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Attempt at a classical cancellation of the cosmological constant

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I examine some classical models for a relaxing cosmological constant. The relaxation occurs by means of a classical field which "rolls" down a potential hill. I suggest a feedback mechanism in which the action of this field depends upon the scalar curvature of space-time. It is shown that Λ_{eff} can fall off faster than the red-shifting radiation density. There are problematic side effects associated with baryonic matter. I discuss these and possible treatments.

I. INTRODUCTION

One of the most puzzling facts of cosmology is the smallness of the present value of the cosmological constant Λ. This fact appears especially mysterious if it is assumed, as is probable, that the Universe underwent several phase transitions that changed the cosmological "constant" in its early history.[1] The amount by which Λ changed depends in a very complicated way on the fundamental parameters of the theory. Yet we find, at the conclusion of these processes, that the present-day value of Λ vanishes at least to 120 decimal places when expressed in natural Planck mass units.

One can envision at least two ways of explaining this fact. One could imagine a symmetry which would require $\Lambda=0$. Or one could imagine a dynamical mechanism which would "relax" Λ to zero over a period of time.

The difficulty with the symmetry approach is that no symmetry is known at present that can do the job. Supersymmetry, if it is a symmetry of nature, must be broken at a large scale, greater than M_W. One would expect then that $\Lambda \geq (M_W)^4$. This is still too big by over 50 orders of magnitude. Perhaps some other hidden symmetry lurks beneath the surface of our apparently "broken" world. If so, either it has always been unbroken so that $\Lambda=0$ at all times (which would be hard to reconcile, say, with what happens at the weak phase transition) or else this symmetry has been "restored" only at a later period in the history of the Universe but was broken when the temperature was higher. While supersymmetry very likely has something to do with the solution of the cosmological-constant problem, precisely how seems as mysterious now as the problem itself.

In this paper I describe a dynamical mechanism to make Λ relax to zero. Before describing it we should mention three other notable attempts to describe such a mechanism.

First, it has been pointed out by Mottola[2] that de Sitter space is quantum-mechanically unstable. He has argued that quantum-gravitational effects will cause Λ to relax to zero in a manner somewhat analogous to the screening of an external electric field by the breakdown of the vacuum. This approach has the great virtue that it does not need to postulate any "new physics" and seems to us consequently to be very appealing. However, it is orthogonal to our approach and we have nothing to say about it here.

The second and third attempts, which to some extent have inspired our own, postulate the existence of a compensating field which "rolls" down a potential hill and as it does so causes the effective value of Λ to decrease. The key question is why this rolling stops when $\Lambda \simeq 0$. There are a few possibilities which suggest themselves. One can imagine that the potential for the compensating field (which henceforth we shall denote by ϕ) "bottoms out" precisely when $\Lambda_{\text{eff}} \leq 10^{-120}$. But this is obviously just equivalent to fine-tuning Λ to be $\leq 10^{-120}$ to begin with. Indeed any theory in which the compensating field sector explicitly depends upon the initial value of Λ, and so "knows" when to stop rolling, involves a fine-tuning as bad as that one is trying to avoid. Another possibility has been suggested by Banks.[3] He supposes a situation in which many bubble universes are nucleated at various stages of the rolling of the compensating field (which he denotes η and dubs the "isachon"). In his approach it is not necessary to stop the rolling at all: we happen to inhabit a bubble formed at a time for which $\Lambda \approx 10^{-120}$. This can be justified using an (unobjectionable) version of the anothropic principle. However, Banks notes that there is a very serious flaw in the particular model he proposes. The fraction of hospitable bubbles that resembles ours in key astrophysical respects is smaller than $\exp(-10^{100})$ and hence the probability of that particular model being true is equally small. It is not clear that this flaw can be remedied.

Finally there is the possibility that a feedback mechanism of some sort could tell ϕ when to stop rolling. A model of this sort was proposed by Abbott.[4] We will not give a detailed explanation of his model, which is quite interesting, except to note that it suffers from the problem (seemingly inherent in the idea) that as ϕ rolls, the Universe must be inflating exponentially, so that by the time Λ has attained its present value the matter density and temperature of the Universe are exponentially small even compared to Λ.

It is this last idea that is closest in spirit to the one we explore in this paper. We also posit a feedback mechanism to stop the compensating field when $\Lambda \simeq 0$. The way this happens is that the action for the compensating field explicitly depends on the scalar curvature R. As $\Lambda \to 0$ so does R, as can be seen by looking at the trace of Einstein's equations:

$$-R = 2\Lambda - \tfrac{1}{2}(T_{\text{matter}})^{\lambda}{}_{\lambda} \ .$$

Our conventions here and throughout are as follows. We set $16\pi G_N = 1$. Our metric is $-+++$. The scalar curvature of a sphere is negative. The cosmological constant Λ is normalized so that the gravitational action is

$$\frac{1}{16\pi G_N} \int d^4x\,(-R - \Lambda_0) \ .$$

We should note here to avoid confusion a very significant difference between the models discussed in this paper and those of Banks and Abbott. In those models, the Universe is undergoing an exponential inflation while ϕ rolls down its hill. In our models, ϕ rolls rapidly enough that Λ_{eff} always is significantly smaller than ρ_{matter}. Thus there is never an exponential "de Sitter" expansion. (One could easily imagine such an inflation occurring before the whole "roll-down" mechanism commences; however, we do not consider such possibilities here.)

The models we study in this paper are put forward in an exploratory spirit. The whole enterprise is too speculative for us to take any particular model very seriously as a theory of the cosmological constant. Rather our hope is to illustrate the problems and possibilities of this approach. Nevertheless, we are encouraged by the results of this reconnaisance. We find models that seem to have a good change of being made consistent with phenomenology.

In Sec. II we look at a couple of very simple models to orient ourselves. We find that there is a tendency for the effective cosmological constant in the class of models we study to relax to zero more slowly than the radiation density (which would give us a universe too cold and empty). In Appendix A we show that this is a feature of a whole class of models. In fact we suspect that this may be an instance of some thermodynamic constraint (which, however, we have not been able to formulate). This notwithstanding, we display in Sec. III a model which does not suffer from this defect. If one looks at the above equation one sees that the presence of matter whose stress-energy tensor is traceful can interfere with the feedback mechanism. One might worry that the presence of such matter (such as baryons, but not photons) could cause problems. Specifically, since our mechanism is one that essentially makes R relax to zero (rather than Λ as such), one might expect it to erase not only the effects of Λ but of the trace of matter stress energy as well. In Sec. IV we show that this particular difficulty does not usually arise. However, other things can go wrong inside baryonic matter and these we discuss also in Sec. IV. These difficulties are model dependent and hence might not arise in some version of this scheme. But in Sec. V we discuss a (desperate?) remedy for these problems if they do arise and suggest as well another version of the basic idea we are investigating in this paper. Finally in Sec. VI we give our conclusions, which are quite hopeful.

