Leading-order perturbative QCD calculation of nucleon Dirac form factors

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(Received 17 September 1986; revised manuscript received 10 February 1987)

The Dirac form factors $F_1^{p,n}$ of nucleons are analyzed to leading order in the strong coupling constant $\alpha_s(Q^2)$ and leading twist using perturbative QCD. The effects of different choices of the nucleon distribution amplitude on the leading-twist result are explored. These results are compared with recent experimental data for the proton. We show that it is possible to fit the data for F_1^p in the range $10 \leq Q^2 \leq 30$ $(\mathrm{GeV}/c)^2$ by evaluating the strong coupling constant $\alpha_s(Q^2)$ at the exact gluon kinematics for each diagram of the process within the integrals over the momentum fraction which govern the perturbative QCD prediction.

I. INTRODUCTION

The study of quantum chromodynamics (QCD) by both perturbative and nonperturbative means has progressed rapidly over the past several years, and QCD now seems compatible with experiment in several areas.[1] Nonetheless convincing evidence for the validity of QCD by precise calculations of strong-interaction dynamics has not yet been forthcoming. One potentially stringent test of any theory of the strong interaction would be to correctly calculate the hadron form factors which control exclusive processes such as hadron pair production, electro- and photoproduction, and elastic lepton-nucleon scattering. Since the proton and neutron are components of ordinary matter, knowledge of their structure is of basic general interest in high-energy physics, and has been the focus of much work in both experiment and theory. Recent experimental measurements[2] of the cross section for elastic electron scattering from the proton have improved the precision of the data in the range of momentum transfer squared $10 \leq Q^2 \leq 30$ $(\mathrm{GeV}/c)^2$. The proton Dirac form factor $F_1^p(Q^2)$ dominates the elastic-scattering cross section at high Q^2 and so can be extracted from these data.

Perturbative QCD calculations for nucleon form factors have previously been done by several authors.[3-10] Taken together, these calculations indicate that the leading-order, leading-twist QCD result for F_1^p is sensitive to the form chosen for the distribution amplitude for the momenta of the quarks in the proton, and to the method used to evaluate the argument of the running strong coupling constant $\alpha_s(Q^2)$. At the same time, nonperturbative methods for calculation of hadron properties have advanced substantially in the past few years. Suggestions have been made for the form of the distribution amplitudes for mesons and nucleons on the basis of the method of QCD sum rules[9-13] and from lattice QCD calculations.[14] There are also hints that distribution amplitudes can be calculated directly from hadron momentum-space wave functions and show the same general features as those calculated from QCD sum rules and lattice techniques, once the basic spinor structure of the theory is taken into account.[15]

Examination of the behavior of the proton Dirac form factor as extracted from the new experimental data[2] bears out the basic QCD expectations of a logarithmic departure of F_1^p from the $1/Q^4$ falloff expected from dimensional-scaling arguments. This encourages the pursuit of a lowest-order perturbative QCD analysis. The dominant logarithmic corrections in the lowest-order come from two powers of the QCD running coupling constant $\alpha_s(Q^2)$, corresponding to the renormalization-group corrections to the propagators of the two exchanged gluons. Other logarithmic corrections come from the evolution of the quark distribution amplitudes, as discussed below, but are suppressed due to the fractional powers of the corresponding anomalous dimensions. This is different from the analogous QED case, which has a well-known boundary condition $\alpha \approx \frac{1}{137}$ at the subtraction point for any renormalization scheme. For form-factor calculations in the strong interaction, it has been shown[16] that the argument of the running coupling constant should be taken as the square of the momentum transfer of the exchanged gluon in order to make the perturbation theory meaningful. This was argued from the convergence of the perturbation series and can be justified in any process which does not involve triple or quartic vertices in the lowest order.

Up to now the evolution of the running coupling constant has only been applied to nucleon form-factor calculations for the process as a whole, ignoring differences between individual subdiagrams. This is unfortunate, as for light quarks the gluons can be exchanged over a wide range of momentum transfer, with four-momenta that in general differ from diagram to diagram. Approximate attempts have been made[9] to account for the distribution of momenta among the gluons by evaluating the running coupling constant $\alpha_s(Q^2)$ at intermediate values of the full Q^2 of the photon. So far, however, there has been no convincing method for making the choice of the intermediate arguments.

The purpose of this paper is to present a more careful analysis of the perturbative QCD approach to calculation of both normalization and dependence on momentum transfer of nucleon form factors. The effects of different choices of the distribution amplitude $\Phi(x,\tilde{Q}^2)$ for the momenta of the quarks in the nucleons are studied. An improved analysis of the argument Q^2 in momentum transfer squared of the running strong coupling constant $\alpha_s(Q^2)$ is given. It is found that agreement with the data for the proton Dirac form factor F_1^p may be obtained by evaluating the coupling constant at the exact gluon kinematics for each diagram of the process within the integrals over momentum fraction which govern the perturbative QCD prediction. It is necessary to introduce a cutoff into the formula for $\alpha_s(Q^2)$ to prevent the coupling constant from becoming infinite for vanishing gluon momenta. The sensitivity of the leading-twist result to different choices of the cutoff is explored. Results for F_1^n for the neutron are also given and compared with previous calculations. Since the neutron and proton have a common hadronic wave function, these predictions for F_1^n are parameter-free and provide a potentially useful tool for discriminating among the various models of the nucleonic wave function by experiment.

II. THEORETICAL FRAMEWORK

We begin with a brief review of the perturbative formalism for exclusive processes. Applying light-cone perturbation theory, the QCD expression for nucleon form factors can be factored[17] in the asymptotic momentum-transfer limit into a convolution of three amplitudes:

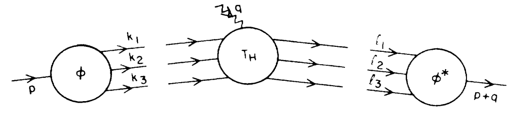

FIG. 1. Factorization of scattering amplitude for exclusive processes involving nucleons. The distribution amplitudes Φ contain the nonperturbative dynamics of the nucleon. The hard-scattering kernel T_H is calculated in perturbation theory.

$$F_{1,2}^{p,n}(Q^2)=\int [dx][dy]\Phi^*(y,\tilde{Q}_y^2)T_H(x,y,Q^2)\Phi(x,\tilde{Q}_x^2) , \tag{1}$$

where $[dx]\equiv dx_1dx_2dx_3\delta(1-\sum_i x_i)$. The photon transfers a four-momentum q to the nucleon. For spacelike processes $Q^2\equiv -q^2$ is a positive quantity, and F_1 and F_2 correspond to total helicity-conserving and total helicity-flip interactions, respectively. The variables x_i and y_i represent the fraction of the nucleon longitudinal momentum carried by each of the quarks in the initial and final states. The function $\Phi(x,\tilde{Q}_x^2)$ is the probability amplitude for the nucleon to exist as three valence quarks with momentum fractions x_i, collinear up to the momentum scale $\tilde{Q}_x\equiv \min_i x_i Q$. The equivalent distribution amplitude for the final state is $\Phi^*(y,\tilde{Q}_y^2)$, with $\tilde{Q}_y\equiv \min_i y_i Q$. (Hereafter we will suppress the subscript on $\tilde{Q}$ whenever it can be understood in context.) These distribution amplitudes contain the nonperturbative dynamics of the nucleon. The hard-scattering amplitude $T_H(x,y,Q^2)$ contains the main dynamical dependence of the perturbative calculation and can be calculated in terms of quark-gluon subprocesses. This factorization is indicated graphically in Fig. 1.

