Model for the dynamical generation of lepton, quark, and intermediate-boson masses

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In an $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge-invariant model without the Higgs sector the fermions are distinguished by different hypercharges of a new renormalizable interaction with an "Abelian" vector boson C of mass M. An interplay of all interactions which contribute to the fermion mass generation yields the fermion mass formulas $m_i(l)=M\exp[-\pi/3\alpha'(\mu_i)]$, $m_i(u)=M\exp\{-\pi/[2\alpha_s(\mu_i)+\frac{2}{3}\alpha'(\mu_i)]\}$, and $m_i(d)=M\exp\{-\pi/[2\alpha_s(\mu_i)-\frac{1}{3}\alpha'(\mu_i)]\}$ for the charged leptons l_i, u_i, and d_i quarks, respectively. Here α' and α_s are the running coupling strengths of the $U(1)_Y$ and $SU(3)_c$ gauge interactions, respectively, and μ_i are the physically preferred renormalization points determined by the C hypercharges. The intermediate-boson masses m_W and m_Z are expressed in terms of the fermion masses by sum rules.

I. INTRODUCTION

In the standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ model,[1] the masses of leptons, quarks, and intermediate bosons are generated by the Higgs mechanism.[2] Although field-theoretically unobjectionable, this mechanism of the mass generation must be regarded as phenomenological by definition: Each mass is determined by its own coupling constant, and its *independent renormalization* simulates the ordinary mass renormalization.

Hope for the calculable mass spectrum of leptons, quarks, and intermediate bosons is provided by theories with a dynamical symmetry breakdown.[3] There, the only parameters in a Lagrangian which undergo the renormalization are the gauge coupling constants. Hence, most of the mass ratios, if dynamically generated, must be the calculable numbers. A theory of this sort should provide a microscopic foundation of the successful phenomenological Higgs approach.

In principle, the strategy is simple. (i) The fermion mass term is a bridge between a left-handed and a right-handed fermion field. In the standard model, such a bridge is built up easily from the Yukawa coupling by an assumption of the nonzero vacuum expectation value of the Higgs field. Without the Higgs field, such a bridge can be built up provided that there exist gauge bosons interacting both with the left-handed and right-handed fermion fields. Technically, one has to find a finite solution of the Schwinger-Dyson (SD) equation for the fermion proper self-energy part Σ. (ii) The gauge-boson mass squared is given by the residue at the single massless pole of the polarization tensor $\Pi_{\alpha\beta}$ of the gauge field. In the standard model, the massless pole in the polarization tensor is due to the "would-be" Nambu-Goldstone (NG) boson described by the Higgs field, and the residue is related to a nonzero vacuum expectation value of this field. Without the Higgs field, the massless pole in the polarization tensor can only be due to a dynamical "would-be" NG boson built up from some fermion fields. The residue is related to a nonzero vacuum expectation value of some fermion-field bilinear combination.

In practice, the situation is sad. A microscopic theory underlying the Higgs theory should, as one always requires from microscopic theories, (1) reproduce good features of the phenomenological theory more economically, (2) be able to calculate the parameters of the phenomenological theory, (3) provide new predictions lying outside the range of validity of the phenomenological theory, and (4) lead to the phenomenological theory by a controllable sequence of approximations. Existing schemes,[3-7] in our opinion, do not satisfy these requirements.

Here we present a model and its solution which, if bona fide, can be regarded as an attempt to approach this difficult task. It is an $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge-invariant model without the Higgs sector supplemented with an "Abelian" vector field C with a mass term $\frac{1}{2}M^2C_\alpha C^\alpha$ interacting with leptons and quarks of both chiralities with a coupling constant h. Such an interaction is renormalizable[8] under certain circumstances and, in general, for any number of fermions it brings only three new ultraviolet renormalizations: an uninteresting renormalization of the field C, the renormalization of the coupling constant h, and the renormalization of M, which fixes the overall mass scale.

We take the liberty of ascribing the different C hypercharges $y(f_L)$ and $y(f_R)$ to the different fermions f_L and f_R, respectively. We emphasize that these hypercharges, which we call the heaviness for the sake of brevity, are the pure numbers not undergoing renormalization. Phenomenologically it is desirable to have a tool for distinguishing otherwise indistinguishable fermions $(e,\mu,\tau,\ldots)$, $(u,c,t,\ldots)$, and $(d,s,b,\ldots)$.

The basic point of this paper is the following. For all fermions (except for the neutrinos) the kernel of the SD equation due to the vector-boson exchanges consists of a massless attraction and a massive repulsion [we take $y(f_L)y(f_R)<0$]. At some physically distinguished renormalization point μ_f at which the corresponding independently running coupling constants become equal, the kernel of the SD equation becomes Fredholm type. The fermion mass calculated at the point μ_f is

$$m_f=M(\mu_f)\exp[8\pi^2/3y(f_L)y(f_R)h^2(\mu_f)] .$$

This mass formula has the appealing property of yielding

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vastly different fermion masses as a response to slightly different heaviness.

Dynamically appearing fermion masses break the gauge $SU(2)_L \times U(1)_Y$ symmetry spontaneously down to $U(1)_{em}$. As a necessary consequence, W and Z bosons acquire masses m_W and m_Z, related to the masses of all fermions present in the theory.

