Towards a general form for the quark mass matrix

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It is shown that the mass matrices for both charges of quarks may be written in a most general form consistent with the data for quark masses and Kobayashi-Maskawa (KM) matrix elements. The most-general form is displayed explicitly for plausible choices of the masses and KM matrix elements. It is shown by direct calculation that in the good-*CP* limit there exists a basis for the quarks in which the squares of the elements of the up-quark mass matrix are proportional to the squares of the elements of the down-quark mass matrix. In this basis, the KM matrix is nontrivial solely because of differing signs of the mass-matrix elements.

I. INTRODUCTION

The problem of the existence of generations is generally considered one of the most significant questions left unsolved by the standard model. No agreement on the fundamental character or even on the appropriate type of labeling of the generations exists. Virtually the only data bearing on this question are the masses of the fermions and the weak-interaction mixing angles of the quarks. It is not surprising that considerable effort has gone into attempting to predict these numbers using symmetry relations to lower the number of parameters. No consensus has come from these efforts.

The problem is not an absence of successful ideas. Many symmetries have been explored as possible descriptions of masses and mixings. Suggestions for such "horizontal" symmetries relating the generations have included both discrete symmetries[1] of various kinds and numerous continuous symmetries.[2] Especially popular are matrices of the Fritzsch[3] type, and there have been efforts to combine grand-unified-theory (GUT) arguments with demands that the matrices be of the Fritzsch form.[4]

The difficulty is that all of these ideas are more or less successful, leaving a choice between them very difficult so that no consensus has arisen. The approaches and comparisons with experiment are sufficiently disparate as to make difficult even a comparison of the relative empirical successes of the various methods. What is needed is some kind of common approach so as to allow one to determine what is common among the methods and what might be done to begin making a choice among them.

This paper is hopefully the first step in an effort to organize in a systematic way how one might describe the generational structure of the quarks. The formalism will also be applicable to leptons but the absence of data on mixing angles makes in unlikely that this method would produce useful results. The approach taken is to use the existing experimental data to write a most-general form of the quark mass matrices consistent with current data in a basis in which the weak interactions are diagonal, the basis in which the coupling of Higgs particles to the fermions is usually described. The determination to reasonable absolute accuracy during the last few years of the last of the Kobayashi-Maskawa (KM) mixing angles makes such an approach viable. The weak-interaction basis is not unique, since a common unitary transformation of the mass matrices of the up-type and of the down-type quarks does not affect the weak-interaction mixing matrices. Thus there are additional parameters describing these transformations which can only be determined from symmetry considerations. We show, however, that there is a basis which is most naturally reached, and this basis can be the starting point for searches for mass matrices with specific desired symmetries. The degree to which the desired symmetry can be achieved may eventually be used as an indication of which mass-matrix symmetries are more or less likely to be valid. For simplicity in this first attempt, we work in the good-*CP* limit using real mass matrices and assuming a real KM matrix. The numerical results therefore might prove to be illustrative rather than definitive.

Ironically, in the process of implementing a procedure by which we hope eventually to reduce the number of viable symmetries of quark mass matrices, we have discovered what appears to be a new one. In the natural basis mentioned above, and using a low but quite viable value for the top-quark mass, the squares of the elements of the up-quark mass matrix are very nearly proportional to the squares of the corresponding elements of the down-quark mass matrix, but with some of the signs of the corresponding matrix elements varying. (Here and in the following, we refer for simplicity to the mass matrix of the charge $\frac{2}{3}$ quarks as the up-quark mass matrix and to that of the charge $-\frac{1}{3}$ quarks as the down-quark mass matrix.) Specifically, if each matrix is scaled by the corresponding third-generation quark mass (which tends to reduce mass-renormalization effects), the squares of the corresponding elements of the two different mass matrices are equal. It is the varying signs of the matrix elements, and not variations in their absolute values, that produce a nontrivial KM matrix. This symmetry is true for only one of the many equivalent weak-interaction bases, and that basis is the one reached most naturally starting from the experimental data. This symmetry is discussed in Sec.

V, which is essentially independent of the earlier sections.