The nucleonic distribution amplitude $\Phi_N(x,\tilde{Q}^2)$ may be written in terms of its spin, flavor, and orbital components in the symmetric form

$$\Phi_N^{\uparrow}(x_1,x_2,x_3,\tilde{Q}^2)=\frac{f_N(\tilde{Q}^2)}{8\sqrt{3}}\{u^{\uparrow}u^{\downarrow}d^{\uparrow}\phi_N(x_1,x_2,x_3,\tilde{Q}^2)+u^{\downarrow}u^{\uparrow}d^{\uparrow}\phi_N(x_2,x_1,x_3,\tilde{Q}^2)$$
$$-u^{\uparrow}u^{\uparrow}d^{\downarrow}[\phi_N(x_1,x_3,x_2,\tilde{Q}^2)+\phi_N(x_2,x_3,x_1,\tilde{Q}^2)]\} \tag{2}$$

for the proton. For the neutron, u should be interchanged with d, with an overall change of sign. (Since the color factor ϵ_{ijk} is antisymmetric, the total nucleon wave function is antisymmetric as required.) This effective representation can be easily derived from the form given in Ref. 3 by gathering terms according to the ordering of the flavors. Equation (2) contains an arbitrary phase which will not affect the final answer. However, it is important to note that the phase of the nucleon spin-down state is opposite to that of the spin-up state. This relative phase difference is important to fix the overall sign of F_2 compared to that of F_1 in calculations to nonleading twist. The dimensional constant f_N is set by the value of the nucleonic wave function at the origin and has been calculated[9,11] to be $f_N(\mu_0^2)=(5.2\pm0.3)\times10^{-3}$ (GeV/c)^2 at $\mu_0^2\approx 1$ (GeV/c)^2.

Diagrams that must be evaluated for the calculation of the hard-scattering kernel for nucleon form factors are shown in Fig. 2. The notation $(x\leftrightarrow y)$ in Fig. 2 indicates that the initial- and final-quark lines are to be exchanged to obtain the remaining diagrams. Only some of these diagrams contribute to the leading-twist result, as shown in Table I below.

Dirac form factors to leading twist. The Dirac form fac-

TABLE I. Leading-twist contributions to terms in the scattering amplitude for nucleon form factors. The diagram indices refer to Fig. 2.

Diagram	Contribution to $\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow \rangle$	Contribution to $\langle \downarrow\uparrow\uparrow \mid T_1 \mid \downarrow\uparrow\uparrow \rangle$
(a)	$\frac{\alpha_s[(1-x_1)(1-y_1)Q^2]\alpha_s(x_3y_3Q^2)}{Q^4(1-x_1)^2(1-y_1)^2x_3y_3}$	0
(b)	$\frac{\alpha_s[(1-x_1)(1-y_1)Q^2]\alpha_s(x_2y_2Q^2)}{Q^4(1-x_1)^2(1-y_1)^2x_2y_2}$	0
(c)	0	0
(d)	0	0
(e)	0	0
(f)	$-\frac{\alpha_s(x_2y_2Q^2)\alpha_s(x_3y_3Q^2)}{Q^4(1-x_1)(1-y_3)x_2y_2x_3y_3}$	0
(g)	0	$\frac{\alpha_s(x_2y_2Q^2)\alpha_s(x_3y_3Q^2)}{Q^4(1-x_3)(1-y_2)x_2y_2x_3y_3}$

tors $F_1^{p,n}$ can be written in a convenient form by expanding Eq. (1) and inserting the distribution amplitudes from Eq. (2). To leading twist, we only need to retain terms in which the helicity of all three quarks remains unchanged during the interaction. Adopting the notation $\phi_{123}(x) \equiv \phi_N(x_1,x_2,x_3,Q^2)$ and $2T_{123}(x) \equiv \phi_{132}(x)+\phi_{231}(x)$, the QCD expression for the proton Dirac form factor is

$$F_1^p(Q^2)=\frac{8\pi^2}{27}\int [dx][dy] f_N(\tilde{Q}_x{}^2) f_N(\tilde{Q}_y{}^2) \times\{[\phi_{123}(x)\phi_{123}(y)(e_u\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow\rangle + e_u\langle \uparrow\downarrow\uparrow \mid T_2 \mid \uparrow\downarrow\uparrow\rangle + e_d\langle \uparrow\downarrow\uparrow \mid T_3 \mid \uparrow\downarrow\uparrow\rangle) + 2T_{123}(x)T_{123}(y)(e_u\langle \uparrow\uparrow\downarrow \mid T_1 \mid \uparrow\uparrow\downarrow\rangle + e_u\langle \uparrow\uparrow\downarrow \mid T_2 \mid \uparrow\uparrow\downarrow\rangle + e_d\langle \uparrow\uparrow\downarrow \mid T_3 \mid \uparrow\uparrow\downarrow\rangle)] + (x\leftrightarrow y)\}\ , \quad (3)$$

where the symbols

$$\langle \uparrow\downarrow\uparrow \mid T_i \mid \uparrow\downarrow\uparrow\rangle \equiv \langle u^\uparrow u^\downarrow d^\uparrow \mid T_i \mid u^\uparrow u^\downarrow d^\uparrow\rangle$$

indicate the initial- and final-quark helicities for an interaction in which the photon couples to the ith quark. The quark charges are $e_u=\frac{2}{3}$ and $e_d=-\frac{1}{3}$ for the proton. For the neutron, $e_u\leftrightarrow e_d$. Equation (3) can be further simplified by the use of permutation identities such as

$$\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow\rangle = \langle \downarrow\uparrow\uparrow \mid T_2 \mid \downarrow\uparrow\uparrow\rangle(1\leftrightarrow 2)\ ,$$

where the symbol $(1\leftrightarrow 2)$ indicates that the variables associated with the first and second quarks are interchanged. By gathering the terms as coefficients of the factors $\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow\rangle$ and $\langle \downarrow\uparrow\uparrow \mid T_1 \mid \downarrow\uparrow\uparrow\rangle$ with appropriate changes of the dummy variables of integration in the distribution amplitudes:

$$F_1^p(Q^2)=\frac{8\pi^2}{27}\int [dx][dy] f_N(\tilde{Q}_x{}^2) f_N(\tilde{Q}_y{}^2) \times[(\{e_u[\phi_{123}(x)\phi_{123}(y)+2T_{132}(x)T_{132}(y)+2T_{312}(x)T_{312}(y)]+e_d\phi_{321}(x)\phi_{321}(y)\}\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow\rangle + [e_u\phi_{213}(x)\phi_{213}(y)+2e_dT_{321}(x)T_{321}(y)]\langle \downarrow\uparrow\uparrow \mid T_1 \mid \downarrow\uparrow\uparrow\rangle)+(x\leftrightarrow y)]\ . \quad (4)$$

Table I gives the leading-order, leading-twist contributions for each diagram in Fig. 2 to the terms $\langle \uparrow\downarrow\uparrow \mid T_1 \mid \uparrow\downarrow\uparrow\rangle$ and $\langle \downarrow\uparrow\uparrow \mid T_1 \mid \downarrow\uparrow\uparrow\rangle$.

