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Radiative-decay systematics and flavor-symmetry breaking from heavy quarks

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Radiative transitions between vector and pseudoscalar mesons are analyzed in a model utilizing SU(N)-flavor symmetry plus breaking from unequal-quark masses, gluon annihilation channels, and particle mixing. Best fits from the light-quark sector propagate terms into the charm sector which require fine-tuning of parameters to remain consistent with data. We conclude that it is unlikely that such models can survive similar extensions to the t- and b-quark sectors without substantial modification.

I. INTRODUCTION

Radiative transitions between vector and pseudoscalar mesons have long been a favorite testing ground for various theoretical models of hadrons.[1] Early formulations in terms of vector-meson dominance of the photon couplings and SU(N)-flavor-symmetric couplings have now largely given way to quarkonium-based models of the mesons.[2] In recent years, a number of new experimental determinations and remeasurements have become available for transitions within the so-called light-quark sector (u,d,s). In addition, some transitions involving mixed heavy (c,b) and light quarks have been seen. Finally, there is a wealth of information on decay modes and upper limits for charmonium into inhibited final states. The purpose of this paper is twofold: first, to reexamine the light-quark sector data and obtain the best possible description using available quark-model and/or symmetry parameters; second, to extend the phenomenology to include the heavy-quark sector, making use of constraints which connect the two sectors (e.g., gluon annihilation contributions). In the next section, we discuss some of the critical tests from individual decay rates and indicate the effects of new experimental information. Section III is devoted to developing a framework for a flavor-SU(N)-symmetric transition amplitude, and adding corrections due to nondegenerate masses, annihilation into gluon channels, and mixed flavor states. The consequences and fits to data are presented in Sec. IV, first for the light-quark sector alone, and then adding the constraints of the mixed- and heavy-quark states. The final section summarizes the results and suggests future possibilities.

II. SURVEY OF SIMPLE RESULTS

We define the $VP\gamma$ coupling g in the usual way, so that the decay width is

$$\Gamma(V\to P\gamma)=\frac{1}{3}\frac{g^2}{4\pi}\frac{k^3}{\Lambda^2} \tag{1}$$

with k the magnitude of the photon momentum in the rest frame of the decaying particle, and Λ a scale factor introduced to make g dimensionless (we take $\Lambda=1.0$ GeV). For the reversed reaction $P\to V\gamma$ an additional factor of 3 appears since there is no averaging over initial spins. The simplest predictions come from assuming a flavor-symmetric SU(N) coupling, so that $g=d_{ij\gamma}$, where the d's are the usual symmetric structure constants for the symmetry group, the ij indices are for VP, and the γ index is taken to be the appropriate combination proportional to the electric charge operator. An explicit realization of this structure appears in the nonrelativistic quarkonium model for the mesons, in which the decay is a magnetic dipole transition between the triplet and singlet s states. The transition amplitude is proportional to the matrix element of the magnetic moment operator μ_{VP}, so that the decay width is

$$\Gamma(V\to P\gamma)=\frac{k^3}{3\pi}(\mu_{VP})^2\,|\,I\,|^2\ , \tag{2}$$

where I is an overlap integral between the wave functions of the vector and pseudoscalar states which should be unity in the nonrelativistic limit ($k\to 0$). The relativistic corrections to this formula can be quite substantial, especially for mesons in the light-quark sector. (One possible correction method results in an extra factor of E_P/M_V, typically a 50% effect.) To avoid these uncertainties, we focus initially on ratios of decay rates for nearly degenerate states, for which phase space is identical and these effects might be expected to cancel.

(1) $(\rho\to\pi\gamma)/(\omega\to\pi\gamma)$. This ratio is predicted to be $\frac{1}{9}$ from SU(3) symmetry, or equivalently from a quark model with equal u and d masses. The $\omega\to\pi\gamma$ rate has long been known to be in the 900-keV range, thus predicting the $\rho\to\pi\gamma$ width of about 100 keV. When the first measurement[3] indicated a width of about 35 keV, many attempts were made to find mechanisms to accommodate the discrepancy. Subsequently, when an updated determination[4] yielded 71 ± 7 keV, it appears as though SU(3) symmetry would only require small corrections. The latest and most accurate determination,[5] however, has found a somewhat smaller value, 59.8 ± 4 keV. It appears now that a ratio of about 14 must be accommodated for the ω/ρ rates. It is obvious that if this were due entirely to differences in the u- and d-quark masses, they would turn out to be much too large to be allowable from other constraints. For example, if one breaks SU(3) by putting in

explicit coupling to quark magnetic moments which are inversely proportional to quark constituent masses, one finds

$$\frac{\Gamma(\rho\to\pi\gamma)}{\Gamma(\omega\to\pi\gamma)}=\left[\frac{2m_d-m_u}{2m_d+m_u}\right]^2 . \quad (3)$$

