PHYSICAL REVIEW D VOLUME 36, NUMBER 6 15 SEPTEMBER 1987

Quantum electrodynamics in the temporal gauge

Kurt Haller
Department of Physics, University of Connecticut, Storrs, Connecticut 06268
(Received 11 August 1986)

A canonical formulation is given for quantum electrodynamics in the temporal gauge. A procedure is developed for implementing the gauge-fixing condition $A_0=0$ and Gauss's law as time-independent constraints. The resulting formalism is shown to be physically equivalent to quantum electrodynamics in the Coulomb and Lorentz gauges. It is demonstrated that photon ghosts are appropriate for the representation of the longitudinal vector potential. Propagators for the longitudinal vector potential are derived and compared to results obtained by other authors. Implications for quantum chromodynamics are discussed.

I. INTRODUCTION

Temporal gauge formulations of gauge theories have long attracted considerable attention. Most recently questions have been raised about the propagator for the longitudinal component of the gauge field A_μ.[1-8] At other times the implementation of the Gauss law constraint and the elimination of the redundant gauge degrees of freedom have been discussed.[4,9,10] In spite of this activity unresolved questions remain about the temporal gauge. One such question deals with accounting for the number of degrees of freedom in the gauge field. A naive counting argument indicates that fixing the gauge at $A_0=0$ eliminates one degree of freedom and leaves open the question of how to eliminate one additional degree of freedom so that only the two transverse modes that propagate energy and momentum remain as dynamical variables. An explicit canonical formulation, in which $A_0=0$ is implemented in the Lagrangian, and the two transverse as well as the longitudinal modes are quantized, illustrates this problem.[11] This work uses a unitary transformation to generate charged states that implement Gauss's law and are subject to the instantaneous Coulomb interaction. The positive-metric longitudinal degree of freedom appears in even the interaction-free Hamiltonian, arising in terms not diagonal in the photon number. These terms take a form in which they mimic the spontaneous generation or destruction of unphysical longitudinal photon pairs. Although devices are available to make such troublesome contributions proportional to a switching parameter which can go to zero, so that these unphysical terms can be attenuated and even be made to vanish, the theory has a singular and problematical limit in that case. To the best of this author's knowledge, the question of how to treat the unobservable degree of freedom, so that the time evolution of state vectors in the temporal gauge is in complete agreement with that in other gauges, has not been given a totally satisfactory answer, even in QED.

A related problem is that Gauss's law is not a consequence of the Euler-Lagrange equations in the temporal gauge, any more than in covariant gauges. It is therefore natural to suspect that the imposition of Gauss's law as a constraint is related to the elimination of the redundant mode left over after the gauge-fixing condition $A_0=0$ has been imposed. But it is still necessary to show how the Gauss law constraint eliminates the degree of freedom left after gauge fixing and, in the case of QED, that the resulting formulation is the properly quantized form of Maxwell's theory. This problem will be addressed and, we believe, resolved in this paper.

It will become apparent in the course of this work that QED in the temporal gauge and QED in covariant gauges have a great deal in common. In particular, in both cases a consistent formulation requires the use of photon ghost states. Some authors have expressed distaste for ghost states and have claimed that they are unphysical, unaesthetic, and responsible for unnecessary complications in the algebraic structure of the Hilbert space in which the gauge theory is embedded. We will argue here the advantages of ghost states in gauge theory. There are some familiar reasons for using ghost modes. It is unlikely that there is any gauge in QCD in which we can dispense with all ghost modes, the redundant gluon polarizations as well as the Faddeev-Popov ghosts. Also, it has been shown that ghosts are necessary for a consistent formulation of QED in covariant gauges.[12] Our contribution to this discussion is to point out that the ghost modes provide a physically realistic and very satisfactory representation for the quantized longitudinal electric field. The quanta of the longitudinal electric field carry neither energy nor momentum; they are not retarded relativistically and hence cannot carry any probability of being detected. Ghost modes provide a natural way of representing these properties. We will see that the use of ghost modes also permits us to give a very satisfactory account of the fate of the redundant degrees of freedom in the temporal gauges.

II. FORMULATION OF THE THEORY

We use the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}F_{ij}F_{ij}+\tfrac{1}{2}(\partial_0 A_i+\partial_i A_0)^2-\partial_0 A_0 G +j_i A_i-j_0 A_0-\bar{\psi}(m+\gamma\cdot\partial)\psi\ , \tag{2.1}$$

where $F_{ij}=\partial_j A_i-\partial_i A_j$ and $j_\mu=ie\bar{\psi}\gamma_\mu\psi$, to describe spinor electrodynamics in the temporal gauge. We could have set $A_0=0$ and $\partial_0 A_0=0$ identically without any harm, but including $-\partial_0 A_0 G$ and the A_0 terms in $\mathcal{L}$ leads to a generalized form of the temporal gauge that is useful in evaluating the photon propagator for cases in which the expectation value of A_0 vanishes in the appropriately defined Hilbert space, even when A_0 does not vanish identically. The choice of a charged spinor field to interact with the electromagnetic field is quite incidental to the main point. The gauge-fixing problem is independent of the nature of the charged particle, and all essential results of this work apply equally well to charged bosons in this as well as other gauges.

The conjugate momenta are given by

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 A_i)}=(\partial_i A_0+\partial_0 A_i)=\Pi_i \tag{2.2}$$

so that Π_i is the negative electric field,

$$\frac{\partial\mathcal{L}}{\partial(\partial_0 A_0)}=-G=\Pi_0 , \tag{2.3}$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_0\psi)}=i\psi^\dagger . \tag{2.4}$$

Every field component has a canonically conjugate momentum to serve as a partner in the equal-time commutation rules,

$$[A_i(\mathbf{x}),\Pi_j(\mathbf{y})]=i\delta_{ij}\delta(\mathbf{x}-\mathbf{y}) , \tag{2.5a}$$

$$[A_0(\mathbf{x}),G(\mathbf{y})]=-i\delta(\mathbf{x}-\mathbf{y}) , \tag{2.5b}$$

and the anticommutation rule

$$\{\psi(\mathbf{x}),\psi^\dagger(\mathbf{y})\}=\delta(\mathbf{x}-\mathbf{y}) . \tag{2.5c}$$

