Multisource model for particle production in high-energy hadron-nucleus collisions

Cai Xu,* Chao Wei-qin,† and Meng Ta-chung
Fachbereich Physik der FU Berlin, Berlin, Germany
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A multisource model for particle-production processes in high-energy hadron-nucleus collisions is presented. The model is an extension of the statistical model proposed some time ago by the Berliner group to describe such processes in hadron-hadron collisions at comparable as well as at higher energies. The concept of the effective target turns out to be very useful for this generalization to nuclear targets. The result shows in particular that the observed characteristic properties of multiplicity distributions, rapidity distributions, and correlations can be understood in terms of a physical picture in which the dynamical details of hadronic interactions do not play a significant role.

I. INTRODUCTION

Experimental results[1–6] for high-energy hadron-nucleus collisions on multiplicity distributions, rapidity distributions, and correlations are now available for a large number of nuclear targets. In these experiments a number of new features have been observed and it is expected,[1–6] in particular, that they can be used as critical tests for the existing hadron-nucleus collision models.[7] The most striking new features are the following.

(a) The ratio $D/\langle n \rangle$ between the dispersion $D=\langle (n-\langle n \rangle)^2 \rangle^{1/2}$ and the average multiplicity $\langle n \rangle$ is a function of the number N_p of identified protons[2] which correspond to the grey tracks measured in emulsion experiments.[6] For small N_p this ratio is close to the value for proton-proton reactions, but it decreases with increasing N_p. (The data points are shown in Fig. 1.)

(b) The ratio $R(y)$ between the rapidity distributions for the produced particles in hadron-nucleus collisions and that in the corresponding hadron-hadron collisions depends not only on the rapidity y, but also on the atomic number of the target nucleus A and/or on the number of identified protons, N_p, mentioned in (a). This ratio is significantly greater than unity in the target-fragmentation region, and these large values are mainly associated with large-N_p events. (The data points[2,3] are shown in Figs. 3–5.) We note that the N_p dependence in the above-mentioned experimental results is of particular interest, because the number N_p is intimately related to the number of nucleons inside the target nucleus which interact with the incident hadron.

(c) The two-particle correlation function

$$R_2(y_1,y_2)=\frac{\rho(y_1,y_2)}{\rho(y_1)\rho(y_2)}-1$$

for $y=y_1=y_2$, measured[2] in proton-proton and proton-xenon reactions at the same incident energy ($p_{\mathrm{lab}}=200$ GeV/c), shows that they are qualitatively different from each other. While $R_2(y,y)$ for p-p reactions is peaked at $y_{\mathrm{lab}}=3.03$ [corresponding to proton-proton center-of-mass-system (c.m.s.) rapidity zero at this incident energy], $R_2(y,y)$ for p-Xe reactions increases for decreasing y in the rapidity region $y_{\mathrm{lab}}<3$. Its value becomes as large as 1.5 in the region $-1<y_{\mathrm{lab}}<0$. (See the data points and the dashed curve in Fig. 6.[2,3])

(d) Measurements of multiplicity distributions of charged particles in different rapidity intervals in proton-proton, proton-argon, and proton-xenon collisions at $p_{\mathrm{lab}}=200$ GeV/c have been reported,[4] where the corresponding results for negatively charged particles are also given. [The data[4] are shown as dashed curves in Figs. 7(a), 7(b), 8(a), and 8(b).] Although the data are given in a special reference frame (c.m.s. of the proton-proton system for proton-nucleus reactions), by using a special parametrization (negative-binomial distributions) a great deal of useful information can be extracted from this experiment. In particular, the similarities and the differences between p-p and p-nucleus collisions in different rapidity intervals have now become more evident.

An attempt has been made to understand these (new), as well as other ("old") characteristic features in high-energy hadron-nucleus collisions. This attempt is motivated by the following observations.

(i) It is now a well-known fact[1] that, in high-energy inelastic hadron-nucleus collisions, the time needed for particle production to take place (after the incident hadron collides with one of the nucleons in the target nucleus) is *much longer* than the time interval for the incident hadron to interact with the nucleons it meets inside the target nucleus. Taken together with the fact that the average binding energy of nucleons in nucleus is negligibly small compared with the incident energy, this implies that the nucleons hit by the incident hadron—they are called the "effective target nucleons"[8]—will be pushed out of the target nucleus and that the production processes take place after these nucleons have left the rest of the nucleus. This implies in particular that *geometrical* concepts are very useful in describing such processes.

(ii) There is strong evidence[9] that high-energy nondiffractive hadron-hadron collisions (which constitute the predominating part of the inelastic hadron-hadron reactions) are *statistical* processes. That is, the dynamical details of such processes do not play a significant role in understanding the characteristic features of such processes.

In this paper we propose a multisource model for particle-production processes in high-energy hadron-nucleus collisions. The model is an extension of the statistical model[9] proposed some time ago by this group to describe such processes in hadron-hadron collisions at comparable as well as at higher energies. The concept of effective target[8] turns out to be very useful for this generalization to nuclear targets. The result shows, in particular, that the observed characteristic properties of multiplicity distributions, rapidity distributions, and correlations can be understood in terms of a physical picture in which the dynamical details of hadronic interactions do not play a significant role.

II. MULTIPLICITY DISTRIBUTIONS

We recall that the basic idea of the proposed statistical model[9] for hadron-hadron (h-h) processes is the following. In a high-energy nondiffractive h-h collision event the projectile (P) and the target (T) go through each other and lose a considerable part of their energies and momenta. A part of the "lost energy" materializes into clusters which later decay into hadrons. This part of energy is distributed in three systems C^*, P^*, and T^* which are located in the central, projectile, and target rapidity regions, respectively. The average size of the three systems relative to one another is determined by an energy-dependent parameter α as follows:

$$\langle n_C \rangle = \alpha \langle n_{hh} \rangle \ , \tag{1}$$

$$\langle n_P \rangle = \langle n_T \rangle = \frac{1-\alpha}{2} \langle n_{hh} \rangle \ , \tag{2}$$

where $\langle n_{hh} \rangle$, $\langle n_C \rangle$, $\langle n_P \rangle$, and $\langle n_T \rangle$ are the average charge multiplicity for the nondiffractive h-h process and the average multiplicities of the charged hadrons produced by the C^*, P^*, and T^* systems, respectively.