In the following sections we elaborate in detail what we have briefly mentioned above. In Sec. II we define the model and discuss its perturbative properties, in particular its anomaly freedom. In Sec. III we describe in detail the mechanism of the fermion mass generation. Its necessary and unique consequence is the appearance of m_W and m_Z. This is elaborated in Sec. IV. Section V is devoted to an analysis of the obtained mass formulas, and Sec. VI contains brief conclusions

II. THE MODEL

Perturbatively, the model is defined by its Lagrangian

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}_{fL} i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_\alpha + ig'\tfrac{1}{2}B_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)\psi_{fL} + \bar{\nu}_{fR} i\gamma^\alpha(\partial_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)\nu_{fR} + \bar{l}_{fR} i\gamma^\alpha(\partial_\alpha - ig'B_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)l_{fR} \\ & + \bar{q}_{fL} i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\boldsymbol{\tau}\cdot\mathbf{A}_\alpha - ig'\tfrac{1}{6}B_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)q_{fL} + \bar{u}_{fR} i\gamma^\alpha(\partial_\alpha - ig'\tfrac{2}{3}B_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)u_{fR} \\ & + \bar{d}_{fR} i\gamma^\alpha(\partial_\alpha + ig'\tfrac{1}{3}B_\alpha - ih\tfrac{1}{2}Y_{fH}C_\alpha)d_{fR} - \tfrac{1}{4}(\partial_\alpha\mathbf{A}_\beta - \partial_\beta\mathbf{A}_\alpha + g\,\mathbf{A}_\alpha\times\mathbf{A}_\beta)^2 - \tfrac{1}{4}(\partial_\alpha B_\beta - \partial_\beta B_\alpha)^2 \\ & - \tfrac{1}{4}(\partial_\alpha C_\beta - \partial_\beta C_\alpha)^2 + \tfrac{1}{2}M^2 C_\alpha C^\alpha + \text{QCD} + \text{analogous contribution from new fermions}\;. \end{aligned} \tag{1}$$

In the standard notation, f is the family index. We note that a possibility of employing the Abelian vector boson with an explicit mass term for the dynamical mass generation was already mentioned in the pioneering work of Ref. 9.

To be defined, the model (1) has to be renormalizable. Since it is naively renormalizable essentially by power counting, the problem reduces to the anomaly freedom. We distinguish several cases discussed in the literature.

(i) Consider only the known fermions plus one right-handed neutrino per family, the case explicitly written up in (1). The theory will be $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_H$ anomaly-free for the heaviness

$$Y_H \equiv (y(q_L), y(u_R), y(d_R); y(\psi_L), y(\nu_R), y(l_R))$$

expressed in terms of two arbitrary real parameters α_f and β_f for each family f (Refs. 10 and 11):

$$Y_{fH} = \alpha_f Y_H^{(1)} + \beta_f Y_H^{(2)}\;, \tag{2}$$

where $Y_H^{(1)} = (\tfrac{1}{3}, \tfrac{4}{3}, -\tfrac{2}{3}; -1, 0, -2)$, and $Y_H^{(2)} = (0, 1, -1; 0, 1, -1)$.

(ii) Consider the same fermions as in (i), but demand only the overall $SU(2)_L \times U(1)_Y \times U(1)_H$ anomaly freedom. This amounts in discriminating the perturbative QCD. A new solution for the heaviness appears in this case:[10]

$$Y_{fH} = \gamma_f Y_H^{(1)} + \delta_f Y_H^{(3)}\;, \tag{3}$$

where γ_f and δ_f are arbitrary real parameters and $Y_H^{(3)} = (0, 5, 1; 0, -(35)^{1/3}, -7)$.

(iii) Reference 11 contains the analysis of the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_H$ anomaly freedom taking into account, aside from the known fermions, the additional ones in real representations and in the complex-conjugate pairs. We will see that our results indicate the necessity of considering even more general possibilities.

Model (1) has to be renormalized at a nonzero $p^2 = -\mu^2$ due to the infrared divergences (all particles except the C boson are perturbatively massless). A nice feature of the Lagrangian (1) is that there is no genuine ultraviolet renormalization of the mass parameter M in it.[8,12] M is renormalized by the same infinite renormalization constant as the coupling constant h: $M = Z_3^{1/2}(h_0, \Lambda/\mu)M_0$. A finite renormalization is necessary to convert the renormalized mass parameter into a physical mass.

Finally we note that the numbers Y_{fH} can in principle be completely fixed (quantized) by embedding properly the model (1) into a grand-unified-theory (GUT) group.[13] We, however, do not make any attempt in this direction, and appreciate that the model (1) is perturbatively well defined in isolation.

III. FERMION MASSES

The gauge $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian (1) guarantees masslessness of fermions and W and Z bosons in every order in the perturbation theory. This symmetry also precludes the presence of the corresponding mass counterterms (in contrast with the massless scalar theories). Consequently, if the masses are found by a genuine nonperturbative technique, they must be finite and calculable.[14]

The fermion mass

$$m \equiv \Sigma(p^2 = m^2)$$

is determined in general by finding the chiral-symmetry-breaking proper self-energy part $\Sigma(p^2)$ as a finite solution of the SD equation for the inverse fermion propagator $S^{-1}(p) = \not{p} - \Sigma(p^2)$ of a chirally invariant Lagrangian.

For the sake of simplicity we assume that the fermion weak-interaction eigenstates are identical with the corresponding mass eigenstates (no fermion mixing).

A. Charged leptons

For Dirac fermions, only those vector bosons in (1) contribute to Σ, which interact both with the left- and right-handed fermion fields. In the case of the charged leptons, these are the massless $U(1)_Y$ boson B and the new massive boson C. The corresponding SD equation for $\Sigma_i(p^2)$, $i=e,\mu,\tau,\ldots$, with Wick's rotation already performed, is[14]

$$\Sigma_i(p^2)=3\int\frac{d^4k}{(2\pi)^4}\left[\frac{2g'^2(\mu)}{(p-k)^2}+\frac{y(\psi_{iL})y(l_{iR})h^2(\mu)}{(p-k)^2+M^2(\mu)}\right]\frac{\Sigma_i(k^2)}{k^2+\Sigma_i^2(k^2)} . \tag{4}$$

The fermion propagator in (4) is full, the vertices and the boson propagators (in the Landau gauge) are taken bare. This (ladder) approximation is justified for small coupling constants, and it is the first step in a systematic iteration procedure.[15] Σ is the only quantity which must be found nonperturbatively.