II. SOME UNSUCCESSFUL EXAMPLES

We consider Lagrangian densities of the form

$$\mathcal{L} = \mathcal{L}_{\text{matter}} - \Lambda_0 - R + \mathcal{L}_\phi(\phi(x), R) \ ,$$

where R is the scalar curvature of space-time and $\phi(x)$ is our compensating field. There are many interesting choices for $\mathcal{L}_\phi$. We will briefly examine three simple models for illustration, none of which will be satisfactory, but whose shortcomings will suggest appropriate modifications. These three are

$$\text{(A)}\quad \mathcal{L}_\phi = -\tfrac{1}{2}(\partial_\lambda \phi)^2 + \alpha\phi \ ,$$

$$\text{(B)}\quad \mathcal{L}_\phi = -\tfrac{1}{2}(\partial_\lambda \phi)^2 + \alpha\phi(-R)^\eta, \quad \eta > 0 \ ,$$

$$\text{(C)}\quad \mathcal{L}_\phi = -\tfrac{1}{2}(\partial_\lambda \phi)^2(-R)^{-\eta} + \alpha\phi, \quad \eta > 0 \ .$$

In case A the potential for ϕ is just $V(\phi) = -\alpha\phi$. The effective cosmological constant, henceforth denoted λ, is given by $\lambda \equiv \Lambda_0 - \alpha\phi$. The classical field $\phi(x)$ will just "roll" to ever more positive values and λ will plunge past zero and become increasingly negative. This is what happens without feedback. Clearly this is no good.

In case B we try an obvious feedback mechanism. The potential for ϕ is now $-\alpha\phi(-R)^\eta$ and $\lambda \equiv \Lambda_0 - \alpha\phi(-R)^\eta$. We will not solve the equations of motion here, but will just describe the result. As ϕ rolls to greater positive values λ decreases toward zero. The trace of Einstein's equation tells us that consequently $|R|$ also decreases toward zero. As a result the force tending to cause ϕ to roll diminishes. (We should note that the expansion of the Universe gives rise to a friction term for ϕ.) There are solutions when λ approaches zero asymptotically. However, a rather obvious disaster occurs. When the action is varied with respect to the metric to obtain Einstein's equations a term is found of the form $\eta\alpha\phi(-R)^{\eta-1}R^{\lambda\rho}$. As $\lambda \to 0$, $\alpha\phi(-R)^\eta \to \Lambda_0$, and this term goes to $[\eta\Lambda_0/(-R)]R^{\lambda\rho}$. This essentially amounts to a contribution to $(16\pi G_N)^{-1}$ of order Λ_0/R which blows up as $R \to 0$. Hence, the feedback mechanism of case B fails miserably. The essential lesson is that if $\mathcal{L}_\phi$ depends on R so as to provide the necessary feedback, then $(G_N)_{\text{eff}}$ depends on $\phi(x)$. One must ensure that $(G_N)_{\text{eff}}$ is not affected to an unacceptable degree.

Model C is a vast improvement over B in this respect, but still (barely) fails this test. The form of the kinetic energy term for ϕ is motivated by the following notion. As λ, and hence $-R$, approach zero the "inertial mass" and "friction" of ϕ will increase without bound, and its terminal velocity will also approach zero. Again, we shall find solutions in which λ approaches zero asymptotically.

The equations of motion for $\phi(x)$ and $g_{\lambda\rho}(x)$ are

$$\frac{-1}{\sqrt{g}}\partial_\sigma[\sqrt{g}\,(-R)^{-\eta}\partial^\sigma\phi]=\alpha\ , \tag{1}$$

$$G^{\lambda\rho}=-\tfrac{1}{2}T^{\lambda\rho}+\tfrac{1}{2}(\Lambda_0-\alpha\phi)g^{\lambda\rho}-\tfrac{1}{2}(-R)^{-\eta}[\partial^\lambda\phi\partial^\rho\phi-\tfrac{1}{2}g^{\lambda\rho}(\partial_\sigma\phi)^2]$$
$$-\tfrac{1}{2}\eta(-R)^{-\eta-1}(\partial_\sigma\phi)^2R^{\lambda\rho}+\frac{\eta}{2}(g^{\lambda\rho}\Box-\nabla^\lambda\nabla^\rho)[(-R)^{-\eta-1}(\partial_\sigma\phi)^2]\ . \tag{2}$$

Let us break up Einstein's equation [Eq. (2)] into its traceless and trace parts:

$$(R^{\lambda\rho}-\tfrac{1}{4}Rg^{\lambda\rho})[1+\tfrac{1}{2}\eta(-R)^{-\eta-1}(\partial_\sigma\phi)^2]=-\tfrac{1}{2}(T^{\lambda\rho}_{\rm rad}-\tfrac{1}{4}T_{\rm rad}g^{\lambda\rho})-\tfrac{1}{2}(-R)^{-\eta}[\partial^\lambda\phi\partial^\rho\phi-\tfrac{1}{4}g^{\lambda\rho}(\partial_\sigma\phi)^2]$$
$$+\frac{\eta}{2}(\tfrac{1}{4}g^{\lambda\rho}\Box-\nabla^\lambda\nabla^\rho)[(-R)^{-\eta-1}(\partial_\lambda\phi)^2]\ , \tag{3}$$

$$-R=-\tfrac{1}{2}(T_{\rm rad})^\lambda{}_\lambda+2(\Lambda_0-\alpha\phi)+\frac{\eta+1}{2}(-R)^{-\eta}(\partial_\sigma\phi)^2+\frac{3\eta}{2}\Box[(-R)^{-\eta-1}(\partial_\sigma\phi)^2]\ . \tag{4}$$

Here $T^{\lambda\rho}_{\rm rad}$ stands for the stress energy of the matter in $\mathcal{L}_{\rm matter}$. We take this matter to be radiation so that the trace of $T^{\lambda\rho}_{\rm rad}$ appearing in Eq. (4) vanishes. One can see that the effective value of $M_{\rm Pl}{}^2\equiv(16\pi G_N)^{-1}$ in the traceless part of Einstein's equation [Eq. (3)], that is the coefficient of $(R^{\lambda\rho}-\frac{1}{4}Rg^{\lambda\rho})$, is just

$$(M_{\rm Pl}{}^2)_{\rm eff}=1+\frac{\eta}{2}(-R)^{-\eta-1}(\partial_\sigma\phi)^2\ . \tag{5}$$

At first glance it would seem that this model must suffer the same disaster as before. $(M_{\rm Pl}{}^2)_{\rm eff}$ would seem to blow up as $R\to0$. However, recall that as $R\to0$ it will also be true that $(\partial_\sigma\phi)^2\to0$.

Now we will solve the above equations under certain simplifying assumptions. We assume a metric of the Friedman-Robertson-Walker (FRW) form. We also assume that initially the radiation density, $\rho_{\rm rad}$, is considerably larger than the effective cosmological constant, $\lambda\equiv\Lambda_0-\alpha\phi$. In that case, according to Eq. (3), if $(M_{\rm Pl}{}^2)_{\rm eff}>0$ the scale factor, r, in the FRW metric will expand approximately as $t^{1/2}$. If, on the other hand, $(M_{\rm Pl}{}^2)_{\rm eff}<0$, r will expand approximately exponentially. Since we do not wish this to happen, we must require that $(M_{\rm Pl}{}^2)_{\rm eff}>0$ and $r\sim t^{1/2}$. We take ϕ to depend only on t, that is we ignore spatial fluctuations. And finally we define γ by $-R\equiv2\gamma\lambda$.