Each system (C^*, P^*, T^*) obtains its materialized energy (that is, the sum of the masses of the produced clusters) randomly from two energy sources. (For details see the third paper in Ref. 9.) Under the assumption that the number of produced hadrons is proportional to the materialized energy of the system, we obtain the following expression for the multiplicity distribution of the charged hadrons produced in each system:

$$\langle n_i \rangle P(n_i) = 4 \frac{n_i}{\langle n_i \rangle} \exp \left[-2 \frac{n_i}{\langle n_i \rangle} \right] . \tag{3}$$

Here, $i = C^*$, P^*, and T^* [as in Eqs. (1) and (2), the asterisks in the subscripts are omitted]; n_i is the number of charged hadrons produced by the system i and $\langle n_i \rangle$ is the average of n_i. The rapidity distribution of the produced clusters in these emitting systems are assumed to be flat.

In hadron-nucleus (h-A) collisions, the incident hadron, in general, hits more than one nucleon inside the target nucleus (of atomic number A); and the final states depend on *how many nucleons* the incident hadron meets and *how it interacts* with these nucleons. In these connections it is convenient to use the concept of effective target[8] (ET), which is the group of nucleons inside the target nucleus hit by the incident hadron as it "goes through" the target.

Suppose ν of the ν_{ET} nucleons inside the ET interact with the incident hadron h nondiffractively and every one of the ν nondiffractive collisions can be described by the statistical model given in Ref. 9. Then, the multiparticle production process in such an h-ET collision can be envisaged to take place as follows.

After the ν nondiffractive collisions between the incident hadron h and ν of the nucleons in the ET, one P^*-system, one C^* system, and ν T^* systems will be formed. The formation and the decay of the P^* and the T^* systems are exactly the same as those in hadron-hadron collisions. That is, the materialization energy of each system is randomly obtained from two energy sources. Hence, the distribution for the materialization energy and the distribution for the multiplicity of the charged hadrons (taken together with the assumption that the multiplicity of the produced charged hadrons and the materialization energy are proportional to each other) are also given by Eq. (3), where in this case i stands only for P^* and for any one of the ν T^* systems. Since the C^* system obtains its materialization energy randomly from $1+\nu$ sources, the corresponding multiplicity distribution for charged hadrons is

$$P_C(n_C \mid 1+\nu) = \frac{(\nu+1)^{\nu+1}}{\nu!} \frac{1}{\langle n_C \rangle_\nu} \left[\frac{n_C}{\langle n_C \rangle_\nu} \right]^\nu \times \exp \left[-(\nu+1) \frac{n_C}{\langle n_C \rangle_\nu} \right] . \tag{4}$$

Furthermore, since the P^*, the C^*, and the ν T^* systems are independent of one another, the probability for observing n charged particles in such an event is

$$P(n \mid 1+\nu) = {\sum}' P_P(n_P) P_C(n_C \mid 1+\nu) P_T(n_{T1}) P_T(n_{T2}) \cdots P_T(n_{T\nu}) \ , \tag{5}$$

where the prime on the summation means that the sum over $n_P, n_C, n_{T1}, n_{T2}, \ldots, n_{T\nu}$ should be taken such that the condition

$$n = n_P + n_C + n_{T1} + n_{T2} + \cdots + n_{T\nu} \tag{6}$$

is satisfied.

The corresponding mean multiplicity $\langle n \rangle_\nu$ and the dispersion D_ν are

$$\langle n \rangle_\nu = \langle n_P \rangle + \langle n_C \rangle_\nu + \nu \langle n_T \rangle \ , \tag{7}$$

$$D_\nu = [d^2(P) + d_\nu{}^2(C) + \nu d^2(T)]^{1/2} \ , \tag{8}$$

respectively, where $\langle n_P \rangle$, $\langle n_T \rangle$ and $d(P)$, $d(T)$ are the mean multiplicities and dispersions contributed from P^* and T^* systems. They are taken to be the same as those in h-h collisions. The contributions from C^*, $\langle n_C \rangle_\nu$, and $d_\nu(C)$ are

$$\langle n_C \rangle_\nu = \frac{\nu+1}{2} \langle n_C \rangle \tag{9}$$

and

$$d_\nu{}^2(C)=\frac{\nu+1}{2}d^2(C) , \tag{10}$$

respectively. Here, $\langle n_C \rangle$ is the mean multiplicity and $d(C)$ the dispersion of the corresponding C^* systems in h-h collisions. It follows from Eqs. (1), (2), (7), (8), (9), and (10),

$$\langle n \rangle_\nu=\frac{\nu+1}{2}\langle n_{hh} \rangle , \tag{11}$$

$$\frac{D_\nu}{\langle n \rangle_\nu}=\left[\frac{2}{\nu+1}\right]^{1/2}\frac{d}{\langle n_{hh} \rangle} , \tag{12}$$

where $\langle n_{hh} \rangle=\langle n_P \rangle+\langle n_C \rangle+\langle n_T \rangle$ is the mean multiplicity and $d=(d_P{}^2+d_C{}^2+d_T{}^2)^{1/2}$ is the dispersion for h-h collisions.

The results given in Eqs. (11) and (12) are the characteristic properties of the proposed model. In this model, the linear dependence of $\langle n \rangle_\nu$ on ν and the behavior of $D_\nu/\langle n \rangle_\nu$ are due to the increase in the number of energy sources in the C^* system and the increase in the number of T^* systems in hadron-nucleus collision processes.