In contrast with the models without the bare masses where the mass scale is given by the renormalization point μ, the mass scale in our model is provided by the mass M of the C boson. Since one arbitrary mass scale is enough, it seems natural to fix μ in (4). Physically preferred renormalization points μ_i are those at which the two independently running coupling constants $2g'^2(\mu)$ and $y(\psi_{iL})y(l_{iR})h^2(\mu)$ meet, provided that $y(\psi_{iL})y(l_{iR})<0$:

$$2g'^2(\mu)+y(\psi_{iL})y(l_{iR})h^2(\mu)=0 . \tag{5}$$

This condition is reminiscent of the condition imposed on two independent coupling constants e and λ in the model of the dynamical mass generation of Coleman and Weinberg.[16]

With (5) taken into account Eq. (4) becomes[14,15]

$$\Sigma_i(p^2)=-3y(\psi_{iL})y(l_{iR})h^2(\mu_i)M^2(\mu_i)\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2[(p-k)^2+M^2(\mu_i)]}\frac{\Sigma_i(k^2)}{[k^2+\Sigma_i^2(k^2)]} ;$$

i.e., its kernel is Fredholm. Although the exact solution of this equation is not known, the approximate value of the fermion mass $m_i\simeq\Sigma_i(0)$ defined at μ_i is, fortunately, determined with a good accuracy:[14,15]

$$\begin{aligned}m_i(l)&=M(\mu_i)\exp[8\pi^2/3y(\psi_{iL})y(l_{iR})h^2(\mu_i)]\\&=M(\mu_i)\exp[-\pi/3\alpha'(\mu_i)] .\end{aligned} \tag{6}$$

An important point is that the conclusion of Refs. 14 and 15 is about inconsistency of the solution (6) for a small coupling constant which does not apply here, since M is not a dynamically generated mass.

Because the relation $g'=e/\cos\theta_W$ is valid in the present model we express the different lepton masses m_i in terms of the fine-structure constant $\alpha=e^2/4\pi$ and the Weinberg angle $\tan\theta_W=g'/g$, both determined at the different scales μ_i.

B. Neutrinos

In general, neutrinos, being the only electrically neutral fermions, can dynamically acquire both the Dirac and the Majorana masses. However, the Fredholm kernel cannot be constructed either for the Dirac mass term $\bar{\nu}_L\Sigma^D\nu_R$ or for the Majorana mass term $\bar{\nu}_R\Sigma_R^M(\nu_R)^C$ of the right-handed neutrino field. The reason is that the weak hypercharge of ν_R is zero and, consequently, only the C boson contributes to Σ^D and Σ_R^M in these cases. The Fredholm kernel cannot be constructed either for the Majorana mass term $\bar{\nu}_L\Sigma_L^M(\nu_L)^C$ of the left-handed neutrino field, although for a different reason—all contributions to Σ_L^M (i.e., A^3,B,C) have necessarily the same sign:

$$g^2(\mu)/(p-k)^2+g'^2(\mu)/(p-k)^2+y^2(\psi_{iL})h^2(\mu)/[(p-k)^2+M^2]\neq0 .$$

Hence, the neutrinos stay massless in our approach (without mixing).

C. Quarks

At small distances where the strong-interaction coupling constant $g_s(\mu)$ is small due to asymptotic freedom, QCD is treated perturbatively. This means that the current quarks and the gluons at small distances behave like ordinary particles, i.e., like leptons and electroweak gauge bosons. Consequently, we are obliged to include into the dynamical generation of the current-quark masses, besides the B and C exchanges, also the attractive exchange of the massless gluons.

The conditions analogous to (5) that fix the renormalization points μ_i are then

$$\tfrac{4}{3}g_s^2(\mu_i)+\tfrac{4}{9}g'^2(\mu_i)+y(q_{iL})y(u_{iR})h^2(\mu_i)=0 \tag{7}$$

for $i=u,c,t,\ldots$ quarks and

$$\tfrac{4}{3}g_s^2(\mu_i)-\tfrac{2}{9}g'^2(\mu_i)+y(q_{iL})y(d_{iR})h^2(\mu_i)=0 \tag{8}$$

for $i=d,s,b,\ldots$ quarks. The quark mass formulas are, correspondingly,

$$\begin{aligned}m_i(u)&=M(\mu_i)\exp[8\pi^2/3y(q_{iL})y(u_{iR})h^2(\mu_i)]\\&=M(\mu_i)\exp\{-\pi/[2\alpha_s(\mu_i)+\tfrac{2}{3}\alpha'(\mu_i)]\}\end{aligned} \tag{9}$$

for $i=u,c,t,\ldots$ quarks and

$$m_i(d)=M(\mu_i)\exp[8\pi^2/3y(q_{iL})y(d_{iR})h^2(\mu_i)]$$
$$=M(\mu_i)\exp\{-\pi/[2\alpha_s(\mu_i)-\tfrac{1}{3}\alpha'(\mu_i)]\} \quad (10)$$

for $i=d,s,b,\ldots$ quarks.

It seems natural that the quark current masses are expressed not only in terms of the electric charge and the Weinberg angle like the lepton masses, but also in terms of the QCD charge.

D. Fermion mixing

There is no reason besides simplicity for the assumption that the weak-interaction eigenstates are identical with the mass eigenstates. In general, the dynamically generated proper self-energy parts $\Sigma(p^2)$ are nondiagonal matrices in the space of the fermion weak-interaction eigenstates with equal electric charges. It is quite simple to derive the corresponding SD equations. For example, in the case of the charged leptons such an equation takes the form

$$\Sigma_{ij}(p^2)=3\int\frac{d^4k}{(2\pi)^4}\left[\frac{2g'^2(\mu)}{(p-k)^2}+\frac{y(\psi_{iL})y(l_{jR})h^2(\mu)}{(p-k)^2+M^2(\mu)}\right]\{\Sigma(k^2)[k^2+\Sigma^+(k^2)\Sigma(k^2)]^{-1}\}_{ij} . \quad (11)$$

The identity $\Sigma(p^2-\Sigma^+\Sigma)^{-1}=(p^2-\Sigma\Sigma^+)^{-1}\Sigma$ is used whenever it is necessary.

