. 5. The value of $|V_{ab}|$ and $|V_{as}|$ vs v_D/v_U for the two-Higgs-doublet model, where $U_{33}(M_X)=D_{33}(M_X)=3.0$ and $m_t^p=50$ GeV are taken.

$$\begin{aligned}0.164(0.210,0.298) &\lesssim |V_{ab}| \lesssim 0.282(0.339,0.440)\ ,\\ 0.034(0.040,0.050) &\lesssim |V_{as}| \lesssim 0.061(0.068,0.077)\ .\end{aligned} \tag{4.3}$$

The $U_{33}(M_X)$ dependence on both V_{ab} and V_{as} are observed to hold, within a few percent, in the region $U_{33}(M_X)=2-4$, which is in contrast with that in Sec. III. These mixing values are larger than that of the one-Higgs-doublet model, because these mixings increase as the energy scale M goes down from M_X to M_W in contrast with the case of the one-Higgs-doublet model.

V. SUPERSYMMETRIC MODEL

At the last step, we discuss the case of the supersymmetric model with minimum two Higgs doublets. The RGE's are given by Eq. (2.2) with $(a,b,c)=(0,2,-\frac{2}{3})$.[3] The values of G_U, G_D, and G_E are written as

$$\begin{aligned}G_U&=\tfrac{16}{3}g_3{}^2+3g_2{}^2+\tfrac{13}{9}g_1{}^2\ ,\\ G_D&=\tfrac{16}{3}g_3{}^2+3g_2{}^2+\tfrac{7}{9}g_1{}^2\ ,\\ G_E&=3(g_2{}^2+g_1{}^2)\ ,\end{aligned} \tag{5.1}$$

where

$$\begin{aligned}(g_3^0/g_3)^2&=1+2(2N_g-9)g_3^{02}t\ ,\\ (g_2^0/g_2)^2&=1+2(2N_g-5)g_2^{02}t\ ,\\ (g_1^0/g_1)^2&=1+2(\tfrac{10}{3}N_g+1)g_1^{02}t\ .\end{aligned} \tag{5.2}$$

The GUT condition as to the gauge couplings $G_3^0=g_2^0=(\frac{5}{3})^{1/2}g_1^0=0.96$ has been given at $M_X=2\times10^{16}$ GeV[3]. The vacuum expectation values v_U, v_D, and v_E are the same as the ones in the two-Higgs-doublet model.

The evolutions of mixings are shown in Fig. 1, where we have taken that $U_{33}(M_X)=D_{33}(M_X)=3.0$, $m_t^p=50$ GeV, and $v_D/v_U=1.0$. The values of parameters at M_X and the masses at M_W are summarized in Table I. As the energy scale M decreases from M_X to M_W, both magnitudes of the mixings V_{ab} and V_{as} increase to about 200%, which are similar to the two-Higgs-doublet model.

We show the allowed regions of the physical top-quark mass versus the running s-quark mass at M_W in the case of $v_D/v_U=1$ in Fig. 6, which are

$$23\ \text{GeV}\lesssim m_t^p\lesssim 29\ \text{GeV},\quad 35\ \text{GeV}\lesssim m_t^p\lesssim 71\ \text{GeV}\ . \tag{5.3}$$

These regions are also dependent on the value of v_D/v_U. If we take $v_D/v_U=0.42$ instead of $v_D/v_U=1$, we get the allowed region such as

$$23\ \text{GeV}\lesssim m_t^p\lesssim 30\ \text{GeV},\quad 38\ \text{GeV}\lesssim m_t^p\lesssim 82\ \text{GeV}\ . \tag{5.4}$$

The tendency of evolutions of V_{ab}, V_{as}, and V_{cb} are the same as the two-Higgs-doublet model, but somewhat steep. In Table II, the calculated values of $|V_{cb}|$, $|V_{as}|$, and $|V_{ab}|$ at M_X and M_W are summarized to-

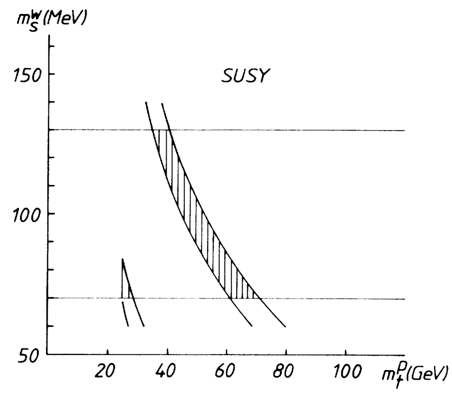

FIG. 6. The allowed region of m_t^P vs m_s^W for the supersymmetric model, where $U_{33}(M_X)=D_{33}(M_X)=3$ and $v_D/v_U=1$ are taken.