To compare these results with experiment, we note that the experimental data are the average of events associated with different ν. Hence, we need to know the distribution of $\nu, w(\nu)$ when we calculate the multiplicity distribution,

$$P(n)=\sum_\nu w(\nu)P(n \mid 1+\nu) , \tag{13}$$

and, from $P(n)$, the corresponding mean multiplicity and the dispersion. In practice we take into account the fact that the number of recoiled protons N_p observed in experiments is related to ν_{ET} in by[8]

$$\nu_{\mathrm{ET}}=\sqrt{N_p} \tag{14}$$

and that the simplest distribution of the number of nondiffractive collisions is a binomial distribution:

$$W_{\mathrm{ET}}(\nu)=B(\nu_{\mathrm{ET}},\nu\,|\,p)=\begin{bmatrix}\nu_{\mathrm{ET}}\\ \nu\end{bmatrix}p^{\nu}(1-p)^{\nu_{\mathrm{ET}}-\nu} , \tag{15}$$

where p (set to be 0.75) is the probability of a collision being nondiffractive. That is, the multiplicity distribution $P(n)$ for events characterized by a fixed number of recoiled protons can be obtained by inserting Eqs. (14), (15), and (5) into Eq. (13).

If we wish to obtain the average over a target nucleus of atomic number A, the geometry of the nucleus has to be taken into account. The number of nucleons in the effective target ν_{ET} is a function of the impact parameter b. Under the assumption that the density distribution in the nucleus can be considered as a constant, we have

$$b(\nu_{\mathrm{ET}})=[(A^{1/3}+x)^2-\tfrac{4}{9}\nu_{\mathrm{ET}}{}^2]^{1/2}r_0 , \tag{16}$$

where r_0 is the nucleon radius and $(A^{1/3}+x)r_0$ is the effective radius of the target nucleus A. (In our calculation we have chosen $r_0=1$ fm, $x=0.3$ to fit the mean number of collisions for both Ar and Xe targets.) The probability of having ν_{ET} nucleons in the effective target is therefore

$$F_A(\nu_{\mathrm{ET}})=\frac{b^2(\nu_{\mathrm{ET}})-b^2(\nu_{\mathrm{ET}}+1)}{b^2(\nu_{\mathrm{ET}}=1)} . \tag{17}$$

Hence, the ν distribution for the target nucleus A is

$$W_A(\nu)=\frac{1}{N}\sum_{\nu_{\mathrm{ET}}=\nu}^{\nu_{\mathrm{ET}}(\max)}F_A(\nu_{\mathrm{ET}})B(\nu_{\mathrm{ET}},\nu) , \tag{18}$$

where $\nu_{\mathrm{ET}}(\max)$ is the integer closest to $\frac{3}{2}(A^{1/3}+x)$, and

$$N=\sum_{\nu_{\mathrm{ET}}=1}^{\nu_{\mathrm{ET}}(\max)}F_A(\nu_{\mathrm{ET}})[1-(1-p)^{\nu_{\mathrm{ET}}}] \tag{19}$$

is the normalization constant. Hence, the multiplicity distribution for a target nucleus with a fixed atomic number A can be obtained by inserting Eqs. (15)–(19) and (5) into Eq. (13).

The results can be summarized as follows.

First, we consider the mean multiplicity. Here, both in the case of fixed N_p and in the case of fixed A, the well-known linear relationship between the mean multiplicities $\langle n \rangle$ and the mean number of collisions $\langle \nu \rangle$ is obtained. That is

$$\langle n \rangle=\frac{\langle \nu \rangle+1}{2}\langle n_{hh} \rangle , \tag{20}$$

where $\langle n \rangle$ and $\langle \nu \rangle$ stand for the average values in the case of a fixed number of recoiled protons N_p as well as for the corresponding average values in the case of a target nucleus of mass number A.

Second, we consider $D/\langle n \rangle$. The calculated dependence of $D/\langle n \rangle$ on N_p is plotted together with the experimental data in Fig. 1. We note that the ratio $D/\langle n \rangle$ shows the same kind of dependence on N_p as those of $D_\nu/\langle n \rangle_\nu$ given in Eq. (12). Furthermore, as we can see in Fig. 2, the calculated multiplicity distributions

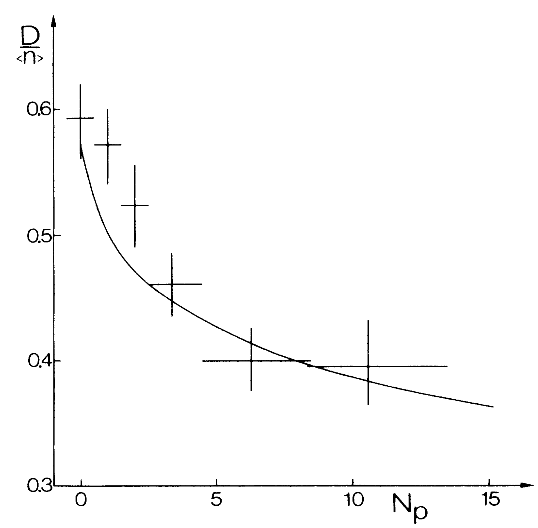

FIG. 1. The ratio of the dispersion D to the mean multiplicity $\langle n \rangle$ as a function of N_p, the number of identified protons. The data are from Ref. 2. The curve is the calculated result.

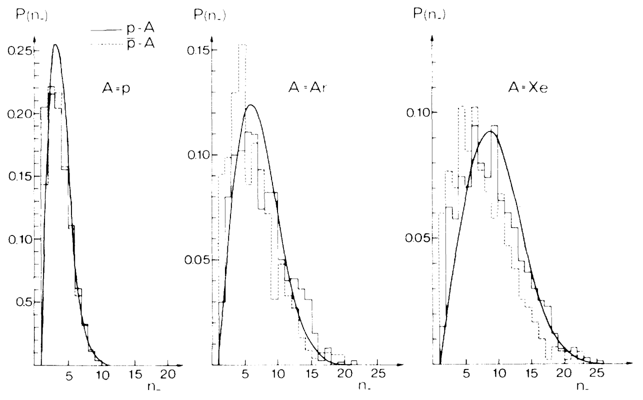

FIG. 2. Multiplicity distributions of the produced negatively charged particles for p-A and $\bar{p}$-A collisions. The data, given by the histograms, are taken from Ref. 2. The solid curves are the calculated results.

of the negatively charged particles for hadron induced reactions on proton, Ar, and Xe targets are in good agreement with the data.[2] Here, the large dispersion of the multiplicity distribution is due to the large fluctuation of the number of nucleons ν_{ET} in the effective target, and this in turn is due to the significant change of the impact parameter in such reactions.