The structure of the paper is as follows. Section II gives the formalism for finding all possible bases in which the weak interactions are diagonal. Section III details the constraints implied by *CP* invariance and obtains real mass matrices. Section IV gives specific numerical results. Section V discusses the possible proportionality of the squares of the elements of the two mass matrices. In all cases we work only with the three currently observed generations.

II. FINDING THE WEAK-INTERACTION BASIS

One of the major difficulties in exploring horizontal symmetry relations is that the physics is likely to be more transparent in a basis in which the weak interactions rather than the mass matrices are diagonal, whereas necessarily all the data are obtained in the basis in which the mass matrices are diagonal. The relationship between the bases is well known. Let $|q_{\rm WI}^{L,R},i\rangle$ be the states in which the weak interactions are diagonal, where L and R denote left- and right-handed helicity, i is the generation label, and q may be replaced by u for charge $\frac{2}{3}$ quarks or d for charge $-\frac{1}{3}$ quarks. Similarly, let $|q_{\rm ph}^{L,R},i\rangle$ be the physical states in which the mass matrices are diagonal. Write the mass operator as M_q and the standard weak Hamiltonian as $H_{\rm WI}$. The relevant terms in the Hamiltonian for up-type and down-type quarks are then

$$\begin{aligned} H &= [\langle u_{\rm WI}^L i \,|\, M \,|\, u_{\rm WI}^R j\rangle + \langle d_{\rm WI}^L i \,|\, M \,|\, d_{\rm WI}^R j\rangle \\ &\quad + \langle u_{\rm WI}^L i \,|\, H_W \,|\, d_{\rm WI}^L j\rangle + \text{H.c.}] + \cdots \\ &= (\langle q_{\rm ph}^L i' \,|\, U_{i'i}^{q\dagger} M_{ij} U_{jj'}^q \,|\, q_{\rm ph}^R j'\rangle \\ &\quad + \langle u_{\rm ph}^L i' \,|\, H_{WK} U_{i'i}^{u\dagger} U_{ij'}^d \,|\, d_{\rm ph}^L j'\rangle + \text{H.c.}) + \cdots \ , \end{aligned} \tag{1}$$

where $U^{q\dagger} M U^q = M_q^D$; a different U must be used for each of the two kinds of quarks, and if M is not Hermitian different U's must be used on the left and the right.

We may incorporate the data on masses and mixings as follows. Choose $|q_{\rm ph},i\rangle = V_{ij}\,|q_{\rm KM},j\rangle$ and use V to diagonalize $U_{\rm KM}$. Because of the unitarity of the KM matrix, the diagonalized matrix U_D must have eigenvalues of $\exp(i\phi)$. With no loss of generality we may write U_D as the product of two diagonal matrices: $U_D = P_u P_d^\dagger$; these phase factors may eventually be absorbed into the phases of the states. After these changes, and suppressing the generation labels, Eq. (1) becomes

$$H = \langle q_{\rm KM}^L \,|\, V^\dagger M_D V \,|\, q_{\rm KM}^R\rangle + \langle u_{\rm KM}^L \,|\, P_u H_w P_d^\dagger \,|\, d_{\rm KM}^L\rangle + \text{H.c.} + \cdots \ , \tag{2}$$

showing explicitly that $H_{\rm WI}$ is diagonal in generation space. It now makes sense to identify the physical states as being, up to phases, the states that make the KM matrix diagonal, i.e.,

$$|q_{\rm WI}\rangle = P_q^\dagger\,|q_{\rm KM}\rangle = P_q^\dagger V^\dagger\,|q_{\rm ph}\rangle \ . \tag{3}$$

Comparing with Eq. (2) we see that the mass matrix is now parametrized as $M_q = P_q^\dagger V^\dagger M_q^D V P_q$. However, any common unitary transformation of the $|q_{\rm WI}\rangle$ will leave $H_{\rm WK}$ diagonal and will therefore produce an equally legitimate weak-interaction basis; in fact separate transformations of left- and right-handed fields would be acceptable. So in general we could have

$$\begin{aligned} M_q &= W_L^\dagger P_q^\dagger V^\dagger M_q^D V P_q W_R \ , \\ U_D &= V^\dagger U_{\rm KM} V = P_u P_d^\dagger \ , \end{aligned} \tag{4}$$

which provides a most-general form of the mass matrices.