First, consider Eq. (1), which with these assumptions becomes

$$t^{-3/2}\frac{\partial}{\partial t}[t^{3/2}\dot\phi(-R)^{-\eta}]=\alpha\ . \tag{6}$$

Asymptotically, for large t, the solution is

$$\begin{aligned}&\dot\phi\simeq\tfrac{2}{5}\alpha t(-R)^\eta\ ,\\ &\therefore\dot\lambda\simeq-\tfrac{2}{5}\alpha^2t(-R)^\eta=-\tfrac{2}{5}\alpha^2t(2\gamma\lambda)^\eta\ ,\\ &\therefore\lambda\simeq\left[\frac{\eta-1}{5}(2\gamma)^\eta\alpha^2t^2\right]^{1/(1-\eta)},\\ &\therefore-R=2\gamma\lambda\simeq\left[\frac{\eta-1}{5}2\gamma\alpha^2t^2\right]^{1/(1-\eta)}.\end{aligned} \tag{7}$$

Observe that the effective cosmological constant λ falls asymptotically to zero as $t^{-2/(\eta-1)}$.

Now we turn to Eq. (4), the trace of Einstein's equations. It is easily seen using Eq. (7) that the $\Box$ term in Eq. (4) vanishes, so that it reduces to

$$\begin{aligned}-R&=2\lambda-\frac{\eta+1}{2}\dot\phi^2(-R)^{-\eta}\ ,\\ \therefore2\gamma\lambda&=2\lambda-\frac{\eta+1}{2}(\tfrac{4}{25}\alpha^2t^2)(2\gamma\lambda)^{+\eta}\ ,\\ \therefore1&=\frac{1}{\gamma}-\frac{\eta+1}{2}\frac{4}{25}\alpha^2t^2(2\gamma\lambda)^{\eta-1}\\ &=\frac{1}{\gamma}-\frac{\eta+1}{2}\frac{4}{25}\alpha^2t^2\left[\frac{5}{\eta-1}\frac{1}{2\gamma}\frac{1}{\alpha^2t^2}\right]\\ &=\frac{1}{\gamma}\left[1-\frac{1}{5}\frac{\eta+1}{\eta-1}\right],\\ \therefore\gamma&=\frac{2}{5}\left[\frac{2\eta-3}{\eta-1}\right].\end{aligned} \tag{8}$$

Finally, we can evaluate $(M_{\rm Pl}{}^2)_{\rm eff}$. Equation (5) gives

$$\begin{aligned}(M_{\rm Pl}{}^2)_{\rm eff}&=1-\frac{\eta}{2}\dot\phi^2(-R)^{-\eta-1}\\ &=1-\frac{\eta}{2}\frac{4}{25}\alpha^2t^2(-R)^{\eta-1}\\ &=1-\frac{\eta}{2}\frac{4}{25}\alpha^2t^2\left[\frac{5}{\eta-1}\frac{1}{2\gamma}\frac{1}{\alpha^2t^2}\right]\\ &=1-\frac{1}{5\gamma}\frac{\eta}{\eta-1}\\ &=\frac{3}{2}\left[\frac{\eta-2}{2\eta-3}\right].\end{aligned} \tag{9}$$

Notice that for $(M_{\rm Pl}{}^2)_{\rm eff}$ to be positive, as required, η must be larger than 2. In that case λ, which falls as $t^{-2/(\eta-1)}$, decreases more slowly than $\rho_{\rm rad}$, which fall as t^{-2}. Eventually λ will come to dominate over $\rho_{\rm rad}$ and r will commence to grow more rapidly than $t^{1/2}$, at which point our approximations become invalid. In any event, this will not give a realistic cosmology; for we know that λ today cannot be much larger than the matter density.

If $\eta < 2$, so that $(M_{\rm Pl}{}^2)_{\rm eff} < 0$, the scale factor grows (and hence $\rho_{\rm rad}$ falls) approximately exponentially with time, while λ only falls as a power. Again, therefore, λ falls more slowly than $\rho_{\rm rad}$. We shall see in the next section and in Appendix A that even for some more complicated actions this remains the case. Why this should be so is not clear to us. Perhaps a thermodynamic argument exists.

Let us make several comments on the foregoing. (1) It appears possible to have a feedback mechanism that works, at least at the classical level. (2) By introducing R into the action for ϕ, we run the risk of changing $(M_{\rm Pl}{}^2)_{\rm eff}$ too much. (3) This effect on gravity is huge and unacceptable if R appears in the potential of ϕ, but is more manageable if R appears in the kinetic energy of ϕ. (4) Nevertheless, to achieve a "realistic" model we will have to worry about making this effect sufficiently small. (5) We must also worry about what happens inside a matter distribution for which $(T_{\rm matter})^{\lambda}{}_{\lambda} \neq 0$. The whole question of what happens inside such a matter distribution will be taken up in Sec. IV where we will see that there are problems, but not necessarily any fatal problems. And (6) it seems very artificial to have negative powers of R appearing in the action. In the next section we will consider models where this is the result of integrating out another field. Even so, it seems peculiar to have a physical effect grow stronger as spacetime becomes flatter. But this, in one way or another, is required by the present approach.

III. A MORE SUCCESSFUL EXAMPLE

Let us consider now a Lagrangian density of the form

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\lambda \phi)^2 X^2 + \alpha\phi - V(X,R) - \tfrac{1}{2}\delta(\partial_\lambda X)^2/X^2 - R - \Lambda_0 + \mathcal{L}_{\rm matter} \, . \tag{10}$$

Here, the potential $V(X,R)$ is to be such that the equation of motion determines X to be of order $(-R)^{-1}$. Then the kinetic energy term for ϕ is essentially equivalent to the $\eta=2$ case of model C considered in the previous section. We recall that $\eta=2$ was a borderline case where $(M_{\rm Pl}{}^2)_{\rm eff}=0$. What we seek here is some form of $V(X,R)$ that will allow $(M_{\rm Pl}{}^2)_{\rm eff}$ to be positive, so as to give $r \sim t^{1/2}$ and $\rho_{\rm rad} \sim t^{-2}$. In this we shall succeed. The form of the kinetic energy for X is determined by the following consideration. As $-R \to 0$ the minimum of $V(X,R)$ will be at $X_{\rm min} \sim -1/R \to \infty$. If X is to be near $X_{\rm min}$ (otherwise the feedback mechanism will not work) it must be that the kinetic energy of X will be well behaved as $X \to \infty$. The coefficient δ appearing in that term is a constant parameter. The form we have chosen for the kinetic energy of X suggests a more natural variable to use would be $e^{\sigma} \equiv X$. Then $\mathcal{L}$ could be rewritten as

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\lambda \phi)^2 e^{2\sigma} + \alpha\phi - V(e^{\sigma},R) - \tfrac{1}{2}\delta(\partial_\lambda \sigma)^2 - R - \Lambda_0 + \mathcal{L}_{\rm matter} \, . \tag{11}$$

A final point to be noted about Eq. (10) is that we need not have chosen a linear potential for ϕ. As long as $V(\phi)$ falls monotonically for a distance of at least Λ_0 it is satisfactory for our purposes.

