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Yang-Mills theory and quantum chromodynamics in the temporal gauge

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A canonical formulation is given for Yang-Mills theory and quantum chromodynamics in the temporal gauge. Operator constraints are avoided so that canonical commutation rules apply throughout. The use of ghost operators for longitudinal and timelike gluons permits the selection of states that obey the gauge-fixing condition and Gauss's law as time-independent constraints. Propagators for longitudinal gluons are evaluated and an argument is given in support of the conclusion that the part of the propagator that violates time-translation invariance has no effect on perturbative S-matrix elements. The validity of asymptotic free-field representations for quarks and gluons in perturbative S-matrix calculations is related to the Lie algebra that characterizes the gauge theory.

I. INTRODUCTION

In the preceding paper we have discussed a canonical formulation of quantum electrodynamics in the temporal gauge in which the gauge-fixing condition $A_0=0$ and Gauss's law are implemented and the unphysical longitudinal and timelike polarization modes of the photon are eliminated.[1] The formulation leads to S-matrix elements which can also be evaluated with a set of perturbative Feynman rules. The propagators included in these rules follow from the canonical apparatus, i.e., momentum representations of $\mathbf{A}$ and A_0 and their conjugate momenta, Hamiltonian, and time displacement operators, etc. However, these propagators are the vacuum expectation values of time-ordered products in a vacuum state $|0\rangle$ that is the lowest-energy eigenstate of only the H_0 part of the Hamiltonian (i.e., the $e\rightarrow 0$ limit of H); and the vacuum state $|0\rangle$ violates the Gauss law constraint. Nevertheless it was shown that the S-matrix elements obtained from these Feynman rules are identical to the ones that follow from the properly formulated theory in which the incident and scattered charged-particle states are coherent superpositions that obey Gauss's law as well as the constraint $A_0=0$.

In this paper we report an extension of this program to non-Abelian gauge theories such as Yang-Mills theory and QCD. We continue to use the canonical apparatus that we implemented in our earlier work, in which primary constraints are avoided and all fields have canonically conjugate momenta. In this way we hope to avoid the ambiguities in propagators obtained by path-integral methods, that have recently been discussed and documented by Cheng and Tsai.[2] We will show that the canonical formulation allows us to implement the gauge-fixing condition and Gauss's law in a manner that parallels the procedure in QED. Because no Faddeev-Popov ghosts are required in this gauge in non-Abelian theories, simultaneous implementation of Gauss's law and gauge fixing is easier than in similar programs in covariant gauges.[3] Moreover, because canonical quantization rules obtain between fields and their conjugate momenta, no operator-ordering ambiguities arise in this gauge. The fact that such operator-ordering problems can arise in noncovariant gauges has been reported;[4,5] it has also been noted that ordering ambiguities do not appear in the temporal gauge.[5] As in QED we generate a set of Feynman rules in an interaction picture in which the "free field" $e\rightarrow 0$ limit of the Hamiltonian time displaces the operators and selects the vacuum state used to evaluate the propagators. Unlike QED, it has not been possible in Yang-Mills theory or in QCD to make a rigorous connection between the S matrix in the theory in which Gauss's law is implemented with the S matrix obtained with Feynman rules. We will postpone further discussion of this point until Sec. III of this paper.

II. CONSTRAINTS AND PROPAGATORS IN YANG-MILLS THEORY

We will develop the formalism for non-Abelian gauge theories in the temporal gauge using Yang-Mills theory as an example. The SU(3) case can be obtained simply by substituting the structure constants of SU(3) for those of SU(2) and extending the gluon triplet to an octet. We postulate the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}\mathbf{f}_{ij}\cdot\mathbf{f}_{ij}+\tfrac{1}{2}\mathbf{f}_{i0}\cdot\mathbf{f}_{i0}+\mathbf{j}_i\cdot\mathbf{b}_i-\mathbf{j}_0\cdot\mathbf{b}_0 -\partial_0\mathbf{b}_0\cdot\mathbf{G}-\bar{\psi}(m+\gamma_\mu\partial_\mu)\psi\ , \tag{2.1}$$

where

$$\mathbf{f}_{ij}=\partial_j\mathbf{b}_i-\partial_i\mathbf{b}_j-2e\,\mathbf{b}_i\times\mathbf{b}_j\ , \tag{2.1a}$$

$$\mathbf{f}_{i0}=\partial_0\mathbf{b}_i+\partial_i\mathbf{b}_0+2e\,\mathbf{b}_i\times\mathbf{b}_0\ , \tag{2.1b}$$

and

$$\mathbf{j}_\mu=ie\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi \tag{2.1c}$$

with $\mathbf{j}_4=i\,\mathbf{j}_0$. We include $\mathbf{b}_0$ components of $\mathbf{f}_{i0}$ and $\mathbf{j}_0\cdot\mathbf{b}_0$ in the Lagrangian, and will later impose the temporal-gauge condition by using the gauge-fixing term $-\partial_0\mathbf{b}_0\cdot\mathbf{G}$. This technique allows us to formulate the temporal gauge with sufficient generality to encompass most of the gluon propagators that have been used in temporal-gauge calculations. The conjugate momenta are

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 b_i^r)} = f_{i0}^r = \Pi_i^r \;, \tag{2.2a}$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 b_0^r)} = -G^r = \Pi_0^r \;, \tag{2.2b}$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_0\psi)} = i\psi^\dagger = \Pi_\psi \;. \tag{2.2c}$$

