PHYSICAL REVIEW D VOLUME 36, NUMBER 5 1 SEPTEMBER 1987

Low-energy theorems for strongly interacting W's and Z's

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(Received 13 April 1987)

Low-energy theorems are proved for the scattering of longitudinally polarized W and Z bosons that hold at a scale intermediate between M_W and the characteristic mass scale of the symmetry-breaking sector. The theorems are proved without assuming a custodial SU(2) symmetry. Three methods are used: a perturbative power-counting analysis in the unitary gauge, a current-algebra derivation in renormalizable gauges, and a nonlinear chiral Lagrangian that is relevant in unitary or renormalizable gauges.

I. INTRODUCTION

Though it is widely believed that the $SU(2)_L \times U(1)_Y$ local symmetry[1] of electroweak interactions is spontaneously broken by asymmetry of the vacuum,[2] the details of the breaking mechanism remain an opaque black box, referred to generically as the "symmetry-breaking sector." We do not know the quanta of the symmetry-breaking sector or even whether the forces between them are of weak or strong magnitude. We do know one scale of this otherwise unknown physics: the vacuum expectation value that is fixed by the Fermi constant to be

$$v=(\sqrt{2}G_F)^{-1/2}\simeq 0.25 \text{ TeV} . \tag{1.1}$$

We expect, as discussed below, that the typical mass scale of the symmetry-breaking sector, denoted M_{SB}, and the strength of the interactions, denoted λ_{SB}, are correlated. For $M_{SB}/v \lesssim 1$ we expect λ_{SB} to be small and the interactions to be amenable to perturbative analysis. For $M_{SB}/v \gg 1$ we expect the interactions to be strong, so that, as for QCD, nonperturbative methods of analysis would be needed to deduce the spectrum from the Lagrangian.

In this paper we derive low-energy theorems for the scattering of longitudinally polarized W and Z bosons, W_L and Z_L, that hold for all strongly interacting symmetry-breaking sectors, provided they contain no quanta (other than W_L and Z_L) that are light compared to the typical scale $M_{SB} \gtrsim 1$ TeV. These low-energy scattering amplitudes are completely determined by v, Eq. (1.1), and by the ρ parameter

$$\rho \equiv (M_W/M_Z \cos\theta_W)^2 . \tag{1.2}$$

Experimental measurements fix $\rho = 1$ to within a few percent, which implies universal values of the low-energy scattering amplitudes for all experimentally viable models of the symmetry-breaking sector with spectra fully at or above 1 TeV. If the spectrum contains bosons much lighter than 1 TeV, e.g., pseudo-Goldstone bosons, they may cause the low-energy amplitudes to be modified.

The extrapolated low-energy scattering amplitudes are the basis of a general probe of the symmetry-breaking sector that could be implemented at a proton-proton collider with the energy and luminosity proposed for the Superconducting Super Collider[3,4] (SSC). The central qualitative point is that WW fusion provides a significantly enhanced yield of longitudinally polarized gauge-boson pairs if and only if the W_L and Z_L scattering amplitudes are strong. This will in turn be true if and only if the symmetry-breaking sector is strongly interacting, since W_L and Z_L are essentially Goldstone bosons associated with the spontaneous breaking of a global symmetry of the symmetry-breaking sector—a statement that is made precise by the equivalence theorem proved to all orders in Ref. 3. The low-energy amplitudes may be extrapolated up to energies at which dynamics sets in (most probably in the form of resonant enhancements), yielding a conservative estimate of the W_L,Z_L pair signal. Given the likely ability to detect W,Z pairs, it seems that the signal could be detected over backgrounds (e.g., $\bar{q}q \rightarrow WW$) at a pp collider with $\sqrt{s}=40$ TeV and $\mathcal{L}=10^{33}$ cm^{-2}sec^{-1} as proposed for the SSC, but not at one with half the energy or one-tenth the luminosity.[4,5] This implies a "no-lose corollary"[4] for a collider with the parameters proposed for the SSC: either we see the gauge-boson pairs signaling a strongly interacting symmetry-breaking sector and/or there are light (compared to 1 TeV) particles from the symmetry-breaking sector that are copiously produced. These phenomenological aspects are discussed briefly in the concluding section and will be more carefully addressed in future work. The emphasis in this paper is on the low-energy theorems themselves.

The correlation between the interaction strength λ_{SB} and the mass scale M_{SB} of the symmetry-breaking sector is exemplified by the minimal Higgs model[2] and can be more generally understood from the perspective of the low-energy theorems. In the Higgs model the scalar interactions are given by the potential

$$V(\mathbf{w},H)=\frac{\lambda}{4}[(\mathbf{w}^2+H^2)^2-v^2]^2 , \tag{1.3}$$

where $\mathbf{w}$ is a triplet of scalar particles and H is a fourth scalar field. Assuming the vacuum state is given by the classical minimum at $\mathbf{w}=0$, $H=v$, and redefining H to have vanishing expectation value, the potential becomes

$$V(\mathbf{w},H)=\frac{\lambda}{4}(\mathbf{w}^2+H^2)^2+\lambda v H(\mathbf{w}^2+H^2)+\frac{m_H{}^2}{2}H^2 , \tag{1.4}$$

where the mass of the Higgs boson is

$$m_H{}^2=2\lambda v^2 \tag{1.5}$$

and the $\mathbf{w}$ remain massless, being the Goldstone bosons associated with the spontaneous symmetry breakdown of the global $SU(2)_L \times SU(2)_R$ symmetry to the diagonal subgroup $SU(2)_{L+R}$ (Ref. 6). Upon including the $SU(2)_L \times U(1)$ gauge interactions, the triplet $\mathbf{w}$ becomes, by virtue of the Higgs mechanism, the longitudinal modes of the gauge bosons, W_L and Z_L, which by the equivalence theorem[3] continue to interact at energies large compared to their masses according to the interactions of Eq. (1.4). The correlation between M_{SB} and λ_{SB} is then exemplified by Eq. (1.5). More precisely the onset of strong coupling may be said to begin at $m_H=1$ TeV where the Born approximation amplitudes for $s \gg m_H{}^2$ saturate partial-wave unitarity.[7] The interpretation of this fact is not that the parameter m_H cannot be larger than 1 TeV or that $\lambda/4\pi$ cannot be larger than $\sim 2/\pi$ but that for larger m_H or λ the quantum corrections become as big as the Born terms, i.e., that the theory becomes strongly interacting. (Of course there is no guarantee in the strong-coupling regime that m_H corresponds to the mass of an observable particle.)