As before, we may fix the subtraction points μ_{ij} by imposing the conditions $2g'^2(\mu_{ij})+y(\psi_{iL})y(l_{jR})h^2(\mu_{ij})=0$ so that Eq. (11) will have the Fredholm kernel. However, in contrast with the unmixed case, we are not able to determine $\Sigma_{ij}(0)$, since Eq. (11) is coupled. Obviously, for the quarks the situation is identical to that of leptons. In the following we simply assume that some nondiagonal solutions $\Sigma^l(0)$, $\Sigma^u(0)$, and $\Sigma^d(0)$ do exist for the charged leptons, $Q=\frac{2}{3}$, and $Q=-\frac{1}{3}$ quarks, respectively.

Further analysis is standard: The matrices Σ are diagonalized by the famous biunitary transformations

$$\Sigma^l(0)=U^\dagger(l_L)m(l)U(l_R) ,$$
$$\Sigma^u(0)=U^\dagger(u_L)m(u)U(u_R) ,$$

and

$$\Sigma^d(0)=U^\dagger(d_L)m(d)U(d_R) ,$$

which simultaneously define the corresponding fermionic mass eigenstates $l'_{L,R}=U(l_{L,R})l_{L,R}$, $u'_{L,R}=U(u_{L,R})u_{L,R}$, and $d'_{L,R}=U(d_{L,R})d_{L,R}$ with the masses $m_i(l)=(m_e,m_\mu,m_\tau,\ldots)$, $m_i(u)=(m_u,m_c,m_t,\ldots)$, and $m_i(d)=(m_d,m_s,m_b,\ldots)$, respectively.

When the Lagrangian (l) is rewritten in terms of the mass eigenstates we see that (i) the electromagnetic current and the weak neutral current remain intact, (ii) the charged quark current acquires the unitary Kobayashi-Maskawa mixing matrix $U(u_L)U^\dagger(d_L)$, (iii) the charged-lepton current remains intact due to arbitrariness in defining the massless neutrino eigenstates provided the neutrinos stay massless even with mixing, and (iv) the neutral current coupled to the C boson becomes

$$J_\alpha^C=\tfrac{1}{2}[\bar\nu'_L V(l_L)\gamma_\alpha\nu'_L+\bar l'_L V(l_L)\gamma_\alpha l'_L+\bar\nu'_R y(\nu_R)\gamma_\alpha\nu'_R+\bar l'_R V(l_R)\gamma_\alpha l'_R$$
$$+\bar u'_L V(u_L)\gamma_\alpha u'_L+\bar d'_L V(d_L)\gamma_\alpha d'_L+\bar u'_R V(u_R)\gamma_\alpha u'_R+\bar d'_R V(d_R)\gamma_\alpha d'_R] , \quad (12)$$

where the Hermitian matrices V are defined as

$$V(l_L)=U(l_L)y(\psi_L)U^\dagger(l_L) ,$$
$$V(l_R)=U(l_R)y(l_R)U^\dagger(l_R) ,$$
$$V(u_L)=U(u_L)y(q_L)U^\dagger(u_L) ,$$
$$V(u_R)=U(u_R)y(u_R)U^\dagger(u_R) ,$$
$$V(d_L)=U(d_L)y(q_L)U^\dagger(d_L) ,$$
$$V(d_R)=U(d_R)y(d_R)U^\dagger(d_R) .$$

Hence, the neutral current (12) is *flavor changing*. This property imposes a severe phenomenological constraint on h^2/M^2. The analyses[17] imply

$$M>10^3 \text{ TeV} . \quad (13)$$

Since the mixing matrices V in (12) are not unitary, the new neutral-current interaction is *not universal* in the usual sense. Another interesting property of (12) is that it provides, in principle, the *observability of the right-handed mixing angles.* Finally, if the phases of the quark fields are fixed so as to reproduce the Kobayashi-Maskawa mixing matrix, the current (12) yields a *new source of the CP violation.*

IV. INTERMEDIATE-BOSON MASSES

The dynamically generated fermion masses Σ break spontaneously the gauge $SU(2)_L\times U(1)_Y$ symmetry down to $U(1)_{em}$. Consequently, the W and Z bosons acquire dynamically the masses. To show this, we must calculate the residue at the massless pole of the polarization tensor of the W and Z gauge fields. The massless poles correspond to the "would-be" NG bosons. They are seen in the proper vertex functions Γ_W^α and Γ_Z^α as necessary consequences of the $SU(2)_L\times U(1)_Y$ Ward-Takahashi identities[5] which we assume renormalizably maintained:

$$\Gamma_W^\alpha(p+q,p)\underset{q\to 0}{=}\frac{g}{2\sqrt2}\gamma^\alpha(1-\gamma_5)-\frac{g}{2\sqrt2}\frac{q^\alpha}{q^2}[(1-\gamma_5)\Sigma_U(p+q)-(1+\gamma_5)\Sigma_D(p)] ,$$

$$\Gamma_Z^\alpha(p+q,p) \underset{q\to 0}{=} \frac{g}{2\cos\theta_W}\left[t_3\gamma^\alpha(1-\gamma_5)-2Q\gamma^\alpha\sin^2\theta_W-\frac{q^\alpha}{q^2}t_3[\Sigma(p+q)+\Sigma(p)]\gamma_5\right].$$

From the pole term of Γ_W^α and Γ_Z^α we extract the effective vertices between fermions and the dynamical "would-be" NG bosons:

$$P_\pm=\frac{(1\mp\gamma_5)m_U-(1\pm\gamma_5)m_D}{(m_U{}^2I_{U;D}+m_D{}^2I_{D;U})^{1/2}},$$

$$P_0=\frac{2\gamma_5t_3m}{(m_U{}^2I_{U;U}+m_D{}^2I_{D;D})^{1/2}}.$$

The indices U and D stand for the up (U) and down (D) fermion in a SU(2) doublet. The dimensionless quantities $I_{F;F'}$ will be defined in the following.