The Euler-Lagrange equations lead to

$$-\Box A_i+\partial_i(\partial_\mu A_\mu)=j_i , \tag{2.6a}$$

$$-\partial_0 G+\partial_i\Pi_i+j_0=0 , \tag{2.6b}$$

and, from $\partial\mathcal{L}/\partial G=0$,

$$\partial_0 A_0=0 . \tag{2.6c}$$

In addition we obtain the spinor equation

$$(m+\gamma\cdot D)\psi=0 , \tag{2.6d}$$

where $D_\mu=\partial_\mu-ieA_\mu$, and the current-conservation equation $\partial_\mu j_\mu=0$. Combining Eqs. (2.6a) and (2.6c) leads to

$$-\Box A_i+\partial_i(\nabla\cdot\mathbf{A})=j_i . \tag{2.7}$$

Equation (2.7) is one of the inhomogeneous Maxwell equations in which $\partial_i(\nabla\cdot\mathbf{A})-\nabla^2 A_i$ is the curl of the magnetic field and $\partial_0\partial_0 A_i$ is the displacement current in the temporal gauge. Taking the divergence of Eq. (2.7) leads to

$$\partial_0[\partial_0(\nabla\cdot A)+j_0]=0 \tag{2.8a}$$

which can be reexpressed as

$$\partial_0(\nabla\cdot\mathbf{E}-j_0)=0 , \tag{2.8b}$$

where $\mathbf{E}=-\mathbf{\Pi}$ is the electric field. The time integral of Eq. (2.8b), i.e., Gauss's law, is not a consequence of the Euler-Lagrange equations in this gauge.

The Hamiltonian density $\mathcal{H}$ is given by

$$\mathcal{H}=\Pi_i\partial_0 A_i+\Pi_0\partial_0 A_0+i\psi^\dagger\partial_0\psi-\mathcal{L} \tag{2.9}$$

and the Hamiltonian by $H=\int\mathcal{H}\,d\mathbf{x}$. We find that $\mathcal{H}$ is given by

$$\mathcal{H}=\tfrac{1}{2}\Pi_i\Pi_i+\tfrac{1}{4}F_{ij}F_{ij}+A_0\partial_i\Pi_i-j_iA_i+j_0A_0 \\ +\psi^\dagger(\beta m-i\boldsymbol{\alpha}\cdot\nabla)\psi \tag{2.10}$$

after integrating $-\Pi_i\partial_i A_0$ by parts to anticipate the freedom to drop a surface term on a spacelike surface when H is evaluated. Next we choose a momentum-space representation of the field variables that is consistent with the equal-time commutation rules. For the transverse parts of $\mathbf{A}$ and $\mathbf{\Pi}$ we use the standard representation

$$\mathbf{A}^T=\sum_{\mathbf{k},i=1,2}\frac{\hat{\boldsymbol{\epsilon}}_i(\mathbf{k})}{(2k)^{1/2}}[a_i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_i^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.11a}$$

and

$$\mathbf{\Pi}^T=-i\sum_{\mathbf{k},i=1,2}\hat{\boldsymbol{\epsilon}}_i(\mathbf{k})\left[\frac{k}{2}\right]^{1/2}[a_i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_i^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.11b}$$

and for the spinor field

$$\psi(\mathbf{x})=\sum_{\mathbf{k},s=1,2}[e_s(\mathbf{k})u_s(k)e^{i\mathbf{k}\cdot\mathbf{x}}+\bar{e}_s^{\,\dagger}(\mathbf{k})v_s(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] . \tag{2.11c}$$

We represent the longitudinal and timelike parts of A_μ, and their canonically conjugate momenta, in terms of the ghost excitations $a_Q(\mathbf{k})$ and $a_R(\mathbf{k})$ and their adjoints in an indefinite-metric space, $a_Q^*(\mathbf{k})$ and $a_R^*(\mathbf{k})$, respectively. We have used these operators previously for QED (Ref. 13) as well as for QCD (Ref. 14). $a_Q(\mathbf{k})$ is the ghost annihilation operator $(\sqrt{2}k)^{-1}[k_\mu a_\mu(\mathbf{k})]$; $a_R(\mathbf{k})$ is the ghost annihilation operator $(\sqrt{2}k)^{-1}[\bar{k}_\mu a_\mu(\mathbf{k})]$ where $\bar{k}_\mu$ differs from k_μ in having $(-k_0)$ instead of k_0 as its timelike component. The algebra of the ghost sector requires a small generalization of the algebra for positive-metric Hilbert spaces. $a_Q(\mathbf{k})$ and its adjoint $a_Q^*(\mathbf{k})$ commute, as do $a_R(\mathbf{k})$ and $a_R^*(\mathbf{k})$. In contrast,

$$[a_Q(\mathbf{k}),a_R^*(\mathbf{k}')]=[a_R(\mathbf{k}),a_Q^*(\mathbf{k}')]=\delta_{\mathbf{k},\mathbf{k}'} .$$

The unit operator in the one-particle ghost-photon sector is

$$\mathbb{1}=\sum_{\mathbf{k}}[a_Q^*(\mathbf{k})\,|\,0\rangle\langle 0\,|\,a_R(\mathbf{k})+a_R^*(\mathbf{k})\,|\,0\rangle\langle 0\,|\,a_Q(\mathbf{k})] \tag{2.12}$$

and, in the n-particle ghost sectors, the obvious generalizations of Eq. (2.12) apply. The norm of any state $a_Q^*(\mathbf{k})\,|\,p\,\rangle$, where $|\,p\,\rangle$ is any normalizable ghost-free state, vanishes because the commutation rules imply that $\langle p \mid a_Q(\mathbf{k}) a_Q^*(\mathbf{k}) \mid p \rangle = 0$. The inner product of $a_Q^*(\mathbf{k}) \mid p \rangle$ and $a_R^*(\mathbf{k}) \mid p \rangle$ however fails to vanish; $\langle p \mid a_R(\mathbf{k}) a_Q^*(\mathbf{k}) \mid p \rangle$ can absorb probability and thereby threaten unitarity in the gauge-independent part of Hilbert space consisting of transverse photons (or gluons) and charged particles. To preserve the consistency of a gauge theory it is essential to eliminate that possibility. Both ghost excitations, the R type as well as the Q type, are necessary to represent the longitudinal and timelike A_μ and Π_μ consistently with the equal-time commutation rules. Even if A_0 and $\partial_0 A_0$ were set equal to zero at the start, the longitudinal components $\mathbf{A}^L$ and $\mathbf{\Pi}^L$ could not be represented with either Q or R operators alone without violating their canonical commutation rule. We therefore cannot eliminate the possibility of unitarity violations by totally rejecting one or the other type of ghost operators, R or Q, as elements used in representing the gauge fields.