Every field component has a corresponding conjugate momentum and the equal-time commutation rules are

$$[b_i^p(\mathbf{x}),\Pi_j^q(\mathbf{y})] = i\delta_{ij}\delta_{p,q}\delta(\mathbf{x}-\mathbf{y}) \;, \tag{2.3a}$$

$$[b_0^p(\mathbf{x}),G^q(\mathbf{y})] = -i\delta_{p,q}\delta(\mathbf{x}-\mathbf{y}) \;, \tag{2.3b}$$

and

$$\{\psi(\mathbf{x}),\psi^\dagger(\mathbf{y})\} = \delta(\mathbf{x}-\mathbf{y}) \;. \tag{2.3c}$$

The Euler-Lagrange equations are

$$\partial_0\mathbf{\Pi}_i - (\partial_j + 2e\mathbf{b}_j\times)\mathbf{f}_{ij} - 2e\mathbf{b}_0\times\mathbf{\Pi}_i - \mathbf{j}_i = 0 \;, \tag{2.4}$$

$$-\partial_0\mathbf{G} + (\partial_i + 2e\mathbf{b}_i\times)\mathbf{\Pi}_i + \mathbf{j}_0 = 0 \;, \tag{2.5}$$

$$\partial_0\mathbf{b}_0 = 0 \;, \tag{2.6}$$

and

$$[\beta m - i\alpha_j(\partial_j - ie\mathbf{b}_j\cdot\boldsymbol{\tau}) - i(\partial_0 + ie\mathbf{b}_0\cdot\boldsymbol{\tau})]\psi = 0 \;. \tag{2.7}$$

Equation (2.7) leads to

$$(\partial_0 - 2e\mathbf{b}_0\times)\mathbf{j}_0 + (\partial_l + 2e\mathbf{b}_l\times)\mathbf{j}_l = 0 \;. \tag{2.7a}$$

To find the equation that allows us to impose Gauss's law as a constraint we take the spatial divergence of Eq. (2.4) and use the lemma

$$\partial_i(\mathbf{b}_j\times\mathbf{f}_{ij}) = -\mathbf{b}_i\times[(\partial_j + 2e\mathbf{b}_j\times)\mathbf{f}_{ij}] \tag{2.8}$$

along with Eqs. (2.4) and (2.7a) to obtain

$$(\partial_0 - 2e\mathbf{b}_0\times)[(\partial_i + 2e\mathbf{b}_i\times)\mathbf{\Pi}_i + \mathbf{j}_0] = 0 \;. \tag{2.9}$$

Equations (2.5) and (2.6) then lead to

$$\partial_0(\partial_0 - 2e\mathbf{b}_0\times)\mathbf{G} = 0 \;. \tag{2.10}$$

We can rewrite Eq. (2.10) in the form

$$\partial_0(\partial_i\mathbf{\Pi}_i + \mathbf{j}_0 + 2e\mathbf{b}_i\times\mathbf{\Pi}_i - 2e\mathbf{b}_0\times\mathbf{G}) = 0 \;. \tag{2.11}$$

The Euler-Lagrange equations can be used to show that $\mathbf{J}_i$ and $\mathbf{J}_0$, given by

$$\mathbf{J}_0 = \mathbf{j}_0 + 2e\mathbf{b}_i\times\mathbf{\Pi}_i - 2e\mathbf{b}_0\times\mathbf{G} \tag{2.12a}$$

and

$$\mathbf{J}_i = \mathbf{j}_i + 2e\mathbf{b}_j\times\mathbf{f}_{ij} + 2e\mathbf{b}_0\times\mathbf{\Pi}_i \;, \tag{2.12b}$$

respectively, form a conserved current, for which

$$\partial_0\mathbf{J}_0 + \partial_i\mathbf{J}_i = 0 \;. \tag{2.13}$$

Equation (2.11) can therefore be expressed as

$$\partial_0(\partial_i\mathbf{\Pi}_i + \mathbf{J}_0) = 0 \;. \tag{2.14}$$

Equations (2.6) and (2.14) fall short of constituting the gauge-fixing condition or Gauss's law. They do however provide a basis for imposing these constraints. If we arrange conditions so that Gauss's law, and the gauge condition $\mathbf{b}_0 = 0$ hold at any one time, Eqs. (2.6) and (2.14) guarantee that both of those conditions will hold at all other times too. In that way the situation in Yang-Mills theory and in QCD exactly parallels circumstances in QED in the temporal gauge.

An important feature of this formulation is the use of gluon ghost modes in the representation of $\mathbf{b}_0$ and $\mathbf{G}$ and in the longitudinal components of the gauge fields and their adjoint momenta, $\mathbf{b}^L$ and $\mathbf{\Pi}^L$, respectively. The relevant momentum-space operators are the ghost annihilation operators $a_Q^p(\mathbf{k})$ and $a_R^p(\mathbf{k})$ and the creation operators that are their respective adjoints in an indefinite-metric space, $a_Q^{p*}(\mathbf{k})$ and $a_R^{p*}(\mathbf{k})$. A Fock space of ghost states consists of the vacuum state $|0\rangle$ for which $a_Q^p(\mathbf{k})|0\rangle = 0$ and $a_R^p(\mathbf{k})|0\rangle = 0$, and the set of n-particle states for which $n_Q a_Q^*$ operators and $n_R a_R^*$ operators act on $|0\rangle$ (with $n_Q + n_R = n$). The commutation rules for these operators are

