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Application of the real-time temperature Green's functions to chiral-symmetry breaking and restoration

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The real-time temperature Green's functions are applied to study spontaneous breaking and restoration of chiral symmetry in quantum chromodynamics. In particular we discuss some salient features of the temperature Green's functions which essentially reflect the Kubo-Martin-Schwinger condition. The gap equation is given in the ladder approximation and solved with a rough approximation.

I. INTRODUCTION

The restoration of spontaneously broken chiral symmetry under the influence of finite temperature and/or finite fermion density has attracted much attention for quite some time.[1,2] Although the following argument may be relevant to a wide class of field theories, we particularly consider its application to quantum chromodynamics (QCD). In this realm, some interesting topics discussed so far include properties of the phase transition,[2-4] cosmological as well as astronomical implications,[5] experimentally accessible phenomena,[4-7] and so on. Lattice gauge theories appear to provide the most powerful approach to studying properties of the chiral phase transition, giving $T_c \approx 3\Lambda_{\overline{\rm MS}}$ ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme) and favoring the first-order transition.[2,3] However, they have a difficult problem[8] and their calculation is still limited owing to a finite size of lattice. Moreover, we would like to obtain other information pertinent to the chiral transition, such as the possible temperature dependence of the decay constant, the pion mass, and other quantities.

Now, since the pioneering work of Nambu and Jona-Lasinio,[9] a traditional approach to chiral-symmetry breaking (CSB) has been to use the Schwinger-Dyson (SD) equation. The temperature and/or quark density effects[7-10] in this approach were thus far studied based on the imaginary-time formalism which uses Matsubara's temperature Green's functions.[11] The purpose of this paper is to present another treatment with the real-time formalism.

Since the development of the thermo field dynamics (TFD) of Umezawa and others,[12,13] especially supplemented with the path-integral formalism of Niemi and Semenoff,[14,15] the real-time formalism has achieved such substantial progress that one can systematically calculate higher-order effects with diagrams analogous to the usual Feynman graphs. Strictly speaking, TFD and the path-integral formalism provide us with different representations of the algebra of operator fields;[13] i.e., they are canonically inequivalent theories. Nevertheless, both theories are equivalent with regard to our perturbative calculation of thermal Green's functions. More recently, it appears that TFD purports to treat nonequilibrium processes, not just equilibrium ones.[16] In this paper we treat only those states in complete thermal equilibrium. Then it turns out that TFD gives substantially equivalent results to the imaginary-time formalism. Nevertheless, we argue that in our treatment there is some novel aspect worth studying which originates from the advantage of TFD, that the certain analytic continuation[11,13,17] required in the imaginary-time formalism is automatically satisfied in TFD.

IN TFD all dynamical freedoms are doubled. Accordingly any two-body Green's function is expressed as a two-by-two matrix. It is remarkable that in the spectral representation,[18] the whole of each matrix can be expressed in terms of a single weight function. It is a direct consequence of the Kubo-Martin-Schwinger (KMS) condition.[19] The spectral representation is quite important in identifying the genuine dynamical part in the two-body Green's functions as well as to establish a close relation between the real-time and the imaginary-time formalisms.[11,13] Therefore, our major task in deriving the gap equation is concerned with retaining the properties imposed by the KMS condition.

This paper is organized as follows. In Sec. II we review the essential arguments to derive the spectral representation in the case of the fermion temperature Green's function. Some expressions we use for the spectral representation are different from the conventional ones.[12,13] Though their contents are the same, our expressions are useful, especially in the case of a finite chemical potential. In Sec. III the gap equation is derived within the ladder approximation and temperature-density effects are studied by perturbation theory. In Sec. IV we treat the gap equation in a crude approximation and give some numerical results for the chiral phase transition. Brief remarks are also given.

II. THE KUBO-MARTIN-SCHWINGER CONDITION AND THE SPECTRAL REPRESENTATION

This section is devoted to elucidating the important consequence of the KMS condition which results in the

spectral representation for two-body temperature Green's functions. The representation was first derived by Matsumoto.[18] Some elaborative discussions are found in Refs. 12 and 13. We treat only the fermion propagator because it plays a central role in the SD equation discussed in the next section. We include both the temperature $T=1/\beta$ and the chemical potential μ.