The two unitary matrices W_L and W_R each have four continuous parameters, so we have altogether eight parameters which cannot be set by data. These parameters must somehow be determined by symmetry relations. The large number of parameters available makes it unsurprising that many different proposals for possible horizontal symmetries seem to give decent results for masses and mixing angles in the quark sector. However, one quick reduction in the number of parameters is possible.

It has already been observed[5] that in the absence of visible effects of right-handed currents, a unitary transformation of the right-handed quarks alone has no observable effects. Thus transformations on the right-handed quarks may be exploited to make the mass matrix Hermitian. We get a Hermitian mass matrix by choosing $W_L = W_R = W$ in Eq. (4) so that there are four parameters instead of eight. It must be remembered, however, that some horizontal symmetries might require a non-Hermitian mass matrix, in which case W_L and W_R must be different. Non-Hermitian mass matrices can be recovered at any point in the following by right multiplying by an arbitrary unitary matrix.

We will make one further simplification in this paper. *CP* violation has a small effect on the values of the matrix elements of the KM matrix. For the moment we will assume *CP* to be a good quantum number. In that case the mass matrices can be chosen to be real, and the KM matrix is a real, orthogonal matrix. Eventually we intend to remove this restriction from the analysis.

III. REAL MASS MATRICES

We first tabulate some easily established properties of orthogonal matrices. Any unitary or orthogonal matrix may be diagonalized by a unitary transformation; the transformation of the orthogonal matrix is generally complex rather than real. The eigenvalues of a 3×3 orthogonal matrix are necessarily $\exp(i\phi)$, $\exp(-i\phi)$, $\lambda=\pm1$. We then choose $P_q = (\exp(i\theta_q), \exp(-i\theta_q), 1)$ with $\theta_u - \theta_d = \phi$. The eigenvectors of the two complex roots may be chosen to be complex conjugates of each other, and the third eigenvector may be chosen real. Thus the matrix V which diagonalizes a real three-generation KM matrix may be written in column form as

$$V = [x^{(1)} \quad x^{(1)*} \quad x^{(3)} = x^{(3)*}] \ . \tag{5}$$

The eigenvalues are completely determined by the trace and determinant of the matrix. Calling the trace T and the determinant D (necessarily ±1), we have

$$\lambda_{1,2}=\exp(\pm i\phi), \quad \lambda_3=-D \ , \qquad \cos\phi=\tfrac{1}{2}(T-D) \ . \tag{6}$$

In the next section we will obtain $\phi=0.0742\pi$ and $\lambda_3=1$ for the KM matrix, so the complexity of the eigenvalues and eigenvectors is significant and may not be ignored.

For the moment let us take W_L and W_R both to be 1. Then we may substitute Eq. (5) into Eq. (4) to get an explicit representation of the mass matrix. To simplify the notation take the complex eigenvector $x^{(1)}$ of V in Eq. (5) to be (x_1,x_2,x_3) and the real eigenvector $x^{(3)}$ to be (y_1,y_2,y_3). Then

$$M=\begin{bmatrix} A & X^* & Y^* \\ X & A & Y \\ Y & Y^* & B \end{bmatrix} , \tag{7}$$

where

$$\begin{aligned} A&=\sum m_i\,|x_i|^2 \ , \\ B&=\sum m_i\,|y_i|^2 \ , \\ X&=\exp(2i\theta_q)\sum m_i {x_i}^2 \ , \\ Y&=\lambda_3\exp(i\theta_q)\sum m_i\,|x_i|^2 \ . \end{aligned} \tag{8}$$

It is amusing to notice that if the first and second generations are interchanged in Eq. (7), the result is the complex conjugate of the original matrix. Thus a horizontal symmetry of a sort follows from no more physics than the imposition of CP invariance. Of course, no useful information comes from this symmetry since it works for arbitrary masses and mixings.