$$[a_Q^p(\mathbf{k}),a_R^{q*}(\mathbf{k}')] = [a_R^p(\mathbf{k}),a_Q^{q*}(\mathbf{k}')] = \delta_{p,q}\delta_{\mathbf{k},\mathbf{k}'} \tag{2.15}$$

and all other combinations commute. The properties of this Hilbert space have been discussed previously in the case of Abelian[1] as well as non-Abelian theories.[3] The representations of the gauge fields are

$$(b_i^p)^T = \sum_{\mathbf{k},s=1,2} \frac{\epsilon_i^s(\mathbf{k})}{(2k)^{1/2}}[a_s^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + a_s^{p\dagger}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}] \tag{2.16a}$$

and

$$(\Pi_i^p)^T = \sum_{\mathbf{k},s=1,2} -i\epsilon_i^s(\mathbf{k})\left[\frac{k}{2}\right]^{1/2}[a_s^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - a_s^{p\dagger}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.16b}$$

for the transversely polarized components of the gluon field,

$$(b_i^p)^L = \sum_{\mathbf{k}} \frac{k_i}{2k^{3/2}}\{[a_R^p(\mathbf{k}) + \gamma a_Q^p(\mathbf{k})]e^{i\mathbf{k}\cdot\mathbf{x}} + [a_R^{p*}(\mathbf{k}) + \gamma a_Q^{p*}(\mathbf{k})]e^{-i\mathbf{k}\cdot\mathbf{x}}\} \tag{2.17a}$$

and

$$(\Pi_i^p)^L = \sum_{\mathbf{k}} \frac{-ik_i}{(k)^{1/2}}[a_Q^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - a_Q^{p*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.17b}$$

for the longitudinal components, and

$$b_0^p = \sum_{\mathbf{k}} \frac{-i\alpha}{(k)^{1/2}}[a_Q^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - a_Q^{p*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.18a}$$

and

$$G^p = \sum_{\mathbf{k}} \frac{(k)^{1/2}}{2\alpha}\{[a_R^p(k) - \gamma a_Q^p(\mathbf{k})]e^{i\mathbf{k}\cdot\mathbf{x}} + [a_R^{p*}(\mathbf{k}) - \gamma a_Q^{p*}(\mathbf{k})]e^{-i\mathbf{k}\cdot\mathbf{x}}\} \;, \tag{2.18b}$$

where $G = -\Pi_0$.

The representations of $\mathbf{b}^L$, $\mathbf{b}_0$, and $\mathbf{G}$ in terms of ghost excitation operators constitutes an implicit choice of gauge appropriate for implementing the applicable constraints in the temporal gauge. α and γ are parameters useful for obtaining gluon propagators of sufficient generality to encompass the forms actually used in perturbative calculations. The commutation rules given in Eqs. (2.3a)–(2.3c) and the equations of motion are independent of α and γ.

The Hamiltonian H is given by $H=\int \mathcal{H}\, d\mathbf{x}$ where $\mathcal{H}$ is the Hamiltonian density

$$\mathcal{H}=\mathbf{\Pi}_i\cdot\partial_0\mathbf{b}_i-\mathbf{G}\cdot\partial_0\mathbf{b}_0+i\psi^\dagger\partial_0\psi-\mathcal{L}\ . \tag{2.19}$$

Since each field component has a canonically conjugate momentum and each field and momentum obey canonical commutation (or anticommutation) rules, there are no primary constraints in this formulation.[6] Use of Eq. (2.1) in (2.19) leads straightforwardly to

$$\mathcal{H}=\tfrac{1}{2}\mathbf{\Pi}_i\cdot\mathbf{\Pi}_i+\tfrac{1}{4}\mathbf{f}_{ij}\cdot\mathbf{f}_{ij}+\mathbf{b}_0\cdot\partial_i\mathbf{\Pi}_i+\psi^\dagger(\beta m-i\alpha_j\partial_j)\psi$$
$$+2e\mathbf{b}_0\cdot(\mathbf{b}_i\times\mathbf{\Pi}_i)-\mathbf{j}_i\cdot\mathbf{b}_i+\mathbf{j}_0\cdot\mathbf{b}_0 \tag{2.20}$$

after $-\partial_i\mathbf{b}_0\cdot\mathbf{\Pi}_i$ has been integrated by parts to anticipate the freedom to drop a surface term on a spacelike surface when H is evaluated. All the operator products that appear in $\mathcal{H}$ involve commuting operators except for the $\psi^\dagger(\beta m-i\alpha_j\partial_j)\psi$ which requires the same operator ordering as does the identical term in QED. The Hamiltonian H properly implements the time-displacement operation. The commutation rules in Eqs. (2.3a)–(2.3c), together with the commutator $i[H,\xi]$ used as an explicit representation of the time derivative $\partial_0\xi$, reproduces Eq. (2.2a) and the Euler-Lagrange equations (2.4)–(2.7).