It is convenient to introduce the tilde operators $\tilde{\psi}(x)$ and $\tilde{\bar{\psi}}(y)$ whose temporal dependences are defined by

$$\tilde{\psi}(x^0,\mathbf{x})=e^{+iKx^0}\psi(0,\mathbf{x})e^{-iKx^0}\ , \tag{2.1a}$$

$$\tilde{\bar{\psi}}(y^0,\mathbf{y})=e^{iKy^0}\bar{\psi}(0,\mathbf{y})e^{-iKy^0}\ , \tag{2.1b}$$

where $K=H-\mu N_F$, N_F denoting the fermion number operator. These are simply related to the usual Heisenberg operators as

$$\tilde{\psi}(x)=e^{i\mu x^0}\psi(x)\ , \tag{2.2a}$$

$$\tilde{\bar{\psi}}(y)=e^{-i\mu y^0}\bar{\psi}(y)\ . \tag{2.2b}$$

In the following we shall concern ourselves with the Green's function defined in terms of the tilde operators. The corresponding Green's function with the Heisenberg operators can easily be obtained by shifting the zeroth component of momenta. It is evident from functional-integration formulations[14,20] that the former type of Green's function, not the latter, is a directly calculable quantity in perturbation theory. Next, in terms of the tilde operators we define two auxiliary functions:

$$[S'_{\beta>}(x,y)]_{\gamma\delta}\equiv \mathrm{Tr}[e^{-\beta K}\tilde{\psi}_\gamma(x)\tilde{\bar{\psi}}_\delta(y)]/\mathrm{Tr}(e^{-\beta K})\ , \tag{2.3a}$$

$$[S'_{\beta<}(x,y)]_{\gamma\delta}\equiv \mathrm{Tr}[e^{-\beta K}\tilde{\bar{\psi}}_\delta(y)\tilde{\psi}_\gamma(x)]/\mathrm{Tr}(e^{-\beta K})\ , \tag{2.3b}$$

together with their Fourier transforms

$$[S'_{\beta\ldots}(x,y)]_{\gamma\delta}\equiv\int\frac{d^4p}{(2\pi)^3}e^{-ip\cdot(x-y)}[\rho_{\ldots}(p^0,\mathbf{p})]_{\gamma\delta}\ . \tag{2.4}$$

The KMS condition follows from the feasibility of cyclic permutations of the operators inside the trace Tr( ).[19] It amounts to

$$\rho_<(p^0,\mathbf{p})=e^{-\beta p^0}\rho_>(p^0,\mathbf{p})\ . \tag{2.5}$$

Hereafter we set

$$\rho_>(p^0,\mathbf{p})=\frac{\rho(p^0,\mathbf{p})}{1+e^{-\beta p^0}}\ , \tag{2.6a}$$

$$\rho_<(p^0,\mathbf{p})=\frac{e^{-\beta p^0}}{1+e^{-\beta p^0}}\rho(p^0,\mathbf{p})\ . \tag{2.6b}$$

The four matrix elements of the fermion temperature Green's function can be expressed as

$$iS'_{\beta 11}(x,y)=\theta(x^0-y^0)S'_{\beta>}(x,y)-\theta(y^0-x^0)S'_{\beta<}(x,y)\ , \tag{2.7a}$$

$$iS'_{\beta 21}(x,y)=S'_{\beta>}\left[x-i\frac{\beta}{2},y\right]\ , \tag{2.7b}$$

$$iS'_{\beta 12}(x,y)=-S'_{\beta<}\left[x,y-i\frac{\beta}{2}\right]\ , \tag{2.7c}$$

$$iS'_{\beta 22}(x,y)=\theta(y^0-x^0)S'_{\beta>}(x,y)-\theta(x^0-y^0)S'_{\beta<}(x,y)\ . \tag{2.7d}$$

Perhaps the easiest justification for these (2.7) can be obtained with a functional-integration method.[20] Their Fourier transforms are

$$S'_{\ldots}(x,y)=\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}S'_{\ldots}(p^0,\mathbf{p})\ . \tag{2.8}$$

