Velocity of sound in hadron matter

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The velocity of sound in hadron matter, in both the confined and deconfined phases, is studied. This velocity of sound appears to be an important tool to distinguish among different bag-model-based thermodynamical descriptions of hadronic matter.

I. INTRODUCTION

There has recently been increased interest in the analysis of the thermodynamical properties of hadron matter at high density and temperature. In particular, one expects a transition from the hadronic phase to a deconfined plasma of quarks and gluons. The eventual existence of this transition and its properties is crucial in our understanding of the matter evolution in the early Universe, neutrons stars, and heavy-ion collisions. The study of the thermodynamics of hadrons certainly implies QCD in the nonperturbative regime. Therefore, Monte Carlo calculations[1] appear as the most unique way of getting information about this kind of phenomena. Nevertheless, from a phenomenological point of view, there exist alternative approaches based on statistical baglike models.[2,3] These models have the clear advantage of providing definite predictions on the finite-temperature behavior of the corresponding hadron gas. Among them we note the existence of a first-order phase transition between hadrons permanently confined in colorless objects and the so-called quark-gluon plasma.

Recently, the temperature behavior of the velocity of sound in hadron matter has been studied by means of Monte Carlo techniques[4] as a complementary study of the previous phase-transition analysis. This calculation gives information not only on a crucial parameter in the solution of the corresponding hydrodynamical equations, but also on the differences among alternative descriptions of the hadronic gas. The main purpose of this work is to report on an analytical study of the thermal behavior of the velocity of sound in a glueball gas described by means of different bag-model-based pictures. In this sense, we have considered three different versions for this glueball gas that are presented in the next section. Afterwards, we summarize the results concerning the temperature behavior of the velocity of sound both below and above the transition temperature. Finally, we comment on the obtained results.

II. MODEL FOR HADRONIC MATTER AND ITS PHASE STRUCTURE

The aim of this section is to briefly describe the different versions of the model for the hadronic matter used in this analysis. All three versions considered consist basically of a gas of MIT bag-model[5] glueballs. Two of them were discussed in detail in Ref. 3, one being a gas of rigid glueballs[2] and the other consisting of glueballs whose volumes are allowed to fluctuate. A slight modification of this last one which includes a new parameter, is the third version considered here. All of them start from the spherical-cavity approximation, where the energy of each state is given in terms of the volume V of the bag by

$$M_i(V)=BV+W_i=BV+\frac{y_i}{V^{1/3}} . \qquad (1)$$

Here B is an energy density simulating the confining forces and W_i are the internal energies of the gluon fields given in terms of the pure numbers y_i, which are determined by the different modes of these eight (Abelian) gluonic fields inside the bags, combined in such a way as to ensure the colorlessness of the glueballs.

A. Standard glueballs

The rigid bags, representing standard glueballs, are obtained by minimization of expansion (1) with respect to V. In this way one obtains, for their masses and volumes,

$$M_{(i)}=4BV_{(i)} \qquad (2a)$$

and

$$V_{(i)}=\left[\frac{y_i}{3B}\right]^{3/4} , \qquad (2b)$$

respectively. The masses and degeneracies d_i of the first members of the spectrum are summarized in Table I. In terms of these parameters one can write the level density

$$\rho(M)=\sum_i d_i\delta(M-M_{(i)}) \qquad (3)$$

needed for the study of the hadron gas. The asymptotic behavior of this density of states can be evaluated assuming that glueballs consist of a gluon gas contained in a volume $V=M/4B$. According to Eqs. (1) and (2), the internal energy of this gas results in $W=\frac{3}{4}M$.

On the other hand, for large W and large V the number of gas states $(W,V)\ dW$ in the internal energy range $(W,W+dW)$ is found to be

$$\sigma(W,V)dW\simeq \text{const}\times W^{-1}(VW^3)^{-3/2}e^{\alpha(VW^3)^{1/4}}dW \qquad (4)$$

with

TABLE I. Masses M_i (in GeV) and spin-color degeneracies d_i of the first 22 states of the glueball spectrum for a value $B=(0.2\ \text{GeV})^4$.