It was shown in Ref. 3 for a particular class of strongly interacting models that current algebra, PCAC (partial conservation of axial-vector current), and the $w-W_L$ equivalence theorem together imply low-energy theorems for W_L, Z_L scattering that are valid to all orders in the strong coupling λ_{SB}. The class of models discussed there had a global $SU(2)_L \times SU(2)_R$ symmetry that breaks spontaneously to the diagonal $SU(2)_{L+R}$ subgroup, with the triplet $\mathbf{w}=w^{\pm},z$ identified as the associated Goldstone bosons. The latter $SU(2)_{L+R}$ invariance, known as the custodial SU(2),[8] is sufficient (though not proven necessary) to guarantee that $\rho=1$ to all orders in the strong coupling λ_{SB}. The minimal Higgs model is one example of such a model. In another context and at a different mass scale, QCD is another example. In fact, if we identify $\mathbf{w}\to\pi$, $H\to\sigma$, and $v=0.25$ TeV$\to F_\pi=93$ MeV, then the minimal Higgs model becomes precisely the pre-QCD σ model that was devised to illustrate the spontaneously broken chiral symmetry of hadronic physics.

Just as Weinberg[9] proved pion-pion scattering low-energy theorems, such as

$$\mathcal{M}(\pi^+\pi^-\to\pi^0\pi^0)\simeq\frac{s}{F_\pi{}^2} \tag{1.6}$$

for all models of hadronic physics in which the pions are Goldstone bosons associated with $SU(2)_L \times SU(2)_R \to SU(2)_{\text{isospin}}$, so for all models of the symmetry-breaking sector with a custodial SU(2) invariance, we have, in the Landau gauge,

$$\mathcal{M}(w^+w^-\to zz)\simeq\frac{s}{v^2} . \tag{1.7}$$

Equation (1.6) is valid for s much smaller than the masses of exchange quanta and much smaller than the scale $4\pi F_\pi \sim 1$ GeV set by quantum corrections.[10] Similarly Eq. (1.7) holds for $s \ll \Lambda_{SB}{}^2$ where

$$\Lambda_{SB}=\min\{M_{SB},4\pi v\} \tag{1.8}$$

provided there are no exchange quanta with masses much lighter than the characteristic scale of the spectrum, M_{SB}. For energies large compared to M_W the equivalence theorem asserts that U-gauge scattering amplitudes for longitudinally polarized W's and Z's are equal to the R-gauge amplitudes of the corresponding w and z Goldstone bosons:

$$\mathcal{M}(W_L(p_1),W_L(p_2),\cdots)_U=\mathcal{M}(w(p_1),w(p_2),\cdots)_R+O\left[\frac{M_W}{E_i}\right] . \tag{1.9}$$

The equivalence theorem was proved to leading order by Cornwall, Levin, and Tiktopoulos.[11] As is essential for applications to strongly coupled theories, it was proved to all orders in Ref. 3.

Combining Eqs. (1.7) and (1.9) we obtain the low-energy theorem for the physical amplitude valid in the energy domain $M_W{}^2 \ll s \ll \Lambda_{SB}{}^2$:

$$\mathcal{M}(W_L^+W_L^-\to Z_LZ_L)\simeq\frac{s}{v^2}\simeq\frac{g^2s}{4M_W{}^2} , \tag{1.10}$$

where we use the relation $M_W=gv/2$ that is valid up to electroweak corrections and corrections of order $M_W{}^2/\Lambda_{SB}{}^2$ (see the discussion of effective Lagrangians, Sec. IV). Similarly we find two other independent amplitudes:

$$\mathcal{M}(W_L^+W_L^-\to W_L^+W_L^-)\simeq-\frac{g^2u}{4M_W{}^2} , \tag{1.11}$$

$$\mathcal{M}(Z_LZ_L\to Z_LZ_L)\simeq 0 . \tag{1.12}$$

The other four scattering amplitudes, elastic scattering of $W_L^{\pm}Z_L$, $W_L^+W_L^+$, and $W_L^-W_L^-$, follow from Eqs. (1.10) and (1.11) by crossing symmetry:

$$\mathcal{M}(W_L^{\pm}Z_L\to W_L^{\pm}Z_L)\simeq\frac{g^2t}{4M_W{}^2} , \tag{1.13}$$

$$\mathcal{M}(W_L^+W_L^+\to W_L^+W_L^+)=\mathcal{M}(W_L^-W_L^-\to W_L^-W_L^-)\simeq-\frac{g^2s}{4M_W{}^2} . \tag{1.14}$$

In this paper we will not assume that the symmetry-breaking sector has a custodial SU(2) invariance, since there is no proof that it is necessary to obtain $\rho=1$. We will show that the low-energy theorems (1.10) and (1.11) are, in general (again for $M_W{}^2 \ll s \ll \Lambda_{SB}{}^2$),

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) \simeq \frac{g^2 s}{4M_W^2} \frac{1}{\rho} , \tag{1.15}$$

$$\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) \simeq -\frac{g^2 u}{4M_W^2} \left[4 - \frac{3}{\rho} \right] , \tag{1.16}$$

while Eq. (1.12) is not modified. Of course, for $\rho=1$, Eqs. (1.15) and (1.16) agree with the low-energy amplitudes that were obtained assuming a custodial SU(2) symmetry. The experimentally established constraint that $\rho=1$, accurate to a few percent, therefore implies that the low-energy theorems are essentially given by Eqs. (1.10)–(1.14) whether or not the symmetry-breaking sector has a custodial SU(2) invariance. In essence the arguments given below show that the condition $\rho=1$ implies that the Goldstone-boson triplet sector must respect an effective custodial SU(2) symmetry at low energies.

We have used three different methods to establish these results: a power-counting analysis carried out in the unitary gauge, a current-algebra derivation in the Landau gauge that uses the equivalence theorem, and a derivation by effective chiral Lagrangian which can be applied to U or R gauges. In a previous Letter[12] we presented the U-gauge power-counting analysis. In this paper we will emphasize the current-algebra and effective-Lagrangian approaches, which are more natural languages for the discussion of dynamical-symmetry-breaking models such as technicolor[13] and ultracolor.[14] The many successes of current algebra and effective Lagrangians in hadronic physics give us confidence that they are appropriate tools as we look toward the possible discovery of a new spectrum of strongly coupled particles. The effective-Lagrangian technique has the additional advantage that we can use it to study the deviations from the low-energy theorems that could be induced by pseudo-Goldstone bosons.

The plan of the paper is as follows. In Sec. II we sketch the U-gauge power-counting analysis, previously presented in Ref. 12. This derivation shows directly that the low-energy amplitudes, Eqs. (1.12), (1.15), and (1.16), are dictated by $SU(2)_L \times U(1)$ gauge invariance alone. The perturbative framework of the analysis is more naturally applied to elementary Higgs models than to dynamical theories.