For the potential $V(X,R)$ we will choose the form

$$V(X,R) = c(XR+1)^2 + \frac{1}{X}(a + b\ln X) = c(e^{\sigma}R+1)^2 + e^{-\sigma}(a+b\sigma) \, . \tag{12}$$

The reason for the $c(XR+1)^2$ term is clear; namely, it has a minimum at $X=-1/R$. In fact any function $V(XR)$ would do just as well as long as $V(z)$ had a minimum at $z \sim 1$. We chose the perfect-square form for simplicity of discussion. It is less obvious why we added the second term in Eq. (12). Without it, as shown in Appendix A, $(M_{\rm Pl}{}^2)_{\rm eff} \le 0$. (This added term is of order $1/X$ so as not to disturb too greatly the minimum at $X \simeq -1/R$, and also so as not to contribute too much to the right-hand side of Einstein's equation.) In fact with $a \neq 0$ but $b=0$ we still find $(M_{\rm Pl}{}^2)_{\rm eff} \le 0$. Thus the logarithmic term plays the critical role of allowing $(M_{\rm Pl}{}^2)_{\rm eff} > 0$. Doubtless other forms of V would work but have not done an exhaustive survey.

We will not present all the details of the solution to the equations of motion, which parallel closely those of the model in the previous section. We again seek solutions where ϕ and X are functions of t only, again assume $r \sim t^{1/2}$, and again define γ by $-R = 2\gamma\lambda$, where $\lambda \equiv \Lambda_0 - \alpha\phi$ is the effective cosmological constant. The equations of motion are given in Appendix B. The analogue of Eq. (4) [and (8)] is

$$-R = -\tfrac{1}{2}(T_{\rm rad})^{\lambda}{}_{\lambda} + 2\lambda + \tfrac{1}{2}\dot\phi^2 X^2 + 2c(XR+1) - 2(a+b\ln X)/X + 6\Box[2cX(XR+1)] - \tfrac{1}{2}\delta \dot X^2/X^2 \, . \tag{13}$$

As before we are assuming that the matter is radiation, so $(T_{\rm rad})^{\lambda}{}_{\lambda}=0$. The analogue of Eqs. (6) and (7) is

$$t^{-3/2}\frac{\partial}{\partial t}(t^{3/2}\dot\phi X^2) = \alpha \, . \tag{14}$$

Asymptotically for large t,

$$\dot\phi \simeq \tfrac{2}{5}\alpha t X^{-2}, \quad \dot\lambda \simeq -\tfrac{2}{5}\alpha^2 t X^{-2} \, . \tag{15}$$

Now, define $\Delta \equiv XR+1$ so that $X = -1/R(1-\Delta)$. It will turn out that Δ is small compared to 1. From Eq. (15) and the definition of γ, it then follows that

$$\dot\lambda \simeq -\frac{8\alpha^2}{5} t \gamma^2 \lambda^2 \, . \tag{16}$$

It will prove convenient to assume that $b \ll 1$ and expand in powers of b. Hence we shall want to expand γ as $\gamma = \gamma_0(1+\beta\ln X)$. Shortly it will appear that $\beta = O(b)$. Solving Eq. (16) gives the result

$$\lambda(1+2\beta-2\beta\ln X)^{-1} \simeq \frac{5}{4\alpha^2\gamma_0{}^2} t^{-2} \, . \tag{17}$$

Up to logarithmic corrections, then, $\lambda \sim t^{-2}$, as expected and desired. From Eq. (17) it follows that R, $\dot{\phi}$, and X are given by

$$R \simeq -\frac{5}{2\alpha^2\gamma_0}t^{-2}(1+2\beta-\beta\ln X) ,$$
$$\dot{\phi} \simeq \frac{5}{2\alpha^3\gamma_0^2}t^{-3}(1+4\beta-2\beta\ln X) , \quad (18)$$
$$X \simeq \frac{2\gamma_0\alpha^2}{5}t^2(1-2\beta+\beta\ln X) .$$

The equation of motion for X [see Eq. (B6) in Appendix B] gives (in the limit $\delta \ll 1$)

$$\Delta \simeq -\frac{1}{2c\gamma_0^2\alpha^2}t^{-2}\left[1+4\beta-2\beta\ln X + \frac{5\gamma_0}{2}(1+2\beta-\beta\ln X) \times(b-a-b\ln X)\right] . \quad (19)$$

Finally, the trace of Einstein's equation [Eq. (13)] gives [the $\Box$ term vanishes to order $b/(\ln X)^2$ and can be neglected when X is large]

$$\gamma_0(1+2\beta-\beta\ln X)(1+b+a+b\ln X) \simeq \tfrac{2}{5}(1-\beta-2\beta\ln X) . \quad (20)$$

This implies (equating equal powers of b)

$$\gamma_0 \simeq \frac{2}{5}\frac{1}{1+a}\left[1+\frac{2b}{1+a}\right] , \quad \beta = -b\frac{1}{1+a} . \quad (21)$$

At last $(M_{\rm Pl}^2)_{\rm eff}$ can be evaluated. From the traceless part of Einstein's equation [Eq. (B4)]

$$(M_{\rm Pl}^2)_{\rm eff} = 1+2c\Delta X . \quad (22)$$

After some algebra, using Eqs. (18), (19), and (21) we find

$$(M_{\rm Pl}^2)_{\rm eff} = -b\left[\frac{\ln X+1}{\ln X+3}(\ln X-2)-(\ln X-1)\right] = 3b+O(b/\ln X) . \quad (23)$$

So, for $b>0$, $(M_{\rm Pl}^2)_{\rm eff}>0$. In order to find the crucial ratio of the effective cosmological constant λ to the radiation density $\rho_{\rm rad}$ we must solve the traceless part of Einstein's equation. If $\rho_{\rm rad}=A/t^2$ we find [see Eq. (B3) in Appendix B]

$$3b\left\{\frac{3}{2}\left[\frac{\ddot{r}}{r}-\left[\frac{\dot{r}}{r}\right]^2\right]\right\} \simeq -\frac{1}{2}\left[\frac{A}{t^2}+\frac{3}{4}\frac{1}{\alpha^2\gamma_0^2t^2}\right] , \quad (24)$$

$$\therefore A \simeq \frac{9}{2}b-\frac{3}{4\alpha^2\gamma_0^2} = \frac{9}{2}b-\frac{75}{16\alpha^2} .$$

Since, from Eqs. (17) and (21), $\lambda=\frac{5}{4}(1/\alpha^2\gamma_0^2t^2) = 125/16\alpha^2t^2$, it follows that

$$\frac{\lambda}{\rho_{\rm rad}} \simeq \frac{5}{(\frac{72}{25}\alpha^2 b-3)} . \quad (25)$$

For sufficiently large α^2 we can have λ small compared to $\rho_{\rm rad}$, consistent with our assumptions.