i	M_i	d_i
1	1.80	6
2	2.07	15
3	2.22	9
4	2.32	21
5	2.33	15
6	2.44	11
7	2.47	15
8	2.50	15
9	2.57	27
10	2.57	35
11	2.58	9
12	2.61	6
13	2.69	45
14	2.71	21
15	2.74	25
16	2.77	21
17	2.78	33
18	2.81	45
19	2.81	28
20	2.82	27
21	2.82	15
22	2.86	15

$$\alpha=\frac{4}{3}\left[\frac{8\pi^2}{15}\right]^{1/4} . \tag{5}$$

For the W and V values corresponding to the static bag, Eq. (4) allows us to write

$$\rho_{\text{as}}(M)=\tfrac{3}{4}\sigma_{\text{as}}\left[\frac{3M}{4},\frac{M}{4B}\right]_{M\to\infty}\!\!\sim \text{const}\times M^{-7}e^{M/T_0} \tag{6}$$

with

$$T_0=\frac{1}{\alpha}\left[\frac{256}{27}\right]^{1/4}B^{1/4} . \tag{7}$$

Then one ends with the following expression for the level density:

$$\rho(M)=\sum_{i=1}^{I} d_i\delta(M-M_{(i)})+\theta(M-M_{(I)})\rho_{\text{as}}(M) , \tag{8}$$

where I indicates the highest discrete level considered.

The final step in the thermodynamical analysis is to calculate the partition function for a free glueball gas from the knowledge of the density (8) and by treating the finite size of the glueballs within the van der Waals approximation. The result[3] can be written as an inverse Laplace transform in the variable conjugate to the volume Ω containing the gas:

$$Z(T,\Omega)=\frac{1}{2\pi i}\int_{s-i\infty}^{s+i\infty}dz\frac{e^{z\Omega}}{z-F(T,z)} \tag{9a}$$

with

$$F(T,z)=\left[\frac{T}{2\pi}\right]^{3/2}\int_{M_{(0)}}^{\infty}dM\,M^{3/2}\rho(M)\times\exp\left[-M\left[\frac{1}{T}+\frac{z}{4B}\right]\right] . \tag{9b}$$

These last expressions are the starting point for the study of the velocity-of-sound behavior of the first version of the hadronic model.

B. Glueballs with thermal fluctuations

In the previous version of the model, when we considered glueballs with fixed volumes, we froze their eventual vibrational degrees of freedom, which could be important in the deconfinement mechanism. This constraint can be relaxed in order to take into account volume fluctuations in a phenomenological way. To this end, one expands the bag energy around its minimum attained at the volume given by Eq. (2b) to obtain

$$M_{(i)}(V)=M_{(i)}+\frac{8B^2}{3M_{(i)}}\left[V-\frac{M_{(i)}}{4B}\right]^2 . \tag{10}$$

The extra energy, coming from the volume variations around the minimum, is used to Boltzmann weight the fluctuations. In this way, the temperature-dependent distribution of mass and volume is

$$\Sigma_T(M,V)=\tfrac{3}{4}\sigma(M-BV,V)\times\frac{\exp\left[-\frac{1}{T}\frac{8}{3}\frac{B^2}{M}\left[V-\frac{M}{4B}\right]^2\right]}{\left[\frac{3\pi TM}{8B^2}\right]^{1/2}} , \tag{11}$$

where

$$\sigma(M-BV,V)=\tfrac{4}{3}\sum_{i=1}^{I}d_i\delta(M-M_{(i)}(V))+\sigma_{\text{as}}(M-BV,V) \tag{12}$$

with the mass spectrum and degeneracies appearing in Table I. Notice that Eq. (11) is normalized in such a way as to ensure that its $T=0$ limit gives the standard glueball distribution (8).

In the present case, the thermodynamics of the hadron gas is contained in the partition function (9a) but with $F(T,z)$ now given by

$$F(T,z)=\left[\frac{T}{2\pi}\right]^{3/2}\int dM\int dV\,\Sigma_T(M,V)\times\exp\left[-\left[\frac{M}{T}+zV\right]\right] \tag{13}$$

instead of (8b).

C. Glueballs with fluctuations measured by a new parameter μ

This version is a slight modification of the previous one in the sense that the dispersion of the Gaussian distribution for the volume fluctuations is not induced by the bag model itself, but is measured by a new parameter μ. In any case, the dispersion is proportional to the particular hadron mass, giving rise to the distribution of mass and volume:

$$\Sigma(M,V)=\tfrac{3}{4}\sigma(M-BV,V)\frac{\exp\left[-\left[V-\frac{M}{4B}\right]^2\Big/2\mu^2\left[\frac{M}{4B}\right]^2\right]}{\left[2\pi\mu\frac{M}{4B}\right]^{1/2}} . \tag{14}$$

On the other hand, the partition function, corresponding to this version of the model, has the same form as Eq. (9a) with $F(T,z)$ given by Eq. (13), where $\Sigma_T(M,V)$ is replaced by $\Sigma(M,V)$ given above.