This would lead to a u-d mass splitting of about 50 MeV, which is much larger than allowable electromagnetic splittings in any realistic constituent-quark model. Within the philosophy of our approach, the only possibility remaining is to exploit the isospin difference between the ρ and ω. This will be done in the next section, where we allow transitions mediated by gluons in the $q\bar{q}$ annihilation channels.

(2) $(K^{*+}\to K^+\gamma)/(K^{*0}\to K^0\gamma)$. This ratio should be $\frac{1}{4}$ in the SU(3) limit. New data on the charged-mode decay[6] width $\Gamma=51.1\pm5.2$ keV (the old upper limit was 80 keV) and the neutral decay width[7] $\Gamma=116\pm11$ keV (the old value was 75 ± 35 keV) give 0.44 ± 0.06 for this ratio. If we allow symmetry breaking with unequal nonstrange and strange-quark masses in the magnetic dipole moments, the result is

$$\frac{\Gamma(K^{*+}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)}=\left[\frac{2m_s-m}{m_s+m}\right]^2 , \quad (4)$$

which predicts $m_s/m=1.24\pm0.08$. This is somewhat smaller than the ratio one would expect from quark constituent masses implied by either mass fits or baryon magnetic moments (typically 500 MeV/330 MeV ≈ 1.5). In this case we have no additional freedom to modify the couplings in annihilation channels, so that if the observed symmetry-breaking is to be correlated with quark masses, it must be more complicated than merely adjusting the Dirac moments. Again, we examine this possibility in the following section.

(3) $(D^{*+}\to D^+\gamma)/(D^{*0}\to D^0\gamma)$. This obvious extension to the charm sector would be $\frac{1}{16}$ if SU(4)-flavor symmetry were exact. Branching ratios for these decays have now been measured,[8] but the absolute rates are unknown, since the total D^* widths only have experimental upper limits. One can get a rough estimate by using SU(4) for the hadronic $D\pi$ decay rates, and normalizing to $\rho\to\pi\pi$. This leads to predictions[9] of approximately 0.5 for the D^* ratio, with uncertainties which could allow it to be as low as 0.1 or as high as 1.0. Even this wide variation, however, does not overlap the symmetry prediction. For magnetic moment expressions involving the charm-quark constituent mass, one obtains

$$\frac{\Gamma(D^{*+}\to D^+\gamma)}{\Gamma(D^{*0}\to D^0\gamma)}=\left[\frac{m_c-2m}{2m_c+2m}\right]^2 . \quad (5)$$

Nominal values of quark mass $m_c\approx1500$ MeV predict a ratio of $\frac{1}{20}$, even lower than the symmetry value, and further away from the allowed experimental region. (Note that the theoretical expression has a maximum value of $\frac{1}{4}$ for $m_c>m$, and even to reach the lower limit of 0.1 for the decay width ratio would require a charm-quark mass of about 2400 MeV.) Thus the situation is similar to that in the strange-quark sector: a more general quark-mass dependence than that for magnetic moments is required to fit even those radiative decay rate ratios which are free of ambiguities from unequal phase-space factors.

(4) $\phi\to\eta_c\gamma$. The measured decay width is 0.80 ± 0.25 keV. Using the $\omega\to\pi\gamma$ for normalization, one would predict about 40 keV in the SU(4) limit, which implies that the charm quark-mass suppression needed in the magnetic moment formula is again larger than expected from mass formulas (m_c of about 2300 MeV in this case).