Section III contains the current algebra derivation, which is similar to Weinberg's derivation[9] of the pion-pion scattering lengths. Where Weinberg's derivation uses PCAC and the $SU(2)_L \times SU(2)_R$ algebra of charges, our derivation uses PCLC (partially conserved left-handed current) and just the $SU(2)_L$ algebra of charges that must exist because of the $SU(2)_L$ gauge invariance.

In Sec. IV we show that the most general effective Lagrangian also implies the amplitudes, Eqs. (1.12), (1.15), and (1.16). This is perhaps the most elegant means to the low-energy theorems. The chiral Lagrangian is gauge invariant. In an R gauge its tree amplitudes reproduce the current-algebra results, while in the U gauge it provides a justification of the U-gauge power-counting analysis.

Section V contains some concluding remarks. We give an incomplete discussion of the effect of light pseudo-Goldstone bosons: for amplitudes involving only $W^\pm$ bosons the sign of their effect is easily deduced but for mixed amplitudes of W and Z bosons the question is under study. The section concludes with a very brief description of the phenomenological implications of the low-energy theorems.

II. PERTURBATIVE ANALYSIS

In this section we sketch a derivation of the low-energy theorems using a power-counting analysis carried out in a perturbative framework. This argument has been presented elsewhere[12] and the discussion here will be very brief. The essence of the argument is to divide the U-gauge scattering amplitude into a gauge sector term, involving only vertices and exchange quanta from the gauge sector, and a symmetry-breaking sector term, including exchange quanta from the symmetry-breaking sector. The first term has "bad" high-energy behavior that is canceled by the second term. However, at low energy the second term is negligible if all exchange quanta from the symmetry-breaking sector are heavy. Then the low-energy amplitude is just given by the gauge sector terms, which are precisely the low-energy theorem amplitudes Eqs. (1.12), (1.15), and (1.16). This derivation shows that the low-energy theorems in the form of Eqs. (1.12), (1.15), and (1.16) are determined by gauge invariance alone. As discussed in the previous section, the perturbative framework of the analysis may be less appropriate than the current-algebra and effective-Lagrangian approaches for dynamical models in which the observable spectrum of the symmetry-breaking sector are composites of the quanta that appear in the Lagrangian.

Consider first the minimal Higgs model. Though we will deduce the low-energy amplitudes to all orders in the strong coupling $\lambda = m_H^2/2v^2$, we begin by computing the U-gauge tree approximation amplitude for $W_L W_L \to Z_L Z_L$ at $s \gg M_W^2$:

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) = \mathcal{M}_{\text{gauge}} + \mathcal{M}_{\text{SB}} , \tag{2.1}$$

where

$$\mathcal{M}_{\text{gauge}} = \frac{g^2 s}{4M_W^2} \frac{1}{\rho} \tag{2.2}$$

is given by u- and t-channel W exchange and by the four-point gauge-boson coupling and

$$\mathcal{M}_{\text{SB}} = -\frac{g^2 s}{4M_W^2} \frac{s}{s - M_H^2} \tag{2.3}$$

is just the s-channel Higgs pole. Since at the tree level in this model we have $\rho=1$, the terms linear in s cancel for $s \gg m_H^2$, as they must to ensure renormalizability. But for $s \ll m_H^2$ the amplitude is dominated by $\mathcal{M}_{\text{gauge}}$ alone. The power-counting analysis that is sketched below shows that the decoupling of $\mathcal{M}_{\text{SB}}$ at low energy is a general result provided all exchange quanta from the symmetry-breaking sector are heavy. Then the only strong corrections that do not decouple at low-energy are incorporated into the renormalized physical values of M_W and ρ that appear in Eq. (2.2).

We illustrate this conclusion by considering a general Higgs model, in which all physical scalars are assumed heavy, of order $M_{\rm SB} \gtrsim 1$ TeV. One-loop corrections from the scalar sector are shown in Fig. 1. Figures 1(a)–1(c) are quadratically divergent contributions to the vacuum-polarization tensor. For instance, after minimal subtraction of the divergences, the finite contribution of Fig. 1(a) is

$$g^2\int d^4p \left.\frac{(2p-q)^\mu(2p-q)^\nu}{(p^2-M_{\rm SB}^2)[(p-q)^2-M_{\rm SB}^2]}\right|_{\rm subtracted} = g^{\mu\nu}(g^2AM_{\rm SB}^2+g^2Bq^2+\cdots) + q^\mu q^\nu\left[g^2C+g^2D\frac{q^2}{M_{\rm SB}^2}+\cdots\right], \quad (2.4)$$

where A,B,C,D are dimensionless numbers and logarithmic dependence on $M_{\rm SB}$ is neglected here and elsewhere. The $g^2AM_{\rm SB}^2$ term contributes to the gauge-boson self-energy. As a fraction of the tree-level mass it is just an $O(\lambda_{\rm SB})$ correction:

$$\frac{\delta M_W^2}{M_W^2}\sim\frac{g^2M_{\rm SB}^2}{g^2v^2}\sim\lambda_{\rm SB}$$

[for instance, see Eq. (1.5)]. The g^2Bq^2 term gives an $O(\alpha_W/\pi)$ contribution to the wave-function renormalization. Adding internal scalar lines to the loops of Figs. 1(a)–1(c) modifies these one-loop results by additional powers of $\lambda_{\rm SB}$. Thus the gauge-boson masses are strongly renormalized while the corrections to the wave-function renormalization are screened (modulo logarithms) by one power of $\alpha_W/\pi=g^2/4\pi^2$.

Like the wave-function renormalization, the one-loop scalar corrections to the three-gauge-boson vertex [Fig. 1(d) and other diagrams with one or two internal scalars replaced by gauge bosons] are only logarithmically divergent and on dimensional grounds the leading finite contribution to the gauge coupling constant renormalization is at most $O((\alpha_W/\pi)\ln M_{\rm SB})$. Again, to higher orders in the strong scalar interactions this correction is multiplied by additional powers of $\lambda_{\rm SB}$. Finite-momentum

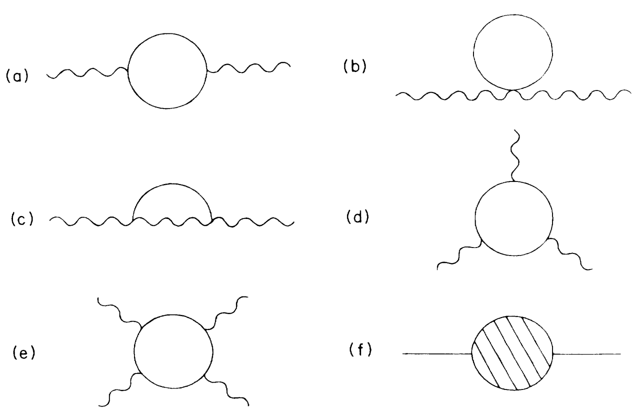

FIG. 1. Quantum corrections from symmetry-breaking sector. Curly lines represent gauge bosons and straight lines represent physical scalar or pseudoscalar bosons from the symmetry-breaking sector.

form-factor effects are suppressed by powers of $Q^2/M_{\rm SB}^2$ where Q is the external momentum scale.