To impose Gauss's law and the gauge condition we define the operators

$$\Omega^p(\mathbf{k})=a_Q^p(\mathbf{k})+J_0^p(\mathbf{k})/(2k^{3/2}) \tag{2.21a}$$

and

$$\Omega^{p*}(\mathbf{k})=a_Q^{p*}(\mathbf{k})+J_0^p(-\mathbf{k})/(2k^{3/2})\ . \tag{2.21b}$$

We note that we can express $\mathbf{b}_0$ and $\partial_i\mathbf{\Pi}_i+\mathbf{J}_0$ in the form

$$b_0^p=-i\alpha\sum_{\mathbf{k}}(k)^{-1/2}[\Omega^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-\Omega^{p*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.22}$$

and

$$\partial_i\Pi_i^p+J_0^p=\sum_{\mathbf{k}}k^{3/2}[\Omega^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\Omega^{p*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\ . \tag{2.23}$$

With these representation we can define a subspace $\{\,|\nu\rangle\}$ of an indefinite-metric space with the condition

$$\Omega^p(\mathbf{k})\,|\nu\rangle=0 \tag{2.24}$$

for all $|\nu\rangle$ in $\{\,|\nu\rangle\}$. In that subspace

$$\langle\nu'\,|\,\mathbf{b}_0\,|\,\nu\rangle=0 \tag{2.25a}$$

and

$$\langle\nu'\,|\,(\partial_i\mathbf{\Pi}_i+\mathbf{J}_0)\,|\,\nu\rangle=0\ . \tag{2.25b}$$

The identities $\partial_0\mathbf{b}_0=0$ and $\partial_0(\partial_i\mathbf{\Pi}_i+\mathbf{J}_0)=0$ then preserve Eqs. (2.25a) and (2.25b) for all time, even though the constraint represented by Eq. (2.24) has been imposed at one time only. Independent verification of this fact is obtained from $[H,\Omega^p(\mathbf{k})]=0$, which follows from a direct calculation using Eq. (2.20). $\Omega^p(\mathbf{k})\,|\,\nu\rangle=0$ then implies that $\Omega^p(\mathbf{k})\exp(-iHt)\,|\,\nu\rangle=0$ and the constraint, imposed at one time holds at all other times too.

In order to evaluate the propagators for the gluon field in the temporal gauge we establish an interaction picture by separating H into H_0, which is the $e\to 0$ limit of H, and H_1 such that $H=H_0+H_1$. We find that H_0 is given by

$$H_0=\int d\mathbf{x}[\tfrac{1}{2}\mathbf{\Pi}_i\cdot\mathbf{\Pi}_i+\tfrac{1}{4}(\partial_j\mathbf{b}_i-\partial_i\mathbf{b}_j)\cdot(\partial_j\mathbf{b}_i-\partial_i\mathbf{b}_j)+\mathbf{b}_0\cdot\partial_i\mathbf{\Pi}_i+\psi^\dagger(\beta m-i\alpha_j\partial_j)\psi] \tag{2.26}$$

and H_1 by

$$H_1=\int d\mathbf{x}[2e(\mathbf{b}_i\times\mathbf{b}_j)\cdot\partial_i\mathbf{b}_j+e^2(\mathbf{b}_i\times\mathbf{b}_j)\cdot(\mathbf{b}_i\times\mathbf{b}_j)+2e\mathbf{b}_0\cdot(\mathbf{b}_i\times\mathbf{\Pi}_i)-\mathbf{j}_i\cdot\mathbf{b}_i+\mathbf{j}_0\cdot\mathbf{b}_0]\ . \tag{2.27}$$

H_0 can be expressed in terms of momentum-space operators and then has the form

$$H_0=\sum_{\mathbf{k},p}|\mathbf{k}|\left[\sum_{s=1,2}a_s^{p\dagger}(\mathbf{k})a_s^p(\mathbf{k})+a_Q^{p*}(\mathbf{k})a_Q^p(\mathbf{k})+\tfrac{1}{2}(1-2i\alpha)a_Q^p(\mathbf{k})a_Q^p(-\mathbf{k})+\tfrac{1}{2}(1+2i\alpha)a_Q^{p*}(\mathbf{k})a_Q^{p*}(-\mathbf{k})\right]$$
$$+\sum_{\mathbf{k},n}(m^2+|\mathbf{k}|^2)^{1/2}[q_n^\dagger(\mathbf{k})q_n(\mathbf{k})+\bar{q}_n^{\,\dagger}(\mathbf{k})\bar{q}_n(\mathbf{k})]\ , \tag{2.26a}$$

where q_n, $q_n^\dagger$, $\bar{q}_n$, and $\bar{q}_n^{\,\dagger}$ designate quark and antiquark operators with spin and isospin indices lumped into n. The longitudinal $[b_i^p(\mathbf{x},t)]^L$ in the interaction picture has the form

$$[b_i^p(\mathbf{x},t)]^L=\exp(iH_0t)[b_i^p(\mathbf{x})]^L\exp(-iH_0t) \tag{2.28}$$

which can be written as

$$[b_i^p(\mathbf{x},t)]^L=[b_i^p(\mathbf{x})]^L+it[H_0,[b_i^p(\mathbf{x})]^L]\ . \tag{2.28a}$$

Because of the commutation rules of the ghost operators $[H_0,[b_i^p(\mathbf{x})]^L]$ commutes with H_0 so that the series in Eq. (2.28a) is complete. The result that the longitudinal $\mathbf{b}_i^L$ in the interaction picture, is a linear function of the time coordinate is closely linked to the use of ghost operators in representing $\mathbf{b}_i^L$.