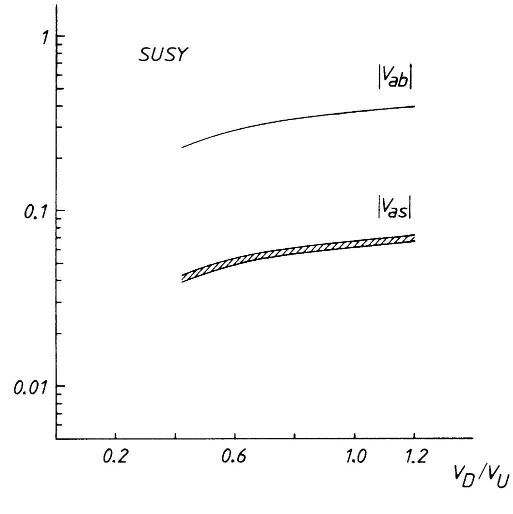

FIG. 7. The value of $|V_{ab}|$ and $|V_{as}|$ vs v_D/v_U for the supersymmetric model, where $U_{33}(M_X)=D_{33}(M_X)=3.0$ and $m_t^P=50$ GeV are taken.

gether with the results of other models. In Fig. 7, $|V_{ab}|$ and $|V_{as}|$ at M_W are shown versus $v_D/v_U(\approx 0.37-1.2)$ in keeping $U_{33}(M_X)=D_{33}(M_x)=3.0$ and $m_t^P=50$ GeV. The lower and upper limits are taken due to the same reason as in Sec. IV. The obtained mixings are, for $m_t^P=40$ (50,70) GeV,

$$\begin{aligned} 0.185(0.245,0.341) &\lesssim |V_{ab}| \\ &\lesssim 0.298(0.364,0.479)\,, \\ 0.036(0.043,0.051) &\lesssim |V_{as}| \\ &\lesssim 0.061(0.067,0.075)\,. \end{aligned} \tag{5.5}$$

The $U_{33}(M_X)$ dependence of both $|V_{ab}|$ and $|V_{as}|$ are very small as well as in the case of the two-Higgs-doublet model. The obtained values of the mixings are almost the same as the ones in Sec. IV, because the evolution of the mixings are similar to the two-Higgs model.

VI. B_s^0-$\bar{B}_s^0$ MIXING

For three models as to the Higgs sector, we get rather large mixings V_{ab} and V_{as} in the previous section. Then it is interesting to see how these mixings have an effect on the low-energy physics. One of the interesting phenomena is the B_s^0-$\bar{B}_s^0$ mixing as already discussed by many authors.[24] The well-known physical mixing parameter r is given as[24]

$$r=\frac{x^2}{2+x^2}\,, \tag{6.1}$$

where

$$\begin{aligned} x &= \Delta M_B/\Gamma_B \\ &\approx (3.2\pm 0.4)\times 10^3 \left|\sum_{t,a} E(x_i,x_j)\lambda_i\lambda_j\right| \frac{B_B f_B{}^2}{(0.15)^2}\,, \end{aligned} \tag{6.2}$$

where $\lambda_i=V_{ib}^*V_{is}$, and $E(x_i,x_j)$ (Ref. 25) is the usual dimensionless box-diagram function. In terms of $x_i=(m_i^P)^2/M_W{}^2$, B_B is the bag parameter and f_B is the decay constant of B_s^0. In deriving Eq. (6.2), we take the B_s^0 meson mass as 5.5 GeV and $1/\Gamma_B=\tau_B=(1.26\pm 0.16)\times 10^{-12}$ sec[5], and QCD correction factor as 0.85. Since B_B and f_B are ambiguous, we cannot exactly predict the parameter r even if we knew the numerical value of m_t^P, m_a^P, λ_t, and λ_a. However, we believe that the value of $B_B f_B{}^2=(0.15\text{ GeV})^2$ ($f_B=0.2$ GeV and $B_B=0.6$) is not far from the true value,[26] and so we start to discuss the value of r under this value of $B_B f_B{}^2$. It is found that the fourth-generation quark contributes to the value of x in the same sign as the contribution of the top quark in the case of the Fritzsch ma-