We have found a model which has the following behavior classically. Given an initial positive Λ_0 (which is small enough initially compared to $\rho_{\rm rad}$ that the Universe expands approximately as $t^{1/2}$) then the effective cosmological constant λ declines in tandem with the redshifting radiation density, always remaining smaller by an approximately constant fraction. This is very much what one would have hoped for. It even allows for various phase transitions to occur which change the cosmological constant. As long as the change $\Delta\lambda$ from one of these transitions is less than λ (which in turn is less than $\rho_{\rm rad}\sim T^4$), so that λ remains positive, the mechanism will operate as desired.

This model alone cannot give a satisfactory cosmology, however. There are two reasons for this. The first has to do with the fact that $(M_{\rm Pl}^2)_{\rm eff}=1+2c\Delta X\simeq 3b$. If we look at the full Einstein's equation [Eq. (B2)] instead of $G^{\lambda\rho}$ we find

$$G^{\lambda\rho}-(2c\Delta X)R^{\lambda\rho}=(1-2c\Delta X)G^{\lambda\rho}-(c\Delta X)Rg^{\lambda\rho} .$$

In order to reproduce classical general relativity it must be that $c\Delta X \ll 1$. Instead, $c\Delta X\simeq(3b-1)/2\simeq-\frac{1}{2}$. We will discuss ways to obviate this difficulty later. The second problem has to do with the solution of Einstein's equation inside baryonic matter. This is the subject of the next section. Again, this problem or set of problems does not appear to be fatal.

IV. INSIDE MATTER

There are two sets of potential difficulties that we might anticipate for models of this type that arise when we consider the interior of matter. In empty space or in, say, a nonzero electromagnetic field the stress-energy tensor has zero trace. However, inside matter whose stress-energy tensor has nonzero trace (such as nuclear matter) there are contributions to $-R$ which can be huge compared to those produced by λ [see Eq. (B3)].

The first potential difficulty is that the compensating field ϕ might be expected to continue rolling down its potential hill until $2\lambda(x)$ is sufficiently negative to cancel off $-\frac{1}{2}(T_{\rm matter})^{\lambda}{}_{\lambda}$ in the trace of Einstein's equation [Eq. (B3)]; that is, until $-R\simeq 0$ inside the matter. In other words, our feedback mechanism tells ϕ to keep rolling until $-R\simeq 0$. So it would seem that ϕ would always act to make $-R\simeq 0$ inside any matter, and thus effectively to cause all matter to gravitate like radiation, which has a traceless stress-energy tensor. Fortunately, this does not (generally) occur, as we shall see.

The second potential problem is that the rapid spatial variation of $-R$, and hence of the field called X, inside matter can itself give rise to unacceptably large contributions to the stress energy. We will illustrate and discuss these two issues in turn.

We want to consider a sphere of nuclear-matter density surrounded by "empty space." What we mean by empty space is just space filled by the cosmic blackbody radiation. We are thinking in particular of a time late enough in the history of the Universe that the blackbody radiation density is negligible compared to nuclear-matter densities. Since the compensating field mechanism is operating, it is also then true that λ in the "empty space" around the sphere is negligible compared to the density of the sphere. So we will, in fact, set λ and R equal to zero outside the sphere. Let the sphere have radius l, density ρ_0, and pressure $p_0 \ll \rho_0$. We would like to solve for λ and $-R$ inside the sphere. We are hoping that the "sag" of λ inside the sphere is much less than ρ_0. Let us examine the model of the previous section. The equation of motion for ϕ inside the sphere reduces in the case of a spherically symmetric ansatz to

$$-\nabla\cdot[(\nabla\phi)X^2]=\alpha \ , \tag{26}$$

or, since $\lambda=\Lambda-\alpha\phi$,

$$\nabla\cdot[(\nabla\lambda)X^2]=\alpha^2 \ .$$

In spherical coordinates r,θ,ϕ, we have

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial\lambda(r)}{\partial r}X^2\right]=\alpha^2 \ . \tag{27}$$

And, if $\partial\lambda/\partial r=0$ at $r=0$,

$$\frac{\partial\lambda(r)}{\partial r}=\tfrac{1}{3}\alpha^2 rX^{-2} \ . \tag{28}$$

The boundary condition at $r=l$ is $\lambda=0$. Now, suppose we can ignore all contributions to the stress energy except those of λ and ρ_0 inside the sphere. (This means we are ignoring the second of the potential difficulties referred to at the beginning of this section. In fact, in this model, this will turn out to be an unjustified assumption. However, in any realistic model this must turn out to be justified. And for such a realistic model our present arguments would, *mutatis mutandis,* apply.) Then inside the sphere

$$X^{-1}\simeq -R\simeq 2\lambda+\tfrac{1}{2}\rho_0 \tag{29}$$

and

$$\frac{\partial\lambda}{\partial r}\simeq\tfrac{1}{3}\alpha^2 r(\tfrac{1}{2}\rho_0+2\lambda)^2 \ . \tag{30}$$

If $|\lambda|\ll\rho_0$, as desired,

$$\frac{\partial\lambda}{\partial r}\simeq\tfrac{1}{12}\alpha^2 r\rho_0{}^2 \ ,$$

$$\lambda\simeq\tfrac{1}{24}\alpha^2\rho_0{}^2(r^2-l^2) \ , \tag{31}$$

$$|\lambda|_{\max}=|\lambda_{(r=0)}|\simeq\tfrac{1}{24}\alpha^2\rho_0{}^2 l^2 \ .$$

Since the mass of the sphere is just $\frac{4}{3}\pi l^3\rho_0\equiv m$,

$$\frac{|\lambda|}{\rho_0}\lesssim\frac{1}{24}\alpha^2\left[\frac{3}{4\pi}m\right]\Big/ l\simeq\frac{+\alpha^2}{4}\frac{r_{\rm Schw}}{l} \ , \tag{32}$$

where $r_{\rm Schw}$ is the Schwarzschild radius of a sphere of mass m with $16\pi G_N=1$. Now, for a proton, say, $r_{\rm Schw}/l\sim 10^{-38}$. So that the amount by which ϕ and λ "sag" inside a proton is totally negligible. What happens is that ϕ is "pinned" outside the proton to its "empty space" value, and hence cannot fall very much inside the proton. In a matter-dominated universe, the compensating field will not act to cancel off the trace part of the matter stress energy. In this context it is wrong to use spatial averages of ρ and R in studying the behavior of ϕ; overwise one would come to very different conclusions. For example, for a star it would be wrong to use the above calculation with l set equal to the radius of the star and ρ_0 set equal to its average density. Rather, in the space between the nuclei, $-R\simeq 0$ (though the Riemann curvature is of order $\langle\rho\rangle_{\rm star}$), so that there the field is "pinned" as before. ϕ can only sag inside the individual nuclei. (One must also take into account the electrons, but the qualitative conclusion is unaffected.) However, for a neutron star the situation is quite different. There the relevant quantity in Eq. (32) would be $r_{\rm Schw}/l$ for the whole star, which can be significant.