III. DISTRIBUTION AMPLITUDES

A. Overview and history

Detailed analyses for exclusive processes require knowledge of the valence-quark distribution amplitudes $\phi(x_i,\tilde{Q}^2)$ of the hadrons.[3] The distribution amplitude is defined as the integral of the light-cone momentum-space wave function $\Psi(x_i,\mathbf{k}_\perp)$ up to a maximum four-momentum scale $\tilde{Q}$ in $\mathbf{k}_\perp$ such that $\tilde{Q}=\min_i(x_iQ)$ is the minimum momentum transfer in the process. The function $\Phi(x,\tilde{Q}^2)$ represents the probability amplitude for finding the valence quarks to be collinear up to the momentum scale $\tilde{Q}$ with longitudinal-momentum fractions x_i of the total momentum of the hadron. Unfortunately the distribution amplitude for nucleons is not yet well known, but suggestions have recently been made for its form based on the use of QCD sum rules.[9-11] Although these suggestions may be preliminary, they provide a useful starting point for perturbative calculations and can be used to generate testable predictions of the theory. In this way, electromagnetic form factors can provide a sensitive method for investigating the valence-quark distribution amplitudes of nucleons.

B. Model forms for ϕ_N: Evolution

In the present work we take three different models for the nucleon distribution amplitude from recent suggestions in the literature. Chernyak and Zhitnitsky[9] proposed a form for $\phi_N(x,\tilde{Q}^2)$ designed to have the moments predicted by their analysis of QCD sum rules. Gari and Stefanis[10] proposed a different form based on their calculations of the leading-twist QCD results for F_1^p and F_1^n at $Q^2=20$ $(\text{GeV}/c)^2$. Most of the moments of the distribution amplitude of Gari and Stefanis are in agreement with the sum-rule predictions, but some moments are far outside the range of allowed values calculated in Ref. 9. The QCD sum-rule constraints on the moments of ϕ_N were recalculated by King and Sachrajda,[11] who obtained results substantially similar to those of Chernyak and Zhitnitsky, but different in some details. King and Sachrajda also proposed a model form for ϕ_N to match their moment predictions.

The three models for ϕ_N are given in Table II as coefficients a_i of a decomposition in terms of the first six Appel polynomials $A_i(x)$. The Appel polynomials are eigenfunction solutions of the evolution equation[3] for $\phi_N(x,\tilde{Q}^2)$. The model forms are given at a squared-momentum-transfer scale $\mu_0{}^2 \approx 1$ $(\text{GeV}/c)^2$, which corresponds to a typical value of Q^2 in the interaction[9] of roughly 20 $(\text{GeV}/c)^2$. To evaluate the distribution amplitudes or evolve them to other values of Q^2, the formula

$$\phi(x,\tilde{Q}^2)=\phi_{as}(x)\sum_{i=0}^{5} N_i \left[\frac{\alpha_s(\tilde{Q}^2)}{\alpha_s(\mu_0{}^2)}\right]^{b_i/\beta} a_i A_i(x)\ , \quad (5)$$

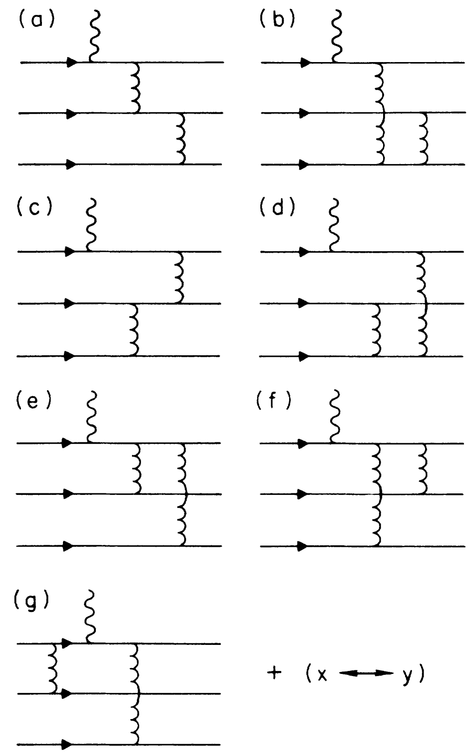

FIG. 2. Diagrams which must be evaluated in the leading-order calculation of nucleon form factors. The contributions of each of these diagrams to the leading-twist result depend on the helicities of the quarks and are listed in Table I. The photon is shown attaching to the first quark in each of these diagrams. The contributions of diagrams in which the photon attaches one of the other two quarks can be obtained through the use of permutation symmetries on the expressions for the diagrams given above.

where $\phi_{as}(x) \equiv 120x_1x_2x_3$ and $\tilde{Q}^2=\min_i(x_iQ)^2$, is used, along with the corresponding evolution in f_N:

$$f_N(\tilde{Q}^2)=f_N(\mu_0{}^2)\left[\frac{\alpha_s(\tilde{Q}^2)}{\alpha_s(\mu_0{}^2)}\right]^{2/(3\beta)} . \quad (6)$$

TABLE II. Coefficients of Appel polynomial decompositions of the nucleon distribution amplitude $\phi_n(x,\tilde{Q}^2)$ for the models of Chernyak and Zhitnitsky (Ref. 9) (CZ), Gari and Stefanis (Ref. 10) (GS), and King and Sachrajda (Ref. 11) (KS). The decompositions given are for the model forms at an effective scale $\mu_0{}^2 \approx 1$ $(\text{GeV}/c)^2$. Evolution of the distribution amplitudes to other values of $\tilde{Q}^2$ is described in the text.

i	a_i (CZ)	a_i (GS)	a_i (KS)	N_i	b_i	$A_i(x)$
0	1.00	1.00	1.00	1	0	1
1	0.410	0.391	0.310	21/2	20/9	x_1-x_3
2	−0.550	−0.588	−0.370	7/2	24/9	$2-3(x_1+x_3)$
3	0.357	−0.749	0.630	63/10	32/9	$2-7(x_1+x_3)+8(x_1^2+x_3^2)+4x_1x_3$
4	−0.012 2	0.017 6	0.003 33	567/2	40/9	$x_1-x_3-(4/3)(x_1^2-x_3^2)$
5	0.001 06	0.574	0.063 2	81/5	42/9	$2-7(x_1+x_3)+(14/3)(x_1^2+x_3^2)+14x_1x_3$

Here $\beta=11-2n_f/3$ has the value 9 for $n_f=3$ flavors. The coefficients of the Appel polynomial decomposition of the model forms were obtained by direct numerical integration using the expression[3,9]

$$a_i(\mu_0^2)=\int [dx]\phi_N(x,\mu_0^2)A_i(x) \ . \tag{7}$$

The coefficients listed in Table II are consistent with the moments listed in Refs. 9–11 within the precision to which the moments were presented by those authors. The Appel coefficients are given here to three significant digits in order to accurately present the proposed polynomial forms for the distribution amplitude. The QCD sum-rule predictions for the moments are in general only known to one or two significant digits.[9,11]

1. *Main features of model distribution amplitudes*

The models decomposed in Table II are all very asymmetric in the distribution of momentum among the valence quarks of the nucleon. In the distribution amplitude of Chernyak and Zhitnitsky (CZ), roughly 70% of the nucleon momentum is carried by the first quark. [See Eq. (2) for interpretation in terms of spin and flavor ordering.] The model of King and Sachrajda (KS) is slightly more asymmetric than that of CZ, but is otherwise similar. As will be seen shortly, this increased asymmetry in momentum balance leads to a larger value of F_1^p than that calculated with the other models. The distribution amplitude of Gari and Stefanis (GS) is qualitatively different from those of CZ and KS. In the model of GS, most of the nucleon momentum is distributed roughly equally between the first and third quarks, and only about 15% of the momentum is carried by the remaining quark.