TABLE III. The values of the $B_s{}^0$-$\bar{B}_s{}^0$ mixing for the three models and the used masses of the quarks. Here x_a/x denotes the relative ratio of the a quark contribution to the total x.

	m_a^P (GeV)	m_t^P (GeV)	m_s^W (MeV)	x	r	x_a/x
One Higgs	200	40	130	1.1 ± 0.1	0.38	0.14
		50	109	1.9 ± 0.2	0.64	0.16
Two Higgs	150	40	126	2.9 ± 0.4	0.80	0.85
($v_D/v_U=1$)		50	98	4.4 ± 0.6	0.91	0.91
SUSY	150	40	125	3.1 ± 0.4	0.83	0.86
($v_D/v_U=1$)		50	97	4.8 ± 0.6	0.92	0.92

trix with $m_t^P \geq 40$ GeV. We summarize in Table III the predicted value of r for $m_t^P = 40$ and 50 GeV in the case of three models as to the Higgs sector. We roughly take the value of the physical a-quark mass as 200, 150, and 150 GeV, respectively, as seen in Table I. The value of m_s^W is taken in order to reproduce the observed $|V_{cb}|$. The value of x_a denotes the contribution of the a quark to total x and then the ratio x_a/x means the relative contribution of the fourth-generation quark.

As shown in Table III, the contribution of the fourth-generation quark is very large for the B_s^0-$\bar{B}_s^0$ mixing. In the three-generation scheme, Altarelli has given $r = 0.2-0.6 (x = 0.7-1.7)$ (Ref. 5) by use of the same parameters in our paper. Hence, it is found that the fourth-generation quark is important for the B_s^0-$\bar{B}_s^0$ mixing in both the two-Higgs-doublet model and the supersymmetric one. Thus the measurement of the r parameter is very interesting for our scheme, although the problem of ambiguous B_B and f_B still remains.

VII. CONCLUSIONS

We have studied the effect of the fourth-generation quarks on the lighter generations using RGE's. For the initial conditions of the Yukawa couplings at M_X, we have used the Fritzsch form with the simple phases conjectured by Shin, which is phenomenologically successful in the three-generation scheme. We find that the renormalization effect is very important if the heavy fourth-generation quarks exist. It is noticed that the magnitude of the mixings V_{ab} and V_{as} depends on the models under the consideration of the Higgs sector. In the case of large Yukawa couplings, they reach the fixed point of RGE's and the evolutions of the mixings V_{ab} and V_{as} are remarkable. The order of the mixing V_{ab} is $O(\lambda)-O(\lambda^2)$ in the one-Higgs-doublet model, and $O(\lambda)$ in both the two-Higgs-doublet model and the supersymmetric model, where $\lambda(=0.23)$ is the Cabibbo angle. On the other hand, the mixing V_{as} is $O(\lambda^2)-O(\lambda^3)$ in the one Higgs-doublet-model and $O(\lambda^2)$ in both the two-Higgs-doublet model and the supersymmetric model. Our result is, of course, not general in the four-generation scheme because it depends on the specific form of the mass matrix. However, we believe that the successful model in the three-generation scheme could be enlarged to the four-generation scheme. So, assuming the fourth generation, we have examined to what extent it gives the mixings V_{ab} and V_{as}, and then have obtained indeed, the large mixings according to expectation. It is worth noticing that these mixings have a significant effect on the B_s^0-$\bar{B}_s^0$ mixing. Also, since the mass value of the top quark is restricted very tightly in the Fritzsch matrix, the observation of the top quark is very important for our scheme. We expect to observe the top quark and the fourth-generation fermions at KEK TRISTAN, the Stanford Linear Collider, and CERN LEP in the near future.

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