Certainly, the analysis of the partition function (9a), specialized to each version, allows one to obtain the corresponding phase diagrams. To this end, one has to study the leading singularity of the integrand in Eq. (9a). This expression has, in all cases under consideration, two types of singularities.[3] One type is poles, defined by

$$z_0(T)=F(T,z_0(T)) . \tag{15}$$

The other one is a branch point coming from F itself, located at $z_1(T)$ depending on each version considered. These singularities will determine different phases which arise when one singularity takes over the other one.

A detailed analysis of the thermodynamical properties of the model under consideration was carried out in Ref. 3. Here, we summarize these conclusions when applied to the three different versions under consideration.

(i) In this first version the model presents either the hadronic phase for any temperature or two first-order transitions, one at T_c and the other at $T_{c'}$. In this last case, after an intermediate region, the model returns to the hadronic phase. For that reason, this version loses effectiveness above T_c.

(ii) and (iii) In these two versions, the model has a first-order phase transition. In both cases, the corresponding transition temperature is a little larger than T_0 given in Eq. (7). In the low-temperature phase, one has the hadronic phase in which in the phase above T_c the bags explode and all the features of the system correspond to a deconfined gas of gluons. In fact, for very high temperatures, the number of bags decreases to a single one, occupying all of the volume available and allowing the gas of free gluons contained therein to manifest. It is also worth mentioning that the Stefan law $P\sim T^4$ is also asymptotically obeyed and one can easily find that, in version (iii), for example,

$$\epsilon \underset{T\to\infty}{\sim} \frac{8\pi^2}{15}T^4 \tag{16}$$

as one should expect for an ideal SU(3) gluon gas.[6]

III. VELOCITY OF SOUND

The velocity of sound in our gas of hadronic matter can be evaluated from the general expression[7]

$$v_s{}^2=\frac{\partial P}{\partial \epsilon} , \tag{17}$$

where P is the pressure of the gas and ϵ its energy density. These magnitudes can be obtained from the partition function resulting in

$$P(T)=T\frac{\partial}{\partial\Omega}\ln Z=Tz(T) \tag{18}$$

and

$$\epsilon(T)=\frac{1}{\Omega}T^2\frac{\partial}{\partial T}\ln Z=T^2\frac{\partial}{\partial T}z(T)=T^2z'(T) , \tag{19}$$

where the function $z(T)$ stands for z_0 or z_1, depending on which singularity dominates the integrand in Eq. (9a) when the thermodynamical limit is considered. Coming back to Eq. (17), for the velocity of sound one obtains

$$v_s{}^2=\frac{z(T)+Tz'(T)}{2Tz'(T)+T^2z''(T)} , \tag{20}$$

giving rise to the following results for the three different versions of the hadronic matter.

$T<T_c$. In this low-temperature phase, as was already mentioned, the partition function is dominated in all the three cases by the corresponding pole $z_0(T)$. The general properties of the pressure and the energy density of the three versions of the model are entirely similar. The corresponding curves are very smooth and as an example we present in Fig. 1 the quantitative behavior of these magni-

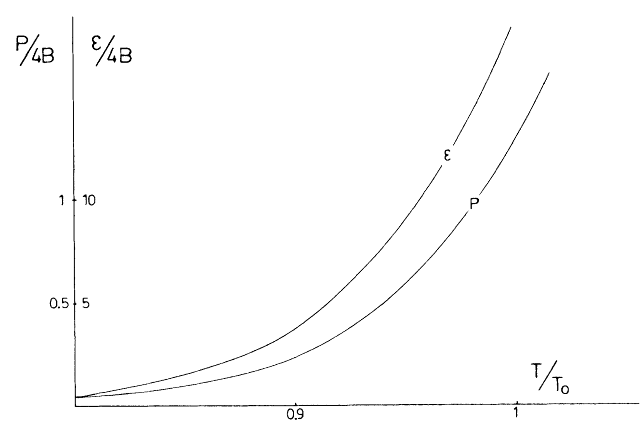

FIG. 1. Pressure and energy density behavior of version (ii) below T_c.