2. *Dependence of F_1^p on the form of ϕ_N*

Before proceeding to the numerical results, we give here a qualitative description of the way in which the leading-order QCD result depends on the form of the distribution amplitude.

Inspection of the leading-twist formula in Table I for the contributions to F_1 shows that there are roughly six powers of terms like x_i, $(1-x_i)$, y_i, or $(1-y_i)$ in the denominator of $T_H(x,y,Q^2)$. To leading twist, the masses and transverse momenta of the quarks are ignored and the integration limits are 0 and 1 in the momentum fraction variables. This situation would lead to divergent integrals if the end-point singularities were not canceled by sufficient powers of x_i, etc., in the numerator from $\phi_N(x)$ and $\phi_N(y)$. The asymptotic form $\phi_{as}(x)\equiv 120x_1x_2x_3$ introduces just enough powers of x_i and y_i to prevent the end-point singularities in T_H from causing the integrals to be divergent.

When the symmetric asymptotic form is modified by multiplicative asymmetric factors as in Eq. (5), the integrals are still convergent, but the presence of additional probability amplitude near the end points produces an increase in the result for F_1. In fact, it has been shown[3–5] that use of the asymptotic distribution amplitude $\phi_{as}(x)$ by itself yields $F_1^p=0$ if the difference between the arguments of $\alpha_s(Q^2)$ is ignored. With highly asymmetric distribution amplitudes such as the ones listed above, it is possible to obtain results for F_1^p in agreement with experiment. Numerical results for this calculation and for F_1^n of the neutron are given in the next section.

The model forms for ϕ_N given above are also all highly relativistic in the sense that no constraints are placed on the range of allowed values of the momentum fraction variables x_i. In contrast, the variables x_i take on only discrete values in the nonrelativistic distribution amplitude $\phi_{NR}(x)=\delta(x_1-m_1/M)\delta(x_2-m_2/M)$ appropriate for nucleons made of very massive quarks. The nonrelativistic distribution amplitude completely avoids the singular region of T_H and in fact produces a negative leading-twist result for F_1^p in the asymptotic Q^2 limit.[4,8,9] When higher-twist effects[18] due to the quark masses are included, the nonrelativistic result for F_1^p becomes

$$F_1^p(\tau)=\frac{C}{\tau^4}[\tfrac{729}{128}(6+23\tau-6\tau^2)] \quad \text{where } \tau\equiv\frac{Q^2}{4M^2} \ . \tag{8}$$

The constant C can be estimated by requiring $F_1^p(0)$ to be the charge of the proton according to the method given in Ref. 18, yielding the replacement

$$C/\tau^4\rightarrow\tfrac{1}{3}\{8\epsilon^2/[27(\tau+\epsilon)^2]\}^2$$

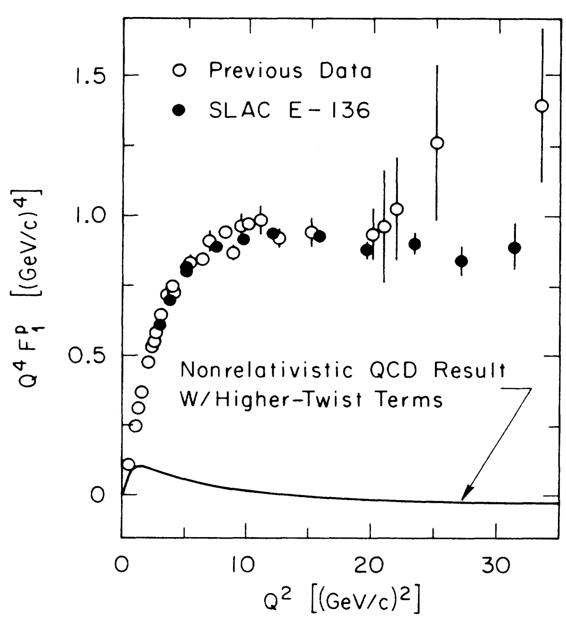

FIG. 3. Comparison of leading-order QCD calculation using nonrelativistic distribution amplitude $\phi_{NR}\equiv\delta(x_1-m_1/M)\delta(x_2-m_2/M)$ with data for $Q^4F_1^p(Q^2)$. Higher-twist terms proportional to the quark masses are included using the formulas of Ref. 18. The data are from Refs. 2, 19, and 20. When binding-energy effects are included as described in the text, the nonrelativistic QCD result is roughly consistent with the trend of the data at very low momentum transfer, but does not match the data above $Q^2=1$ (GeV/c)^2.

with

$$\epsilon \equiv 9\pi f_N \alpha_s/(2\sqrt{2}M^2) \approx \tfrac{1}{4} .$$

With this formula, F_1^p becomes positive at low momentum transfer, crosses zero at $Q^2 \approx 15$ (GeV/c)^2, and is negative at high momentum transfer, but remains 1 or 2 orders of magnitude below the experimental data for values of Q^2 above about 1 (GeV/c)^2. Figure 3 shows the nonrelativistic QCD result in comparison with the data for $Q^4F_1^p(Q^2)$ as extracted from elastic electron-proton cross-section data assuming $G_E^p = G_M^p/\mu_p$. Other higher-twist effects due to transverse momentum can be neglected in the massive-quark limit, so Eq. (8) represents the complete leading-order result for F_1^p when the nonrelativistic distribution amplitude $\phi_{\rm NR}(x)$ is used. These considerations show that a highly relativistic form such as the ones given above must be used to obtain results for F_1^p that are consistent with experiment.

IV. NUMERICAL RESULTS

A. $\alpha_s(Q^2)$ outside integral

As can be seen by inspection of the terms in Table I, a correct treatment of the leading-twist calculation would require $\alpha_s(Q^2)$ to be evaluated separately for each exchanged gluon in each diagram that contributes to the result. Up to now the leading-twist calculation has only been done with the strong coupling constant $\alpha_s(Q^2)$ evaluated outside of the integral, generally at some intermediate argument of Q^2. The integration of Eq. (1) can be done analytically in that case.[21] To check and verify the previous results,[3,9,10] we have repeated the previous calculations and also present a new result for F_1^p as calculated using the distribution amplitude of King and Sachrajda.[11]

To treat the calculations with different distribution amplitudes consistently, we evaluated the two leading powers of $\alpha_s(Q^2)$ at intermediate values of the argument using the formula

$$\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9) .$$

This formula was proposed by the authors of Ref. 9 to roughly match the gluon momenta expected for the most probable values of the quark momentum fractions in their model for ϕ_N. The basic features of the model form of KS are very similar to those of the model form of CZ, although (contrary to a statement in their paper) the KS model is slightly more asymmetric. Gari and Stefanis[10] used the same formula for $\bar{\alpha}_s{}^2(Q^2)$ as proposed by CZ, even though the model they propose for the distribution amplitude has a substantially different shape. For consistency, we have used the same definition for $\bar{\alpha}_s{}^2(Q^2)$ for all three models in the treatment given in the following section. In a later section, an improved treatment is given which eliminates the ambiguity regarding the proper arguments with which to evaluate $\alpha_s(Q^2)$.