We will use the following representation of the gauge field operators:

$$A_i^L(\mathbf{x}) = \sum_{\mathbf{k}} \frac{k_i}{(2k^{3/2})} \{ [a_R(\mathbf{k}) + \gamma a_Q(\mathbf{k})] e^{i\mathbf{k}\cdot\mathbf{x}} + [a_R^*(\mathbf{k}) + \gamma a_Q^*(\mathbf{k})] e^{-i\mathbf{k}\cdot\mathbf{x}} \} \quad (2.13a)$$

and

$$\Pi_i^L(\mathbf{x}) \sum_{\mathbf{k}} = \frac{-ik_i}{(k)^{1/2}} [a_Q(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} - a_Q^*(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}] \quad (2.13b)$$

for the longitudinal components, and

$$A_0(\mathbf{x}) = \sum_{\mathbf{k}} \frac{-i\alpha}{(k)^{1/2}} [a_Q(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} - a_Q^*(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}] \quad (2.14a)$$

and

$$\Pi_0(\mathbf{x}) = \sum_{\mathbf{k}} \frac{-(k)^{1/2}}{2\alpha} \{ [a_R(\mathbf{k}) - \gamma a_Q(\mathbf{k})] e^{i\mathbf{k}\cdot\mathbf{x}} + [a_R^*(\mathbf{k}) - \gamma a_Q^*(\mathbf{k})] e^{-i\mathbf{k}\cdot\mathbf{x}} \} \quad (2.14b)$$

for the timelike components. α and γ are useful parameters in these operator representations because the equations of motion as well as the commutation rules are invariant to changes in α and γ.

The momentum-space representations in Eqs. (2.13a)–(2.14b) allow us to express the Hamiltonian H in the form

$$H = H_0^T + H_0^{(e)} - \int \mathbf{j}\cdot\mathbf{A}^T d\mathbf{x} + \sum_k \Bigg[k[a_Q^*(\mathbf{k}) a_Q(\mathbf{k}) + \tfrac{1}{2} a_Q(\mathbf{k}) a_Q(-\mathbf{k})(1-2i\alpha) + \tfrac{1}{2} a_Q^*(\mathbf{k}) a_Q^*(-\mathbf{k})(1+2i\alpha)] - \left[[a_R(\mathbf{k}) + \gamma a_Q(\mathbf{k})] \frac{\mathbf{k}\cdot\mathbf{j}(-\mathbf{k})}{2k^{3/2}} + [a_R^*(\mathbf{k}) + \gamma a_Q^*(\mathbf{k})] \frac{\mathbf{k}\cdot\mathbf{j}(\mathbf{k})}{2k^{3/2}} \right] - i\alpha \left[a_Q(\mathbf{k}) \frac{j_0(-\mathbf{k})}{k^{1/2}} - a_Q^*(\mathbf{k}) \frac{j_0(\mathbf{k})}{k^{1/2}} \right] \Bigg] , \quad (2.15)$$

where

$$H_0^T = \sum_{\mathbf{k}, i=1,2} |\mathbf{k}| a_i^\dagger(\mathbf{k}) a_i(\mathbf{k}) \quad (2.16a)$$

and

$$H_0^{(e)} = \sum_{\mathbf{k}, s=1,2} (m^2 + |\mathbf{k}|^2)^{1/2} [e_s^\dagger(\mathbf{k}) e_s(\mathbf{k}) + \bar{e}_s^\dagger(\mathbf{k}) \bar{e}_s(\mathbf{k})] . \quad (2.16b)$$

It is easy to verify that setting $\partial_0 \xi = i[H, \xi]$, where ξ represents any operator, reproduces the identities in Eqs. (2.6a)–(2.6d) as well as

$$\partial_0 \partial_0 G = 0 . \quad (2.17)$$

A number of features of H are worth noting. One is that the equations of motion are independent of the parameters α and γ and that the $\alpha \to 0$ limit of H is the Hamiltonian we would have obtained had we set $A_0 = 0$ and $\partial_0 A_0 = 0$ initially in the Lagrangian. It is also important to realize that, as arbitrarily chosen state vectors evolve in time under the influence of $\exp(-iHt)$, Q and R-type excitations will both arise. It is possible to view Q and R ghost modes as linear combinations of longitudinal and timelike modes, and the form of H clearly shows that all 4 degrees of freedom in A_μ can be excited by the interaction Hamiltonian. It is not at all obvious from the form of H alone how the theory avoids the violations of unitarity previously discussed, in which linear superpositions of Q and R photon ghost-containing states drain probability from the states consisting only of transverse photons and charged particles.

It is also worth noting that H, given in Eq. (2.15) differs conspicuously from the Hamiltonians that describe QED in the Coulomb[15] and the manifestly covariant Lorentz gauges.[16] We need to verify therefore that all these Hamiltonians describe the same physical theory.

We begin the resolution of these questions with the observation that if we set the expectation value $\langle A_0 \rangle = 0$ in some suitably chosen subspace at $t = 0$, then the identity $\partial_0 A_0 = 0$ guarantees the continued validity of $\langle A_0 \rangle = 0$ at all times. Similarly, since $\partial_0 \partial_0 G$ is an identity, if we set $\langle \partial_0 G \rangle = 0$ at $t = 0$, Gauss's law too will hold at all times. The conditions $\langle A_0 \rangle = 0$ and $\langle \partial_0 G \rangle = 0$, imposed simultaneously, imply the validity of both, gauge fixing in the temporal gauge and Gauss's law, for all times.

We note that we can represent $\partial_0 G$ as

$$\partial_0 G = \sum_{\mathbf{k}} [\Omega(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \Omega^*(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}] k^{3/2} , \quad (2.18)$$

where

$$\Omega(\mathbf{k})=a_Q(\mathbf{k})+j_0(k)/(2k^{3/2})$$

and $\Omega^*(\mathbf{k})$ is its adjoint $a_Q^*(\mathbf{k})+j_0(-\mathbf{k})/(2k^{3/2})$. Choosing a subspace $\{|\nu\rangle\}$, whose states obey

$$\Omega(\mathbf{k})|\nu\rangle=0\ , \tag{2.19}$$

therefore guarantees the validity of Gauss's law:

$$\langle\nu'|\partial_0 G|\nu\rangle=\langle\nu'|(\nabla\cdot\mathbf{\Pi}+j_0)|\nu\rangle=0 \tag{2.19a}$$

for any $|\nu\rangle$ and $|\nu'\rangle$ in $\{|\nu\rangle\}$. Equation (2.14a) shows that A_0 can be represented as

$$A_0(\mathbf{x})=-i\alpha\sum_{\mathbf{k}}(k)^{-1/2}[\Omega(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-\Omega^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{2.20}$$

so that confining a state vector to the subspace $\{|\nu\rangle\}$ at $t=0$ guarantees both, $\langle A_0\rangle=0$ and $\langle\partial_0 G\rangle=0$ for all other times. The same fact can also be verified independently of these considerations. Explicit calculations show that $[H,\Omega(\mathbf{k})]=[H,\Omega^*(\mathbf{k})]=0$. Therefore if $\Omega(\mathbf{k})|\nu\rangle=0$,

$$\Omega(\mathbf{k})\exp(-iHt)|\nu\rangle=\exp(-iHt)\Omega(\mathbf{k})|\nu\rangle$$
$$=0$$

so that if a state vector is within $\{|\nu\rangle\}$ initially it will remain there forever.