In the foregoing calculation we assumed we could neglect the contributions to the stress energy inside a nucleon except from λ and ρ_0. This is wrong for the model of Sec. III which we used. An inspection of the trace of Einstein's equation [Eq. (B3)] reveals several dangerous terms:

$$\begin{aligned}-R &= 2\lambda+\tfrac{1}{2}(\partial_\sigma\phi)^2X^2+2c(XR+1)\\ &\quad -2(a+b\ln X)/X+\frac{\delta}{2}(\partial_\sigma X)^2/X^2\\ &\quad +6\Box[2c(XR+1)X] \ .\end{aligned} \tag{33}$$

The terms that cause concern are $(\delta/2)(\partial_\sigma X)^2/X^2$, and $6\Box[2c(XR+1)X]$. Returning to our analysis of a spherical distribution of matter we have, using the equation of motion for X [see Eq. (B6)],

$$2\eta\Delta R=-(\nabla\phi)^2X+\frac{1}{X^2}(b-a-b\ln X)+O(\delta) \ . \tag{34}$$

We are looking for nearly static solutions, and setting $\delta=0$ for simplicity. Also [see Eq. (28)]

$$\frac{\partial\lambda}{\partial r}=\tfrac{1}{3}\alpha^2 rX^{-2}, \quad \therefore |\nabla\phi|=\tfrac{1}{3}\alpha rX^{-2} \ . \tag{35}$$

So

$$\begin{aligned}2c\Delta X &= -(\nabla\phi)^2X^2/(XR)-(b-a-b\ln X)/(XR)\\ &= \left[\frac{1}{1-\Delta}\right](\tfrac{1}{9}\alpha^2r^2X^{-1}-b+a+b\ln X)\end{aligned} \tag{36}$$

and

$$6\Box(2c\Delta X)=6\Box[(1-\Delta)^{-1}(\tfrac{1}{9}\alpha^2r^2X^{-1}-b+a+b\ln X)] \ . \tag{37}$$

The term proportional to α^2 is of order $\alpha^2 R$. Since we have $\alpha^2>1/b>\ln X\gg 1$, this contribution is much greater than the left-hand side of Eq. (33). Far worse, however, is the contribution from $\Box(b\ln X)$ which is of order b/l^2. Since $1/l^2\rho$ for a nucleon is roughly 10^{38}, this term completely overwhelms the ordinary stress energy of the nuclear matter itself. We have set $\delta=0$, but if $\delta\neq 0$ then the term $\frac{1}{2}\delta(\partial_\sigma X)^2/X^2$ also contributes of order δ/l^2.

Thus we have identified some potentially serious difficulties with our approach. The gravitational equations inside a nucleon can receive anomalous and huge contributions. To some extent this may be a peculiar feature of the particular model we have studied. For example, the really severe troublesome contribution from the $\Box$ term as we saw came from the $b \ln X/X$ term in the potential $V(X,R)$. The rest of the terms in that potential led only to contributions that were of order unity times ρ_0 rather than order 10^{38} times ρ_0. This suggests that other potentials for X might be found that do not have this problem, though we have not found any. Furthermore, if instead of $(\delta/2)(\partial_\sigma X)^2/X^2$ the kinetic energy of X had been of the form $(\delta/2)(\partial_\sigma X)/X^n$, with $n \geq \frac{5}{2}$, this term would have given no trouble. It would seem then that, while these effects are a danger, there is no reason to conclude that they will be fatal to the whole approach.

We can summarize this section in three statements. (1) The second difficulty that we anticipated might arise inside baryonic matter does arise in the model of Sec. III. That is, the rapid spatial variation of R gives rise to huge and unacceptable contributions to the stress energy. (2) These objectionable terms are very dependent on the details of the model, and there is reason to suspect that models can be found where this trouble does not arise. And (3) the first difficulty we anticipated does not arise—at least in models where the second difficulty is absent—because of the pinning of λ to its empty space value outside of the nucleon.

V. A REMEDY FOR SIDE EFFECTS AND ANOTHER (LESS PROMISING) APPROACH

So far we have shown that it is possible to have the effective cosmological constant relax toward zero as fast as the red-shifting blackbody radiation density. We can imagine the Universe starting (when $T \sim M_{\rm Pl}$) with a cosmological constant Λ_0 near $(M_{\rm Pl})^4$, and ending up at the present epoch with an effective cosmological constant of less than $(3K)^4$. And, as noted above, this can still hold even if the Universe has undergone in the meantime various phase transitions [such as the grand-unified-theory (GUT), weak, and chiral and confinement phase transitions]. Were there no baryonic matter around but only radiation we would have in hand a fairly successful phenomenology. The two main negative side effects for phenomenology are, first, that there is an extra term of the form $(c\Delta X)Rg^{\lambda\rho}$ in Einstein's equation, where $c\Delta X \sim 1$ (see Sec. III), and, second, that the stress energy inside nucleons or nuclei can be badly distorted by anomalous contributions (see Sec. IV). These problems will not be too serious until later in the evolution of the Universe when baryonic matter comes to be important. Therefore one can imagine that a mechanism similar to that embodied in the model of Sec. III operates in the very early Universe, but that at some critical temperature (or curvature[5]) a phase transition occurs which has the effect of changing the character of the effective action in such a way that the diseases mentioned above are cured. In the particular model of Sec. III what would have to happen in such a phase transition is that what we called Δ would have to become exceedingly small. If Δ were zero then the extra term in Einstein's equation, $(c\Delta X)Rg^{\lambda\rho}$, would simply vanish, and the troublesome term $6\Box(2c\Delta X)$ would do so as well. Now, Δ is given by Eq. (B6). It could therefore be suppressed if $(\partial_\lambda\phi)^2X^2$, $(a+b\ln X)/X$, and $\frac{1}{2}\delta(\partial X/X)^2$ were all suppressed. One could imagine, for example, that each of these terms depended upon another "dilaton" field, $Y \equiv e^\tau$, analogous to $X=e^\sigma$. If, say, the kinetic energy term for ϕ were $-\frac{1}{2}(\partial_\lambda\phi)^2e^{2\sigma}e^{+2\tau}$ and the potential for $X(\equiv e^\sigma)$ were $e^{-\sigma}e^{-\tau}(a+b\sigma)$; these terms would be exponentially suppressed if a phase transition were to occur in which τ suddenly sharply increased. This device adds another layer of artificiality to the model of Sec. III. But at least it shows that these difficulties are not unsuperable in principle, and perhaps an effect such as we have described could ultimately form a part of a simple and more realistic theory,