Then it is remarkable that all four quantities in (2.7a)–(2.7d) can be expressed with a single weight function $\rho(p^0,\mathbf{p})$:

$$iS'_{\beta 11}(p^0,\mathbf{p})=\int_{-\infty}^{+\infty}d\omega\left[\frac{i}{p^0-\omega+i\epsilon}-2\pi\delta(p^0-\omega)f_D(p^0)\right]\rho(\omega,\mathbf{p})\ , \tag{2.9a}$$

$$iS'_{\beta 21}(p^0,\mathbf{p})=e^{\beta p^0/2}f_D(p^0)\rho(p^0,\mathbf{p})\ , \tag{2.9b}$$

$$iS'_{\beta 12}(p^0,\mathbf{p})=-e^{\beta p^0/2}f_D(p^0)\rho(p^0,\mathbf{p})\ , \tag{2.9c}$$

$$iS'_{\beta 22}(p^0,\mathbf{p})=\int_{-\infty}^{+\infty}d\omega\left[\frac{-i}{p^0-\omega-i\epsilon}-2\pi\delta(p^0-\omega)f_D(p^0)\right]\rho(\omega,\mathbf{p})\ , \tag{2.9d}$$

where ϵ denotes an infinitesimal positive quantity and f_D the Fermi-Dirac statistical factor:

$$f_D(p^0)=\frac{1}{e^{\beta p^0}+1}\ . \tag{2.10}$$

The expressions (2.9a) and (2.9d) in the spectral representation are somewhat different from the conventional ones,[12,13] though their contents are the same. The above expressions can be arranged as a matrix in quite a suggestive way:[12,13]

$$\begin{bmatrix}S'_{\beta 11} & S'_{\beta 12}\\ S'_{\beta 21} & S'_{\beta 22}\end{bmatrix}(p^0,\mathbf{p})=U_F\begin{bmatrix}G_+(p) & 0\\ 0 & G_-(p)\end{bmatrix}U_F\ , \tag{2.11}$$

where

$$U_F=\begin{bmatrix}\cos\theta_F & \sin\theta_F\\ -\sin\theta_F & \cos\theta_F\end{bmatrix} \tag{2.12}$$

with $(\sin\theta_F)^2=f_D(p^0)$ and

$$G_{\pm}(p)=\pm\int_{-\infty}^{+\infty}d\omega\frac{\rho(\omega,\mathbf{p})}{p^0-\omega\pm i\epsilon}\ . \tag{2.13}$$

As was emphasized by Matsumoto,[16] the genuine dynamical part of the fermion propagator is $G_{\pm}(p)$. Furthermore, at this point a close relation between the real- and imaginary-time formalisms becomes evident. We note that $iG_+(p)$ is just the Fourier transform of the retarded Green's function:

$$i[G_+(x,y)]_{\gamma\delta}=\theta(x^0-y^0)\mathrm{Tr}[e^{-\beta K}\{\tilde{\psi}_\gamma(x),\bar{\tilde{\psi}}_\delta(y)\}]/\mathrm{Tr}(e^{-\beta K})\ . \tag{2.14}$$

It is well known that this function and the imaginary-time temperature Green's function share the same weight function.[11,12,17] Indeed the Fourier transform of the latter can in general be written as

$$G_\beta(\omega_n,p)=\int d\omega\frac{\rho(\omega,p)}{i\omega_n-\omega}\ , \tag{2.15}$$

where $\omega_n=(2n+1)\pi/\beta$, n being integers.

III. GAP EQUATION IN THE LADDER APPROXIMATION

Let us calculate the self-energy correction to the quark propagator in the ladder approximation depicted in Fig. 1. Insofar as vertex corrections are neglected, it is straightforward to incorporate manifestly the important properties of the temperature Green's functions, which are discussed in the preceding section. Inclusion of vertex corrections will require a much more complicated calculation.