Thus, we have shown that the striking N_p dependence of $D/\langle n\rangle$, mentioned in Sec. I [point (a)], can be understood in terms of the proposed multisource model.

III. RAPIDITY DISTRIBUTIONS AND RAPIDITY CORRELATIONS

In this section we show that the measured rapidity distributions and the observed short-range rapidity correlations, in particular the striking features mentioned in point (b) and point (c) in Sec. I, can also be understood in terms of the proposed model. In order to see how the observed phenomena are related to the physical picture of this model, we think it is useful to point out the following facts.

First, the generalization of the statistical model[9] for hadron-hadron collisions to hadron-nucleus collisions has been made in such a way that when the target nucleus is simply a nucleon (that is, when $A=\nu_{\mathrm{ET}}=1$), everything that has been said in the preceding sections about the target nucleus and/or about the effective target should reduce to that of a hadron in the hadron-hadron case.

Second, once we accept the interpretation that[1] the observed particles are produced after the nucleons hit by the incident hadron have already left the rest of the target, then the c.m.s. of the h-ET system should be a very useful reference frame in such production processes. This point is of particular importance when we compare the results obtained for different nuclear targets (different N_p and/or different A). In fact, the observed[1] shift of the peak position in rapidity distributions for a different "average number of collisions $\bar{\nu}$" can be understood by identifying $\bar{\nu}$ with the average of ν_{ET} in this picture.[8]

Third, a large number of experiments[10] on multiparticle production processes in nondiffractive hadron-hadron collisions suggest that a dominating part of the observed hadrons are produced via clusters and that there are different kinds of clusters. The result of more recent and more detailed studies[11] of the cluster problems strongly indicates that an appreciable amount of clusters decay into more than two charged hadrons; this is particularly the case in the central rapidity region, where the contributions from gluon-gluon interaction are supposed to dominate. In fact, this physical picture is in accordance with the observed behavior of

$$R_2(y,y')=\frac{\rho_2(y,y')}{\rho_1(y)\rho_1(y')}-1 \tag{21}$$

at $y=y'$. Hence $\rho_1(y)$ and $\rho_2(y,y')$ are the one- and two-particle inclusive distributions, respectively. In particular, it is seen[2] that $\rho_2(y,y)/[\rho_1(y)]^2$ in the central rapidity region for pp collisions at $p_{\mathrm{lab}}=200$ GeV/c has approximately a Gaussian-type distribution.

Fourth, since each of the nucleons in the effective target obtains, in general, a small amount (compared to the momentum transfer in the longitudinal direction) of transverse momentum from the incident hadron, it will usually leave the target nucleus with a small scattering angle in the forward direction. Hence, the first nucleon in the effective target of ν_{ET} nucleons will meet on the average $\nu_{\mathrm{ET}}-1$, the second will meet $\nu_{\mathrm{ET}}-2$ secondary nucleons, etc., on their way out. That is, there will on the average be

$$(\nu_{\mathrm{ET}}-1)+(\nu_{\mathrm{ET}}-2)+\cdots+2+1=\tfrac{1}{2}\nu_{\mathrm{ET}}(\nu_{\mathrm{ET}}-1) \tag{22}$$

secondary nucleons for an ET with ν_{ET} nucleons. Since the secondary nucleons do not obtain their energy and momentum transfer directly from the energetic incident hadron, they are—in contrast with the effective target nucleons—less excited and rather slow in the laboratory frame. Hence, we expect to see, in addition to those emitted by C^*, P^*, T^*, other charged particles in the neighborhood of $y_{\mathrm{lab}}=0$. The number of such particles is directly proportional to $\nu_{\mathrm{ET}}(\nu_{\mathrm{ET}}-1)$.

A quantitative study in the framework of the multisource model has been carried out where these points are taken into account. The method of calculation and the result can be summarized as follows.

The rapidity distribution for nondiffractive hadron-hadron collisions can be expressed as[9]

$$\rho_1(y\mid h)=\sum_i \langle n_i\rangle f(y-\Delta_i)\,, \tag{23}$$

where $\langle n_i\rangle$, $i=C^*$, P^*, and T^*, are the average multiplicities of the charged hadrons produced by the emitting systems C^*, P^*, and T^*, respectively. The function $f(y-\Delta_i)$ can be obtained in different ways.[12] Here we adopt the expression used in Ref. 13:

$$f(y)=K\left[\xi+\frac{\cosh y}{T}\right]^{-2}, \tag{24}$$

where ξ and T are parameters and K is the normalization constant.[12] The Δ_i's ($i = C^*$, P^*, and T^*) are associated with the positions of these emitting systems in the rapidity space. In practice, we chose $\Delta_C = 0$ and determined Δ_p ($= -\Delta_T$) from energy and momentum conservation.

The rapidity distribution, due to the corresponding C^*, P^*, and T^* systems in collision processes between a hadron and an effective target consisting of $\nu_{\rm ET}$ nucleons where ν of these nucleons interact with the incident hadron nondiffractively, can be written as

$$\rho_1(y \mid \nu_{\rm ET},\nu) = \nu\langle n_T\rangle f(y-y_T) + \frac{\nu+1}{2}\langle n_{C\nu}\rangle f(y-y_C) + \langle n_p\rangle f(y-y_P) \,, \tag{25}$$

Here, y_C, the rapidity of C^*, is taken to be the same as the c.m.s. of the h-ET c.m.s.; and, the positions of the P^* and the T^* systems are again determined by energy and momentum conservation.

Hence, taken together with the contributions from the excited secondary nucleons, the rapidity distribution for collision events with a fixed number (N_p) of identified protons is

$$\rho_1(y \mid \nu_{\rm ET} = \sqrt{N_p}) = \sum_{\nu=1}^{\nu_{\rm ET}} B(\nu_{\rm ET},\nu)\rho_1(y \mid \nu_{\rm ET},\nu) + \tfrac{1}{2}\nu_{\rm ET}(\nu_{\rm ET}-1)f(y) \,, \tag{26}$$

where $B(\nu_{\rm ET},\nu)$ is the binomial distribution given in Eq. (15), $\rho_1(y \mid \nu_{\rm ET},\nu)$ and $f(y)$ are the rapidity distributions given in Eqs. (25) and (24), respectively. A comparison between data[2] and the calculated result, in terms of the ratio $\rho_1(y \mid \nu_{\rm ET} = \sqrt{N_p})/\rho_1(y/h)$ which we call R, is shown in Fig. 3.