The explicit form of $[b_i^p(\mathbf{x},t)]^L$ is

$$[b_i^p(\mathbf{x},t)]^L=\sum_{\mathbf{k}}\frac{k_i}{2k^{3/2}}(\{a_R^p(\mathbf{k})+a_Q^p(\mathbf{k})[\gamma-2itk(1-i\alpha)]\}e^{i\mathbf{k}\cdot\mathbf{x}}+\{a_R^{p*}(\mathbf{k})+a_Q^{p*}(\mathbf{k})[\gamma+2itk(1+i\alpha)]\}e^{-i\mathbf{k}\cdot\mathbf{x}}) . \tag{2.28b}$$

The propagator for the longitudinal gluon field is the vacuum expectation value of the time-ordered product $T\{[b_i^p(\mathbf{x}_1,t_1)]^L[b_j^q(\mathbf{x}_2,t_2)]^L\}$ in the vacuum state $|0\rangle$ for which $a_Q^p(\mathbf{k})|0\rangle=a_R^p(\mathbf{k})|0\rangle=0$. $|0\rangle$ is the lowest-energy eigenstate of H_0. It is straightforward to use Eq. (2.28b) to evaluate the vacuum expectation value

$$\langle 0|T\{[b_i^p(\mathbf{x}_1,t_1)]^L[b_j^q(\mathbf{x}_2,t_2)]^L\}|0\rangle=\delta_{p,q}D_{ij}^L(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2) \tag{2.29a}$$

to find that

$$D_{ij}^L(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)=-\left[\frac{i}{2}|t_1-t_2|+\frac{\alpha}{2}(t_1+t_2)\right]\frac{\partial_i\partial_j}{\nabla^2}\delta(\mathbf{x}_1-\mathbf{x}_2)+i\gamma\frac{\partial_i\partial_j}{\nabla^2}\Delta(\mathbf{x}_2-\mathbf{x}_1) , \tag{2.29b}$$

where

$$\Delta(\mathbf{x})=-i(2\pi)^{-3}\int d\mathbf{k}(2k)^{-1}\exp(i\mathbf{k}\cdot\mathbf{x}) .$$

In addition to $\delta_{p,q}D_{ij}^L(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)$, there is another nonvanishing propagator in this gauge, consisting of the time-ordered products of A_0 and A^L. We find that

$$\langle 0|T\{[b_i^p(\mathbf{x}_1,t_1)]^L b_0^q(\mathbf{x}_2,t_2)\}|0\rangle =\delta_{p,q}\left[\frac{\alpha}{2}\right]\frac{\partial_i}{\nabla^2}\delta_3(\mathbf{x}_1-\mathbf{x}_2) . \tag{2.30}$$

III. DISCUSSION

In this section we will discuss questions on which the material presented in earlier parts of this paper has bearing. One such question deals with the implications of our work to some unresolved issues about the part of the propagator for longitudinal gluons that is not time-translationally invariant. Another question deals with the relation of the scattering amplitude for the particle states $|\nu\rangle$ that obey Eq. (2.24) to the perturbative S matrix evaluated with the propagators for the temporal gauge. The latter of these two questions is of central importance because only the states that obey Gauss's law have a legitimate claim to be particle states for QCD. The theory available for the temporal gauge is, in some sense, not Yang-Mills theory (or QCD) at all unless we take the important step of implementing Gauss's law at one point in time. We have every reason therefore to be extremely cautious about using Feynman rules, since they include propagators that are based on a Fock space whose states violate Gauss's law. Our incomplete understanding of confinement in QCD and the limitations of perturbation theory in an asymptotically free theory are of course generally recognized. Here we want to suggest that the failure of the vacuum state used to calculate vacuum expectation values for Feynman rules to obey Gauss's law may play an important role in these aspects of non-Abelian gauge theories. Not much concern has been expressed about the use of such a Fock space in evaluating propagators in QCD because the substitution of just such a Fock space for the coherent states that obey Gauss's law is as much a feature of QED as of QCD. And in QED we have extensive experience with the fact that this substitution is entirely harmless and does not threaten the validity of the Feynman rules.[1,7,8] However the use of such a substitute Fock space has a much more precarious basis in QCD than in QED. In QED there is sound theoretical support for this practice. In that case the states $|\nu\rangle$ that obey Gauss's law and the Fock states $|n\rangle$ used in deriving Feynman rules are related by a unitary transformation $|n\rangle=U|\nu\rangle$ with $U^*=U^{-1}$. When such a unitary equivalence obtains between $|\nu\rangle$ and $|n\rangle$ the latter may be substituted for the former in representations of incident and scattered particle states without affecting S-matrix elements provided U satisfies some fairly general requirements, which it does in QED. This accounts for the fact that in QED the S matrix is not harmed by the practice of representing asymptotic charged particle states as solutions of a free field equation (for example, the Dirac equation), and in the process amputating the electric field that must accompany charged particles to preserve consistency with Gauss's law.

In QCD the situation is somewhat similar, but with some crucial differences. We observe that in QCD, as well as in QED, in implementing Gauss's law we take the operator $\partial_i\Pi_i+J_0$ and divide it into two non-Hermitian parts, that are each others Hermitian adjoints: namely,

$$\Omega(\mathbf{x})=\sum_{\mathbf{k}}k^{3/2}\Omega(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{x})$$

and

$$\Omega^*(\mathbf{x})=\sum_{\mathbf{k}}k^{3/2}\Omega^*(\mathbf{k})\exp(-i\mathbf{k}\cdot\mathbf{x}) .$$