III. SU(N) SYMMETRY AND SYMMETRY BREAKING

In the SU(N)-flavor-symmetric model, we place the $I=0$ pseudoscalar and vector mesons into (N^2-1)-plets and singlets and represent them by $N\times N$ matrices which we denote by P and V. The properties of the radiative decay can be introduced by representing the photon in an analogous fashion, in a diagonal matrix proportional to the charge operator. In terms of the Cartesian index $i=1,\ldots,(N^2-1)$, one has

$$\text{SU(3)}:\quad Q=(3)+\frac{1}{\sqrt{3}}(8) ,$$

$$\text{SU(4)}:\quad Q=\frac{\sqrt{2}}{3}(0)+(3)+\frac{1}{\sqrt{3}}(8)-(\tfrac{2}{3})^{1/2}(15) , \quad (6)$$

$$\text{SU(5)}:\quad Q=\frac{\sqrt{10}}{15}(0)+(3)+\frac{1}{\sqrt{3}}(8) -\frac{\sqrt{6}}{3}(15)+\frac{\sqrt{10}}{5}(24) .$$

There is only one independent coupling for $VP\gamma$ invariant under SU(N) rotations. We write an effective interaction Lagrangian

$$L=g_1\mathrm{Tr}(P\{Q,V\}) , \quad (7)$$

where the normalization is such that $g_1/3$ is the coupling which appears in (1) for the $\rho\to\pi\gamma$ decay width. The second column of Table I shows the predicted decay widths for SU(4)-symmetric couplings, arbitrarily normalized to the $\rho\pi\gamma$ rate. The deficiencies of the previous discussion are evident, along with the inability to deal with the small but nonzero rates for flavor-changing decays.

As a first step toward symmetry breaking, the charge matrix Q can be changed to be proportional to the magnetic dipole moments of the effective (constituent) quarks. We write the matrix μ as

$$\mu=\frac{m}{m_i}Q , \quad (8)$$

where m_i is the quark constituent mass and m is some SU(N)-symmetric value, and substitute it for Q in (7). A more general way of relating symmetry breaking in the coupling to that observed in the mass spectrum was proposed in Ref. 10. One employs the current mass matrix $M_{ij}=M_i\delta_{ij}$ as an independent component available to construct the interaction Lagrangian, and works to lowest order. Then two additional terms can be constructed:

TABLE I. Decay widths for $VP\gamma$ processes.

Process	Γ(expt.)[a]	Γ(SU(4)) (keV)	Broken SU(3)	Broken SU(4)-A	Broken SU(4)-B
$\rho\to\pi\gamma$	59.8± 4.0[b]	59.8	67.5	64.5	68.5
$\rho\to\eta\gamma$	55±14	23.5	51.1	35.8	49.0
$\eta'\to\rho\gamma$	72± 9.8	103	62.5	73.4	69.1
$\omega\to\pi\gamma$	853±56	574	692	788	680
$\omega\to\eta\gamma$	3.0± 2.5	3.0	5.4	9.6	6.2
$\eta'\to\omega\gamma$	6.5± 1.4	9.3	9.1	6.3	7.2
$\phi\to\pi\gamma$	5.5± 0.6	0	5.5	5.5	5.5
$\phi\to\eta\gamma$	55± 4.6	147	54.5	56.1	55.3
$K^{*+}\to K^+\gamma$	51± 5.2	33.6	51.1	51.1	51.1
$K^{*0}\to K^0\gamma$	116±11[c]	137	115	103	114
$\psi\to\pi\gamma$	0.0025± 0.0007	0		0.0025	0.0025
$\psi\to\eta\gamma$	0.054± 0.009	0		0.054	0.054
$\psi\to\eta'\gamma$	0.265± 0.049	0		0.264	0.264
$\psi\to\eta_c\gamma$	0.800± 0.254	27.5		0.737	0.808
$D^{*+}\to D^+\gamma$	4.6± 3.0[d]	2.9		0.47	1.93
$D^{*0}\to D^0\gamma$	9.7± 3.7[d]	47.8		16.0	9.47
$\eta_c\to\rho\gamma$		0		3011	50.2
$\eta_c\to\omega\gamma$		0		877	16.1
$\eta_c\to\phi\gamma$		0		2290	1.0

[a]All data from the Particle Data Group properties, except where noted.
[b]From Ref. 5.
[c]From Ref. 7.
[d]From Ref. 9.