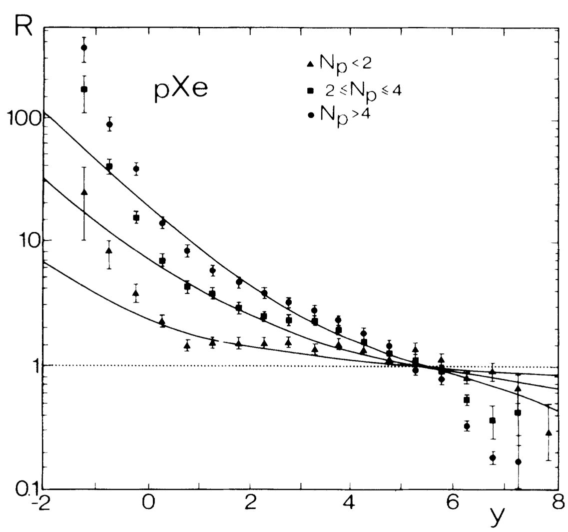

FIG. 3. The ratio $R(y \mid N_p) = \rho_1(y \mid \nu_{\rm ET} = N_p)/\rho_1(y \mid h)$ as a function of y, for different sets of events characterized by different values of N_p. The single-particle inclusive rapidity distributions $\rho_1(y/h)$ and $\rho_1(y \mid \nu_{\rm ET})$ are obtained from Eqs. (23) and (26), respectively. The data, taken from Ref. 2, are for p-Xe collisions at $p_{\rm lab} = 200$ GeV/c.

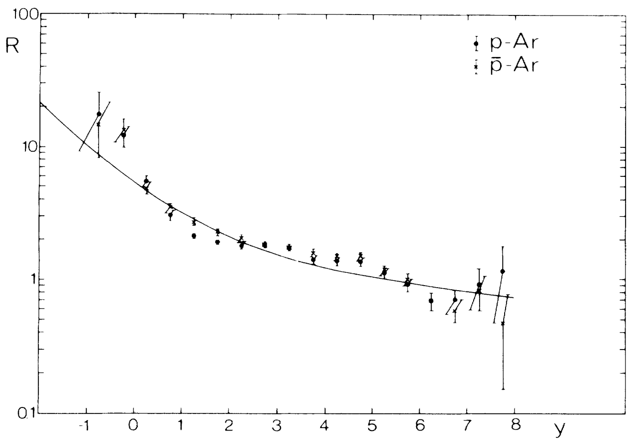

FIG. 4. The ratio $R(y \mid A) = \rho_1(y \mid A)/\rho_1(y \mid h)$ for $p+{\rm Ar}$ collisions at $p_{\rm lab} = 200$ GeV/c. The data are from Ref. 2. The rapidity distributions $\rho_1(y \mid A)$ and $\rho_1(y \mid h)$ are obtained from Eqs. (27) and (23), respectively.

The rapidity distribution for target nuclei with a given atomic number A is then

$$\rho_1(y \mid A) = \frac{1}{N}\sum_{\nu_{\rm ET}=1}^{\nu_{\rm ET}(\max)} F_A(\nu_{\rm ET})\rho_1(y \mid \nu_{\rm ET}) \,, \tag{27}$$

where $F_A(\nu_{\rm ET})$, N, and $\rho_1(y \mid \nu_{\rm ET})$ are given in Eqs. (17), (19), and (26), respectively. The results of our calculation for Ar and Xe are shown in Figs. 4 and 5, together with the data.[2] They are given in terms of the ratio $\rho_1(y \mid A)/\rho(y \mid h)$ which is also denoted by R in the figures.

To calculate the two-particle correlation function $R_2(y,y \mid \nu_{\rm ET} = \sqrt{N_p})$ for hadron-nucleus collision events associated with a fixed number of identified protons, we make use of the empirical fact, mentioned at the beginning of this section, regarding the corresponding correla-

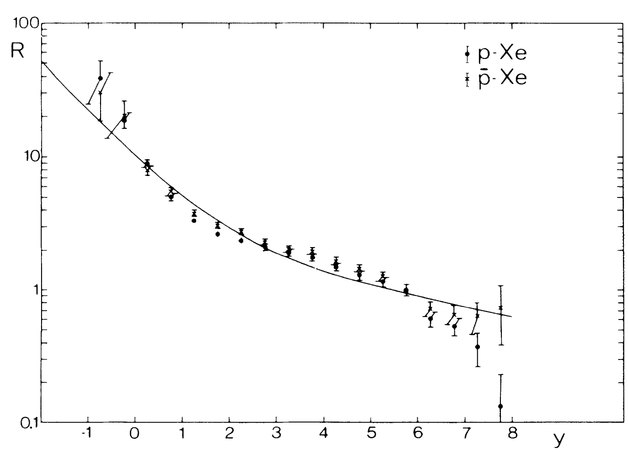

FIG. 5. The ratio $R(y \mid A) = \rho_1(y \mid A)/\rho_1(y \mid h)$ for $p+{\rm Xe}$ collisions. The data are from Ref. 2.

tion function $R_2(y,y \mid h)$ for hadron-hadron processes, namely, that it is approximately a Gaussian distribution in y. By assuming that the correlations among the charged hadrons emitted from each system C^*, P^*, and T^* are such that the corresponding $R_2(y,y \mid h \mid i)$, $i = C^*, P^*, T^*$, are Gaussian, that is,

$$R_2(y,y \mid h \mid i) = \lambda g(y - y_i) \ , \tag{28}$$

where λ is a constant and $g(y)$ is a Gaussian distribution in y; we have

$$R_2(y,y \mid h) = [\rho_1(y \mid h)]^{-2} \sum_i [\lambda g(y - \Delta_i) - 1] \times [\langle n_i \rangle f(y - \Delta_i)]^2 \ , \tag{29}$$

where the summation over i means taking into account the contributions from the emitting systems C^*, P^* and T^*.