The identical procedure is applied in QED and QCD with the exceptions that $\Omega(\mathbf{x})$ carries a Lie group index in QCD but not in QED, and that the charge density in QED is independent of photon fields while the gluons carry color and contribute to $\mathbf{J}_0$ in QCD. In the temporal gauge $\Omega(\mathbf{x})$ and $\Omega^*(\mathbf{x})$ are time independent in QED as well as in QCD; but $\Omega(\mathbf{x})$ and $\Omega^*(\mathbf{x})$ commute in QED only. The more general situation in gauge theories is that the operators $\Omega(\mathbf{x})$ and $\Omega^*(\mathbf{x})$ reflect the commutation rules imposed on $\partial_i\Pi_i+J_0$ by the Lie algebras of U(1) in QED, of SU(2) in Yang-Mills theory, and of SU(3) in QCD. In the case of Yang-Mills theory we find the following identities among commutators:

$$[\Omega^i(\mathbf{x}),\Omega^j(\mathbf{y})] =\frac{ie}{2}\epsilon_{ijp}[\partial_n\Pi_n^p(\tfrac{1}{2}(\mathbf{x}-\mathbf{y}))+J_0^p(\tfrac{1}{2}(\mathbf{x}-\mathbf{y}))]\delta(\mathbf{x}+\mathbf{y}) \tag{3.1}$$

and

$$[\Omega^i(\mathbf{x}),\Omega^{j*}(\mathbf{y})] =\frac{ie}{2}\epsilon_{ijp}[\partial_n\Pi_n^p(\tfrac{1}{2}(\mathbf{x}+\mathbf{y}))+J_0^p(\tfrac{1}{2}(\mathbf{x}+\mathbf{y}))]\delta(\mathbf{x}-\mathbf{y})\ . \tag{3.2}$$

We see that the commutation rule for $\Omega(\mathbf{x})$ and $\Omega^*(\mathbf{y})$ in QED, $[\Omega(\mathbf{x}),\Omega^*(\mathbf{y})]=0$, is included in that rule as a degenerate case since all structure constants for U(1) vanish.

It is nontrivial that the non-Hermitian $\Omega(\mathbf{x})$ and $\Omega^*(\mathbf{y})$ reflect the commutation rules of the Hermitian $\partial_l\Pi_l+J_0$ in QED and QCD. In general it is not possible to divide a Hermitian field of the form $\sum_\mathbf{k} c(\mathbf{k})[\alpha(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\alpha^\dagger(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}]+\psi$, where $\alpha(\mathbf{k})$ and $\alpha^\dagger(\mathbf{k})$ are annihilation and creation operators, respectively, and $c(\mathbf{k})$ is a c number, into non-Hermitian parts that commute, because $\alpha(\mathbf{k})$ and $\alpha^\dagger(\mathbf{k})$ cannot commute in a positive-metric Hilbert space. It is only because the $a_Q(\mathbf{k})$ in $\Omega(\mathbf{k})$ and the $a_Q^*(\mathbf{k})$ in $\Omega^*(\mathbf{k})$ are ghost operators that they commute with each other and that the commutation rules $[\Omega(\mathbf{k}),\Omega^*(\mathbf{k}')]=0$ for QED and those of Eqs. (3.1) and (3.2) for Yang-Mills theory are possible.

In contrast with Eqs. (3.1) and (3.2) the non-Abelian operators $a_Q^i(\mathbf{k})$ and $a_Q^{i*}(\mathbf{k})$, which are the limiting forms of $\Omega^i(\mathbf{k})$ and $\Omega^{i*}(\mathbf{k})$, respectively, as $e\to 0$, do not obey the commutator algebras of SU(2) for Yang-Mills theory, or of SU(3) for QCD. As was discussed in Sec. II,

$$[a_Q^i(\mathbf{k}),a_Q^j(\mathbf{k}')]=[a_Q^i(\mathbf{k}),a_Q^{j*}(\mathbf{k}')]=0$$

and the corresponding $\sum_\mathbf{k} k^{3/2}a_Q^i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$ and $\sum_\mathbf{k} k^{3/2}a_Q^{j*}(\mathbf{k})e^{-i\mathbf{k}'\cdot\mathbf{x}}$ also commute. The conditions $a_Q^i(\mathbf{k})\,|\,n\,\rangle=0$ and $(H_0-E_n)\,|\,n\,\rangle=0$ define and select a Fock space whose elements represent multiparticle states of wholly noninteracting quarks and gluons, that serve as incident and scattered states in perturbative calculations. These noninteracting multiparticle states, and the vacuum state on which they are built, are sometimes explicitly referred to in deriving propagators, as in this work; or their use is implicit as when path-integral techniques are applied. But whether their use is explicit or implicit, it is these Fock states whose scattering amplitudes are perturbatively evaluated when Feynman rules are used. The fact that the $\Omega^i(\mathbf{k})$ and their corresponding $e\to 0$ limits, $a_Q^i(k)$, do not obey identical algebras in Yang-Mills theory and in QCD implies that they cannot be unitarily equivalent to each other, and that the coherent states $|\nu\rangle$ that obey Eq. (2.24) and the members of the Fock space $\{\,|\,n\,\rangle\}$ also cannot be related by a unitary transformation. In contrast the $|\,\nu\,\rangle$ and $|\,n\,\rangle$ states are unitarily equivalent in QED because in that case, and in that case only, all structure constants vanish so that $\Omega(\mathbf{k})$ and $a_Q(\mathbf{k})$ obey the same degenerate Lie algebra; thus we observe that, in the Abelian QED, the commutation rules $[\Omega(\mathbf{k}),\Omega(\mathbf{k}')]=0$ and $[\Omega(\mathbf{k}),\Omega^*(\mathbf{k}')]=0$ are exactly the same as the corresponding $[a_Q(\mathbf{k}),a_Q(\mathbf{k}')]=0$ and $[a_Q(\mathbf{k}),a_Q^*(\mathbf{k}')]=0$, respectively. As was previously shown, the unitary equivalence between $|\,\nu\rangle$ and $|\,n\,\rangle$ states in QED permits us to make the more or less *ad hoc* substitution of $|\,n\,\rangle$ states for $|\,\nu\rangle$ states in perturbative calculations even though the $|\,n\,\rangle$ states do not obey Gauss's law. The fact that such a unitary equivalence between $|\,\nu\rangle$ and $|\,n\,\rangle$ states does not obtain in Yang-Mills theory or in QCD has a significant implication for perturbative QCD: The perturbative S matrix for QCD, evaluated with Feynman rules, may significantly misrepresent the correct scattering amplitudes for the theory in which Gauss's law is implemented.