$$L'=\frac{g_2}{\Lambda}\mathrm{Tr}(\{M,P\}\{V,Q\})+\frac{g_3}{\Lambda}\mathrm{Tr}([M,P][V,Q]) , \quad (9)$$

where we use $\Lambda=1.0$ GeV again to make the g_i dimensionless. For SU(3) no additional parameters come in, since a common mass $M_u=M_d$ can be absorbed into the g_1 terms and the strange-nonstrange mass difference always appears in a dimensionless combination multiplied by g_i/Λ. Starting with SU(4), one additional independent mass ratio per flavor is present in the effective couplings. For this parametrization to be equivalent to the magnetic moment scheme, one finds the constraints $g_2=g_3$ and

$$\frac{m}{m_i}=1+2\frac{g_2M_i}{g_1\Lambda} \quad (10)$$

must be valid in any SU(N). When fitting data in the next section, we use the more general coupling (9).

For the flavor-changing decays, we parametrize the quark annihilation into gluons by flavor-independent couplings g_4 (for 2 gluons) and g_5 (for 3 gluons). Then we can write additional interaction terms

$$L''=g_4\mathrm{Tr}(P)\mathrm{Tr}(VQ)+g_5\mathrm{Tr}(V)\mathrm{Tr}(PQ) . \quad (11)$$

These will provide nonzero couplings for the flavor-changing decays, as well as modify the ratio of singlet to (N^2-1)-plet couplings within the allowed sectors. One additional source of such couplings is through explicit particle mixing, i.e., the quark-antiquark states in the isoscalar sector mix through these same gluon annihilation processes to produce the physical particle states. In each SU(N) multiplet, mixing of the $N-1$ isoscalar mesons can be parametrized with $(N-1)(N-2)/2$ independent real mixing angles. For a definite example, we write the mixing of the η, η', and η_c pseudoscalar states in SU(4), in terms of the "pure" quark-antiquark states 1 = nonstrange, 2 = strange, and 3 = charm:

$$\begin{aligned}\eta&=(\cos\beta\cos\alpha-\cos\gamma\sin\alpha\sin\beta)\,|\,1\,\rangle\\&\quad+(\cos\beta\sin\alpha+\cos\gamma\cos\alpha\sin\beta)\,|\,2\,\rangle\\&\quad+\sin\gamma\sin\beta\,|\,3\,\rangle ,\\\eta'&=(-\sin\beta\cos\alpha-\cos\gamma\sin\alpha\cos\beta)\,|\,1\,\rangle\\&\quad+(-\sin\beta\sin\alpha+\cos\gamma\cos\alpha\cos\beta)\,|\,2\,\rangle\\&\quad+\sin\gamma\cos\beta\,|\,3\,\rangle ,\\\eta_c&=\sin\gamma\sin\alpha\,|\,1\,\rangle-\sin\gamma\cos\alpha\,|\,2\,\rangle+\cos\gamma\,|\,3\,\rangle .\end{aligned} \quad (12)$$

A similar expression holds for the vector mesons ω, ϕ, and ψ in terms of mixing angles α', β', and γ'. To decouple the charm part, we set $\gamma=\gamma'=0$ and define the usual SU(3) mixing angles $\chi=\alpha+\beta$ and $\chi'=\alpha'+\beta'$. We treat these mixing angles as independent parameters for now, although they must of course be related in some manner to the annihilation parameters g_4 and g_5.

IV. DATA FITTING AND PREDICTIONS

We start in the light-quark SU(3) sector alone. We perform a least-squares fit to the ten measured decay widths, using the five couplings g_i and the two mixing angles χ and χ'. The parameter search was limited to values of g_2 through g_5 small compared with g_1, consistent with the philosophy that these represent corrections to a basic SU(3)-symmetric coupling. The results are shown in the third column of Table I. The overall χ^2 of 4.3 for the three degrees of freedom is not bad, but the ω/ρ puzzle remains difficult. The overall coupling g_1 is the only contribution to the ρ decay, so that it is tightly constrained. To account for the factor of 14 in ω decay, the term involving g_5 tries to become large, but it is prevented from getting too large by its contribution to the $\phi\to\pi\gamma$ rate. The ratio of K^* decays is fit quite nicely, but only by using both the g_2 and g_3 couplings independently. Their ratio turns out to be ≈ 1.5, so that an interpretation in terms of magnetic moments alone is not possible. Note that the K^{*+} rate is fit exactly, since the g_3 term appears only in that coupling. The vector mixing angle χ' comes out very small, again a compromise between large ω and small ϕ decay widths. The pseudoscalar mixing angle χ was fitted at about $-39°$, very close to that which comes out of broken-symmetry mass formulas and strong decay amplitudes.