The pole term of Γ_W^α can be decomposed[3,9] according to Fig. 1. The only quantity to be calculated is the loop integral

$$J_W^\alpha(q)=\frac{g}{2\sqrt{2}}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}P_-S_U(k)\gamma^\alpha(1-\gamma_5)S_D(k-q)\,. \tag{14}$$

We evaluate it with the fermion propagators approximated by $S_F(p)=[\not{p}+\Sigma_F(p^2)]/(p^2-m_F{}^2)$. The most divergent part of the integral identically vanishes due to the trace, while the rest is finite due to the fact[14,15] that $\Sigma_F(p^2)\sim p^{-4}$ for large p. If we define $\Sigma_F(p^2)=m_F\sigma_F(p^2)$, and

$$I_{U;D}^\alpha(q)=8n_c\int\frac{d^4k}{(2\pi)^4}\frac{(k-q)^\alpha\sigma_U(k^2)}{(k^2-m_U{}^2)[(k-q)^2-m_D{}^2]}\equiv(-iq^\alpha)I_{U;D}(q^2)$$

($n_c=1$ for leptons and $n_c=3$ for quarks), we can rewrite the integral (14) as

$$J_W^\alpha(q)=\frac{g}{2\sqrt{2}}(-iq^\alpha)\frac{m_U{}^2I_{U;D}(q^2)+m_D{}^2I_{D;U}(q^2)}{[m_U{}^2I_{U;D}(0)+m_D{}^2I_{D;U}(0)]^{1/2}}.$$

$J_W^\alpha(q)$ represents the direct coupling of the charged "would-be" NG boson with the W boson (see Fig. 1). Its knowledge is crucial for the determination of $m_W{}^2$ as a residue at the massless pole of the longitudinal part of the polarization tensor of the W boson (see Fig. 2). The rest of $\Pi_{\alpha\beta}$ follows automatically[3] from the property of transversality. Since both lepton and quark doublets operate in this mechanism incoherently, we arrive at the sum rule for $m_W{}^2$:

$$m_W{}^2=\tfrac{1}{4}g^2\sum_{U,D}[m_U{}^2I_{U;D}(0)+m_D{}^2I_{D;U}(0)]\,. \tag{15}$$

The evaluation of $m_Z{}^2$ proceeds quite analogously. The basic loop integral (direct coupling of the neutral "would-be" NG boson with the Z boson) is

$$J_Z^\alpha(q)=\frac{g}{2\cos\theta_W}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}P_0S(k)[t_3\gamma^\alpha(1-\gamma_5)-2Q\gamma^\alpha\sin^2\theta_W]S(k-q)$$
$$=\frac{g}{2\cos\theta_W}(-iq^\alpha)\frac{m_U{}^2I_{U;U}(q^2)+m_D{}^2I_{D;D}(q^2)}{[m_U{}^2I_{U;U}(0)+m_D{}^2I_{D;D}(0)]^{1/2}},$$

where

$$I_{F;F}^\alpha(q)=8n_c\int\frac{d^4k}{(2\pi)^4}\frac{(k-q)^\alpha\sigma_F(k^2)}{[(k-q)^2-m_F{}^2](k^2-m_F{}^2)}\equiv(-iq^\alpha)I_{F;F}(q^2)\,.$$

The knowledge of $J_Z^\alpha(q)$ leads immediately to the sum rule for $m_Z{}^2$:

$$m_Z{}^2=\tfrac{1}{4}(g^2+g'^2)\sum_{U,D}[m_U{}^2I_{U;U}(0)+m_D{}^2I_{D;D}(0)]\,. \tag{16}$$

This completes our program of the dynamical calculation of the elementary-particle masses. To the reader's satisfaction the knowledge of the function $\Sigma_i(p^2)$ is, however, clearly missing.

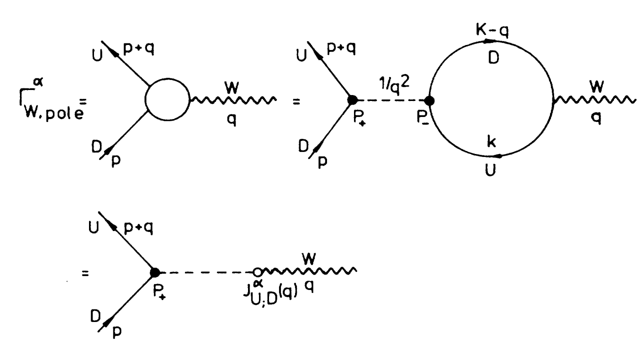

FIG. 1. The effective coupling of charged NG bosons with fermions and W boson.

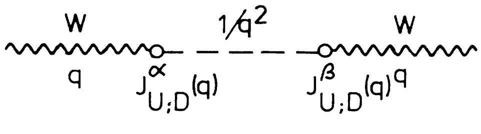

FIG. 2. The pole term in the vacuum-polarization tensor of the *W* boson.

V. ANALYSIS OF THE RESULTS

We start by answering the question of how many parameters we are dealing with in the Lagrangian (1). We must distinguish between those which undergo an infinite renormalization, and between the pure finite numbers.

The former category belongs to the coupling constants g, g', g_s, h, and the mass M of the vector boson C. These must be fixed once forever from an experiment at some point μ_0, say $\mu_0 = m_W = 83$ GeV. Three of these five parameters are already known (g, g', g_s), although g_s not with a satisfactory accuracy.