One possible interpretation of the phenomenon that we call "confinement" is the following: When two hadrons (for example, $p-\bar{p}$) collide, only color-singlet combinations of quarks and gluons emerge from the interaction region as observable scattered particles. But when the two colliding hadrons are treated as combinations of "free" quarks that are eigenstates of H_0, the perturbative Feynman rules predict color-bearing final scattered states, such as free quarks and gluons. The speculation, to which our work naturally leads, is that if we were to use the states $|\,\nu\rangle$, that implement Gauss's law and the gauge-fixing constraint, that discrepancy might be avoided.

The preceding considerations make it very pertinent and desirable to identify as many of the properties of the $|\,\nu\rangle$ states as possible. From

$$[\Omega^i(\mathbf{k}),\Omega^j(\mathbf{k}')] =\frac{ie}{2}\epsilon_{ijp}\frac{|\,\mathbf{k}+\mathbf{k}'\,|^{3/2}}{k^{3/2}k'^{3/2}}[\Omega^p(\mathbf{k}+\mathbf{k}')+\Omega^{p*}(-(\mathbf{k}+\mathbf{k}'))] \tag{3.3}$$

we can infer that the set of states that obey $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$ for all i and $\mathbf{k}$ also obey $[\Omega^i(\mathbf{k}),\Omega^j(\mathbf{k}')]\,|\,\nu\rangle=0$ and therefore also obey $\Omega^{i*}(\mathbf{k})\,|\,\nu\rangle=0$ as well as $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$. This leads to the interesting conclusion that, because of the Lie algebras of Yang-Mills theory and QCD, the states $|\,\nu\rangle$ for which Gauss's law as well as the gauge-fixing condition are implemented obey $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$, $\langle\nu\,|\,\Omega^i(\mathbf{k})=0$, $\Omega^{i*}(\mathbf{k})\,|\,\nu\rangle=0$, and $\langle\nu\,|\,\Omega^{i*}(\mathbf{k})=0$. These conditions impose very severe constraints on the states in $\{\,|\,\nu\rangle\}$. One such constraint is that the states $|\,\nu\rangle$ cannot be normalizable. From $\langle\nu\,|\,\Omega^i(\mathbf{k})=0$ and $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$ we can infer that for any operator ξ, $\langle\nu\,|\,[\Omega^i(\mathbf{k}),\xi]\,|\,\nu\rangle=0$. Since there are numerous choices for ξ for which $[\Omega^i(\mathbf{k}),\xi]=\chi$ so that $\langle\nu\,|\,\chi\,|\,\nu\rangle\neq 0$, $|\,\nu\rangle$ cannot be normalizable. It is worth noting that the simultaneous validity of $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$ and $\Omega^{i*}(\mathbf{k})\,|\,\nu\rangle=0$ is more constraining on the state $|\,\nu\rangle$ than is the condition

$$(\partial_i\Pi_i^p+J_0^p)\,|\,\Phi\rangle=0 \tag{3.4}$$

which has been proposed in other work.[9] Equation (3.4) can also be expressed as

$$\sum_{\mathbf{k}} k^{3/2}[\Omega^p(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\Omega^{p*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\,|\,\Phi\rangle=0\ . \tag{3.4a}$$

Equation (3.4a) is a consequence of the two separate conditions $\Omega^p(\mathbf{k})\,|\,\Phi\rangle=0$ and $\Omega^{p*}(\mathbf{k})\,|\,\Phi\rangle=0$ but it does not, in turn, imply them both. Equation (3.4) is often used in a formulation of the temporal gauge in which $\mathbf{b}_0$ and all space-time derivatives of $\mathbf{b}_0$ are set $=0$ in the Lagrangian and $\mathbf{G}$ never appears. The Hamiltonian that follows from this procedure can be obtained by setting $\mathbf{b}_0=0$ in our Eq. (2.20). The propagator that follows from the resulting formalism is our (2.29b) with α and γ restricted to the values $\alpha=\gamma=0$. The option of choosing nonvanishing values of α and γ is therefore precluded by implementing $\mathbf{b}_0=0$ as an operator identity. It is also worth noting that this Hamiltonian [i.e., Eq. (2.20) with $\mathbf{b}_0=0$] implies $i[H,\mathbf{b}_0]=\partial_0\mathbf{b}_0=0$, but does not automatically constrain expectation values of $\mathbf{b}_0$ to vanish.