We write the inverse of the quark propagator as

$$S'^{-1}_\beta(p)=U_F{}^{-1}\begin{bmatrix}\gamma^0(p^0_\epsilon+\mu)-\boldsymbol{\gamma}\cdot\mathbf{p}-\Sigma_\beta(p_\epsilon) & 0\\ 0 & -\gamma^0(p^0_{-\epsilon}+\mu)+\boldsymbol{\gamma}\cdot\mathbf{p}+\Sigma_\beta(p_{-\epsilon})\end{bmatrix}U_F{}^{-1}\ . \tag{3.1}$$

$\Sigma_\beta(p_\epsilon)$ denotes the proper self-energy part of the whole contribution in Fig. 1. Hereafter we use the notation $p^\mu_{\pm\epsilon}=(p^0\pm i\epsilon,\mathbf{p})$. We write the gauge-boson propagator in the Feynman gauge as

$$\begin{bmatrix}(D^{\mu\nu}_\beta)_{11} & (D^{\mu\nu}_\beta)_{12}\\ (D^{\mu\nu}_\beta)_{21} & (D^{\mu\nu}_\beta)_{22}\end{bmatrix}(k^0,\mathbf{k})=(-g^{\mu\nu})U_B\begin{bmatrix}\int\frac{\sigma(\omega,\mathbf{k})}{k^0-\omega+i\epsilon}d\omega & 0\\ 0 & (-1)\int\frac{\sigma(\omega,\mathbf{k})}{k^0-\omega-i\epsilon}d\omega\end{bmatrix}U_B\ , \tag{3.2a}$$

where

$$U_B=\begin{bmatrix}\cosh\theta_B & \sinh\theta_B\\ \sinh\theta_B & \cosh\theta_B\end{bmatrix} \tag{3.2b}$$

with

$$(\sinh\theta_B)^2=\frac{1}{e^{\beta k^0}-1}\ . \tag{3.2c}$$

We use the free boson propagator for which

$$\sigma(\omega,\mathbf{k})=\frac{1}{2|\mathbf{k}|}[\delta(\omega-|\mathbf{k}|)-\delta(\omega+|\mathbf{k}|)]\ .$$

It is enough to calculate only the (11) element of the proper self-energy part of Eq. (3.1), because all the other elements can automatically be fixed by it. We find that

$$\cos^2\theta_F\Sigma_\beta(p_\epsilon)+\sin^2\theta_F\Sigma_\beta(p_{-\epsilon})=\frac{ig^2C_F}{(2\pi)^4}\int\gamma_\mu[S'_\beta(p-k)]_{11}\gamma_\nu[D^{\mu\nu}_\beta(k)]_{11}d^4k\ , \tag{3.3}$$

where C_F is the Casimir factor $C_F=(N_c{}^2-1)/2N_c=\frac{4}{3}$ and

$$i[D_\beta^{\mu\nu}(k)]_{11}=(-g^{\mu\nu})\int d\omega\left[\frac{i}{k^0-\omega+i\epsilon}+2\pi\delta(k^0-\omega)f_B(k^0)\right]\sigma(\omega,\mathbf{k}) .$$

The left-hand side (LHS) of Eq. (3.3) consists of two parts, each characterized by different analytic properties of the variable p^0. To be consistent, we must similarly distinguish two parts on the RHS. The separation of two parts has crucial importance in summing up the contributions in Fig. 1. This separation can be achieved with the aid of the formula[12,13]

$$J\equiv\int\frac{dk^0}{2\pi}\left[\frac{i}{p^0-k^0-\omega+i\epsilon}-2\pi f_D(p^0-k^0)\delta(p^0-k^0-\omega)\right]\left[\frac{i}{k^0-\omega'+i\epsilon}+2\pi f_B(k^0)\delta(k^0-\omega')\right]$$
$$=\left[\frac{i}{p^0-\omega-\omega'+i\epsilon}-2\pi f_D(p^0)\delta(p^0-\omega-\omega')\right]\frac{f_D(\omega)f_B(\omega')}{f_D(\omega+\omega')} . \tag{3.4}$$

Thus one arrives at

$$\Sigma_\beta(p_\epsilon)=-\frac{g^2C_f}{(2\pi)^3}\int d^3k\int d\omega\int d\omega'\frac{g^{\mu\nu}\gamma_\mu\rho(\omega,\mathbf{p}-\mathbf{k})\gamma_\nu\sigma(\omega',\mathbf{k})}{p^0-\omega-\omega'+i\epsilon}\frac{f_D(\omega)f_B(\omega')}{f_D(\omega+\omega')} . \tag{3.5}$$