This line of thought suggests a variant of the ideas we have been exploring up to now. The models we have so far examined all had feedback mechanisms which operated continuously, as it were. That is, as $-R$ changed smoothly the action for the compensating field, ϕ, also changed smoothly in such a way as to slow down the motion of ϕ when λ approached zero. Alternatively, one could envision a situation in which ϕ rolls freely, without any significant feedback from the metric, until a critical value of the scalar curvature is reached, at which point a sudden phase transition[5] causes the motion of the compensating field to be quickly damped. For example, if the Lagrangian for ϕ is as before $-\frac{1}{2}(\partial_\lambda\phi)^2e^\sigma+\alpha\phi$, then the equation of motion for ϕ is $-(1/\sqrt{g})\partial_\lambda \times[\sqrt{g}(\partial^\lambda\phi)e^\sigma]=\alpha$. If a phase transition causes σ suddenly to increase then $(\partial^\lambda\phi)$ will be squeezed to an exponentially small value. All of this sounds very simple (and as we observed some such effect could cure the side effects of the other mechanism). However, by itself, considered as a mechanism to solve the cosmological-constant problem; it has severe difficulties. Without going into quantitative detail we will just list and describe qualitatively the chief difficulties we have found. (1) If the kinetic energy of the compensating field ϕ were $-\frac{1}{2}(\partial_\lambda\phi)^2e^\sigma$ then, as ϕ rolls freely down its potential hill with σ remaining nearly constant, the scalar curvature $-R$ will not approach zero as λ does. This is simply because much of the potential energy of ϕ is converted into kinetic energy (of order Λ_0 if ϕ starts initially at $\phi=0$), which contributes to $-R$ through Einstein's equation. A possible, if radical, remedy would be to have a kinetic energy term for ϕ which has a traceless stress energy, such as $((\partial_\lambda\phi)^2)^2$. Another possibility would be some kind of extra damping for ϕ. (2) The kinetic term for ϕ acts as a steep bottomless potential,

$$V=\tfrac{1}{2}(\partial_\lambda\phi)^2e^\sigma=-\tfrac{1}{2}[\dot\phi^2-(\nabla\phi)^2]e^\sigma ,$$

for the field σ. Even if there were some potential barrier in place to keep σ roughly constant until the phase transition occurred, one must take into account the possibility that σ would tunnel through this barrier prematurely. (3) Finally, if ϕ rolls freely down its potential hill fast enough that λ stays small compared to the red-shifting $\rho_{\rm rad}$, it will take a very small increment of time for λ to race past $\lambda \sim 10^{-120}$ (which is the value we would like it to have today), cross zero, and become hugely negative. There may not be sufficient time for the metric to respond or for the phase transition to occur.

All of these considerations led us to prefer rather to search for examples of the other "continuous feedback" mechanism. Perhaps with more imagination a simple way to implement successfully this other approach could be found.

VI. CONCLUSION

As emphasized in the Introduction we have advanced these ideas in an exploratory spirit, recognizing that our models are somewhat artificial. However we are encouraged by the fact that in its main outlines the approach seems to work. There are some problems, but they are not as severe as one might have expected, and seem remediable. Indeed, on the hopeful side, it is good that there is some area of phenomenological difficulty as there is then some chance that a successful model will have something interesting to say which can be checked by experiment.

The whole approach we are exploring is in spirit akin to the study of Higgs-boson potentials or to the search for a satisfactory model of inflation. The positing of some complicated Higgs-boson potential does not really give a completely dynamical understanding of a pattern of spontaneous symmetry breaking, but it shows that it is possible, and can serve as a phenomenological description of the process which one can study and even use to make predictions. Similarly, the invention of various scenarios for inflation (old, new, supersymmetric, Kaluza-Klein, and so forth) and of various forms of the "inflaton" potential has, at least up to now, just shown us what some of the problems and possibilities are, without giving us anything approximating a "theory" of inflation.

From the point of view of real physical understanding, the approaches of supersymmetry or that of Mottola[2] seem somewhat more principled than ours. On the other hand, the approach here may sooner give an easily studied and phenomenologically successful model.

We have neglected completely quantum effects. In particular one should worry about radiative corrections that might induce terms that would destroy the feedback mechanism, tunneling, particle creation, and, generally, about how we have made the problems of quantum gravity worse. We do not see any reason, as yet, why quantum effects would prove harmful to the present approach.

At least at the classical level, then, we believe we have shown that it is possible to find models in which the cosmological "constant" relaxes to zero rapidly enough that one ends up with a matter-filled universe similar to our own. There are some painful side effects associated with the presence of baryonic matter; but there seem to be ways to palliate and perhaps even cure these. We hope that with more work better mechanisms and simpler models may be found.

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APPENDIX A

To show how general the phenomenon remarked on in the text whereby the ratio $\lambda/\rho_{\rm rad}$ tends to increase for a wide class of models of this type, we look at the case

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\lambda\phi)^2 X^\eta + \alpha\phi - \tfrac{1}{2}\delta(\partial_\lambda X)^2/X^2 - V(X,R) - R - \Lambda_0 + \mathcal{L}_{\rm matter} , \tag{A1}$$

where we take

$$V(X,R) = \sum_n a_n (XR)^n = f(XR) . \tag{A2}$$

Einstein's equation is

$$G^{\lambda\rho} = -\tfrac{1}{2}T^{\lambda\rho}_{\rm matter} + \tfrac{1}{2}(\Lambda_0 - \alpha\phi)g^{\lambda\rho} - \tfrac{1}{2}[\partial^\lambda\phi\partial^\rho\phi - \tfrac{1}{2}g^{\lambda\rho}(\partial_\sigma\phi)^2]X^\eta - \tfrac{1}{2}V(XR)g^{\lambda\rho} - \frac{\delta}{2}[\partial^\lambda X\partial^\rho X - \tfrac{1}{2}g^{\lambda\rho}(\partial_\lambda X)^2]/X^2$$
$$- (\partial V/\partial R)R^{\lambda\rho} - 2[(g^{\lambda\rho}\Box - \nabla^\lambda\nabla^\rho)(\partial V/\partial R)] . \tag{A3}$$

The trace is

$$-R = -\tfrac{1}{2}T_{\rm matter} + 2\lambda + \tfrac{1}{2}(\partial_\sigma\phi)^2 X^\eta - 2V(XR) + \frac{\delta}{2}(\partial_\sigma X)^2/X^2 - (\partial V/\partial R)R - 6\Box(\partial V/\partial R) . \tag{A4}$$

The traceless part is

$$(R^{\lambda\rho}-\tfrac{1}{4}Rg^{\lambda\rho})\left[1+\frac{\partial V}{\partial R}\right]=-\tfrac{1}{2}(T^{\text{traceless}}_{\text{matter}})^{\lambda\rho}-\tfrac{1}{2}[\partial^\lambda\phi\partial^\rho\phi-\tfrac{1}{4}g^{\lambda\rho}(\partial_\sigma X)^2]X^\eta$$
$$-\frac{\delta}{2}[\partial^\lambda X\partial^\rho X-\tfrac{1}{4}g^{\lambda\rho}(\partial_\sigma X)^2]/X^2-2[(\tfrac{1}{4}g^{\lambda\rho}\Box-\nabla^\lambda\nabla^\rho)(\partial V/\partial R)]\ . \qquad \text{(A5)}$$