The one-particle-irreducible (1PI) scalar loop contributions to the W_LW_L scattering amplitude [Fig. 1(e) and other diagrams with one, two, or three scalar propagators replaced by gauge bosons] make an at most logarithmically-divergent contribution to terms in the amplitudes that are linear in s, t, or u. Dimensionally the dependence on $M_{\rm SB}$ of this contribution to the low-energy amplitudes is at most logarithmic, so that Fig. 1(e) also makes no $O(\lambda_{\rm SB})$ correction to the tree approximation. Relative to the tree amplitudes, Eqs. (1.15) and (1.16), it contributes corrections of $O(\alpha_W/\pi)$ or $O((\alpha_W/\pi)(s/M_W^2))$, multiplied by factors $(s/M_{\rm SB}^2)^n$ for integer $n>0$. The $O((\alpha/\pi)s/M_W^2)$ terms correspond to the $O(s/(4\pi v)^2)$ terms expected from loop corrections in the phenomenological Lagrangian approach.[10]

Finally there are strong renormalizations of the scalar-boson self-energies, indicated generically in Fig. 1(f). These could affect the leading threshold behavior of the W_LW_L scattering amplitudes if they were to induce a pole in the complex energy plane with real and imaginary parts small compared to $M_{\rm SB}$. This possibility is excluded by the assumption that there be no physical particles in the spectrum of the symmetry-breaking sector that are light compared to the typical scale $M_{\rm SB}$. If light scalars do exist, they can in different cases increase, decrease or leave unaffected the low-energy amplitudes Eqs. (1.15) and (1.16).

The conclusion is that M_W and M_Z are strongly renormalized by order $\lambda_{\rm SB}$ effects while all other corrections to the low-energy W_LW_L amplitudes from a strongly interacting Higgs sector are screened by a power of α_W/π or suppressed by powers of $s/\Lambda_{\rm SB}^2$. This establishes the validity of the low-energy amplitudes [(1.12), (1.15), and (1.16)] where ρ and M_W are the physical values, incorporating the strong quantum corrections from the Higgs sector.

The same power-counting analysis applies if the symmetry-breaking sector also contains heavy fermions. The one-fermion-loop contributions to the vacuum-polarization tensor induce $O(\lambda_{\rm SB})$ renormalizations of M_W and M_Z [where now $\lambda_{\rm SB}\sim(gM_F/2M_W)^2\sim(M_{\rm SB}/v)^2$ with $M_F\sim M_{\rm SB}$ the heavy-fermion mass], while the one-fermion-loop contributions to the three- and four-point gauge-boson amplitudes are screened by α_W/π or by $s/M_{\rm SB}^2$ (Ref. 15). These conclusions also hold if the fermions carry the charge of a strong unbroken non-Abelian gauge symmetry as in technicolor models. Higher-order corrections due to the strong gauge interaction then modify the one-loop corrections by powers of the strong gauge coupling constant. However, for theories of this kind, which are probably confining, we prefer the current-algebra and effective-Lagrangian methods that we turn to next.

III. CURRENT-ALGEBRA DERIVATION

In this section we present a current-algebra derivation of the low-energy theorems. As in Ref. 3 we work in a renormalizable gauge and use the equivalence theorem, discussed in the Introduction, to obtain the W_L, Z_L amplitudes. The present derivation differs from Ref. 3 in that it does not make use of the full $SU(2)_L \times SU(2)_R$ global symmetry

$$[L_a, L_b] = i\epsilon_{abc} L_c \ , \tag{3.1}$$

$$[R_a, R_b] = i\epsilon_{abc} R_c \ , \tag{3.2}$$

$$[L_a, R_b] = 0 \ , \tag{3.3}$$

used by Weinberg[9] to obtain the pion-pion scattering lengths. Instead we use only the $SU(2)_L$ charge algebra, Eq. (3.1), which is necessarily satisfied in the symmetry-breaking sector in order to satisfy electroweak $SU(2)_L$ gauge invariance. Consequently our derivation is valid whether or not there is a custodial SU(2) symmetry and applies for all values of the ρ parameter.

The currents L_a^μ can in general be expanded as

$$L_a^\mu = -\tfrac{1}{2} f_a \partial^\mu w_a + \tfrac{1}{2} r_a \epsilon_{abc} w_b \partial^\mu w_c + \cdots \ , \tag{3.4}$$

where f_a and r_a are constants (no sum on a) and the omitted terms involve the non-Goldstone fields and/or carry higher operator dimension and are suppressed by powers of the large parameter Λ_{SB}. The w_a are just the three Goldstone bosons which mix with the W and Z gauge bosons. Signs and factors of two are chosen to make contact with the usual $L = (V - A)/2$ current. The charges appearing in Eq. (3.1) are

$$L^a \equiv \int d^3x L^0(\mathbf{x}, 0) \ . \tag{3.5}$$

The unbroken U(1) of electromagnetism requires

$$f_1 = f_2 \tag{3.6}$$

and

$$r_1 = r_2 \ . \tag{3.7}$$

If there were a custodial SU(2) we would also have $f_1 = f_3$ and $r_1 = r_3$. In general f_1/f_3 is related to ρ by considering the contribution of the Goldstone bosons to the vacuum-polarization tensor $\langle L_a^\mu L_b^\nu \rangle$. Up to corrections of order α and/or $(M_W/\Lambda_{SB})^2$ the gauge-boson masses are

$$M_W = \tfrac{1}{2} g f_1 \ , \tag{3.8a}$$

$$M_Z = \tfrac{1}{2} g f_3 / \cos\theta_W \ , \tag{3.8b}$$

from which we deduce

$$f_1 = v \simeq 0.25 \text{ TeV} \tag{3.9}$$

and

$$\rho = (f_1/f_3)^2 \ . \tag{3.10}$$

The r_a are determined by the $SU(2)_L$ charge algebra, Eq. (3.1). In particular, they are determined by the commutator of the $f\partial^0 w$ term in L_a^0 with the $rw \times \partial^0 w$ term in L_b^0 to yield the $f\partial^0 w$ term in L_c^0 on the right-hand side of Eq. (3.1).