Results of our calculation of the proton and neutron Dirac form factors with the replacement of $\bar{\alpha}_s{}^2$ for the two powers of α_s in the terms of Table I are given in Table III under the heading "α_s outside integral." The published results of Refs. 9 and 10 are also listed in Table III for comparison. We verify the numerical results given in Ref. 10, but find small numerical differences[22] with those of Ref. 9. The remainder of the table gives our results for F_1^p and F_1^n at $Q^2=20$ (GeV/c)^2 with an improved treatment of the argument of $\alpha_s(Q^2)$ as discussed below. The integration uncertainty on our results is approximately ± 0.02 (GeV/c)^4.

1. Comparison with proton data, $\alpha_s(Q^2)$ outside integral

Figure 4 shows the leading-twist QCD results for the three different models for $\phi(x,\tilde{Q}^2)$ when the coupling constant is evaluated outside the integral. Although the magnitude of F_1^p is approximately correct at $Q^2 \approx 20$ (GeV/c)^2 with these models, the dependence of the result on Q^2 does not agree with that indicated by the data.

It would be possible to improve slightly the apparent agreement of these predictions with the proton data by

TABLE III. Results of numerical integration of Eq. (4) for the proton and neutron Dirac form factors $F_1^{p,n}$ at $Q^2=20$ (GeV/c)^2 under various assumptions. The two columns on the left-hand side are previous calculations and are listed here for comparison. The middle three columns give our results using $\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9) \approx (0.3)^2$ evaluated outside the integral. The one-loop formula $\alpha_s(Q^2)=4\pi/[\beta \ln(Q^2/\Lambda^2)]$ was used for these calculations, with the value $\Lambda=0.1$ GeV for the QCD scale parameter. The right-most columns are our results with an improved treatment of the running coupling constant as described in the text, using an effective dynamical gluon mass squared of $m_g{}^2=0.3$ (GeV/c^2)^2.

	Previous calculations $\alpha_s(Q^2)$ outside integral		α_s outside integral $\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9)$			α_s inside integral $m_g{}^2=0.3$ (GeV/c)^2		
Model	CZ (Refs. 9 and 21)	GS (Ref. 10)	CZ	GS	KS	CZ	GS	KS
$Q^4F_1^p$ [(GeV/c)^4]	1.10	0.86	0.88	0.89	1.28	0.86	0.94	1.28
$-Q^4F_1^n$ [(GeV/c)^4]	0.57	0.07	0.43	0.09	0.52	0.42	0.10	0.53
$\dfrac{-F_1^n}{F_1^p}$	0.5	0.08	0.49	0.10	0.41	0.49	0.10	0.41

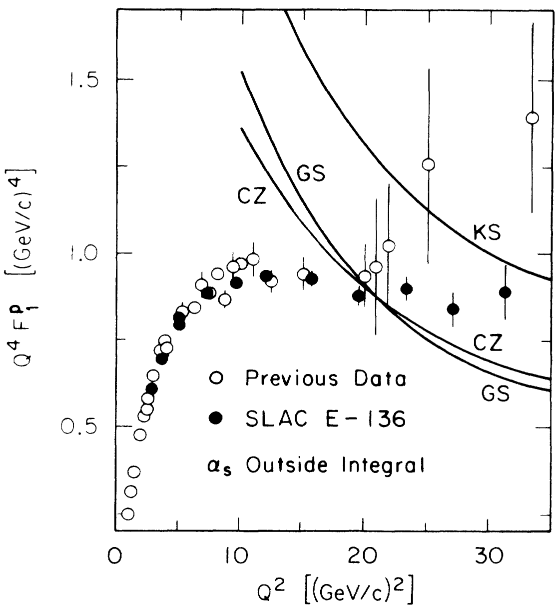

FIG. 4. Leading-twist QCD calculation of $F_1^p(Q^2)$ with the strong coupling constant $\alpha_s(Q^2)$ evaluated outside of the integrals using the formula $\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9)$ and $\Lambda = 0.1$ GeV. The magnitude of the QCD result is approximately correct within the factor-of-2 uncertainty quoted by the authors of Refs. 9 and 11. The notation is explained in the text.

comparing them instead with values of the *magnetic* form factor $G_M^p(Q^2) \equiv F_1^p(Q^2) + F_2^p(Q^2)$. (Here F_2^p is the form factor corresponding to total helicity flip of the proton in spacelike processes.) To leading twist, $F_2^p(Q^2)=0$ and it is impossible to distinguish the calculation of G_M^p from that of F_1^p. Nonetheless the leading-twist calculation seems to be more readily identified as F_1^p, since it includes only interactions in which there is total helicity conservation. The comparisons with the data are given here under that assumption.

We note in particular the disagreement of the results using the distribution amplitude of Chernyak and Zhitnitsky[9] (labeled CZ in Fig. 4) with the results published by those authors. The slope with Q^2 of the curve they give does not match the slope given above, though the calculation was done using equivalent assumptions. To reproduce the CZ curve as shown in their publication[9] one would have to use the quantity

$$\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9)$$

to calculate the magnitude of the form factor, but use $\alpha_s{}^2(Q^2)$ with the full Q^2 argument to obtain the Q^2 evolution, which would clearly be inconsistent.

2. *Calculation for neutron F_1, $\alpha_s(Q^2)$ outside integral*

Unfortunately it is difficult to attempt a comparison of the leading-twist QCD results for F_1^n with data, since no experimental separation of the neutron form factors has been performed above $Q^2 \approx 3$ (GeV/c)^2. Experimental data do exist[23] for electron scattering from deuterium from which the elastic electron-neutron cross section can be extracted up to $Q^2 = 10$ (GeV/c)^2. References 24–26 contain examples of fits to proton and neutron form-factor data in terms of the parameters of vector-meson-dominance models for the photon-nucleon coupling. The data do not yet allow a model-independent separation of F_1^n and F_2^n to be made at high Q^2.

We present the leading-twist QCD results for F_1^n in Fig. 5. The values of F_1^n calculated using the model distribution amplitude of GS are about 4 times smaller in magnitude than those calculated with the models of KS or CZ. The parameters of the GS model (Ref. 10) for $\phi(x,\tilde{Q}^2)$ were chosen in order to yield such a small result for F_1^n.