It is interesting that the constraint equations that define the subspace, and the dynamical effects that keep the state vector within the subspace, are very similar in the temporal gauge and in covariant gauges. In the temporal gauge, as well as in the covariant gauges, the gauge constraints and Gauss's law are implemented by confining state vectors within an appropriately chosen subspace.[13] In both gauges that constraint is imposed by the same equation: i.e., Eq. (2.19). In both gauges two initial conditions specify the initial-value problem completely. It does appear, from configuration space considerations, that in the manifestly covariant Lorentz gauges (we choose the Feynman gauge as an illustrative example) the initial-value problem takes on a very different form. There we find that $\partial_\mu A_\mu$ obeys the second-order equation $\Box(\partial_\mu A_\mu)=0$, so that, if we define a subspace in which both $\langle\partial_\mu A_\mu\rangle=0$ and $\langle\partial_0(\partial_\mu A_\mu)\rangle=0$ at $t=0$, $\langle\partial_\mu A_\mu\rangle=0$ at all times. But when we study the momentum-space representations of these operators the similarity between the convariant gauges and the temporal gauge becomes clear. $\partial_\mu A_\mu$, in the Feynman gauge, can be represented as

$$\partial_\mu A_\mu=i\sum_{\mathbf{k}}k^{1/2}[\Omega(\mathbf{k})e^{ik\cdot x}-\Omega^*(\mathbf{k})e^{-ik\cdot x}] \tag{2.21}$$

and $\partial_0(\partial_\mu A_\mu)$ is

$$\partial_0(\partial_\mu A_\mu)=\sum_{\mathbf{k}}k^{3/2}[\Omega(\mathbf{k})e^{ik\cdot x}+\Omega^*(\mathbf{k})e^{-ik\cdot x}]\ , \tag{2.22}$$

where $\Omega(\mathbf{k})$ is the same operator as in the temporal gauge. The identical subspace $\{|\nu\rangle\}$, defined by Eq. (2.19), confines state vectors for both the temporal and the covariant gauges; and the gauge constraint as well as Gauss's law hold in $\{|\nu\rangle\}$ for the temporal as well as for convariant gauges. In the Feynman gauge $[H,\Omega(\mathbf{k})]=-k\Omega(\mathbf{k})$ and $[H,\Omega^*(\mathbf{k})]=k\Omega^*(\mathbf{k})$, whereas in the temporal gauge both these commutators vanish identically. But in the Feynman gauge we still obtain the result

$$\Omega(\mathbf{k})\exp(-iHt)|\nu\rangle=\exp(-iHt)e^{-ikt}\Omega(\mathbf{k})|\nu\rangle=0\ ,$$

so that, if a state vector is in $\{|\nu\rangle\}$ initially, it remains there forever in covariant gauges as well as in the temporal gauge.

To study further consequences of this formulation of the temporal gauge we will apply a procedure introduced in earlier work.[13] We carry out a similarity transformation to a new set of states, given by

$$|n\rangle=e^{D}|\nu\rangle\ , \tag{2.23}$$

where e^D is unitary and D is given by

$$D=-\sum_{\mathbf{k}}(2k^{3/2})^{-1}[a_R(\mathbf{k})j_0(-\mathbf{k})-a_R^*(\mathbf{k})j_0(\mathbf{k})] \tag{2.24a}$$

and we carry out the corresponding transformations on all operators ξ, which in the new representation have the form

$$\hat{\xi}=e^D\xi e^{-D}\ . \tag{2.24b}$$

The transformed operators take the form

$$\hat{A}_i^L(\mathbf{x})=A_i^L(\mathbf{x})\ , \tag{2.25a}$$

$$\hat{\Pi}_i^L(\mathbf{x})=\Pi_i^L(\mathbf{x})+\frac{\partial}{\partial x_i}\int\frac{j_0(\mathbf{y})}{4\pi|\mathbf{x}-\mathbf{y}|}d\mathbf{y}\ , \tag{2.25b}$$

$$\hat{A}_0(\mathbf{x})=A_0(\mathbf{x})\ , \tag{2.25c}$$

$$\hat{\Pi}_0(\mathbf{x})=\Pi_0(\mathbf{x})+\frac{\gamma}{\alpha}\int d\mathbf{y}\left[(2\pi)^{-3}\int\frac{d\mathbf{k}}{2k}\exp[i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})]\right]\times j_0(\mathbf{y})\ , \tag{2.25d}$$

$$\hat{\Omega}(\mathbf{k})=a_Q(\mathbf{k})\ , \tag{2.25e}$$

and

$$\hat{\psi}(\mathbf{x})=\exp\left[-\sum_{\mathbf{k}}(2k^{3/2})^{-1}[a_R(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}-a_R^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]\right]\times\psi(\mathbf{x}) \tag{2.25f}$$

as well as

$$\hat{j}_\mu(\mathbf{x})=j_\mu(\mathbf{x})\ . \tag{2.25g}$$

The transformed Hamiltonian is given by

$$\hat{H}=H_0^T+H_0^{(e)}-\int \mathbf{j}\cdot\mathbf{A}^T d_{\mathbf{x}}+\sum_{\mathbf{k}}\Bigg[k[a_Q^*(\mathbf{k})a_Q(\mathbf{k})+\tfrac{1}{2}a_Q(\mathbf{k})a_Q(-\mathbf{k})(1-2i\alpha)+\tfrac{1}{2}a_Q^*(\mathbf{k})a_Q^*(-\mathbf{k})(1+2i\alpha)] -(k)^{-1/2}\left[j_0(-\mathbf{k})+\gamma\frac{\mathbf{k}\cdot\mathbf{j}(-\mathbf{k})}{2|\mathbf{k}|}\right]a_Q(\mathbf{k})-(k)^{-1/2}\left[j_0(\mathbf{k})+\gamma\frac{\mathbf{k}\cdot\mathbf{j}(\mathbf{k})}{2|\mathbf{k}|}\right]a_Q^*(\mathbf{k})\Bigg] + \int d\mathbf{x}\,d\mathbf{y}\,\frac{j_0(\mathbf{x})j_0(\mathbf{y})}{8\pi|\mathbf{x}-\mathbf{y}|}\ . \tag{2.26}$$