The next step was to include the four measured ψ decays into π, η, η', and η_c. This involves adding four additional mixing angles for SU(4), plus one additional mass matrix parameter, which we take as the ratio of charm to strange. In the first fits, we use the limit of charm sector decoupling for vector mesons, so that only $\chi'=\alpha'+\beta'$ occurs, and we remain with three degrees of freedom, fitting 14 decays with 11 parameters. The results are shown in the next column in Table I, labeled broken SU(4)-A. In this fit, the ω decay does a little better, at the expense of some of the other light-quark decays. The ψ decays are fit almost exactly, using the additional freedom from the mixing angles. For the D^* decays, we have calculated the predictions from this set of parameters. Even though the additional freedom of two independent couplings proportional to the quark mass was present, we find the ratio of charged to neutral rates is still very small compared with the indirect experimental evidence. The situation is even more extreme for the suppressed η_c decays into ρ, ω, and ϕ. In this particular fit, these rates are predicted to be huge, adding up to more than 50% of the total η_c width. The reasons for this can be traced directly back to the ω rate, where we have encouraged the annihilation term g_5 to be as large as possible. This flavor-independent quantity propagates into the charm sector, where the large available phase space enhances the effect. In order to accommodate these constraints, additional fits were performed with upper limit branching ratios of one percent for each of the η_c modes, and rejecting solutions for which the D^* ratio fell outside the range from 0.1 to 1.0. We also reinserted the additional mixing angle for the vectors as a free parameter, resulting in fits to 17 experimental constraints with 12 parameters. In each case a moderately acceptable fit was obtained, but at the expected price of a higher χ^2 due to worse fits in the light-quark sector, especially $\omega\to\pi\gamma$. In addition, all of the upper limit branching ratios for the η_c were saturated, indicating the desire of the fitting program to make them as large as possible. To explore this further, sets of "manufactured" experimental values were assigned to the η_c decays, and the same fitting procedure used. Results of a representative fit are shown in the last column in Table I, labeled broken SU(4)-B. The fitted values for the η_c decay widths are all close to the values assigned, and the $\omega\to\pi\gamma$ rate is seen to suffer, as does the quality overall in the light-quark sector. An examination of the individual terms in decay couplings for the charm sector is quite revealing. It shows that a typical suppressed (by gluon annihilation and mixing) decay is made up of several different terms, each with magnitude of 100 or so times the experimental value. The final fitted value is obtained by an incredible fine-tuning of the parameters which produces the desired cancellation.

V. CONCLUSIONS

Results of these fits lead to the following general statements on the present understanding of quarkonium radiative decays and prospects for the future.

(1) Ratios of different charge modes within the K^* and D^* sectors require symmetry-breaking mechanisms more general than simple magnetic moments using constituent-quark masses.

(2) The ω/ρ ratio appears to be settling down to a final number which is not quite close enough to simple broken-symmetry predictions to claim satisfactory agreement.

(3) Propagation of parameters from fits in the light-quark sector into the charm sector give disastrously large predictions for some of the flavor-changing decays. These can be fit to reasonable experimental numbers, but at the expense of an intricate fine-tuning of parameters.

(4) It is likely that extension to the t- and b-quark sectors will involve similar problems, even with the additional freedom of more mixing angles. It appears that some strongly mass-dependent mechanism must exist to avoid propagation of correction factors from the light-quark sector.

(5) A measurement of absolute decay rates for the D^* and D_s^* allowed radiative decays would place severe constraints on parameters within the charm sector alone.

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ment is for the positive charge state, while the previous measurements were for the ρ^-. We use this latest value in our fits rather than a weighted average, in order to test the flexibility of various symmetry-breaking mechanisms to their extremes. For the same reason, we do not use the alternate $\omega \to \pi\gamma$ width of 789±92 keV extracted by T. Oshima [Phys. Rev. D **22**, 707 (1980)], based on a possible adjustment of relative rates in the overall fit.

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