If one recalls that $\Sigma_\beta(p_\epsilon)$ is the proper self-energy part of the retarded Green's function (2.14), Eq. (3.5) clearly corresponds to Dzyaloshinski's formula in the imaginary-time formalism.[13,17]

Now the self-consistency requirement in the Hartree-Fock approximation[9] is to impose the condition

$$\frac{1}{\gamma^0(p_\epsilon^0+\mu)-\boldsymbol{\gamma}\cdot\mathbf{p}-\Sigma_\beta(p_\epsilon)}=\int_{-\infty}^{+\infty}d\omega\frac{\rho(\omega,\mathbf{p})}{p^0-\omega+i\epsilon} . \tag{3.6}$$

Equations (3.5) and (3.6) together give us the gap equation to determine the dynamical mass of quarks at a finite temperature and finite quark density.

Let us evaluate the temperature and density effects up to the lowest order. We start from the dynamical solution of the SD equation at $T=\mu=0$. It was estimated by many authors.[21] We assume that the solution is expressed as

$$S_F'(p)^{-1}=\gamma\cdot p-A(p^2)+i\epsilon . \tag{3.7}$$

Strictly speaking, the above form is valid only for the Landau gauge. The solution in the Feynman gauge appears a bit complicated. However, as the following procedure is essentially unchanged, we dare to use Eq. (3.7).

As is evident from Eq. (2.11), we need the retarded Green's function in order to get the temperature Green's function. For this purpose we cast the inverse of Eq. (3.7) into the famous spectral representation of Gell-Mann, Low, and Lehmann:[22]

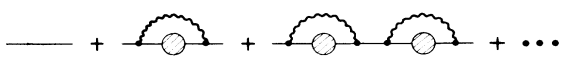

FIG. 1. Ladder-type diagrams.

$$S_F'(p)=\int_0^\infty\left[\frac{g(\kappa)}{\gamma\cdot p-\kappa+i\epsilon}+\frac{h(\kappa)}{\gamma\cdot p+\kappa-i\epsilon}\right]\frac{d\kappa}{\kappa} . \tag{3.8}$$

Now it is easy to confirm that the retarded Green's function at $T=\mu=0$ can be expressed as

$$G_R(p_\epsilon)=\int_0^\infty\left[\frac{g(\kappa)}{\gamma\cdot p-\kappa+i\epsilon}+\frac{h(\kappa)}{\gamma\cdot p+\kappa+i\epsilon}\right]\frac{d\kappa}{\kappa} . \tag{3.9}$$

Let us set

$$w(z)\equiv z-A(z^2) . \tag{3.10}$$

If and only if $w(z)$ is a generalized R function, it is possible to write the inverse of Eq. (3.7) in the spectral representation.[23] If this is the case, the weight functions are given by

$$g(\kappa)/\kappa=\frac{i}{2\pi}\lim_{\epsilon\to0}\left[\frac{1}{w(\kappa+i\epsilon)}-\frac{1}{w(\kappa-i\epsilon)}\right]\quad\text{for }\kappa>0 , \tag{3.11a}$$

$$h(\kappa)/\kappa=\frac{i}{2\pi}\lim_{\epsilon\to0}\left[\frac{1}{w(-\kappa+i\epsilon)}-\frac{1}{w(-\kappa-i\epsilon)}\right]\text{ for }\kappa>0 . \tag{3.11b}$$

We now substitute $\pm G_R(p_{\pm\epsilon})$ for $G_\pm(p)$ after shifting the zeroth component of the momenta: $p^0\to p^0+\mu$. Thus, we are provided with an approximate temperature Green's function, which in turn is substituted into Eq. (3.5). In this way we obtain the dynamical mass term modified by temperature and chemical potentials to the lowest order:

$$A_\beta(p_\epsilon)=A[(p_\epsilon^0+\mu)^2-\mathbf{p}^2]-\frac{2g^2C_F}{(2\pi)^3}\int_0^\infty d\kappa[g(\kappa)-h(\kappa)][J_1(p_\epsilon,\kappa)-J_2(p_\epsilon,\kappa)]\ , \tag{3.12a}$$