The transform of Eq. (2.19), which defines the subspace in which the constraints $\langle A_0\rangle=0$ and $\langle\nabla\cdot\Pi+j_0\rangle=0$ hold, in the new representation is

$$a_Q(\mathbf{k})|n\rangle=0\ . \tag{2.27}$$

The set of states $\{|n\rangle\}$, the "allowed subspace," includes the set of states $\{|p\rangle\}$, the "quotient space," which consists of the Fock states for all charged particles (electrons), and the transverse photon Fock states. It is important to note however that in this new representation $\langle e_p|\nabla\cdot\hat{\Pi}+\hat{j}_0)|e_p\rangle=0$, so that the electron Fock states carry their static electric field with them and obey Gauss's law. That is because these electron Fock states are unitarily equivalent, in the old representation, to coherent states of electrons and photon ghosts which constitute the electron's Coulomb field.

In addition to the set $\{|p\rangle\}$, the allowed subspace $\{|n\rangle\}$ includes all states of the form $a_Q^*(k)|p\rangle,\ldots,[a_Q^*(k_1)\cdots a_Q^*(k_i)]|p\rangle$; but it excludes any state of the form $a_R^*(k)|n\rangle$. The allowed subspace $\{|n\rangle\}$, in the new representation, is unitarily equivalent to the set $\{|\nu\rangle\}$ in the old representation. Part of the Hamiltonian $\hat{H}$ for QED in the temporal gauge, in the new representation, is given by H_c where

$$H_c=H_0^T+H_0^{(e)}-\int \mathbf{j}\cdot\mathbf{A}^T d\mathbf{x}+\int d\mathbf{x}\,d\mathbf{y}\,\frac{j_0(\mathbf{x})j_0(\mathbf{y})}{8\pi|\mathbf{x}-\mathbf{y}|}\ . \tag{2.28a}$$

H_c is the Hamiltonian for QED in the Coulomb gauge. The remaining parts of $\hat{H}$ are given by

$$H_Q=\sum_{\mathbf{k}}\Bigg[k[a_Q^*(\mathbf{k})a_Q(\mathbf{k})+\tfrac{1}{2}a_Q(\mathbf{k})a_Q(-\mathbf{k})(1-2i\alpha)+\tfrac{1}{2}a_Q^*(\mathbf{k})a_Q^*(-\mathbf{k})(1+2i\alpha)] -(k)^{-1/2}\left[j_0(-\mathbf{k})+\gamma\frac{\mathbf{k}\cdot\mathbf{j}(-\mathbf{k})}{2|\mathbf{k}|}\right]a_Q(\mathbf{k})-(k)^{-1/2}\left[j_0(\mathbf{k})+\gamma\frac{\mathbf{k}\cdot j(\mathbf{k})}{2|\mathbf{k}|}\right]a_Q^*(\mathbf{k})\Bigg]\ . \tag{2.28b}$$

All parts of H_Q contain either the factor $a_Q(\mathbf{k})$, or its adjoint $a_Q^*(\mathbf{k})$, or bilinear combinations of either or both. Since $a_Q(\mathbf{k})$ and $a_Q^*(\mathbf{k})$ commute with each other, and with all other operators appearing in $\hat{H}$, H_Q cannot contribute to any internal loop processes. The only effect H_Q can have on states in the allowed subspace $\{|n\rangle\}$ is to produce further states of the form $a_Q^*(k_1)\cdots a_Q^*(k_i)|n\rangle$, which are in $\{|n\rangle\}$ again. All states that H_Q can generate have zero norm and zero inner product with all other states in $\{|n\rangle\}$. They cannot absorb probability, can never carry energy or momentum, and can never affect the unitarity of the theory adversely. It is a trivial consequence of the form of $\hat{H}$ that all state vectors $|p\rangle$, consisting of electrons and transverse photons, have precisely the same time evolution in the quotient space $\{|p\rangle\}$, and therefore lead to identical physical predictions, as they would in the Coulomb gauge. Whatever components state vectors develop in $\{|n\rangle\}$ outside the quotient space $\{|p\rangle\}$ are physically irrelevant. Because H_Q is included in $\hat{H}$, time derivates, and with them equations of motion, for gauge-dependent parts of A_μ change with the gauge. But the presence of H_Q in $\hat{H}$ is totally without consequence for the time evolution of state vectors that represent physically realizable configurations of photons and charged particles.

It is much easier to clarify how gauge-fixing eliminates redundant degrees of freedom in the transformed representation of the temporal gauge, in which $\exp(-i\hat{H}t)$ is the time-evolution operator and $\{|n\rangle\}$ the set of states in which constraints are implemented. Both $\langle A_0\rangle=0$ and $\langle\partial_0 G\rangle=0$ are required to eliminate the $a_R^*(\mathbf{k})|n\rangle,\ldots,a_R^*(\mathbf{k}_1)\cdots a_R^*(\mathbf{k}_i)|n\rangle$ set of ghost states from the allowed subspace by implying Eq. (2.19) in the original, and Eq. (2.27) in the transformed representation. But once the degree of freedom that corresponds to the presence of the R ghost is eliminated, both degrees of freedom, Q as well as R, are effectively removed from the theory, because as we have shown, the Q-ghost degree of freedom, without the simultaneous presence of R-ghost states, is irrelevant to the predictions of this theory.

III. THE INTERACTION PICTURE AND THE PROPAGATOR FOR THE GAUGE FIELD

In the preceding section we have discussed two unitarily equivalent representations of QED in the temporal gauge. In one (the "original" representation) the time evolution operator $\exp(-iHt)$ acts on a set of states $\{|\nu\rangle\}$. In the other (the "transformed" representation) the time-evolution operator $\exp(-i\hat{H}t)$ acts on a set of states $\{|n\rangle\}$. Both of these representations afford us the basis for a consistent description in which state vectors originally confined to $\{|\nu\rangle\}$ in the original, or $\{|n\rangle\}$ in the transformed representation, are kept within that set of states (the allowed subspace) by the equations of motion. In contrast with these two consistent representations, the perturbative expansion of S-matrix elements with Feynman rules makes implicit use of the time-evolution operator $\exp(-iHt)$ together with the set of states $\{|n\rangle\}$ instead of $\{|\nu\rangle\}$. The time-evolution operator $\exp(-iHt)$ appears in the disguised but entirely equivalent form in which the interaction Hamiltonian $(-\int j\cdot \mathbf{A}\, d\mathbf{x} + \int j_0 A_0 d\mathbf{x})$ is used in an interaction picture driven by $\exp(-iH_0t)$. The $\int j_0 A_0 d\mathbf{x}$ term vanishes in the $\alpha\to 0$ limit, and H_0 is the $e\to 0$ limit of H in Eq. (2.10), and is given by

$$H_0=H_0^T+H_0^{(e)}+\sum_{\mathbf{k}}\{k[a_Q^*(\mathbf{k})a_Q(\mathbf{k})+\tfrac{1}{2}a_Q(\mathbf{k})a_Q(-k)(1-2i\alpha)+\tfrac{1}{2}a_Q^*(\mathbf{k})a_Q^*(-\mathbf{k})(1+2i\alpha)]\} \; . \tag{3.1}$$