The result is

$$r_1 = r_2 = \frac{1}{\sqrt{\rho}} \ , \tag{3.11a}$$

$$r_3 = 2 - \frac{1}{\rho} \ . \tag{3.11b}$$

These results establish the claim made in Sec. I that $\rho = 1$ implies an effective low-energy custodial SU(2) for the Goldstone-boson triplet. For $\rho = 1$ we have $f_1 = f_2 = f_3$ and $r_1 = r_2 = r_3 = 1$, so that the purely Goldstone-boson components of the current L_a^μ can be decomposed into vector and axial-vector terms, $L^\mu = \tfrac{1}{2}(V_a^\mu - A_a^\mu) + \ldots$, where the vector component $V_a^\mu = \tfrac{1}{2}\epsilon_{abc} W_b \partial^\mu w_c$ generates the custodial SU(2) for the Goldstone sector under which the axial-vector component $A_a^\mu = \tfrac{1}{2} f \partial^\mu w_a$ forms an isotriplet.

Next we use the standard current-algebra soft-pion method, similar to Weinberg's derivation of the $\pi\pi$ low-energy theorems, except that we work in the Goldstone limit with $\partial_\mu L_a^\mu = 0$. The fundamental equation is then

$$\int d^4x\, d^4y\, e^{i(p_a \cdot y + p_c \cdot x)} \langle d \mid T \partial L_a(y) \partial L_c(x) \mid b \rangle = 0 \ . \tag{3.12}$$

Integrating twice by parts and taking $p_a, p_c \to 0$ we find a term proportional to $\mathcal{M}_{a,b;c,d}$, the amplitude for $w_a w_b \to w_c w_d$ scattering, that arises from pole diagrams in which the currents L_a^μ and L_c^ν create w_a and w_c bosons. Using the form of the current in Eq. (3.4), we find

$$\lim_{p_a, p_c \to 0} \mathcal{M}_{a,b;c,d} = 2is \frac{r_e \epsilon_{ace} \epsilon_{bde}}{f_a f_c} + \frac{4}{f_a f_c} p_a^\mu p_c^\nu \int d^4x\, e^{-ip_c \cdot x} \langle d \mid T L_a^\mu(0) L_c^\nu(x) \mid b \rangle \ , \tag{3.13}$$

where $s = (p_a + p_b)^2$. The first term arises from the commutator Eq. (3.1) and the second contributes in leading order only if there are s-, t-, or u-channel pole contributions from massless particle exchanges.

In Weinberg's derivation there are no pole terms because the $J_5^\mu \pi\pi$ vertex is forbidden by G parity, but in our case there is an Lww vertex and pole terms do contribute. Assuming w and z are the only light particles we evaluate explicitly their contribution to the pole terms. The result for the sum of the equal-time commutator term and the pole term is

$$\lim_{p_a p_c \to 0} \mathcal{M}_{a,b;c,d} = \frac{is}{f_a f_c} (2r_e - r_a r_c) \epsilon_{ace} \epsilon_{bde} \ . \tag{3.14}$$

Bose symmetry, U(1) invariance, and crossing symmetry constrain the low-energy expansion of the off-shell scattering amplitude to have the form

$$\begin{aligned}\mathcal{M}_{a,b;c,d}=&(\delta^{a3}\delta^{b3}\delta^{c3}\delta^{d3})(A_1)+(\delta^{a3}\delta^{b3}\Delta^{cd}+\delta^{c3}\delta^{d3}\Delta^{ab})[A_2+B_2(t+u)+C_2s]\\&+(\delta^{a3}\delta^{c3}\Delta^{bd}+\delta^{b3}\delta^{d3}\Delta^{ac})[A_2+B_2(s+u)+C_2t]+(\delta^{b3}\delta^{c3}\Delta^{ad}+\delta^{a3}\delta^{d3}\Delta^{bc})[A_2+B_2(s+t)+C_2u]\\&+(\Delta^{ab}\Delta^{cd})[A_3+B_3(t+u)+C_3s]+(\Delta^{ac}\Delta^{bd})[A_3+B_3(s+u)+C_3t]\\&+(\Delta^{ad}\Delta^{bc})[A_3+B_3(s+t)+C_3u]+\cdots\ ,\end{aligned}\tag{3.15}$$

where $s=(p_a+p_b)^2$, $t=(p_a+p_c)^2$, $u=(p_a+p_d)^2$, and A_i,B_i,C_i are constants. The tensor Δ^{ab} is defined by

$$\Delta^{ab}=\begin{cases}1 & \text{if } i=j\neq 3,\\ 0 & \text{otherwise}\end{cases}\tag{3.16}$$

and δ^{ab} is the usual Kronecker δ.

From Eq. (3.14) we see that

$$A_1=A_2=A_3=0\ .\tag{3.17}$$

Since on mass shell $s+t+u=0$ the leading low-energy behavior of the amplitude is determined by just two constants $D_i\equiv C_i-B_i$, $i=2,3$:

$$\begin{aligned}\mathcal{M}_{a,b;c,d}\simeq&(\delta^{a3}\delta^{b3}\Delta^{cd}+\delta^{c3}\delta^{d3}\Delta^{ab})D_2s+(\delta^{a3}\delta^{c3}\Delta^{bd}+\delta^{b3}\delta^{d3}\Delta^{ac})D_2t\\&+(\delta^{b3}\delta^{c3}\Delta^{ad}+\delta^{a3}\delta^{d3}\Delta^{bc})D_2u+\Delta^{ab}\Delta^{cd}D_3s+\Delta^{ac}\Delta^{bd}D_3t+\Delta^{ad}\Delta^{bc}D_3u\ .\end{aligned}\tag{3.18}$$

By comparing Eq. (3.18) with Eq. (3.14) we extract the full content of the current-algebra result. In the limit $p_{a,c}\to 0$, Eq. (3.18) becomes

$$\begin{aligned}\lim_{p_a,p_c\to 0}\mathcal{M}_{a,b;c,d}=&(\delta^{a3}\delta^{b3}\Delta^{cd}+\delta^{c3}\delta^{d3}\Delta^{ab}\\&-\delta^{b3}\delta^{c3}\Delta^{ad}-\delta^{a3}\delta^{d3}\Delta^{bc})D_2s\\&+(\Delta^{ab}\Delta^{cd}-\Delta^{ad}\Delta^{bc})D_3s\ .\end{aligned}\tag{3.19}$$

Comparing Eqs. (3.19) and (3.14) for various values of a,b,c,d we determine D_2 and D_3 to be

$$D_2=\frac{i}{f_1f_3}(2r_1-r_1r_3)\ ,\tag{3.20a}$$

$$D_3=\frac{i}{f_1{}^2}(2r_3-r_1{}^2)\ ,\tag{3.20b}$$

or, using (3.9)–(3.11),

$$D_2=\frac{i}{v^2}\frac{1}{\rho}\ ,\tag{3.21a}$$

$$D_3=\frac{i}{v^2}\left[4-\frac{3}{\rho}\right]\ .\tag{3.21b}$$

Substituting these values of D_2 and D_3 into Eq. (3.18) and using the equivalence theorem it is easy to verify that we have recovered (up to an overall phase convention) precisely the low-energy theorems Eqs. (1.12), (1.15), and (1.16). Since the equivalence theorem requires $E_W\gg M_W$, the W_L,Z_L scattering theorems derived in this way hold for the intermediate domain between M_W and $\Lambda_{\rm SB}$, just as was found in the U-gauge derivation of Sec. II.