where

$$J_1(p_\epsilon,\kappa)=\int\frac{d^3k}{E_{\mathbf{p}-\mathbf{k}}}\left[\frac{f_D(E_{\mathbf{p}-\mathbf{k}}-\mu)}{\mathbf{k}^2-(p^0+\mu-E_{\mathbf{p}-\mathbf{k}}+i\epsilon)^2}+\frac{f_D(E_{\mathbf{p}-\mathbf{k}}-\mu)}{\mathbf{k}^2-(p^0+\mu+E_{\mathbf{p}-\mathbf{k}}+i\epsilon)^2}\right]\ , \tag{3.12b}$$

$$J_2(p_\epsilon,\kappa)=\int\frac{d^3k}{|\mathbf{k}|}f_B(|\mathbf{k}|)\left[\frac{1}{E_{\mathbf{p}-\mathbf{k}}{}^2-(p^0+\mu-|\mathbf{k}|+i\epsilon)^2}+\frac{1}{E_{\mathbf{p}-\mathbf{k}}{}^2-(p^0+\mu+|\mathbf{k}|+i\epsilon)^2}\right]\ , \tag{3.12c}$$

with $E_\mathbf{p}=(\mathbf{p}^2+\kappa^2)^{1/2}$. In order to get a rough figure for the second term in (3.12a), we use a crude approximation $A(p^2)\approx$const, setting $g(\kappa)=M\delta(\kappa-M)$ and $h(\kappa)=0$. Then one obtains

$$A_\beta(p^0+\mu=M,\mathbf{p}=0)=M-\frac{g^2C_FM}{2\pi^2}\int_0^\infty\frac{dk}{E_k}\left[\left[\frac{E_k}{M}+1\right]f_D(E_k-\mu)-\left[\frac{E_k}{M}-1\right]f_D(E_k+\mu)\right]\ , \tag{3.13}$$

where $E_k=(k^2+M^2)^{1/2}$. This result implies that the dynamical mass tends to decrease under the influence of the temperature and chemical potentials.

We remark that the confinement effect may invalidate the representation (3.8) (Ref. 24). Thus, in order to apply the above argument, it is presumably a requirement to cut off infrared singularities. The singularities themselves can be treated, at least, to some approximation.[10]

IV. SOME NUMERICAL RESULTS AND REMARKS

The dynamical mass of a quark at finite temperature and density may be defined by the condition

$$\Sigma_\beta(p^0=M-\mu,\mathbf{p}=0)=M\ . \tag{4.1}$$

This quantity plays the role of an order parameter which distinguishes between the Wigner and the Nambu-Goldstone phase of chiral symmetry. Let us make a rough calculation to find the critical values of T and μ for which M changes its value from zero to nonzero. We approximate the weight function $\rho(\omega,\mathbf{p})$ on the RHS of Eq. (3.5) by

$$\rho(\omega,\mathbf{p})=\frac{1}{2E_\mathbf{p}}[(\gamma\cdot p_+ +M)\delta(\omega+\mu-E_\mathbf{p})-(\gamma\cdot p_- +M)\delta(\omega+\mu+E_\mathbf{p})]\ , \tag{4.2}$$

where $E_\mathbf{p}=(\mathbf{p}^2+M^2)^{1/2}$ and $p_\pm^\mu=(\pm E_\mathbf{p},\mathbf{p})$. This approximation utilizes the free retarded Green's function

$$1/[\gamma^0(p^0+\mu+i\epsilon)-\boldsymbol{\gamma}\cdot\mathbf{p}-M] \tag{4.3}$$

to all quark lines in Fig. 1.

The resulting equation involves ultraviolet as well as infrared divergences. In fact, the ultraviolet divergence is false; there is no divergence of this kind, if one takes into account the correct asymptotic behavior of the dynamical mass term.[25] Even if one includes current-quark masses as explicit symmetry-breaking terms, the ultraviolet divergence should offer no problem, once it is renormalized at $T=\mu=0$. In the present approximation we merely introduce two cutoff parameters Λ and λ, respectively, for ultraviolet and infrared divergences.