3. *Effect of evolution on F_1*

Using Eqs. (5) and (6), we studied the effect of evolution on the leading-twist result for F_1^p as given by integration of Eq. (4). With either the model of CZ or that of KS, evolution of ϕ_N as given by Eq. (5) produced no effect on the results to within the numerical integration accuracy of approximately $\pm 2\%$ in the range of interest for comparison with the experimental data [$10 \le Q^2 \le 30$ (GeV/c)^2]. The Q^2 dependence of F_1^p calculated with the model of GS differed slightly from that calculated with the other two models, as shown in Fig. 4. Inspection of the coefficients in Table II shows that the model of GS requires a large coefficient for $A_5(x)$, which is the highest-order Appel polynomial in the decomposition. Presumably this is the reason for the increased Q^2 variation of the form-factor results when the model form of GS is used. On the other hand, the presence of such a large coefficient for the last term of the truncated expansion would indicate that the expansion should properly be carried to a larger number of terms. Thus, the evolution of

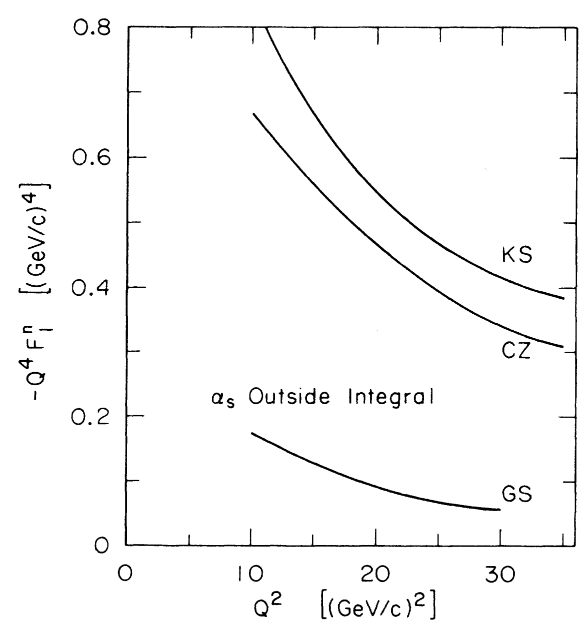

FIG. 5. Leading-twist QCD calculation of $F_1^n(Q^2)$ with the strong coupling constant $\alpha_s(Q^2)$ evaluated outside of the integrals using the formula $\bar{\alpha}_s{}^2(Q^2) \equiv \alpha_s(Q^2/36)\alpha_s(Q^2/9)$ and $\Lambda = 0.1$ GeV, as in Fig. 4.

the model form for $\phi(x,\tilde{Q}^2)$ of GS may be less reliable than that of the other two models.

The variation of f_N goes as $\frac{2}{27}$ powers of the ratio of coupling constants $\alpha_s(\tilde{Q}^2)/\alpha_s(\mu_0^2)$ and can be calculated outside of the integral to a good approximation, since the main Q^2 dependence of the integral in Eq. (4) is contained in the two powers of α_s in T_H, as listed in Table I.

B. Improved treatment of $\alpha_s(Q^2)$

More reliable results for the magnitude and Q^2 dependence of the nucleon form factors should be obtained when the arguments of the strong coupling constant are correctly evaluated as indicated in Table I. An immediate problem arises if this is attempted with the usual one-loop formula

$$\alpha_s(Q^2)=4\pi/[\beta\ln(Q^2/\Lambda^2)] ,$$

since the leading-twist formulas given in Table I allow α_s to be evaluated at zero momentum transfer. Conceptually this is disastrous from the point of view of perturbative QCD, since as $Q^2\rightarrow 0$ the assumption that the interaction can be modeled by minimal gluon coupling breaks down. In this case, however, the scale in Q^2 is set by the photon momentum, and if the momentum carried by one of the gluons is very small, then that of the other gluon must be large. In such a situation other techniques may be applied to evaluate the nonperturbative modification to the contribution from the end points of the integration. In particular, Cornwall[27] has proposed the introduction of a cutoff in the formula for α_s in the form

$$\alpha_s(Q^2)\equiv 4\pi/\{\beta\ln[(Q^2+4m_g^2)/\Lambda^2]\} , \tag{9}$$

where m_g is interpreted as a dynamical gluon mass, with a value of typically about 0.5 GeV/c^2. For $Q^2 \gg m_g^2$, this formula coincides with the one-loop version, but at very low momentum transfers, this formula "freezes" the coupling constant to some finite (not necessarily small) value.[27]

With such a formula, the integrations of Eq. (4) become possible, and $\alpha_s(Q^2)$ may be evaluated within the integrals. We emphasize that similar results should be obtained using any form of cutoff which prevents $\alpha_s(Q^2)$ from becoming infinite. We chose to use the formula of Eq. (9) because of its simple analytical form and because it has arisen in a higher-order analysis.[27]

1. Comparison with proton data, $\alpha_s(Q^2)$ inside integral

Figure 6 shows the results of the improved calculation for three values of the dynamical gluon mass m_g, using the distribution amplitude of Chernyak and Zhitnitsky. We see that the calculation agrees very nicely with the data for the value $m_g^2=0.3$ (GeV/c^2)^2. Figure 7 shows the results of the same calculation for the three models for $\phi(x,\tilde{Q}^2)$ discussed above, using the values $m_g^2=0.3$ (GeV/c^2)^2 and $\Lambda=0.1$ GeV in the Cornwall formula for $\alpha_s(Q^2)$. The normalization of the leading-twist result for F_1^p is different for each of the three models for $\phi(x,\tilde{Q}^2)$,

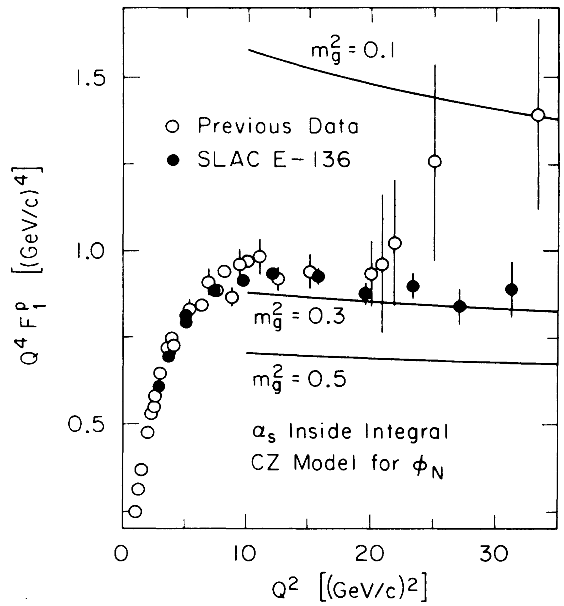

FIG. 6. Leading-twist calculation of $F_1^p(Q^2)$ with the distribution amplitude of Chernyak and Zhitnitsky (Ref. 9) and with the arguments of the running strong coupling constant $\alpha_s(Q^2)$ evaluated at the correct values for each diagram as listed in Table I. The results are shown with three choices for the dynamical gluon mass as applied using the formula for α_s proposed by Cornwall (Ref. 27) and $\Lambda=0.1$ GeV.

but within the factor-of-2 range of accuracy suggested by the authors of Refs. 9 and 11 for the normalization uncertainty presently expected from the QCD sum rules. The slope of the result with Q^2 is compatible with the trend of the data for all three models.