To the latter category belong the Y and H hypercharges, and the number of the fermion families, F. We should notice the trivial fact that the group structure $SU(2)_L \times U(1)_Y \times U(1)_H$ admits arbitrary Y and H hypercharge assignments. The weak hypercharges are fixed by agreement with the experimentally observed fermion electric charges, and the heaviness will be fixed by agreement with the experimentally observed fermion masses. Both the charge quantization and the mass quantization, together with unique determination of F, are in principle possible within a grand-unification scheme.

A. Fermion masses

The fermion mass formulas to be analyzed are

$$m_i(l) = M \exp[-\pi/3\alpha'(\mu_i)] , \tag{17a}$$

$$m_i(u) = M \exp\{-\pi/[2\alpha_s(\mu_i) + \tfrac{2}{3}\alpha'(\mu_i)]\} , \tag{17b}$$

$$m_i(d) = M \exp\{-\pi/[2\alpha_s(\mu_i) - \tfrac{1}{3}\alpha'(\mu_i)]\} , \tag{17c}$$

where the points μ_i are the solutions of the corresponding equations:

$$3\alpha'(\mu_i) = -\tfrac{3}{2} y(\psi_{iL}) y(l_{iR}) \alpha_h(\mu_i) , \tag{18a}$$

$$2\alpha_s(\mu_i) + \tfrac{2}{3}\alpha'(\mu_i) = -\tfrac{3}{2} y(q_{iL}) y(u_{iR}) \alpha_h(\mu_i) , \tag{18b}$$

$$2\alpha_s(\mu_i) - \tfrac{1}{3}\alpha'(\mu_i) = -\tfrac{3}{2} y(q_{iL}) y(d_{iR}) \alpha_h(\mu_i) . \tag{18c}$$

The coupling constants $\alpha' = g'^2/4\pi$, $\alpha_s = g_s{}^2/4\pi$, and $\alpha_h = h^2/4\pi$ are assumed to run according to the one-loop renormalization-group formulas

$$\frac{1}{\alpha'(\mu)} = \frac{1}{\alpha'(\mu_0)} - \frac{1}{\pi}\frac{10}{9} F \ln(\mu/\mu_0) , \tag{19}$$

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{1}{\pi}(\tfrac{11}{2} - \tfrac{2}{3}F) \ln(\mu/\mu_0) , \tag{20}$$

$$\frac{1}{\alpha_h(\mu)} = \frac{1}{\alpha_h(\mu_0)} - \frac{1}{\pi} b(F) \ln(\mu/\mu_0) , \tag{21}$$

where[18]

$$b(F) = \tfrac{1}{3} \sum_{f=1}^{F} [6y^2(q_{fL}) + 3y^2(u_{fR}) + 3y^2(d_{fR}) + 2y^2(\psi_{fL}) + y^2(\nu_{fR}) + y^2(l_{fR})] .$$

Since we are interested merely in an order-of-magnitude agreement with reality, we do not take into account all possible subtleties such as the two-loop and threshold effects, etc. In particular, we ignore the logarithmic dependence of M upon μ, and take everywhere M as a renormalized constant.

First observation is that the conditions $y(\psi_{iL})y(l_{iR}) < 0$, $y(q_{iL})y(u_{iR}) < 0$, and $y(q_{iL})y(d_{iR}) < 0$, which follow from Eqs. (18) severely restrict the available heaviness assignments. In particular, the solution (2) for Y_{fH} is excluded by these conditions. The solution (3) which, however, corresponds only to the $SU(2)_L \times U(1)_Y \times U(1)_H$ anomaly freedom, survives. New solutions hopefully appear with assuming the existence of new fermions.[11,19]

Another observation is the following. Since the renormalization-group analysis with (19)–(21) is reasonably justified only for the momentum region below the Planck mass, we consider $\mu < 10^{19}$ GeV, $M < 10^{19}$ GeV. An immediate consequence of this restriction is that *more fermions with nonzero standard interactions than those presently known are necessary.* For simplicity, consider only the conventional fermion content of the model. With $F=3$ and $\alpha'(\mu_0) = 0.0099$ (Ref. 20) $\alpha'(\mu)$ remains too small up to $\mu = 10^{19}$ GeV. To get the electron mass $m_e = 5\times 10^{-4}$ GeV from Eq. (17a) would require $M \simeq 10^{24}$ GeV. With $F = 6,7,8$, $\alpha'(\mu)$ grows faster, and the situation looks reasonable. In fact, F is restricted by the requirement of the asymptotic freedom of QCD to $F \leq 8$.

The curves $3\alpha'(\mu)$, $2\alpha_s(\mu) + \tfrac{2}{3}\alpha'(\mu)$, and $2\alpha_s(\mu) - \tfrac{1}{3}\alpha'(\mu)$ relevant to the fermion masses $m_i(l)$, $m_i(u)$, and $m_i(d)$, respectively, are shown in Fig. 3 for $F=6$. Their shapes depend upon $\alpha'(\mu_0)$ and $\alpha_s(\mu_0)$. We fix these values according to Ref. 20: $\alpha'(\mu_0) = 0.0099$ and $\alpha_s(\mu_0) = 0.15$. For the fermion masses we take the values $m_e = 5\times 10^{-4}$ GeV, $m_\mu = 0.105$ GeV, $m_\tau = 1.785$ GeV, $m_u = 5\times 10^{-3}$ GeV, $m_c = 1.35$ GeV, $m_t = 50$ GeV, $m_d = 9\times 10^{-3}$ GeV, $m_s = 0.175$ GeV, and $m_b = 5.6$ GeV.