Equation (7) is not in an entirely satisfactory form, since we expect to have real mass matrices when CP is a good symmetry whereas the mass matrix in Eq. (7) is complex. To cure this problem we make use of the freedom to make a common unitary transformation on the two mass matrices, using

$$W_0=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i & 0 \\ 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} . \tag{9}$$

The resulting matrix is our intended starting point for searches for mass matrices satisfying desired symmetries:

$$M=W^\dagger \tilde{M} W \ , \qquad \tilde{M}=\begin{bmatrix} A+\mathrm{Re}X & -\mathrm{Im}X & \sqrt{2}\,\mathrm{Re}Y \\ -\mathrm{Im}X & A-\mathrm{Re}X & -\sqrt{2}\,\mathrm{Im}Y \\ \sqrt{2}\,\mathrm{Re}Y & -\sqrt{2}\,\mathrm{Im}Y & B \end{bmatrix} . \tag{10}$$

If at this point we do a further orthogonal transformation with the matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

the only effect on M will be to replace θ_q by $\theta_q+\theta$. Thus the arbitrariness of the factorization of the KM matrix into $P_u P_d^\dagger$, which requires $\phi=\theta_u-\theta_d$ but determines neither θ individually, may be absorbed into the remaining arbitrary orthogonal transformation and does not represent an additional degree of freedom. Ordinarily we will choose

$$\theta_u=-\theta_d=\tfrac{1}{2}\phi \tag{12}$$

in order to preserve the maximum possible symmetry between the charge $\frac{2}{3}$ and charge $-\frac{1}{3}$ quark mass matrices.

IV. SOME ILLUSTRATIVE NUMERICAL RESULTS

For the KM matrix we adopt the values[6]

$$U_{\mathrm{KM}}=\begin{bmatrix} 0.974 & 0.227 & 0.001 \\ -0.226 & 0.973 & -0.045 \\ -0.012 & 0.044 & 0.999 \end{bmatrix} . \tag{13}$$

For simplicity in a first attempt we have, as discussed above, chosen the KM matrix real, specifically by taking the CP-violating phase to vanish. We choose this procedure in order to retain the magnitudes of the matrix elements unchanged. Strictly speaking we should take the parametrization of Chau and Keung[7] (or Fritzsch[8]) and take $\sin\theta_z=0$. The resulting matrix differs by at most by an amount of 2×10^{-3} in any element from the matrix adopted. Since these amounts (and indeed the imaginary parts themselves) are small, we do not anticipate appreciable effects from taking them into account.

For the masses of the quarks[9,10] we have adopted, in GeV,

$$\begin{aligned} M_u&=36\,\mathrm{diag}(1.11\times10^{-4},3.19\times10^{-2},1) \ , \\ M_d&=4.7\,\mathrm{diag}(1.70\times10^{-3},3.19\times10^{-2},1) \ , \end{aligned} \tag{14}$$

where diag means a diagonal matrix and where the masses are evaluated at the conventional scale of $q^2=(1\ \mathrm{GeV})^2$. In general, since the top-quark mass is not well known and might be much higher than the figure used here it should be treated as a parameter rather than a known quantity. The s-quark mass is also uncertain by at least 20% and many of the results below are likely to be insensitive to a simultaneous increase of the top- and strange-quark masses. We choose the top-quark mass to be 36 GeV, using the relation[11]

$$m_t=m_b(m_c/m_s) \tag{15}$$

which for the mass values we have adopted gives $m_t=36$ GeV at a scale of $(1\ \mathrm{GeV})^2$. Using the methods of Gasser and Leutwyler[9] to change the scale to $q^2=(m_t)^2$ for comparison with experiment, we get a value of 24 GeV, slightly above the current experimental[12] lower limit of 23 GeV. Equation (15) will be discussed further in the next section.

With the values for the elements given in Eq. (13) it is straightforward to find the eigenvalues and eigenvectors of the KM matrix. The eigenvalues are $\exp(\pm\phi),1$ with

$$\phi=0.0742\pi \ . \tag{16}$$

Since the eigenvalues are determined by the values of the diagonal elements alone, they are insensitive to the adopted values of the off-diagonal elements. The matrix of eigenvectors is

TABLE I. Parameters obtained from Eqs. (7) and (16) for the mass matrix for charge $\frac{2}{3}$ quarks. The signs tabulated are for m_1 and m_2; m_3 may always be chosen positive. The parameters quoted are defined in Eq. (8).