The combination of the Hamiltonian H appropriate for the original representation, together with the set of states $\{|n\rangle\}$ appropriate for the transformed representation, falls short of guaranteeing that state vectors originally confined to $\{|n\rangle\}$ will permanently remain in that subspace. In fact state vectors in $\{|n\rangle\}$ do develop $a_R^*(\mathbf{k}_1)\cdots a_R^*(\mathbf{k}_i)|n\rangle$ components under the influence of the time-evolution operator $\exp(-iHt)$. This identical situation prevails also in the covariant gauges.

In spite of the questionable consistency of mixing a time-evolution operator from one representation with a set of states from another, the S matrix that results from this combination is substantially correct. If a wave function is entirely within the subspace $\{|n\rangle\}$ in the limit $t\to -\infty$ then, even though it will not remain entirely within $\{|n\rangle\}$ at all times under the effect of the time-evolution operator $\exp(-iHt)$, it will again be entirely within $\{|n\rangle\}$ in the limit $t\to +\infty$.

The theoretical support for this use of Feynman rules for a gauge theory in a formulation that fails to implement the gauge constraint and Gauss's law, can be given in terms of what we have called a "hybrid transformation."[17] In a hybrid transformation the states of a theory are unitarily transformed but the operators, in particular the time-evolution operator, are left untransformed. It is possible to derive an expression for the change in a transition amplitude that results from a hybrid transformation. This derivation relates the following two transition amplitudes for transitions from an initial state $|i\rangle$ to a final state $|f\rangle$: One is $\overline{T}_{f,i}$, the transition amplitude based on the Hamiltonian $\hat{H}$ and the subspace $\{|n\rangle\}$ (or equivalently on H and $\{|\nu\rangle\}$) and is given by

$$\overline{T}_{f,i}=\langle f\,|\tilde{H}_1+\tilde{H}_1(E_i-\hat{H}+i\epsilon)^{-1}\tilde{H}_1\,|\,i\rangle \; , \tag{3.2}$$

where $\tilde{H}_1=\hat{H}-H_0$. The other is $T_{f,i}$, the transition amplitude that corresponds to Feynman rules for the perturbative S matrix, and is given by

$$T_{f,i}=\langle f\,|H_1+H_1(E_i-H+i\epsilon)^{-1}H_1\,|\,i\rangle \; , \tag{3.3}$$

where $H_1=H-H_0$. In earlier work we have obtained the equation[13,17]

$$\overline{T}_{f,i}=T_{f,i}+(E_f-E_i)\mathcal{T}_{f,i}{}^{(\alpha)}+i\epsilon\mathcal{T}_{f,i}{}^{(\beta)} \; , \tag{3.4}$$

where

$$\mathcal{T}_{f,i}^{(\alpha)}=\langle f\,|(1-e^D)\,|\,i\rangle + \langle f\,|(1-e^D)(E_i-H+i\epsilon)^{-1}H_1\,|\,i\rangle \tag{3.4a}$$

and

$$\mathcal{T}_{f,i}^{(\beta)}=\langle f\,|[(e^D-1)(E_i-H+i\epsilon)^{-1}H_1 - \tilde{H}_1(E_i-\hat{H}+i\epsilon)^{-1}(e^D-1)]i\rangle \; . \tag{3.4b}$$

For the S-matrix elements only the $E_f\to E_i$ and the $i\epsilon\to 0$ limits of $T_{f,i}$ and $\hat{T}_{f,i}$ are significant so that, unless $\mathcal{T}_{f,i}^{(\alpha)}$ develops an $(E_f-E_i)^{-1}$ or $\mathcal{T}_{f,i}^{(\beta)}$ an $(i\epsilon)^{-1}$ singularity, the S-matrix elements in the consistent formulation and in the hybrid-transformed representation will agree. e^D transforms "bare" charged-particle Fock states, devoid of any electric fields, into coherent states that obey Gauss's law. It does not endow them with the complement of transverse photons that an actual charged particle with nonvanishing momentum requires. For that reason its effect is relatively benign. Because of the inclusion of only a single variety of ghost, and the absence of transverse photons in the unitary e^D, there are no infrared singularities that keep the $|\nu\rangle$ and $|n\rangle$ charged-particle states from differing by infinite normalization constants. In fact $|\nu\rangle$ and $|n\rangle$ states have the identical norms. Moreover the only cases in which $\mathcal{T}_{f,i}^{(\alpha)}$ can develop an $(E_f-E_i)^{-1}$ or $\mathcal{T}_{f,i}^{(\beta)}$ an $(i\epsilon)^{-1}$ singularity, are self-energy correction to external electron lines and these only affect the electron wave-function renormalization constant. The fact that these singularities arise is responsible for the fact that renormalized rather than unrenormalized S-matrix elements are identical in different gauges.[13,18]

Once we have established on-shell identity of $\overline{T}_{f,i}$ and $T_{f,i}$ we have also established identity of the perturbative S matrix, based on Feynman rules for the temporal gauge, with the S matrix in the Coulomb gauge. Since in earlier work the same proof was given for perturbative Feynman rules for covariant gauges and the Coulomb gauge, we can infer the identity of the perturbative S matrix in the temporal gauge and in covariant gauges.[13]