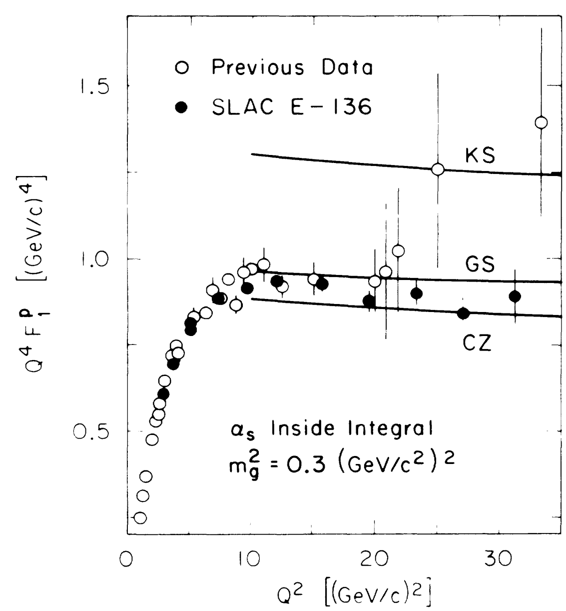

FIG. 7. Leading-twist calculation of the proton Dirac form factor with three different models for the distribution amplitude $\phi(x,\tilde{Q}^2)$ and with the arguments of the running strong coupling constant $\alpha_s(Q^2)$ evaluated as in Table I. The results are shown for the value $m_g^2=0.3$ (GeV/c^2)^2.

We conclude that the inclusion of α_s in the integral with the kinematically correct arguments as discussed above dramatically improves the agreement of the leading-twist QCD result with the experimentally observed values of $F_1^p(Q^2)$. The QCD results become sensitive to the value chosen for the dynamical gluon mass m_g and to the exact choice of the model for the nucleon distribution amplitude $\phi(x,\tilde{Q}^2)$. With the distribution amplitudes of Chernyak and Zhitnitsky[9] and Gari and Stefanis, optimum agreement with the data is obtained for $m_g{}^2 \approx 0.3$ $(\text{GeV}/c^2)^2$ when $\Lambda = 0.1$ GeV. If the model of King and Sachrajda is chosen instead, then the data support the choice $m_g{}^2 \approx 0.6$ $(\text{GeV}/c^2)^2$.

To check whether the observed sensitivity of the results to the choice of m_g was due to the neglect of higher-twist terms, we repeated some of the calculations of F_1^p using the arguments of $\alpha_s(Q^2)$ that correspond to the gluon propagators with the inclusion of terms involving the quark masses. The effect on the magnitude of F_1^p was less than 5%, so higher-twist terms were neglected in the argument of $\alpha_s(Q^2)$.

It is clear from Figs. 6 and 7 that the leading-twist QCD prediction for F_1^p would begin to deviate from the trend of the data below $Q^2 \approx 10$ $(\text{GeV}/c)^2$. At low momentum transfer we would expect effects due to the intrinsic transverse momentum of the quarks to become important, as discussed in Ref. 28.

2. *Calculation for neutron F_1, $\alpha_s(Q^2)$ inside integral*

The results of equivalent calculations for the neutron Dirac form factor with the improved treatment of $\alpha_s(Q^2)$ are shown in Fig. 8 as calculated using the different models for the nucleon distribution amplitude. We emphasize that the perturbative QCD calculation of F_1^n is exactly the same as that for F_1^p; only the charge factors in Eq. (4) change. A comparison with experimental values of F_1^n would provide an independent test of the calculations, since in principle there are no free parameters left which could be adjusted to obtain agreement with the data once the values of m_g and Λ have been fixed by comparison with the proton data. As Fig. 8 shows, the magnitude of the leading-twist QCD result for F_1^n is substantially smaller when the distribution amplitude of GS is used than when the model forms of CZ and KS are used. New experimental data for the neutron form factors could help to distinguish between the models for the nucleon distribution amplitude.

3. *High-Q^2 predictions*

Although the measurement of nucleon form factors at very high momentum transfer [above $Q^2 \sim 100$ $(\text{GeV}/c)^2$] may be inaccessible experimentally for many years, we give the QCD predictions for F_1^p and F_1^n in Figs. 9 and 10 as obtained using the three models for $\phi(x,\tilde{Q}^2)$ and the procedure discussed above.

4. *Dependence of results for $\alpha_s(Q^2)$ inside integral on Λ*

The choice of m_g which fits the data is dependent on the value of the QCD scale parameter Λ used. The results reported here have been given for the value $\Lambda = 0.1$ GeV. When the running coupling constant $\alpha_s(Q^2)$ is evaluated inside the integral, the results indicate that most of the logarithmic dependence on Q^2 disappears. The slope of the results with Q^2 is thus only slightly depen-

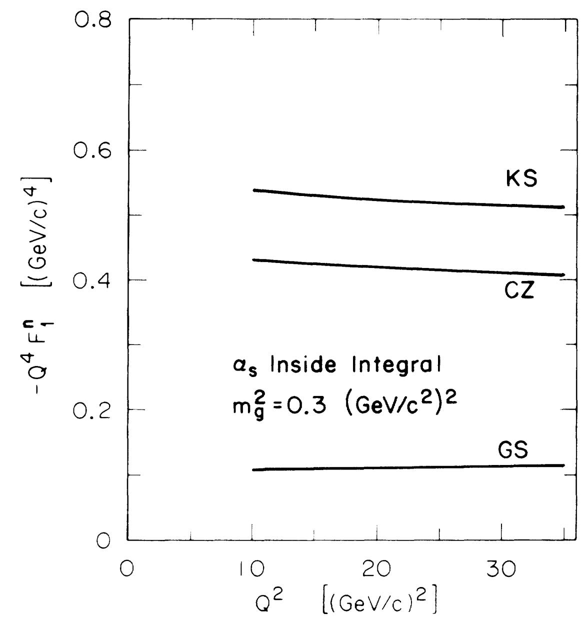

FIG. 8. Leading-twist calculation of the neutron Dirac form factor with three different models for the distribution amplitude $\phi(x,\tilde{Q}^2)$ and with the arguments of the running strong coupling constant $\alpha_s(Q^2)$ evaluated as in Table I. The results are shown for the value $m_g{}^2 = 0.3$ $(\text{GeV}/c^2)^2$. Experimental data on the neutron form factors at high momentum transfer would help to distinguish between different models for $\phi(x,\tilde{Q}^2)$.

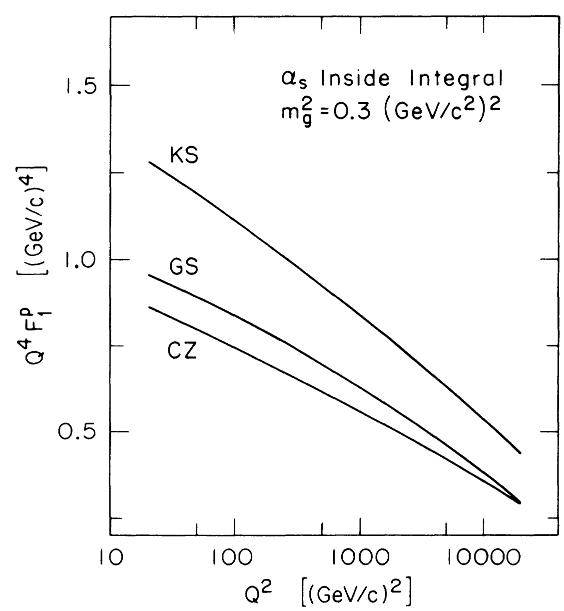

FIG. 9. Perturbative QCD predictions for F_1^p at very high momentum transfer with improved treatment of the arguments of $\alpha_s(Q^2)$. The notation and conditions are the same as in Fig. 7.