tudes in the case of version (ii). The limit $T \to 0$ can be analytically analyzed (see the Appendix). In this way, for the pressure one finds the behavior

$$P(T) = T z_0(T) \underset{T\to 0}{\sim} T^{5/2} e^{-m_G/T} ,$$

which for the energy density $\epsilon(T) \sim_{T\to 0} T^{3/2} e^{-mG/T}$. Then, for the versions considered one obtains

$$v_s{}^2 \underset{T\to 0}{\sim} T/m_G , \qquad (21)$$

corresponding to the velocity of sound in a gas of glueballs of mass m_G, where m_G stands for the lowest mass of the glueball spectrum which for $B = (0.2 \text{ GeV})^4$ is $m_G = 1.8$ GeV. The behavior of $v_s{}^2$, linear with T as given by Eq. (21), persists up to $T \sim 0.8 T_c$. In the cases with volume fluctuations, versions (ii) and (iii), $v_s{}^2$ reaches a maximum near $T \sim 0.9 T_c$ and then decreases to a nonvanishing value at the transition temperature T_c. The quantitative results for the three versions considered are summarized in Fig. 2. There we have included the values corresponding to a particular electron of $\mu = 0.1$ because the results are quite insensitive to it. Notice once more that the value of $v_s{}^2$ is finite at T_c when this point is reached from below.

$T > T_c$. This high-temperature regime is dominated by the singularity given by $z_1(T)$. We consider now each version of the model for hadronic matter separately. In version (i) after a frustrated deconfining transition, the system returns to a hadron phase at high temperature. The intermediate phase is dominated by the singularity

$$z_1(T) = 4B \left[\frac{1}{T} - \frac{1}{T_0} \right] , \qquad (22)$$

clearly showing that the pressure of the gas grows linearly with T, whereas the energy density remains constant. As a consequence the velocity of sound becomes infinite in this phase. This result can also be understood by noticing that for standard glueballs the radiation pressure of gluons is exactly compensated by the bag pressure B. Standard glueballs behave then as rigid objects avoiding a real deconfined gluons phase.

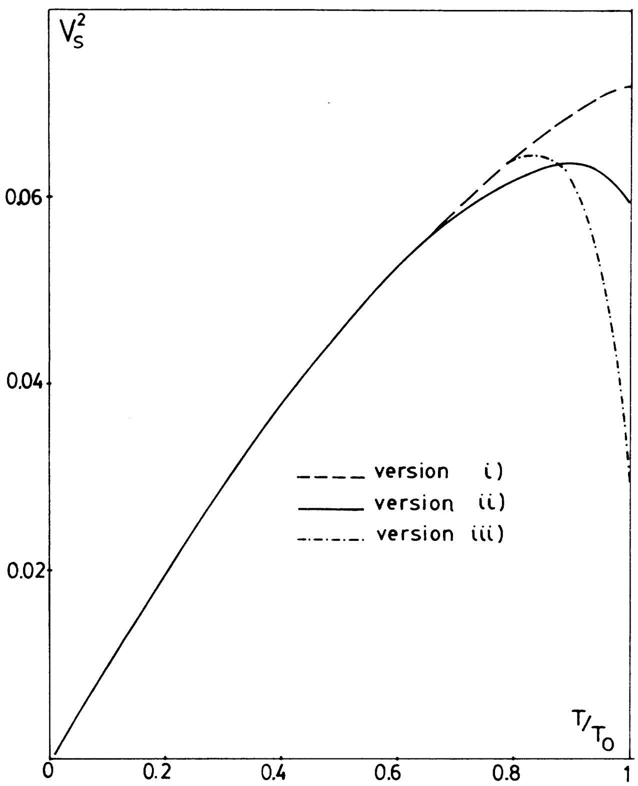

FIG. 2. Velocity of sound vs temperature below the critical temperature for the three versions.