We now turn to the Feynman rules in the temporal gauge. The propagators are given by the vacuum expectation value of time-ordered products in the $\{|n\rangle\}$ space vacuum. The vacuum expectation values for the transverse parts of $\mathbf{A}$ and for the spinor field are the same in all gauges and need not be reevaluated here. The longitudinal part of $\mathbf{A}$, in the interaction picture, is given by

$$A_i^L(\mathbf{x},t)=\exp(iH_0t)A_i^L(\mathbf{x})\exp(-iH_0t) \; . \tag{3.5a}$$

Expansion of Eq. (3.5a) leads to

$$A_i^L(\mathbf{x},t)=A_i^L(\mathbf{x})+it[H_0,A_i^L(\mathbf{x})] \tag{3.5b}$$

and all higher-order commutators vanish. Equation (3.1) and the ghost operator commutation rules lead to

$$A_i^L(\mathbf{x},t)=\sum_{\mathbf{k}}\frac{k_i}{2k^{3/2}}(\{a_R(\mathbf{k})+a_Q(\mathbf{k})[\gamma-2itk(1-i\alpha)]\}e^{i\mathbf{k}\cdot\mathbf{x}}+\{a_R^*(\mathbf{k})+a_Q^*(\mathbf{k})[\gamma+2itk(1+i\alpha)]\}e^{-i\mathbf{k}\cdot\mathbf{x}}) \tag{3.6}$$

and to the vacuum expectation value

$$\langle 0\,|\,T(A_i^L(\mathbf{x}_1,t_1)A_j^L(\mathbf{x}_2,t_2))\,|\,0\rangle=\left[-\frac{i}{2}\,|\,t_1-t_2\,|\,-\frac{\alpha}{2}(t_1+t_2)\right]\frac{\partial_i\partial_j}{\nabla^2}\delta(\mathbf{x}_2-\mathbf{x}_1)+i\gamma\frac{\partial_i\partial_j}{\nabla^2}\Delta(\mathbf{x}_2-\mathbf{x}_1)\ , \tag{3.7}$$

where $\Delta(\mathbf{x}_2-\mathbf{x}_1)$ is the massless Feynman propagator for the interval $\mathbf{x}=\mathbf{x}_i-\mathbf{x}_2$, and $t_1-t_2=0$. It is given by

$$\Delta(\mathbf{x})=\frac{-i}{(2\pi)^3}\int\frac{d\mathbf{k}}{(2k)}\exp(i\mathbf{k}\cdot\mathbf{x})\ . \tag{3.7a}$$

The time-ordered product

$$\langle 0\,|\,T(A_0(\mathbf{x}_1,t_1)A_0(\mathbf{x}_2,t_2))\,|\,0\rangle$$

vanishes identically, but

$$\langle 0\,|\,T(A_i^L(\mathbf{x}_1,t_1)A_0(\mathbf{x}_2,t_2))\,|\,0\rangle$$

gives the contribution

$$\langle 0\,|\,T(A_i^L(\mathbf{x}_1,t_1)A_0(\mathbf{x}_2,t_2))\,|\,0\rangle=\frac{\alpha}{2}\frac{\partial_i}{\nabla^2}\delta(\mathbf{x}_1-\mathbf{x}_2) \tag{3.8}$$

which vanishes in the $\alpha\to 0$ limit.

The α and γ dependence of propagators does not interfere with the validity of Eq. (3.4), so that the S matrix evaluated with different values of α and γ will still agree with the S matrix evaluated in the Coulomb or in the covariant gauges. The Hamiltonian, the Green's functions and the off-shell transition amplitudes may depend on α and γ, but the scattering amplitudes are free from any α or γ dependence.

It is straightforward to use Eqs. (2.13a)–(2.14b) to evaluate the four-point function in the interaction picture. We represent $\langle 0\,|\,T(A_i^L(\mathbf{x}_1,t_1)A_j^L(\mathbf{x}_2,t_2))\,|\,0\rangle$ in the temporal gauge, given in Eq. (3.7) as $D_{ij}^L(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2 0$ and find that

$$\langle 0\,|\,T(A_i^L(\mathbf{x}_1,t_1)A_j^L(\mathbf{x}_2,t_2)A_k^L(\mathbf{x}_3,t_3)A_n^L(\mathbf{x}_4,t_4))\,|\,0\rangle=D_{ij}^L(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)D_{kn}^L(\mathbf{x}_3,t_3;\mathbf{x}_4,t_4)+D_{ik}^L(\mathbf{x}_1,t_1;\mathbf{x}_3,t_3)D_{jn}^L(\mathbf{x}_2,t_2;\mathbf{x}_4,t_4)$$
$$+D_{in}^L(\mathbf{x}_1,t_1;\mathbf{x}_4,t_4)D_{jk}^L(\mathbf{x}_2,t_2;\mathbf{x}_3,t_3)\ . \tag{3.9}$$

IV. DISCUSSION

In this section we will discuss the relationship of our result to work based on different approaches to the temporal gauge. For example, the propagator for longitudinal photons in the temporal gauge, given in our Eq. (3.7), is also evaluated by Caracciolo, Curci, and Menotti[1] (CCM). In their work CCM makes a comparison, to fourth-order perturbation theory, of the Wilson loop in the $A_0=0$ gauge, with the results obtained in the Coulomb and Feynman gauges. Their propagator has the form

$$D_{ij}^{CCM}(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)=-\frac{i}{2}[\,|\,t_1-t_2\,|\,+\alpha'(t_1+t_2)+\gamma']\times\left[\frac{\partial_i\partial_j}{\nabla^2}\right]\delta_3(\mathbf{x}_2-\mathbf{x}_1) \tag{4.1}$$

and they do not report any A^L-A_0 propagator.

Our propagator has a part proportional to $|\,t_1-t_2\,|$ and devoid of arbitrary parameters. This part is identical to the temporal-gauge propagator obtained by a number of workers with a principal-value prescription,[6] and agrees with a corresponding part in the CCM propagator. There is another part of our propagator that lacks time-translation invariance and is proportional to $\alpha(t_1+t_2)$, where α is a real parameter that can be varied at will, and in particular may be set equal to zero. CCM find a similar arbitrarily variable part of the propagator for QED in the temporal gauge, but conclude that it is imaginary (in the case of QCD they report that, in their notation, $\alpha'=\pm 1$ is required for consistency with other gauges). Finally our propagator has a time-independent part proportional to an additional variable parameter γ, which also may assume any real value, including zero. CCM also have a time-independent part, proportional to a variable parameter, in their propagator, but whereas in our case the spatial dependence of the time-independent part is given by $(\partial_i\partial_j)\Delta(\mathbf{x}_2-x_1)$, CCM give it as

$$(\partial_i\partial_j/\nabla^2)\delta_3(\mathbf{x}_2-\mathbf{x}_1)\ .$$

The functional form $(\partial_1\partial_j/\nabla^2)\Delta(\mathbf{x}_2-\mathbf{x}_1)$ that we obtain in Eq. (3.7) also appears in the time-independent part of the propagator evaluated by Dahmen, Schulz, and Steiner[3] (DSS); their propagator is not a special case of the CCM propagator and needs to be regularized by a limiting procedure that cannot be carried out until later stages of the S-matrix calculations have been completed. Their effective propagator, which incorporates the results of that limiting procedure, has a (t_1+t_2)-dependent part that agrees with ours in being imaginary with respect to the principal-value part proportional to $|\,t_1-t_2\,|$. Landshoff has also proposed an alternate form of the propagator;[8] his form retains time-translation invariance and still reproduces CCM's Wilson loop calculation in perturbation theory up to fourth order in the coupling constant. Landshoff stresses the importance of not neglecting the nonlinear time dependence that stems from finite displacements of k_0 poles from the real axis in the limiting process in which these k_0 poles approach the real axis.