For h-ET collision processes, in which ν out of the $\nu_{\rm ET}$ effective target nucleons interact with the incident hadron nondiffractively, we have

$$R_2(y,y \mid \nu_{\rm ET},\nu) = [\rho_1(y \mid \nu_{\rm ET},\nu)]^{-2} \left[[\lambda g(y - y_C) - 1] \left[\langle n_c \rangle_\nu \frac{\nu+1}{2} f(y - y_C) \right]^2 + [\lambda g(y - y_P) - 1][\langle n_P \rangle f(y - y_P)]^2 + [\lambda g(y - y_T) - 1][\nu \langle n_T \rangle f(y - y_T)]^2 \right] . \tag{30}$$

By taking the contribution from the secondary nucleons into account we have

$$R_2(y,y \mid \nu_{\rm ET} = \sqrt{N_p}) = [\rho_1(y \mid \nu_{\rm ET} = \sqrt{N_p})]^{-2} \left\{ \sum_\nu B(\nu_{\rm ET},\nu) R_2(y,y \mid \nu_{\rm ET},\nu)[\rho_1(y \mid \nu_{\rm ET},\nu)]^2 + [\kappa g(y) - 1][\tfrac{1}{2}\nu_{\rm ET}(\nu_{\rm ET} - 1) f(y)]^2 \right\} , \tag{31}$$

where κ is a parameter. The correlation function $R_2(y,y \mid A)$ for target nuclei with given atomic number A can be obtained in a way similar to that for the rapidity distribution. It is

$$R_2(y,y \mid A) = N^{-1}[\rho_1(y \mid A)]^{-2} \sum_{\nu_{\rm ET}} F_A(\nu_{\rm ET})[\rho_1(y \mid \nu_{\rm ET})]^2 R_2(y,y \mid \nu_{\rm ET}) \ . \tag{32}$$

The calculated result is shown in Fig. 6, together with the data taken from Ref. 2. The following remarks should be made in connection with the numerical results obtained in this section.

The numerical result depends on several parameters. But, all parameters except one—the ratio between κ and λ in Eqs. (30) and (31)—are determined by comparison with experimental data *independent* from those we wish to understand in this paper. Besides, explicit calculations have shown that the parameters that cannot be accurately determined in hadron-hadron experiments are *not sensitive* to the data for hadron-nucleus collisions either. For example, α in Eqs. (1) and (2) is determined by fitting the multiplicity distribution data in nondiffractive hadron-hadron collisions. But, as we have already pointed out in Ref. 9, the multiplicity and rapidity-distribution data in this energy region (total c.m.s. energy $\sqrt{s} < 100$ GeV, say) is rather insensitive to the α values. That is, we are not sure whether the relative weight of the contribution from the central system C^* should be exactly $\frac{1}{3}$ at $p_{\rm lab} = 200$ GeV ($\sqrt{2} \approx 20$ GeV). Analyses[9] of the CERN-$\bar{p}p$-collider data[14] show, however, that the C^* system should become the dominating part at higher energies (for example, $\sqrt{s} > 100$ GeV). What we found through this calculation is that the description of the hadron-nucleus collision data is not sensitive to the α value either. What we expect to see is that the central system C^* will play the dominating role

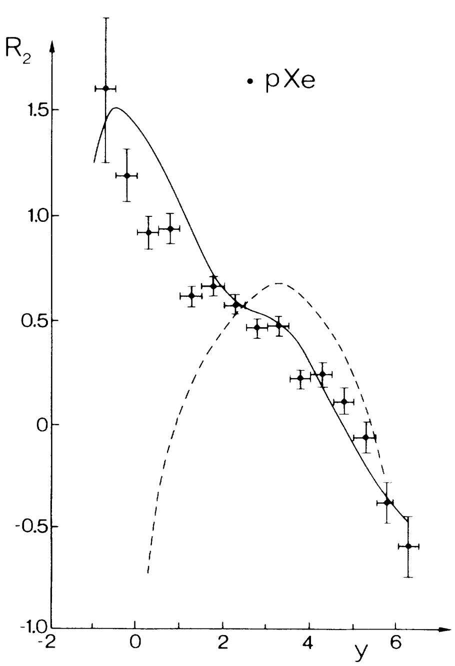

FIG. 6. The two-particle correlation function $R_2(y,y \mid A)$ for p-Xe collisions; and that for p-p collisions. The data are from Ref. 2. (Those for p-p collisions are shown as dashed curve.) The solid curve is the calculated result.

also in hadron-nucleus collisions at higher energies.

The parameters λ and κ in Eqs. (28), (30), and (31) characterize the size of the clusters. The values are chosen under the assumption that only charge-neutral clusters that decay into two charged hadrons exist. While λ is determined by the short-range correlation data on R_2 for proton-proton collisions, the relative magnitude $\kappa/\lambda=2$ is determined[15] by the fact that the secondary nucleons (on the average 50% of them are protons) and the produced charged hadrons (almost all of them are pions) are located in the neighborhood of $y_{\rm lab}=0$ in rapidity space. Hence, not only the produced pions, but also the secondary protons should be taken into account when we consider the rapidity correlation of two charged hadrons near $y_{\rm lab}=0$.

In summary, we have demonstrated in this section that the striking new features observed[2] in rapidity distributions and correlations [in particular those mentioned in (b) and (c) in Sec. I] can be understood in this model.

IV. DEPENDENCE OF MULTIPLICITY DISTRIBUTIONS AND MULTIPLICITY CORRELATIONS ON RAPIDITY WINDOWS

In this section we show that the observed[4] dependence of multiplicity distributions on the rapidity intervals in hadron-nucleus collisions is also in agreement with the proposed model.