A further step is to fix M. Since M cannot be taken from experiment at present, we proceed as follows. We place (rather arbitrarily) the lightest $Q = \tfrac{2}{3}$ quark into the minimum of the curve $2\alpha_s + \tfrac{2}{3}\alpha'$, and from the experimental value $m_u = 5\times 10^{-3}$ GeV, using Eq. (17b) we fix the mass scale of the world: $M \simeq 1.14\times 10^{10}$ GeV. The other fermions are placed on their respective curves (there are two possibilities for $Q = \tfrac{2}{3}$ quarks) according to their experimentally known masses using formulas (17). Hence, $\mu_i(l)$, $\mu_i(u)$, and $\mu_i(d)$ become known (Fig. 3).

The last point is to fix the heaviness (3) in such a way as to satisfy Eqs. (18). The shape of the curve $\alpha_h(\mu)$ alone depends only upon two parameters $\alpha_h(\mu_0)$ and $b(F=6)$. Having fixed them, we can calculate from (3) and (18):

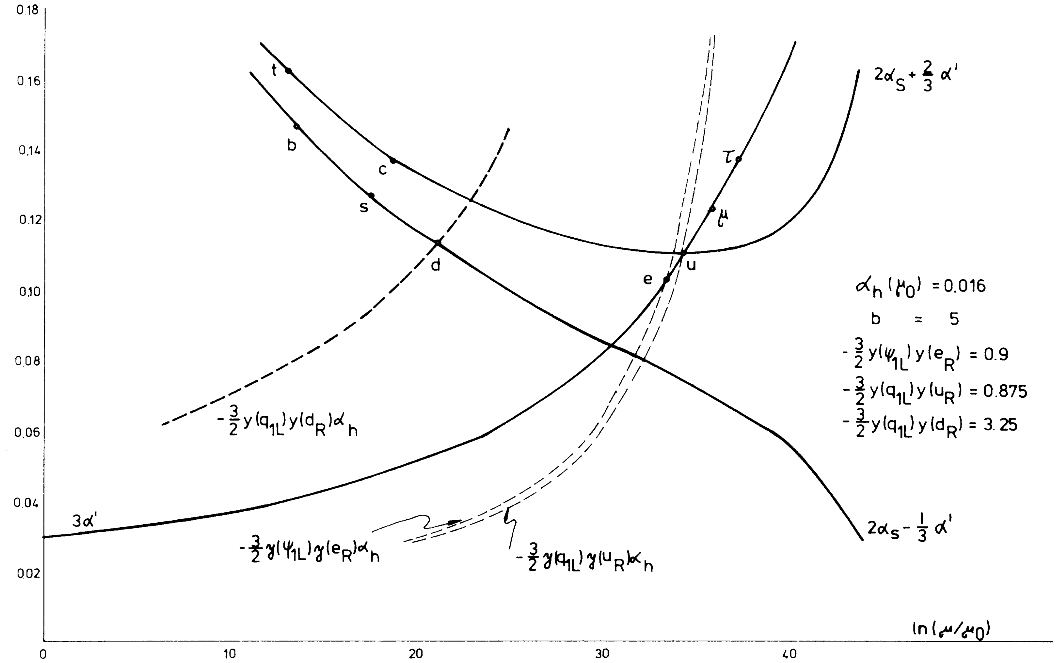

FIG. 3. Typical behavior of the coupling constants which determine the fermion masses.

$$
\begin{aligned}
-\tfrac{3}{2}y(\psi_{iL})y(l_{iR}) &= -\tfrac{3}{2}\gamma_i(2\gamma_i+7\delta_i)=3\alpha'[\mu_i(l)]/\alpha_h[\mu_i(l)]\ ,\\
-\tfrac{3}{2}y(q_{iL})y(u_{iR}) &= -\tfrac{1}{2}\gamma_i(\tfrac{4}{3}\gamma_i+5\delta_i)=\{2\alpha_s[\mu_i(u)]+\tfrac{2}{3}\alpha'[\mu_i(u)]\}/\alpha_h[\mu_i(u)]\ ,\\
-\tfrac{3}{2}y(q_{iL})y(d_{iR}) &= -\tfrac{1}{2}\gamma_i(-\tfrac{2}{3}\gamma_i+\delta_i)=\{2\alpha_s[\mu_i(d)]-\tfrac{1}{3}\alpha'[\mu_i(d)]\}/\alpha_h[\mu_i(d)]\ .
\end{aligned}
\tag{22}
$$

This system of equations for γ_i and δ_i is overdetermined. With γ_i and δ_i calculated from any two equations (22), the remaining equation should be satisfied identically. If verified, this property could be used for "predicting" the mass of the third fermion in each family.

Typical possible values of the parameters $\alpha_h(\mu_0)$, and $b(F=6)$ can be guessed from Fig. 3. We have done the analysis described above with two of them. Neither for $\alpha_h(\mu_0)=0.016$, $b=5$, nor for $\alpha_h(\mu_0)=0.8$, $b=0.1$ do we get close in "predictions" to the experimental values of the masses of arbitrarily selected fermions in each family. We are inclined to interpret this disagreement as a good reason for the existence of new fermions.

Reference 19 suggests yet another possibility: Ignoring entirely the constraints imposed by $U(1)_H$ anomaly freedom we can fix the heaviness $y(f_L)y(f_R)$ from the experimental values of the fermion masses without any difficulty for any reasonable value of $\alpha_h(\mu_0)$ (see Fig. 3).

B. Intermediate-boson masses

Sum rules for the gauge-boson masses follow from the dynamical mechanism of the fermion mass generation in many schemes.[5,6] In our case, the sum rules (15) and (16) imply the following.

(i) The canonical tree-level ratio of the standard model

$$m_W{}^2/m_Z{}^2\cos^2\theta_W=1 \tag{23}$$

is obtained only provided the fermion masses in the weak doublets are degenerate. In reality this is not the case. Hence, we predict a definite (hopefully small) departure from the relation (23). The reason for this is clear. Using the Nambu–Jona-Lasinio-type approach to the present model[21] it is easy to show that the dynamical "would-be" NG bosons form the SU(2) doublets. In contrast with the technicolor approach[4] these doublets are built up from fermions with different masses, and this fact is reflected in the formulas (15) and (16).