The equation of motion for X is

$$-\partial V/\partial X-\frac{\eta}{2}(\partial_\lambda\phi)^2X^{\eta-1}=-\delta(\partial_\lambda X)^2/X^2-\delta\frac{1}{\sqrt{g}}\partial_\sigma[\sqrt{g}\,(\partial^\sigma X)X^{-2}]\ , \qquad \text{(A6)}$$

while that for ϕ is

$$-\frac{1}{\sqrt{g}}\partial_\sigma[\sqrt{g}\,(\partial^\sigma\phi)X^\eta]=\alpha\ . \qquad \text{(A7)}$$

We assume ϕ, R, X, etc., depend only on t and that $(M_{\text{Pl}}{}^2)_{\text{eff}}=1+\partial V/\partial R>0$ so that $r\sim t^{1/2}$. Then

$$t^{-3/2}\frac{\partial}{\partial t}(t^{3/2}\dot\phi X^\eta)=\alpha,\quad \dot\phi\simeq\tfrac{2}{5}\alpha tX^{-\eta},\quad \therefore\dot\lambda\simeq-\tfrac{2}{5}\alpha^2tX^{-\eta}\ . \qquad \text{(A8)}$$

We define β and γ by $X\simeq-(1/R)\beta$ and $-R\simeq2\gamma\lambda$, so that

$$\dot\lambda\simeq-\tfrac{2}{5}\alpha^2\beta^{-\eta}t(2\gamma)^\eta\lambda^\eta\ . \qquad \text{(A9)}$$

For large t

$$\lambda\simeq\left[\frac{5}{\eta-1}\left[\frac{\beta}{2\gamma}\right]^\eta\frac{1}{\alpha^2t^2}\right]^{1/(\eta-1)}\ . \qquad \text{(A10)}$$

The equation for X becomes, in the limit we can neglect δ,

$$-\partial V/\partial X+\frac{\eta}{2}\dot\phi^2X^{\eta-1}=0,\quad \partial V/\partial X=\frac{2\gamma}{\beta}\frac{2\eta}{5(\eta-1)}\lambda^2\ . \qquad \text{(A11)}$$

Equation (A4) becomes, if we assume the matter is radiation,

$$+2\gamma\lambda=2\lambda-\frac{2}{5(\eta-1)}\lambda-2V+\frac{\delta}{2}\dot X^2/X^2-(\partial V/\partial R)R-6\Box(\partial V/\partial R)\ . \qquad \text{(A12)}$$

Now if X satisfies $XR\simeq-\beta$ (and if we neglect δ), then V is approximately a constant in [Eq. (A12)] which can be absorbed into Λ_0. Moreover one can check that the last term in [Eq. (A12)] vanishes. Thus

$$+2\gamma\lambda=\frac{2}{5}\left[\frac{5\eta-6}{\eta-1}\right]\lambda-\frac{\beta}{2\gamma\lambda}\frac{\partial V}{\partial X}\ . \qquad \text{(A13)}$$

Using Eq. (A11)

$$\gamma=\frac{1}{5}\left[\frac{5\eta-6}{\eta-1}\right]-\frac{\eta}{5(\eta-1)}=\frac{4\eta-6}{5(\eta-1)}\ . \qquad \text{(A14)}$$

Then

$$(M_{\text{Pl}}{}^2)_{\text{eff}}=1+\partial V/\partial R=1-\frac{\eta}{5(\eta-1)}\gamma^{-1}=\frac{3(\eta-2)}{2(2\eta-3)}\ . \qquad \text{(A15)}$$

We see that $(M_{\text{Pl}}{}^2)_{\text{eff}}$ is only positive if $\eta>2$. However, when $\lambda\sim t^{-2(\eta-1)}$ falls more slowly than ρ which goes as t^{-2} (assuming that initially $\lambda\ll\rho$).

APPENDIX B

Here we present the equations of motion for the model of Sec. III. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{2}(\partial_\lambda\phi)^2X^2+\alpha\phi-c(XR+1)^2+\frac{1}{X}(a+b\ln X)-\frac{1}{2}\delta(\partial_\lambda X)^2/X^2-R-\Lambda-\mathcal{L}_{\text{matter}}\ . \qquad \text{(B1)}$$

Einstein's equation is

$$G^{\lambda\rho}=-\tfrac{1}{2}T^{\lambda\rho}_{\text{matter}}+\tfrac{1}{2}(\Lambda-\alpha\phi)g^{\lambda\rho}-\tfrac{1}{2}[\partial^\lambda\phi\partial^\rho\phi-\tfrac{1}{2}g^{\lambda\rho}(\partial_\sigma\phi)^2]X^2-\frac{1}{2}\left[-c(XR+1)^2+\frac{1}{X}(a+b\ln X)\right]g^{\lambda\rho}$$
$$-2c(XR+1)XR^{\lambda\rho}-\frac{\delta}{2}[\partial^\lambda X\partial^\rho X-\tfrac{1}{2}g^{\lambda\rho}(\partial_\sigma X)^2]/X^2+2\{(g^{\lambda\rho}\Box-\nabla^\lambda\nabla^\rho)[2c(XR+1)X]\}\ . \quad \text{(B2)}$$

The trace of this is

$$-R=2\lambda+\tfrac{1}{2}(\partial_\sigma\phi)^2X^2+2c(XR+1)-2(a+b\ln X)/X+\frac{\delta}{2}(\partial_\sigma X)^2/X^2+6\Box[2c(XR+1)X]-\tfrac{1}{2}(T_{\text{matter}})^\lambda{}_\lambda\ . \quad \text{(B3)}$$

The traceless part of Eq. (B2) obtained by subtracting $\frac{1}{4}g^{\lambda\rho}$ trace is (assuming the matter is radiation now)

$$(R^{\lambda\rho}-\tfrac{1}{4}Rg^{\lambda\rho})[1+2c(XR+1)X]=-\tfrac{1}{2}[\partial^\lambda\phi\partial^\rho\phi-\tfrac{1}{4}g^{\lambda\rho}(\partial_\sigma\phi)^2]X^2+2\{(\tfrac{1}{4}g^{\lambda\rho}\Box-\nabla^\lambda\nabla^\rho)[2c(XR+1)X]\}\ . \quad \text{(B4)}$$

The equation of motion of ϕ is

$$-\frac{1}{\sqrt{g}}\partial_\sigma[\sqrt{g}\,(\partial^\sigma\phi)X^2]=\alpha\ . \quad \text{(B5)}$$

The equation of motion of X is

$$-2c(XR+1)R+\frac{1}{X^2}(b-a-b\ln X)-(\partial_\lambda\phi)^2X=-\delta(\partial_\lambda X)^2/X^3-\delta\frac{1}{\sqrt{g}}\partial_\sigma[\sqrt{g}\,(\partial^\sigma X)X^{-2}]\ . \quad \text{(B6)}$$

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