We recall that multiplicity distributions of charged hadrons in limited rapidity intervals (windows) have been measured by the UA5 Collaboration[14] for $\bar{p}p$ collisions in the energy range $\sqrt{s}=200$–900 GeV. Similar measurements have been made for e^+e^- annihilation processes $\sqrt{s}=29$ GeV by the High Resolution Spectrometer (HRS) PEP group,[16] for pp and πp reactions at $\sqrt{s}=22$ GeV by the NA22 Collaboration,[17] and for μ^+p reactions for total hadron final-state energies up to 20 GeV by the European Muon Collaboration[18] (EMC). We performed an analysis[19] of these data some time ago. The result has led us to the conclusion that standard statistical methods, in particular the binomial distribution law, can be used to describe all of the observed rapidity dependence of multiplicity distributions mentioned above.

To be more precise, it is shown that the multiplicity distribution $P_w(n_w)$ for charged hadrons (produced at a given total c.m.s. energy $\sqrt{s}$), in a given rapidity window w, can be written as

$$P_w(n_w=2N_w)=\sum_{N=0,1,\ldots} P(N)N!/[N_w!(N-N_w)!] \times(1-q_w)^{N-N_w}q_w^{\,N_w} , \quad (33)$$

provided that there is only one emitting system that contributes to this window. Here, $P(N)$ is the multiplicity distribution of the neutral clusters that decay into one positively and one negatively charged hadron, $q_w=\langle N_w\rangle/\langle N\rangle=\langle n_w\rangle/\langle n\rangle$, where $\langle n_w\rangle$ is the average multiplicity of charged hadrons observed in the rapidity window w, and $\langle n\rangle$ is the average number of charged hadrons produced by the emitting system.

The fact that the above-mentioned hadron-hadron, lepton-lepton, and lepton-hadron collision data can be described by such a simple standard distribution has to be considered as a strong indication of the *stochastic nature* of these processes. It is therefore of some interest to see whether the corresponding hadron-nucleus data can be understood in a similar manner.

We recall that the presently available data on rapidity dependence of multiplicity distributions in hadron-nucleus collisions are those given by Dengler *et al.*[4] for p-Ar and p-Xe processes at incident laboratory momentum $p_{\rm lab}=200$ GeV/c. All charged particles and the negatively charged particles produced in selected rapidity windows have been analyzed separately in the "forward" and in the "backward hemisphere," where the two hemispheres are defined with respect to the p-p c.m.s. The results[4] are presented in terms of empirical fits. (Values with error bars for the parameters of negative-binomial fits for the rapidity intervals $\Delta y=0.5$, 1.0, 1.5, 2.0, 2.5, 3.5 are given.)

Now, if the observed particles in the "forward" ("backward") hemisphere can indeed be considered as products of one emitting system (which emits neutral clusters decaying into one positively and one negatively charged hadrons) then we should expect to see separately for the "forward" and for the "backward hemisphere" the following: The multiplicity distribution $P_w(n_w^-)$ for negatively charged hadrons observed in the rapidity window w can be calculated from Eq. (23) by inserting the experimental value of q_w (that is, take $\langle n_w^-\rangle$ and $\langle n^-\rangle$ from Ref. 4 and calculate $q_w=\langle n_w^-\rangle/\langle n^-\rangle$) for all given rapidity windows and by inserting the empirical curve for $P(N)$ [that is, identify the measured multiplicity distribution for negatively charged hadrons observed in the corresponding hemisphere with $P(N)=P(n_w^-)$ because each cluster has only one negatively charged hadron].

The calculated results are shown in Figs. 7(a), 7(b), 8(a), and 8(b). As we can see, they are in fairly good agreement with the data.[4] The discrepancies are expected. They are caused by the following facts.

First, the division of the entire rapidity space of proton-nucleus collision processes at a given incident momentum into the "forward" and the "backward hemisphere" is made with respect to the c.m.s of proton-proton collisions at the same incident momentum. In our model, such a division is only a rough approximation which is valid when the central emitting system C^* is negligibly small. In fact, we know[9] that at $p_{\rm lab}=200$ GeV/c, the corresponding α value [which is a measure of the average relative importance of C^* see Eqs. (1) and (2)] can indeed be small. Hence, the observed discrepancy between the data[4] and our calculation simply reflects the limitation of this approximation.

Second, for the sake of simplicity, the calculation was performed under the assumption that in these multiparticle production processes there are only neutral clusters which decay into one positively and one negatively charged hadron. Hence, the number of observed negatively charged hadrons was set to be identical to the number of produced clusters. Also this approximation has to be considered as rather crude. In fact, a recent systematic