IV. EFFECTIVE LAGRANGIAN

An effective field theory analysis of the physics below $M_{\rm SB}$ begins with a specification of the symmetry structure of the symmetry-breaking sector. The physics of SU(2)×U(1) breaking in the absence of the electroweak gauge interactions, whatever it is, has some initial global symmetry G that is spontaneously broken by the symmetry-breaking dynamics to a global-symmetry group H that describes the symmetry of the vacuum state. Associated with this spontaneous breakdown is a Goldstone-boson manifold G/H. When the electroweak interactions are gauged, some or all of these Goldstone bosons are absorbed by the Higgs mechanism. The properties of the absorbed Goldstone bosons determine the properties of the longitudinal-polarization states of the massive gauge bosons. The reason that the effective field theory language is useful is that the properties of the Goldstone bosons at energies small compared to $M_{\rm SB}$ are almost completely determined by the structure of the symmetries.

The global-symmetry group G must contain electroweak SU(2)×U(1), generated by T_a for $a=1$ to 3 and S. Furthermore, the unbroken-symmetry group H must contain the combination T_3+S, in order to preserve electromagnetic gauge invariance, but must not contain the other SU(2) generators. If we also demand that the *only* Goldstone bosons associated with the symmetry breaking are the three that are eaten by the $W^{\pm}$ and Z in the Higgs mechanism, then, as shown at the end of this section, there are precisely two possibilities for the symmetries. Either G is SU(2)×SU(2) and H is SU(2), or G is SU(2)×U(1) and H is U(1). The first is the usual scheme in which there is a custodial SU(2) symmetry that ensures $\rho=1$. We will discuss only the SU(2)×U(1)/U(1) case, because SU(2)×SU(2)/SU(2) has been discussed extensively elsewhere.[16]

To construct the effective Lagrangian, we follow the instructions of Coleman, Wess, Zumino, and Callan[17] (CWZC). Because the global-symmetry group G and gauge group are the same, we will not distinguish their generators. Thus we let T_a, $a=1$ to 3 [for the SU(2)], and S [for the U(1)] be the generators of G, and T_3+S be the generator of H. We can let these generators act on a two-dimensional space in which

$$T_a=\tfrac{1}{2}\sigma_a\,, \quad S=\tfrac{1}{2}I\,, \tag{4.1}$$

where the σ are Pauli matrices. We can take the broken generators to be the T_a. Then we construct the effective Lagrangian in terms of fields obtained by exponentiating the broken generators

$$\xi=e^{iT_a w_a/f_a}\,. \tag{4.2}$$

Note that we cannot assume that the decay constants of Goldstone-boson fields are all equal, because we are not imposing any custodial SU(2) symmetry. Under an SU(2)×U(1) transformation, ξ transforms as

$$\xi\rightarrow\xi'=g\xi h^{\dagger}\,, \tag{4.3}$$

where g is an SU(2)×U(1) transformation, $g=e^{i(\epsilon_a T_a+\epsilon_4 S)}$, and h is a transformation in the unbroken U(1), $h=e^{iu(T_3+S)}$ chosen to make the ξ' field an exponential of broken generators, as in Eq. (4.2). In general, u is a nonlinear function of the ϵ's and the Goldstone-boson fields, but in this instance, it is easy to see that we must choose $u=\epsilon_4$. Thus we can write the transformation simply as a linear transformation on the ξ field as

$$\xi\rightarrow\xi'=g'\xi h'^{\dagger}\,, \tag{4.4}$$

where $g'=e^{i\epsilon_a T_a}$ and $h'=e^{i\epsilon_4 T_3}$. It is convenient to use Eq. (4.4) and build our effective Lagrangian out of ξ's.

Consider first, the effective action as a function of ξ and the gauge fields:

$$S(\xi,W_a^{\mu},B^{\mu})\,. \tag{4.5}$$

The action S is invariant under a gauge transformation of the form of Eq. (4.4) on ξ and transformation of the gauge fields of the form

$$\begin{aligned}&W_a^{\mu}\sigma_a\rightarrow g'W_a^{\mu}\sigma_a g'^{\dagger}-i\frac{\sin\theta}{e}g'\partial^{\mu}g'^{\dagger},\\ &B^{\mu}\rightarrow B^{\mu}-\frac{\cos\theta}{e}\partial^{\mu}\epsilon_4\,.\end{aligned} \tag{4.6}$$

Now consider a gauge transformation of this form with $g'=\xi^{\dagger}$ and $h'=I$. This gives

$$S(\xi,W_a^{\mu},B^{\mu})=S(I,\mathcal{W}_a^{\mu},B^{\mu})\,, \tag{4.7}$$

where

$$\mathcal{W}_a^{\mu}=-i\frac{\sin\theta}{e}\mathrm{tr}(\sigma_a\xi^{\dagger}\tilde{D}^{\mu}\xi),\quad \mathcal{W}_{\pm}^{\mu}=\frac{\mathcal{W}_1^{\mu}\mp i\mathcal{W}_2^{\mu}}{\sqrt{2}}\,, \tag{4.8}$$

with the covariant derivative $\tilde{D}^{\mu}$ defined by

$$\tilde{D}^{\mu}\xi=\partial^{\mu}\xi+i\frac{e}{\sin\theta}\mathbf{W}^{\mu}\cdot\mathbf{T}\xi\,. \tag{4.9}$$