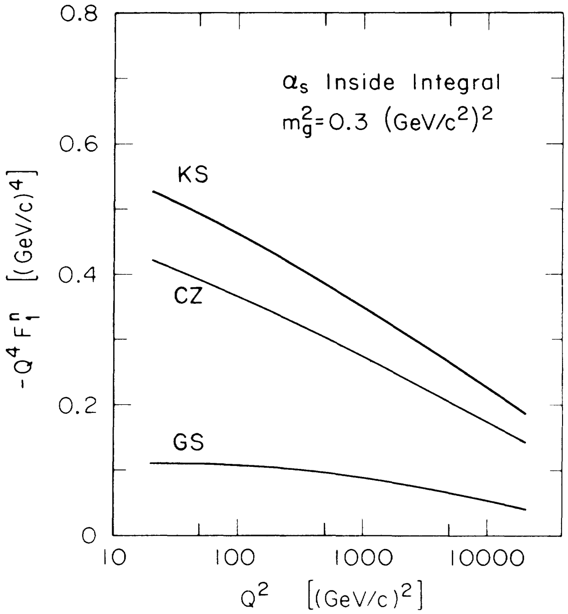

FIG. 10. Perturbative QCD predictions for F_1^n at very high momentum transfer with improved treatment of the arguments of $\alpha_s(Q^2)$. The notation and conditions are the same as in Fig. 7.

dent on the choice of Λ, and the magnitude of the Dirac form factors is roughly proportional to $[\ln(m_g{}^2/\Lambda^2)]^{-2}$, modulo small logarithmic corrections from the evolution of the distribution amplitudes. At $Q^2=20$ (GeV/c)^2, as can be seen in Table III, the overall magnitudes of the nucleon form factors as calculated with $\alpha_s(Q^2)$ inside the integral and the Cornwall form

$$\alpha_s(Q^2)\equiv 4\pi/\{\beta \ln[(Q^2+4m_g{}^2)/\Lambda^2]\}$$

$[m_g{}^2=0.3$ (GeV/c^2)^2, $\Lambda=0.1$ GeV] agree with those calculated using $\alpha_s(Q^2)$ outside the integral and the one-loop formula

$$\alpha_s(Q^2)=4\pi/[\beta \ln(Q^2/\Lambda^2)]$$

with

$$\bar{\alpha}_s{}^2(Q^2)\equiv\alpha_s(Q^2/36)\alpha_s(Q^2/9) .$$

The effective value of the strong coupling constant is approximately 0.3 at $Q^2=20$ (GeV/c)^2 with either of these two formulas.

V. CONCLUSIONS

In this paper, we have explored the calculation of nucleon Dirac form factors using leading-order perturbative QCD. By comparing the leading-twist QCD results for the proton Dirac form factor F_1^p with recent experimental results,[2] one can make the following conclusions. The qualitative prediction of the perturbative QCD for the Q^2 dependence of the form factor is impressively consistent with the experimental data above $Q^2\approx 5-10$ (GeV/c)^2. The normalization of the form factor is very much dependent on the assumed form of the nucleon distribution amplitude $\phi(x,\tilde{Q}^2)$, which describes the momentum distribution of the quarks within the nucleon, while the Q^2 dependence is in general less sensitive to the choice of distribution amplitude. It seems that a highly relativistic form must be used for $\phi(x,\tilde{Q}^2)$, as the nonrelativistic distribution amplitude produces results for F_1^p that are far below the experimental data, and change sign near $Q^2\approx 15$ (GeV/c)^2, in contradiction with the observed behavior of the data. Furthermore, an asymmetric form for $\phi(x,\tilde{Q}^2)$ must be used in order to achieve the observed magnitude of F_1^p. We used the three different distribution amplitudes recently proposed from QCD sum-rule calculations.

Secondly, the slope of the QCD prediction with Q^2 does not match the trend of the data if the argument of $\alpha_s(Q^2)$ is evaluated outside of the integrals over the momentum fraction variables x_i and y_i for the initial- and final-quark momenta. We have attempted to improve the calculation by evaluating the strong coupling constant at the exact gluon kinematics for each diagram of the process according to the procedure dictated by the renormalization-group basis of the theory.[16] Because the leading-twist expressions for the gluon propagators are singular at the end points in x_i and y_i, when the arguments of $\alpha_s(Q^2)$ are evaluated within the integrations it is necessary to modify the one-loop expression for $\alpha_s(Q^2)$ to prevent it from becoming infinite when the gluon momentum transfer ranges to zero during integration. Some form of cutoff of $\alpha_s(Q^2)$ is necessary in order to keep the leading-order perturbation theory sensible. In the calculations presented above, this cutoff was implemented using a form proposed by Cornwall,[27] which postulates introduction of an effective dynamical mass m_g for the gluons. We emphasize that similar results should be obtained using any form of cutoff which keeps $\alpha_s(Q^2)$ finite at low Q^2.

When the improved method is used, agreement with the data for $F_1^p(Q^2)$ in the approximate range $10\le Q^2\le 30$ (GeV/c)^2 can be obtained. The value of m_g which produces the best agreement is dependent on the choice of the model form for $\phi(x,\tilde{Q}^2)$. When the distribution amplitudes proposed by Chernyak and Zhitnitsky[9] or Gari and Stefanis are used, the magnitude and trend of the proton data are well fitted for the choices $[m_g{}^2\approx 0.3$ (GeV/c^2)^2, $\Lambda\approx 0.1$ GeV]. These two model forms yield very different predictions for the neutron Dirac form factor F_1^n, however. Use of the distribution amplitude of King and Sachrajda[11] would require $m_g{}^2\approx 0.6$ (GeV/c^2)^2 in order to fit the proton data.

The range of values of the effective gluon mass ($m_g\approx 0.5$ to 0.8 GeV/c^2) obtained in this analysis is consistent with values given by Cornwall[27] ($m_g=0.5\pm 0.2$ GeV/c^2) and with the results of a lattice QCD calculation[29] ($m_g\approx 0.52$ GeV/c^2) and a recent discussion of dynamical mass generation in QCD[30] ($m_g\approx 0.6$ GeV/c^2). The correctness of the approach presented here could be tested by experimental separation of the form factors of the neutron at high Q^2. Such experimental data would also help to distinguish among the various proposed models[9-11] for the nucleon distribution amplitude.

Finally, it should be observed that the sensitivity of the above results to the choice of cutoff parameter indicates the potential importance of evaluating the effects of higher-twist terms in perturbative QCD calculations for exclusive processes. Although we checked that higher-twist terms were not important in the gluon momentum transfer arguments of $\alpha_s(Q^2)$ (see Sec. IV B 1), such terms could affect the main QCD calculation significantly by modifying the hard-scattering kernel T_H to be less sensitive to the end points of integration. As in the present situation approximately 40% of the result for F_1^p comes from the region within 1% of the end points in x_i and y_i, higher-twist terms which alter the end-point behavior of T_H could have a large effect on the answer and should be investigated.

ACKNOWLEDGMENTS

We are grateful to Professor S. Brodsky for several helpful discussions regarding light-cone kinematics and the basic formulation of this problem, and for his patient and kind support. Professor C. Carlson provided some valuable comments on distribution amplitudes. We would also like to thank Professor J. D. Walecka for instructive discussions and encouragement. This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00515, and by National Science Foundation Grants Nos. PHY-85-08735 (C-R.J.) and PHY-85-10549 (A.F.S.).

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