Our approximate gap equation reads as

$$\frac{3\pi}{8}\left[\frac{1}{\bar\alpha_s}-\frac{1}{\alpha_s}\right]=f\left[\frac{M}{\Lambda},\beta\Lambda,\frac{\mu}{\Lambda}\right]\ . \tag{4.4}$$

On the LHS $\alpha_s=g^2/4\pi$, while on the RHS

$$f\left[\frac{M}{\Lambda},\beta\Lambda,\frac{\mu}{\Lambda}\right]=\ln\left[\frac{\xi+E(\xi)}{2\xi}\right]+\int_\xi^1\frac{dx}{E(x)}\left[\frac{1}{D_+(x)}+\frac{1}{D_-(x)}\right]+\frac{\Lambda}{M}\int_\xi^1 dx\left[\frac{1}{D_+(x)}-\frac{1}{D_-(x)}\right]\ , \tag{4.5a}$$

where

$$\xi=\lambda/\Lambda,\quad E(x)=(x^2+M^2/\Lambda^2)^{1/2}\ , \tag{4.5b}$$

and

$$D_\pm(x)=\exp\{\beta\Lambda[E(x)\mp\mu/\Lambda]\}+1\ . \tag{4.5c}$$

$\bar\alpha_s$ in Eq. (4.4) denotes the critical value of the coupling constant beyond which the Nambu-Goldstone phase is realized at $T=\mu=0$. It is given by

$$\bar\alpha_s=3\pi/[8\ln(\Lambda/\lambda)]\ . \tag{4.6}$$

TABLE I. Parameter values for numerical calculation.

Λ	1000 MeV
λ	100 MeV
$\bar\alpha_s$	0.51
α_s	0.75
M_0 at $T=\mu=0$	300 MeV

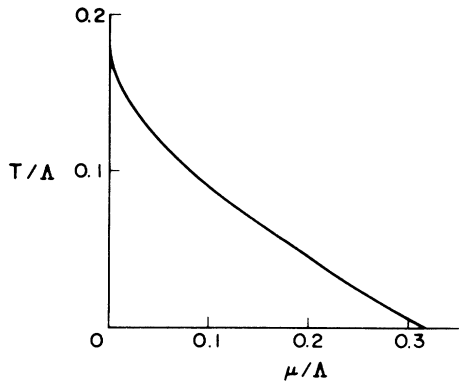

FIG. 2. Phase diagram for chiral transition.

At $T=\mu=0$ the dynamical mass value is expressed as

$$M_0=(2\lambda/\eta)\sqrt{1-\eta}\ , \tag{4.7a}$$

$$\eta=\exp\left[-\frac{3\pi}{8}\left[\frac{1}{\bar{\alpha}_s}-\frac{1}{\alpha_s}\right]\right]\ . \tag{4.7b}$$

Taking the appropriate parameter values listed in Table I we calculate the phase diagram in Fig. 2. Among the parameters in Table I, α_s, Λ, and λ are fixed and the other parameters are calculated from them. The mass values on the critical curve are shown in Fig. 3. The figure shows that Eq. (4.4) implies the chiral transition of first order for $\mu\neq 0$, while of second order for $\mu=0$.

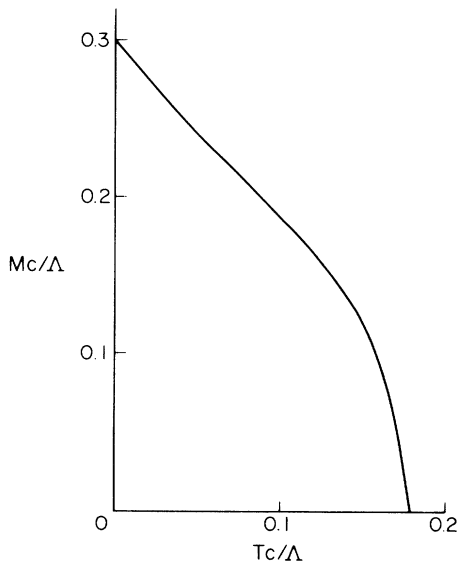

FIG. 3. The dynamical mass of a quark vs the critical temperature.

In conclusion we remark that it will be possible to improve the treatment of the ladder-type gap equation. It is also interesting to treat instanton effects[26] with the real-time formalism; it seems especially suitable to treat the weak effect of T and μ, once solutions are known at $T=\mu=0$, while the imaginary-time formalism is known to yield reliable results in the high-temperature case.[1]

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