In version (ii) the leading singularity of $F(T,z)$, given in Eq. (13), comes from the asymptotic contribution $\sigma_{\text{as}}(M - BV, V)$ to the distribution $\Sigma_T(M,V)$, Eq. (11). This singular part of $F(T,z)$ is

$$F(T,z) \approx T \int_0^\infty dV\, V^{-5} \int_{M_{22}(v)/4BV}^\infty d\lambda\, \lambda (\lambda - \tfrac{1}{4})^{-11/2} e^{-(4BV/T) S(\lambda, T, z)} , \qquad (23)$$

where

$$S(\lambda, T, z) = \lambda + \frac{Tz}{4B} - \frac{T}{T_0} \left[\frac{4\lambda - 1}{3} \right]^{3/4} + \frac{(\lambda - 1)^2}{6\lambda} \qquad (24)$$

and

$$\lambda = \frac{M}{4BV} . \qquad (25)$$

Then, $z_1(T)$, the rightmost singularity, defining the high-temperature phase is determined from the conditions

$$S(\lambda, T, z) = 0 , \qquad (26a)$$

$$\frac{\partial}{\partial \lambda} S(\lambda, T, z) = 0 \qquad (26b)$$

in the parametric form

$$z_1 = \frac{4B}{T_0} \left[\frac{4\lambda - 1}{3} \right]^{-1/4} \frac{(7\lambda - 1)(\lambda^2 - 1)}{3(7\lambda^2 - 1)} , \qquad (27a)$$

$$T = T_0 \left[\frac{4\lambda - 1}{3} \right]^{1/4} \frac{7\lambda^2 - 1}{6\lambda^2} . \qquad (27b)$$

From these expressions, the thermodynamical properties of the system can be immediately computed. In particular, the pressure of the gluon gas and its energy density behave as shown in Fig. 3. Thus, the velocity of sound reads

$$v_s{}^2 = \frac{1}{3} + \frac{2}{3} \frac{4\lambda - 1}{\lambda (7\lambda^2 - 1)} . \qquad (28)$$

This expression takes the value $\frac{2}{3}$ at the transition temper-

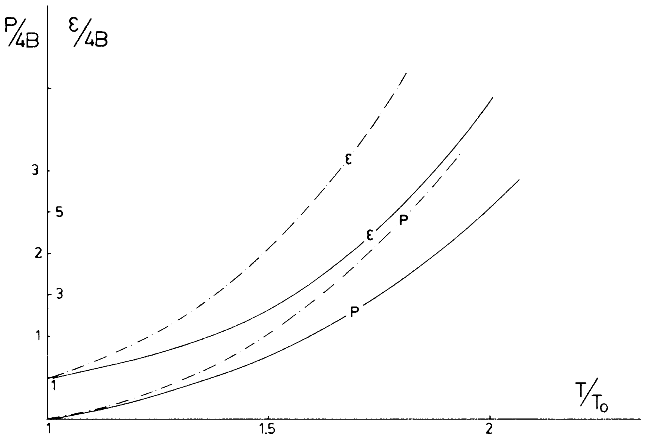

FIG. 3. Pressure and energy density behavior of versions (ii) and (iii) above T_c.

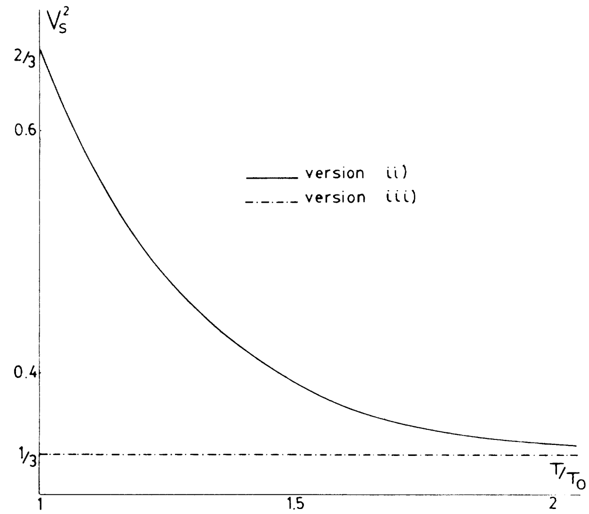

FIG. 4. Velocity of sound vs temperature above the critical temperature for versions (ii) and (iii).

ature T_c. In Fig. 4 we show the entire behavior of this $v_s{}^2$ as a function of T. Notice that only at asymptotic temperatures, $v_s{}^2$ takes the free gluons gas value of $\frac{1}{3}$.