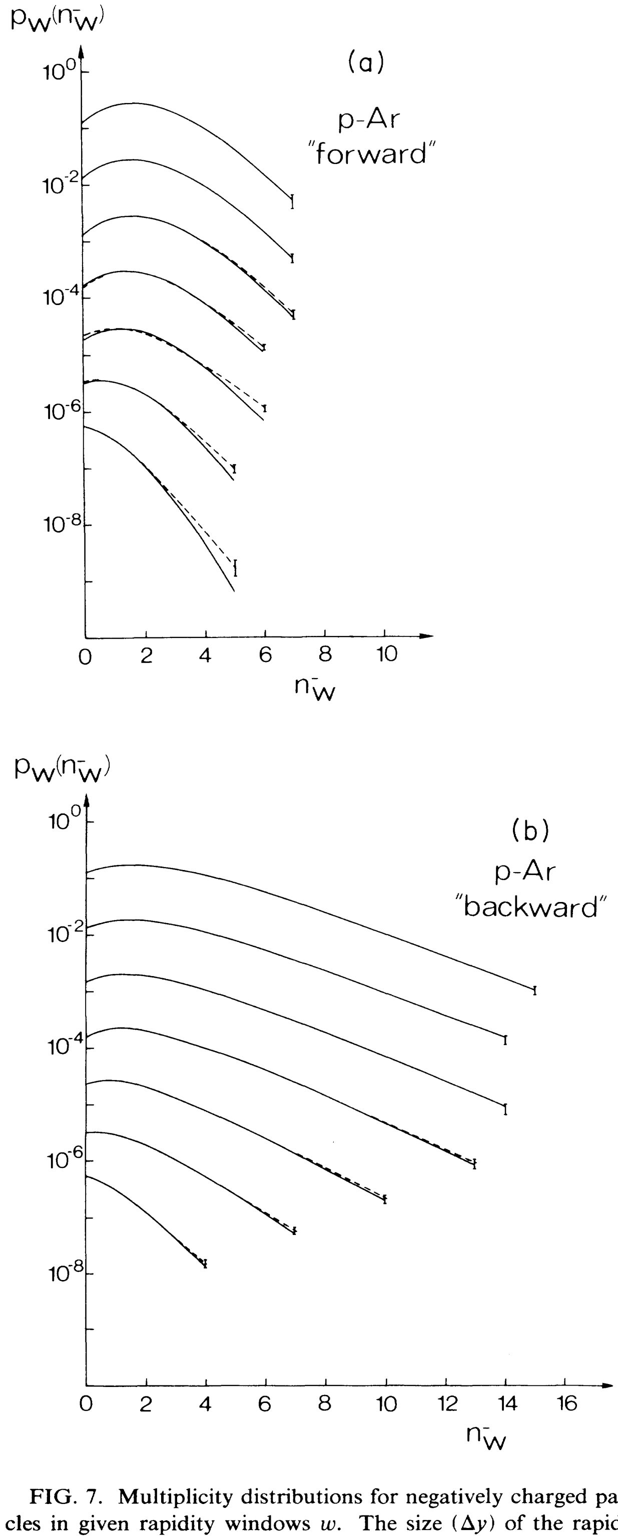

FIG. 7. Multiplicity distributions for negatively charged particles in given rapidity windows w. The size (Δy) of the rapidity windows are (counted from below) 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 3.5, respectively. The data, taken from Ref. 3, are shown as dashed curves when they do not coincide with the corresponding solid curves. The "forward" and the "backward hemisphere" are defined in Ref. 4 as the c.m.s. of the p-p system at $p_{\text{lab}}=200$ GeV/c. The solid curves are the calculated results. The distributions are p + Ar collisions at $p_{\text{lab}}=200$ GeV/c.

study[11] of the cluster problem shows that a considerably large fraction of the clusters produced in nondiffractive hadron-hadron collisions may decay into more than two negatively charged hadrons.

In conclusion, the new experimental results mentioned at the beginning of this paper have led us to a new era in the study of high-energy hadron-nucleus collision process-

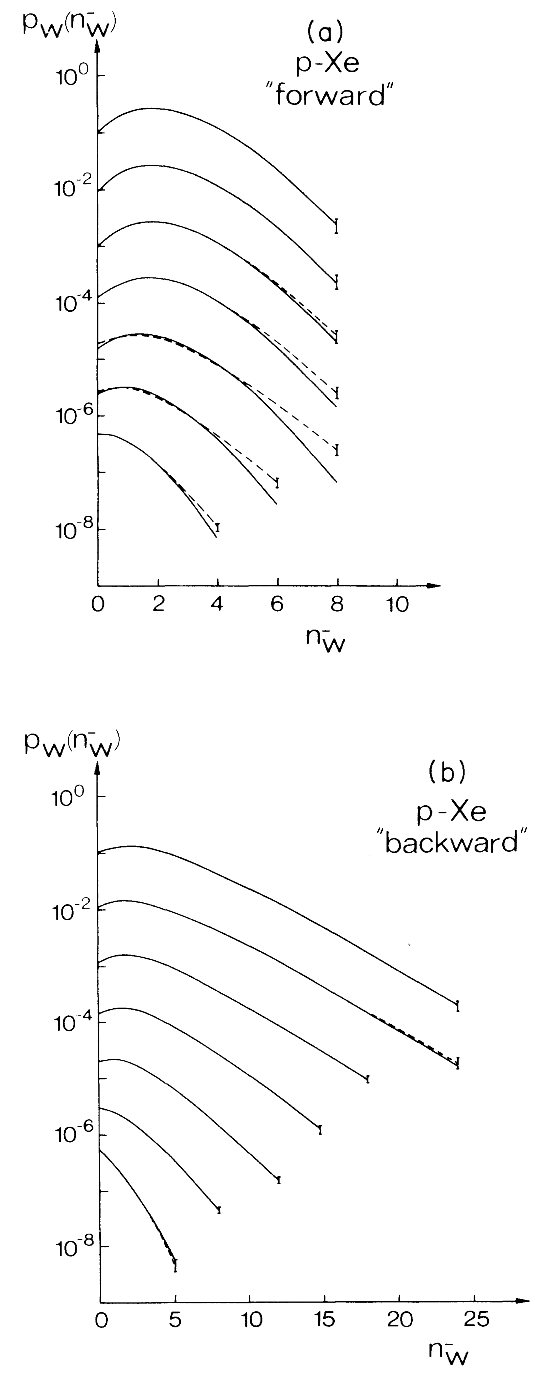

FIG. 8. The corresponding multiplicity distributions for p + Xe collisions at $p_{\text{lab}}=200$ GeV/c.

es. We think it is worthwhile to measure multiplicity distributions in various rapidity intervals at higher incident energies (for example, in Fermilab Tevatron energies). In particular, such studies for different sets of events that are characterized by the numbers (N_p) of the "gray prongs" and/or "identified protons" would be useful.[20] Also correlation measurements, for example, the multiplicity correlations proposed in Refs. 19 and 11 for different N_p sets of events can yield useful information on questions such as whether or not (if yes, how) cluster formation depends on the number of nucleons which interact with the incident hadron in high-energy hadron-nucleus collision processes.

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*Present address: Hua-Zhong Normal University, Wuhan, China.

†On leave from Institute of High-Energy Physics, Academia Sinica, Beijing, China.

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[15]Under the assumption that only charge-neutral clusters which decay into one positively and one negatively charged hadron are produced, we see that there will be two-particle short-range rapidity correlations when the two observed hadrons are decay products of the same cluster. But, when a secondary proton emits a neutral cluster which decays into one positively and one negatively charged pion. Since all three charged hadrons have laboratory rapidity $y_{\rm lab}$ near zero, there will be two-particle short-range rapidity correlations when we observe any two of these three charged hadrons. Simple combinatorics show that the ratio of the "correlation strength" κ/λ should be 2.

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[20]In such cases, we think it would be very interesting to analyze the result when the "forward" and the "backward hemispheres" are defined according to the c.m.s. of the incident hadron and the effective target discussed in this paper.