Signs	A	B	X	Y
$++$	0.034 51	0.9624	$0.005\,118e^{0.3660\pi i}$	$0.1333e^{0.0358\pi i}$
$+-$	0.002 587	0.9623	$0.034\,23e^{0.0361\pi i}$	$0.1335e^{0.0384\pi i}$
$-+$	0.034 40	0.9624	$0.005\,079e^{0.3722\pi i}$	$0.1334e^{0.0357\pi i}$
$--$	0.002 480	0.9623	$0.034\,12e^{0.0362\pi i}$	$0.1335e^{0.0384\pi i}$

$$V=\begin{bmatrix} 0.694\exp(-0.043\pi i) & 0.694\exp(+0.043\pi i) & -0.190 \\ 0.707i\exp(-0.041\pi i) & -0.707i\exp(+0.041\pi i) & -0.025 \\ 0.136 & 0.136 & 0.981 \end{bmatrix}. \tag{17}$$

We are now ready to use Eq. (10) and the mass values in Eq. (14) to get the mass matrix in the natural weak-interaction basis. There is a significant technicality to take account of. Because the mass matrix connects the left-handed fields with the right-handed fields rather than some field with itself, the phases of the mass eigenvalues are not well determined. Having chosen to use Hermitian matrices, we have automatically assured that the mass eigenvalues are real, but we have no way of knowing their sign. No physics can be affected by simultaneously changing the signs of all the mass eigenvalues for either charge of quark, but the relative signs of the masses of different generations of quarks matter. Thus we may fix one mass to be positive (we choose the third generation), but we have four possibilities for the signs of the masses of the remaining two generations. In Tables I and II we tabulate, for all possible signs, the parameters A, B, X, and Y that through Eq. (10) give the mass matrices.

We have quoted four-figure numbers in the tables even though the data are good to three figures at most. The reason for this is that the diagonalization of the mass matrices is ill conditioned because of the large range of magnitudes of the mass eigenvalues, and four figures in the general mass matrix are required to get acceptable first-generation masses. In view of this instability, one might feel more comfortable if some general principle could be invoked for the smallness of the first-generation masses; such a principle could be used to restrict the remaining parameters in the mass matrices.

V. PROPORTIONALITY OF THE SQUARES OF MASS-MATRIX ELEMENTS

In the preceding sections we have shown that by diagonalizing the KM matrix it is possible to find explicit quark-mass matrices in a basis in which the weak interactions are diagonal, with the value of the top-quark mass as a parameter. If these matrices are determined in the approximation that CP is a good quantum number, and if the absence of observed effects of right-handed currents is used to permit the restriction to Hermitian matrices,[5] then there is a three-parameter set of appropriate bases. The basis in which the phases $\theta_{u,d}$ associated with the quarks are handled as symmetrically as possible is the most natural one to start from, and we have chosen to obtain numerical values for that basis. Using the parameters in Tables I and II and Eq. (10) we may write out specific mass matrices once we have chosen the signs we want for the quark masses. We will take m_c and m_d negative and all others positive, in which case

$$m_u=36\begin{bmatrix} 0.0366 & -0.0039 & 0.1874 \\ -0.0039 & -0.0314 & -0.0227 \\ 0.1874 & -0.0227 & 0.9623 \end{bmatrix},$$
$$m_d=4.7\begin{bmatrix} 0.0368 & -0.0036 & 0.1875 \\ -0.0036 & 0.0304 & 0.0228 \\ 0.1875 & 0.0228 & 0.9623 \end{bmatrix}. \tag{18}$$

The similarity of the magnitudes of these matrix elements is striking. This similarity suggests an attempt to determine the mass matrices from the condition that the squares of the elements of the up-quark mass matrices should be proportional to the squares of the corresponding elements of the down-quark mass matrix. We have used Eq. (10) to make a search for Hermitian matrices which best satisfy this criterion; those given in Eq. (18) in fact satisfy this criterion better than any other possible pair. Moreover, for almost all values of the angles of the orthogonal transformation, the transformed matrices satis-

TABLE II. Mass-matrix parameters for the charge $-\frac{1}{3}$ quarks. The notation is the same as for Table I.