The calculation corresponding to version (iii) can be carried out in a similar way as in version (ii) to obtain

$$z_1(T)=\frac{B}{T}\left[\left[\frac{T}{T_0}\right]^4-1\right] . \tag{29}$$

From this expression the pressure and the energy density can be immediately obtained (see Fig. 3). For the velocity of sound, $v_s{}^2$, it results in the constant value $\frac{1}{3}$ which is just the expected value for a deconfined phase immediately after $T=T_c$. We remark that this result is independent of the specific value of the parameter μ that measures the volume fluctuations of the glueballs.

IV. FINAL COMMENTS

We have evaluated the velocity of sound in hadron matter modeled as a simple gas of MIT bags. The results obtained in the hadronic phase, the low-temperature phase, corresponding to version (i) of the model grow monotonically up to the critical temperature T_c. On the other hand, the values for the velocity of sound, corresponding to versions (ii) and (iii), present a maximum near T_c showing a behavior entirely compatible with previous Monte Carlo estimations.[4] The quantitative discrepancies depend essentially upon the mass of the lightest glueball predicted by both calculations whereas our smallest glueball for $B=(200\text{ MeV})^{1/4}$ corresponds to $m_G=1.8$ GeV, the value appearing in Ref. 4 is $m_G=0.9$ GeV. At the critical temperature the velocity of sound presents a discontinuity due to the fact that the transition is a first-order one. Above this transition all three versions of the model considered behave differently. The first version gives an infinite value for the velocity of sound as it was discussed previously. The behavior of the other two versions are shown in Fig. 4. Only version (iii), where the glueballs have volume fluctuations measured by a parameter μ, gives a constant value of $v_s{}^2=\frac{1}{3}$ corresponding to the case of a completely free gluon gas. Then, in this case $T=T_c$ is certainly the deconfining transition temperature.

Finally, we can conclude that our calculations indicate that the velocity of sound is a very interesting parameter in the analysis of the thermodynamical description of hadron matter.

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APPENDIX

Here we study the limit $T\rightarrow 0$ of the function $z_0(T)$. We present only the calculation corresponding to version (i) of the model because for the other two versions the procedure is entirely similar.

For $T<T_c$, the partition function is dominated by the pole $z_0(T)$ defined through

$$z_0(T)=F(T,z_0(T)) . \tag{A1}$$

The function $F(T,z_0)$ can be split as

$$F(T,z)=F_{\text{disc}}(T,z)+F_{\text{as}}(T,z) , \tag{A2}$$

where

$$F_{\text{disc}}(T,z)=\left[\frac{T}{2\pi}\right]^{3/2}\sum_{i=1}^{I} d_i M_{(i)}{}^{3/2}\exp\left[-M_{(i)}\left[\frac{1}{T}+\frac{z}{4B}\right]\right] \tag{A3a}$$

and

$$F_{\rm as}(T,z)=\left[\frac{T}{2\pi}\right]^{3/2}K\int_{M_{(I)}}^{\infty}dM\,M^{-11/2}\exp\left[-M\left[\frac{1}{T}-\frac{1}{T_0}+\frac{z}{4B}\right]\right]. \tag{A3b}$$

This last expression provides, for $T \ll M_{(I)}$, the result is

$$F_{\rm as}(T,z)\approx\frac{K}{(2\pi)^{3/2}}M_{(I)}{}^{-11/2}T^{5/2}e^{-M_{(I)}/T}\exp\left[-M_{(I)}\left[\frac{z}{4B}-\frac{1}{T_0}\right]\right]\left[1-\frac{11}{2}\left[\frac{T}{M_{(I)}}\right]+O\left[\left[\frac{T}{M_{(I)}}\right]^2\right]+\cdots\right], \tag{A4}$$

while for $T \ll M_{(1)}$, (A4) is negligible and from (A3a) one obtains

$$F(T,z)\approx\left[\frac{T}{2\pi}\right]^{3/2}d_1M_{(1)}{}^{3/2}\exp\left[-M_{(1)}\left[\frac{1}{T}+\frac{z}{4B}\right]\right] \tag{A5}$$

that, when used in (A1), gives, for $z_0(T)$,

$$z_0(T)\approx d_1\left[\frac{M_{(1)}}{2\pi}\right]^{3/2}e^{-M_{(1)}/T}T^{3/2}\;. \tag{A6}$$

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$$\epsilon=k\frac{N^2-1}{15}\pi^2T^4$$

with $k=1$ in version (iii) and $k=\frac{4}{3}(\frac{6}{7})^3$ in version (ii).

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