The derivative $\tilde{D}^{\mu}$ is covariant with respect to SU(2) gauge transformations. Thus the $\mathcal{W}$'s in Eq. (4.8) are nonlinear functions of the Goldstone-boson fields that transform as SU(2) singlets, so that the SU(2) gauge invariance imposes no further constraints on the action of Eq. (4.7). Under the SU(2)×U(1) transformation, Eq. (4.4), only the U(1) factor is relevant and the $\mathcal{W}$'s transform as

$$\mathcal{W}_3^{\mu}\rightarrow\mathcal{W}_3^{\mu}-\frac{\sin\theta}{e}\partial^{\mu}\epsilon_4,\quad \mathcal{W}_{\pm}^{\mu}\rightarrow e^{\pm i\epsilon_4}\mathcal{W}_{\pm}^{\mu}\,. \tag{4.10}$$

If we now define

$$\mathcal{A}^{\mu}=B^{\mu}\cos\theta+\mathcal{W}_3^{\mu}\sin\theta,\quad \mathcal{Z}^{\mu}=\mathcal{W}_3^{\mu}\cos\theta-B^{\mu}\sin\theta\,, \tag{4.11}$$

then we can write the SU(2)×U(1) transformations as

$$\mathcal{A}^{\mu}\rightarrow\mathcal{A}^{\mu}-\frac{1}{e}\partial^{\mu}\epsilon_4,\quad \mathcal{Z}^{\mu}\rightarrow\mathcal{Z}^{\mu},\quad \mathcal{W}_{\pm}^{\mu}\rightarrow e^{\pm i\epsilon_4}\mathcal{W}_{\pm}^{\mu}\,. \tag{4.12}$$

This has the form of ordinary electromagnetic gauge invariance. Thus we have reduced the problem to the simple one of finding a U(1)-invariant action of the nonlinear functions $\mathcal{A}$, $\mathcal{Z}$, and $\mathcal{W}_{\pm}$.

The most straightforward use of the effective Lagrangian, in this context, is to justify and make precise the unitary gauge argument of Ref. 12. The above argument shows that the manipulations we do in the unitary gauge argument have exact analogs in the effective Lagrangian in an arbitrary gauge. All we have to do to construct the effective Lagrangian is to build a theory of A's, W's, and Z's consistent with electromagnetic gauge invariance and then make the replacements $A\rightarrow\mathcal{A}$, etc. If we then go to unitary gauge by requiring the Goldstone boson fields to vanish, so that $\xi=1$, the nonlinear fields reduce back to their linear counterparts, $\mathcal{A}\rightarrow A$, etc. Thus we have merely constructed the most general theory of photons, W's, and Z's. The advantage of the effective Lagrangian is twofold. We can work in any gauge and, even more important, we can use the general properties of effective Lagrangian to estimate the corrections to lowest-order relations.

To estimate the size of terms in the effective Lagrangian, we briefly review standard power-counting rules.[10] A typical term in the action of Eq. (4.5) involves the dimensionless field ξ and derivatives and the external (that is, not directly involved in the strong interactions that produce the effective Lagrangian) fields $(e/\sin\theta)W_a^{\mu}$ and $(e/\cos\theta)B^{\mu}$. The power-counting rules state that if each of these dimensional quantities is divided by $M_{\rm SB}$, then the coefficient of the term is of order

$$v^2 M_{\rm SB}{}^2 \quad \text{where } v\approx M_{\rm SB}/4\pi\,. \tag{4.13}$$

It follows from these power-counting rules that the quantity v in Eq. (4.13) is roughly equal to the decay constants of the Goldstone bosons produced by the symmetry breaking, the f_i of Eq. (4.2). In this case, of course, f is the $v\approx 250$ GeV of SU(2)×U(1) breaking.

It follows that we expect the terms in the effective action as a function of $\mathcal{W}$ and B to have the form of coefficients of order 1 times

$$v^2 M_{\rm SB}{}^2\left[\frac{\partial^{\mu}}{M_{\rm SB}}\right]^{n_1}\left[\frac{e\mathcal{W}_a^{\mu}/\sin\theta}{M_{\rm SB}}\right]^{n_2}\left[\frac{eB^{\mu}/\cos\theta}{M_{\rm SB}}\right]^{n_3}. \tag{4.14}$$

The only terms that will not obey this power-counting rule are the kinetic energy terms of the gauge bosons. The conventional contribution

$$\mathcal{L}_{\mathrm{KE}}=-\tfrac{1}{4}W_a^{\mu\nu}W_{a\mu\nu}-\tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{4.15}$$

comes directly from the electroweak gauge theory. It is not produced by the strong symmetry-breaking interactions and thus does not obey the counting rules. Note, however, that $W_a^{\mu\nu}W_{a\mu\nu}=\mathcal{W}_a^{\mu}\mathcal{W}_{a\mu\nu}$ so that Eq. (4.7) is still valid. Similar terms are produced by the effective Lagrangian, but they are much smaller.

We can now systematically analyze the terms produced by the strong symmetry-breaking interactions, using Eq. (4.14). The leading terms are those with $\sum n_i=2$. The only possibilities are the "mass terms:"

$$\mathcal{L}_{\mathrm{mass}}=\frac{e^2v^2}{4\sin^2\theta}\mathcal{W}_-^{\mu}\mathcal{W}_{+\mu}+\frac{e^2v^2}{8\rho\sin^2\theta\cos^2\theta}\mathcal{Z}^{\mu}\mathcal{Z}_{\mu}\ . \tag{4.16}$$

In Eq. (4.16) we have chosen v and ρ to give the leading-order mass relations for the W and Z. This determines the Goldstone-boson decay constants, to leading order, so that f_1 and f_2 are equal to v and $f_3=v/\sqrt{\rho}$.

There are many terms with $\sum n_i=4$. There are terms with two derivatives and two $\mathcal{W}$'s. In unitary gauge, these are corrections to the gauge-boson kinetic energy. These terms are smaller than the terms of Eq. (4.15) by factors of order

$$\frac{v^2}{M_{\mathrm{SB}}{}^2}\frac{e^2}{\sin^2\theta}\approx\frac{\alpha}{4\pi\sin^2\theta}\equiv\varepsilon\ . \tag{4.17}$$

There are also terms with one derivative and three $\mathcal{W}$'s, and other terms with four $\mathcal{W}$'s. In unitary gauge, these are corrections to the gauge couplings of Eq. (4.15), also smaller than the corresponding terms in Eq. (4.15) by factors of order ϵ.

Terms with $\sum n_i>4$ correspond, in unitary gauge, to operators of dimension higher than four in the gauge fields. Their coefficients are of order ϵ times $(1/M_{\mathrm{SB}})$ to the appropriate power.

All of the nonleading terms in the effective Lagrangian produce effects that are suppressed, compared to the leading contributions, by powers of (E/M_{SB}), where E is a typical energy ($\sqrt{s}$, for example). These effects are small in the kinematic region $M_W{}^2\ll s\ll M_{\mathrm{SB}}{}^2$, but they become as big as the leading terms as s approaches $M_{\mathrm{SB}}{}^2$, where the underlying strong interactions begin to show up.