The condition that $\partial_i\mathbf{\Pi}_i+\mathbf{J}_0$ and $\mathbf{b}_0$ both vanish in some suitable defined subspace requires $\Omega^i(\mathbf{k})\,|\,\nu\rangle=0$ as well as $\Omega^{i*}(\mathbf{k})\,|\,\nu\rangle=0$ simultaneously.

In spite of the fact that the perturbative S matrix fails to account for the confinement mechanism, there is evidence that inclusive processes, interpreted as hadronized perturbative S-matrix elements give satisfactory agreement with high-energy collision processes. It is tempting to speculate that this very important feature of inclusive "hadronized" perturbative S-matrix theory can be understood in terms of a relation between the $|\nu\rangle$ states and the Fock states $|\,n\,\rangle$ that is a generalization of unitary equivalence, consistent with the fact that the $\Omega^i(\mathbf{k})$ and $\Omega^{i*}(\mathbf{k})$, and their $e\to 0$ limits, obey different commutator algebras.

The other question we want to address is whether all values of the parameter α in the propagator for longitudinal gluons are equally acceptable in perturbative calculations. There has been considerable discussion of this point in the literature.[10–16] Caracciolo, Curci, and Menotti (CCM) have claimed that the propagator can be used for perturbative calculations in QCD only when the parameter α in Eq. (2.29b) has the value $\alpha=\pm i$, but that all values of α are permissible in QED (Ref. 10). Other authors have argued that special values of α are necessary in QED as well as in QCD and we have discussed that point, as it applies to QED, in earlier work.[1] In a recent paper Landshoff offers evidence that a propagator that lacks time-translation invariance is not necessary even in QCD (Ref. 17)

We will argue here that the restriction to special values of α is neither necessary nor appropriate in perturbative S-matrix calculations.

We begin with the observation that there is a unitary transformation V that shifts the value of α in $\mathbf{b}_0$ and $\mathbf{G}$ and therefore also in the Hamiltonian H. The unitary

$$V=\exp\left[i\frac{\lambda}{2}\int d\mathbf{x}(\mathbf{b}_0\cdot\mathbf{G}+\mathbf{G}\cdot\mathbf{b}_0)\right] \tag{3.5}$$

used to transform $\xi'=V\xi V^{-1}$, where ξ designates any operator, leads to

$$\mathbf{b}_0'=e^{\lambda}\mathbf{b}_0 \tag{3.6a}$$

and

$$\mathbf{G}'=e^{-\lambda}\mathbf{G}\ . \tag{3.6b}$$

For all other fields, $\mathbf{b}_i,\mathbf{\Pi}_i,\psi,\psi^\dagger$, the primed transforms are identical to the original operators. If we choose the parameter $\lambda=\ln(\alpha_2/\alpha_1)$ then $\mathbf{b}_0'$ and $\mathbf{G}'$ correspond to $\mathbf{b}_0$ and $\mathbf{G}$, respectively, with the value of α shifted from α_1 to α_2. We now make use of the equation we discussed in earlier work on QED (Refs. 7 and 18):

$$\bar{T}_{f,i}=T_{f,i}+(E_f-E_i)\mathcal{T}_{f,i}^{(\alpha)}+i\epsilon\mathcal{T}_{f,i}^{(\beta)}\ , \tag{3.7}$$

where $\bar{T}_{f,i}$ and $T_{f,i}$ represent the transition amplitudes when H' and H, respectively, are the Hamiltonians, and $|\,i\,\rangle, |\,f\,\rangle$ are initial and final states that obey $(H_0-E_i)\,|\,i\,\rangle=0$ and $(H_0-E_f)\,|\,f\,\rangle=0$, and where $\mathcal{T}_{f,i}^{(\alpha)}$ and $\mathcal{T}_{f,i}^{(\beta)}$ are given by

$$\mathcal{T}_{f,i}^{(\alpha)}=\langle f\,|\,(1-V)[1+(E_i-H-i\epsilon)^{-1}H_1]\,|\,i\,\rangle \tag{3.7a}$$

and

$$\mathcal{T}_{f,i}^{(\beta)}=\langle f\,|\,[(V-1)(E_i-H+i\epsilon)^{-1}H_1 - H_1'(E_i-H'+i\epsilon)^{-1}(V-1)]\,|\,i\,\rangle\ . \tag{3.7b}$$

Since only ghost operators appear in V, and no ghost operators appear in either $|\,i\,\rangle$ or $|\,f\,\rangle$, $(E_f-E_i)\mathcal{T}_{f,i}^{(\alpha)}$ and $i\epsilon\mathcal{T}_{f,i}^{(\beta)}$ must vanish as $(E_f-E_i)\to 0$ and $i\epsilon\to 0$, respectively, except in graphs including self-energy radiative corrections to external lines. We conclude therefore that on-shell transition amplitudes (and therefore S-matrix elements) do not depend on the value of α in the propagator in Eq. (2.29b), except for graphs with self-energy insertion in external lines which are absorbed in the wave-function renormalization. An argument has been given by Bialynicki-Birula that the renormalized S matrix in QED is invariant to changes in the gauge parameters in the propagator even though the unrenormalized S matrix is not.[19] It is likely that this argument can readily be extended to this case and that any α dependence of the wave-function renormalization constants drops out of the renormalized S matrix. Of course specific calculations with these propagators can be very sensitive to regularization procedures and these can mask the basic α independence of the S matrix. Also it needs to be remembered that unlike QED this α-independent S matrix has not been shown to be the same as the S matrix in the formulation in which incident and scattered particles are represented by coherent superpositions that obey Gauss's law.

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