(ii) There is no fundamental weak-interaction scale in the present model. The value $(\sqrt{2}G_F)^{-1/2}\simeq 250$ GeV is a remnant of the heavy fermions. The only fundamental mass scale is fixed by the mass of the C boson, and it is much higher, see (13). In fact, for $F=6$ we found $M\sim 10^{10}$ GeV.

(iii) We can determine an absolute upper bound on the heaviest fermion in the world, and to study the saturation of the sum rules (15) and (16) with presently known fermions.

For a numerical illustration of these points we use a model,[22] which satisfies the correct behavior of $\Sigma_i(p^2)$ both at $p^2=0$ and $p^2\to\infty$:

$$\Sigma_i(p^2)=m_iM^4(p^2-M^2)^{-2}=m_i\sigma(p^2)\ . \tag{24}$$

With (24) the integrals $I_{F;F'}(0)$ are easily calculated. For $M\gg m_U$, m_D, which is a limit we are really interested in, we get the sum rules for the W- and Z-boson masses explicitly expressed in terms of the experimentally accessible quantities

$$m_W{}^2=\tfrac{1}{4}g^2\sum\frac{n_c}{(2\pi)^2}\left[m_U{}^2\left[\ln\frac{M^2}{m_U{}^2}-[1+\Delta(\xi)]\right]+m_D{}^2\left[\ln\frac{M^2}{m_D{}^2}-[1+\Delta(\xi)]\right]\right], \quad (25)$$

$$m_Z{}^2=\tfrac{1}{4}(g^2+g'^2)\sum\frac{n_c}{(2\pi)^2}\left[m_U{}^2\left[\ln\frac{M^2}{m_U{}^2}-\frac{3}{2}\right]+m_D{}^2\left[\ln\frac{M^2}{m_D{}^2}-\frac{3}{2}\right]\right]. \quad (26)$$

Here $\xi=m_U{}^2/m_D{}^2$, and $\Delta(\xi)=\xi(\xi^2-1)^{-1}\ln\xi$.

In accordance with the general formulas the relation (23) is exact with (25) and (26) at $\xi=1[\Delta(1)=\frac{1}{2}]$. Deviation from this relation cannot be large even at the extreme limit $\xi\to\infty$, since $\Delta(\xi)$ monotonically decreases from $\frac{1}{2}$ to zero at $\xi=\infty$, and the symmetric logarithmic terms always dominate for $M \gg m_U, m_D$.

An absolute upper bound on the heaviest fermion in the world is obtained in the approximation (24) assuming that the sum rules (25) and (26) are saturated by just one quark:

$$m_W{}^2=\tfrac{1}{4}g^2\frac{3m_U{}^2}{(2\pi)^2}\left[\ln\frac{M^2}{m_U{}^2}-1\right],$$

$$m_Z{}^2=\tfrac{1}{4}(g^2+g'^2)\frac{3m_U{}^2}{(2\pi)^2}\left[\ln\frac{M^2}{m_U{}^2}-\frac{3}{2}\right].$$

For $M=10^6$ GeV we get from the first equation $m_{U\max}\simeq 230$ GeV. With this value $m_W{}^2/m_Z{}^2\cos^2\theta_W=1.032$. Similarly, with $M=10^5$ GeV we get $m_{U\max}\simeq 275$ GeV, and $m_W{}^2/m_Z{}^2\cos^2\theta_W=1.048$. It is good that the dependence of $m_{\max}$ upon M is weak. Even for $M=10^{15}$ GeV we get a reasonable value of $m_{\max}\simeq 120$ GeV.

Finally, we check how the presently known fermions saturate the sum rule for $m_Z{}^2$. With the values of the known fermions listed in the text we get (using $M=10^6$ GeV) $m_Z{}^2=\frac{1}{4}(g^2+g'^2)\times(60\text{ GeV})^2$ instead of $m_Z{}^2=\frac{1}{4}(g^2+g'^2)(250\text{ GeV})^2$. Hence, again, new heavy fermions are necessary. These can be either yet unknown members of the first three families, or the standard members of new families, or both.

VI. CONCLUSIONS

We believe that the mechanism of the fermion mass generation driven by a combined massless attraction and a massive repulsion in the SD equation for the fermion proper self-energy part represents a real computational framework. In any case, the concept of heaviness puts the fermion mass on the same footing with the fermion electric charge. We have essentially no doubt in the subsequent intermediate-boson mass generation. The whole machinery should, however, be improved in several respects. On the theoretical side, the main point is to find trustworthy solution of the SD equation for $\Sigma(p^2)$. On the more phenomenological side, a thorough analysis of the fully acceptable anomaly-free heaviness assignments is necessary.

We conclude with some short comments on the points in the Introduction characterizing the relation between microscopic and phenomenological theories.

(1) Strong, weak, and electromagnetic interactions of the known particles are the same in (1) as in the standard model. Besides g_s, g, and g', there is only *one new genuinely renormalized parameter* in the Lagrangian (1). It is the coupling constant h. This is to be contrasted with plenty of independently renormalized parameters in the Higgs sector of the standard model.

(2) We have demonstrated that the fermion mass ratios, determined in the Higgs approach by fitting the Yukawa coupling constants, are the calculable numbers in our model. We did not succeed in demonstrating this also for the mixing angles. The reason is, hopefully, only technical.

(3) As a new prediction lying outside the Higgs approach we mention the sum rules for $m_W{}^2$ and $m_Z{}^2$. They alone imply that there must exist new massive fermions with $SU(2)_L\times U(1)_Y$ interactions.

(4) Formal derivation of the Higgs Lagrangian from (1) does not exist, but the Nambu–Jona-Lasinio–type procedure applied to it[21] provides a clear hint in this direction.

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