Signs	A	B	X	Y
$++$	0.035 26	0.9624	$0.005\,460e^{0.175\pi i}$	$0.1331e^{-0.0384\pi i}$
$+-$	0.003 369	0.9624	$0.034\,97e^{-0.113\pi i}$	$0.1333e^{-0.0357\pi i}$
$-+$	0.003 362	0.9623	$0.004\,861e^{0.267\pi i}$	$0.1336e^{-0.0385\pi i}$
$--$	0.001 728	0.9623	$0.033\,35e^{-0.111\pi i}$	$0.1337e^{-0.0358\pi i}$

fy this criterion much more poorly than those in Eq. (18).

If we then demand that $m_{u,d}$ satisfy this criterion exactly by choosing

$$m_{d,u}=m_{3d,u}\begin{bmatrix} 0.0367 & -0.0038 & 0.1874 \\ -0.0038 & \pm 0.0309 & \pm 0.0228 \\ -0.1874 & \pm 0.0228 & 0.9623 \end{bmatrix} \quad (19)$$

the resulting quark masses are

$$m_u=(7.56\times 10^{-3},-1.13,36.0)\ ,$$
$$m_d=(8.60\times 10^{-3},0.152,4.70)\ , \quad (20)$$

and the KM matrix is

$$U_{\rm KM}=\begin{bmatrix} 0.0975 & 0.224 & 0.001 \\ -0.224 & 0.974 & -0.045 \\ -0.011 & 0.044 & 0.999 \end{bmatrix} \quad (21)$$

which, except for the mass of the up quark, are entirely satisfactory results. Of course, one would prefer to have some fundamental understanding of the origin of such a symmetry and of the unsymmetric magnitudes of the amplitudes.

We should mention one condition on the quark masses which follows from this symmetry. We have written in essence $(M_{ij}^u)^2=(m_t/m_b)^2(M_{ij}^d)^2$. Since the trace of $M^\dagger M$ is invariant, this condition implies that

$$m_u{}^2+m_c{}^2+m_t{}^2=(m_t/m_b)^2(m_d{}^2+m_s{}^2+m_t{}^2)\ . \quad (22)$$

Equation (22) may be solved for the top-quark mass in terms of the other masses; neglecting small terms we recover Eq. (15), which for the mass values we have adopted gives 36 GeV for m_t, corresponding as discussed in the last section to an experimental value for m_t of 24 GeV. In fact, we used Eq. (15) to choose the exact value of m_t before finding the proportionality symmetry. In any event, the requirement that the magnitudes of the elements of the quark mass matrices should be proportional to each other gives a prediction for the top-quark mass which is experimentally acceptable.

VI. SUMMARY

Diagonalizing the KM matrix directly establishes a set of bases in which the weak interactions rather than the mass matrices are diagonal. If the nonobservation of right-handed currents is used to permit the requirement that the mass matrices be Hermitian, then there are three continuous parameters describing the various bases. If non-Hermitian matrices are permitted, then there are three additional continuous parameters not determined by the requirement that the weak interactions be diagonal. Of necessity, these parameters must be specified by symmetry relations, not by comparison with data. It is feasible to perform searches over these parameters seeking mass matrices obeying specified symmetry conditions. Such searches could potentially be used to determine which of the several viable horizontal symmetries gives the best fit to experimental data. There is a single basis in which the up quarks (all charge $\frac{2}{3}$ quarks) and the down quarks (all charge $-\frac{1}{3}$ quarks) are treated as symmetrically as possible given the constraint of a nontrivial KM matrix. For a top-quark Lagrangian mass of 36 GeV, the magnitudes of the matrix elements of the two quark-mass matrices are proportional to each other, and the nontrivial KM matrix comes about solely from the differing signs of corresponding elements of the mass matrix.

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