Other workers, using different methods, obtain the CCM propagator, but with a special value of α (in our notation) for QED as well as for QCD. Thus Lim, by means of what he calls "stochastic quantization" finds that $\alpha=\pm i$ is the correct choice for both QED and QCD (Ref. 2). Girotti and Rothe[7] also maintain the $\alpha=\pm i$ choice for QED as does Yamagishi,[4] the latter by generating solutions of the constraint equation $(\nabla\cdot E-j_0)\,|\,\psi\rangle=0$ by means of the Møller operator applied to eigenstates of $(\nabla\cdot E)\,|\,\phi\rangle=0$. Lim, Yamagishi, and GR attach importance to finding a definite value of α, because, they argue, that only the propagator with a definite value of α corresponds to a form of the theory in which the gauge has been completely fixed and ambiguities in the formulation eliminated.

We find that definite real values of α and γ are neither necessary for QED, nor are they appropriate, because there is no compelling *a priori* reason for preferring one set of values over another. Choosing different values of α and γ is as harmless, and quite analogous to choosing the Feynman,[19] Landau,[20] or Fried-Yennie[21] propagators in covariant QED, or choosing the unitary or the renormalizable forms of propagators in gauge theories with spontaneously broken symmetries.[22] It is indeed true that changes in γ implicitly gauge-transform $\mathbf{A}^L$, and that changes in α change the gauge by determining whether $A_0=0$ holds as an operator identity or whether only expectation values of A_0 in the allowed subspace vanish. But the effect of these changes of gauge in A_μ are without any significance to the dynamical implications of the theory as long as Maxwell's equations hold. We should distinguish between choosing the gauge on the one hand and, on the other, fully implementing the gauge constraint as well as all other time-independent constraints in the appropriate subspace. Dangerous ambiguities in gauge theories can occur when Gauss's law is not fully realized, though that does not threaten the validity of the perturbative S matrix in QED because Eq. (3.4) supports the substitution of $\overline{T}_{f,i}$ for $T_{f,i}$ for all values of α and γ in that case. Inspection of the α and γ dependence of $\hat{H}$ in Eq. (2.26) clearly demonstrates the harmlessness of varying α and γ.

There appears not to be any general acceptance by other workers of the fact that the longitudinal gauge field must involve ghost excitations and require an extension of the kinematic framework beyond that of a positive metric Hilbert space. That ghosts and such a kinematic extension are necessary has been pointed out by Nakanishi,[23] though his fears that their inclusion threatens the consistency of the theory are unfounded. Many other workers who treat the temporal gauge as a canonically quantized theory, explicitly or implicitly treat it as free of ghosts. Zeppenfeld,[5] for example, claims that Nakanishi's arguments in behalf of ghosts are wrong. Interestingly, Zeppenfeld's and DSS (Ref. 3) operators $\chi_{(+)}$ and $\chi_{(-)}$ [see the latter's Eq. (B1), for example] admit easily and naturally of a ghost representation. In our formulation $\chi_{(+)}$ and $\chi_{(-)}$ given by DSS are representable as

$$\chi_{(+)}=\frac{-i}{2}\sum_{\mathbf{k}}\{[a_R(\mathbf{k})+\gamma a_Q(\mathbf{k})]e^{i\mathbf{k}\cdot\mathbf{x}} -[a_R^*(\mathbf{k})+\gamma a_Q^*(\mathbf{k})]e^{-i\mathbf{k}\cdot\mathbf{x}}\} \tag{4.2}$$

and

$$\chi_{(-)}=-\sum_{\mathbf{k}}[(1-i\alpha)a_Q(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} +(1+i\alpha)a_Q^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}] \tag{4.3}$$

and this representation leads to the commutation relation

$$[\chi_{(+)}(\mathbf{x}),\chi_{(+)}(\mathbf{y})]=[\chi_{(-)}(\mathbf{x}),\chi_{(-)}(\mathbf{y})]=0 \tag{4.4}$$

and

$$[\chi_{(+)}(\mathbf{x}),\chi_{(-)}(\mathbf{y})]=i\delta(\mathbf{x}-\mathbf{y})\ . \tag{4.5}$$

The process of transforming a state $|\,\phi\rangle$ into $|\,\psi\rangle$ by a unitary transformation, in which photon creation and annihilation operators obey the positive metric rules $[a,a^*]=1$, would require that $|\,\phi\rangle$ and $|\,\psi\rangle$ differ by infinite normalization factors and threaten the consistency of the theory.[24] It is possible that the discrepancy between the DSS and the CCM propagator stems from the efforts of DSS to implicitly use the formalism of a positive-metric space of longitudinal photons, though this author does not know of any proofs to that effect.

What implication does this formulation have for QCD in the temporal gauge? Intuition alone suggests that ghost mode repreentations of longitudinal gauge fields and the form of the Lagrangian generalized to SU(n) from the U(1) case are appropriate extensions from QED to QCD. But it is apparent that the dynamics of non-Abelian fields makes it harder to find the proper form of $\Omega(\mathbf{k})$ and to carry out the complete analysis of Sec. II. In particular it is unclear whether the unitary transformation $|\,n\rangle=\exp(D)\,|\,\nu\rangle$ has a suitable non-Abelian analog. There is reason to suspect that the transformation, whose validity is required to justify the non-Abelian Feynman rules, must be critically reexamined.

The canonical formalism we are using serves as a useful framework for such an examination of the non-Abelian case. It provides an apparatus for developing a perturbative S matrix, and connecting it with a well-defined Fock space. At the same time it permits analysis of particle states consistent with constraints. The advantages of a canonical theory, and the ambiguities inherent in path-integral derivations have been discussed and documented by Cheng and Tsai.[25] An extension of this work to Yang-Mills theory and QCD is given in the following paper.[26]

ACKNOWLEDGMENTS

The author wishes to thank Dr. J. Mandula for a useful conversation. This research was supported by the Department of Energy under Grant No. DE-AC02-79ER 10336.A.

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