One of the most important morals of the above analysis is that effective-Lagrangian arguments can be used to make sense of calculations in unitary gauge. The nonrenormalizable interactions in the unitary gauge theory introduce cutoff dependence in the infinite tower of nonleading terms. However, when all that cutoff dependence is absorbed into the physical parameters, the remaining effects of the nonleading terms are small, of order E/M_{SB}, in the kinematic region in which the effective-Lagrangian analysis is valid. Thus we conclude that the unitary gauge argument of Ref. 12 is perfectly valid.

We can also obtain the desired result for longitudinal WW scattering from the effective Lagrangian by working in Landau gauge and using the "equivalence theorem" of Chanowitz and Gaillard.[3] We simply calculate the invariant scattering matrix element for the Goldstone bosons and identify this with the matrix element for the corresponding longitudinal gauge bosons:

$$\begin{aligned}&\mathcal{M}_{U\ \mathrm{gauge}}(W_L^iW_L^j\to W_L^kW_L^l)\\&\quad=\mathcal{M}_{\mathrm{Landau\ gauge}}(w^iw^j\to w^kw^l)+O(M_W{}^2/s)\ ,\end{aligned} \tag{4.18}$$

where W_L is the longitudinally polarized W.

In the Landau gauge, the "mass term" of Eq. (4.16) comes apart into a W mass term, a w kinetic energy term and an infinite series of terms describing the derivative interactions of the w fields. To calculate the leading contribution to the scattering we need the 3- and 4-w vertices. Note that in the absence of a custodial SU(2) symmetry, both the 3- and 4-w vertices are present. The G parity that forbids the 3-w couplings in the SU(2)$\times$SU(2)/SU(2) calculation is explicitly broken for $\rho\neq 1$. If we put Eqs. (4.2), (4.8), and (4.11) into Eq. (4.16) and expand the exponential, we can calculate these vertices explicitly. Then a straightforward calculation gives the result

$$\begin{aligned}&\mathcal{M}(w_1w_1\to w_3w_3)=\frac{ig^2}{4M_W{}^2\rho}\ ,\\&\mathcal{M}(w_1w_1\to w_2w_2)=\frac{ig^2}{4M_W{}^2}\left[4-\frac{3}{\rho}\right]\ ,\end{aligned} \tag{4.19}$$

in agreement with the unitary-gauge result and the equivalence theorem.

The discussion in this section has treated the SU(2)$\times$U(1)/U(1) case. Similar treatment of SU(2)$\times$SU(2)/SU(2) can be found in many places. Let us close by showing that these are the only two possibilities, as long as we assume that there are no other light particles. The important point is that if there are no extra light particles, then the mechanism responsible for SU(2)$\times$U(1) breaking cannot produce any Goldstone or pseudo-Goldstone bosons except those absorbed by the Higgs mechanism.

The generators T_a and S cannot be part of the same simple subalgebra of G, because no simple algebra except SU(2) can be broken down to a subalgebra with only three fewer generators. Let t be the simple [or U(1)] factor of G that contains the T_a, s be the simple factor that contains S, and r be everything else. Because the W and Z correspond to broken generators in t and s, and because we have assumed that only three Goldstone bosons are produced by the symmetry breaking, r cannot be broken at all. Thus r is a common factor of G and H, irrelevant to the structure of the Goldstone-boson manifold, and we can ignore it. Now if either t or s is larger than SU(2), it must remain unbroken, or else there would be more than three Goldstone bosons. But then the other factor must be completely broken and T_3+S will not be in H. Thus the only possibilities are those we have considered. Of course, it may be that there are actually additional pseudo-Goldstone bosons produced, in which case there may be additional light particles below M_{SB}. This possibility is briefly discussed in the next section.

V. CONCLUSION

We have derived low-energy theorems for the longitudinal modes W_L, Z_L of the $SU(2)\times U(1)$ electroweak gauge theory that are valid for W,Z energies in the domain $M_W{}^2 \ll E^2 \ll \Lambda_{SB}{}^2$ where Λ_{SB} is defined in Eq. (1.8). The theorems are derived for any value of ρ and therefore do not assume the existence of a custodial SU(2) symmetry. We have demonstrated the result by three methods. The perturbative U-gauge analysis shows that the low-energy amplitudes are a consequence of the gauge sector interactions and the decoupling of the heavy-symmetry-breaking sector as indicated by naive power counting. The current-algebra R-gauge derivation stresses the analogy with pion low-energy theorems, with a technical difference due to the absence of an isospin counterpart [custodial SU(2)] and G parity in the general case. Finally, the effective Lagrangian provides a succinct formulation in any gauge: applied to the U gauge it justifies the power counting of the perturbative analysis while in the R gauge it is the analogue of the pion effective Lagrangians and reproduces the current-algebra results.

The low-energy amplitudes given in the text assume, as do the pion low-energy theorems, that there are no light spin-zero exchange particles which could contribute to the low-energy scattering. While some special cases are easily understood, we have not obtained a general formulation of the effect of light particles, such as pseudo-Goldstone bosons, on the low-energy W_L, Z_L scattering amplitudes. A trivial example is given by the global symmetry $G = SU(3)_L \times SU(3)_R$, as in three-flavor QCD, which would result in five pseudo-Goldstone bosons, the counterparts of K and η. Just as in QCD where K and η do not modify the pion low-energy theorems, in the electroweak theory the W_L, Z_L amplitudes would also be unaffected.

It is also easy to see that the sign of the effect of light particles on amplitudes involving only $W^\pm$ but not Z, $WW \rightarrow WW$, is trivially fixed by the electric charge of the light exchanged particle, since only the square of the absolute value of the WW coupling appears in the amplitude. For neutral scalars, as for the ordinary Higgs boson, the effect is to diminish the magnitude of the amplitude, while for $Q = 2$ scalars the amplitude is increased. Similar rules will apply to $WW \rightarrow ZZ$ and $WZ \rightarrow WZ$ if there is a custodial SU(2) symmetry relating the couplings of the W and Z bosons to the light exchange particles, but not in general. While the results of the preceding sections show that the low-energy W_L, Z_L interactions with one another do respect an effective low-energy custodial SU(2) if $\rho = 1$, we have not yet investigated the question of their interactions with possible pseudo-Goldstone bosons. An SU(4)/SU(2) model proposed recently by Chivukula and Georgi[18] should be a useful theoretical laboratory for studying this question.

The principal phenomenological use of the low-energy theorems is to estimate the magnitude of the longitudinal gauge-boson-pair signal that would be observed due to the WW fusion mechanism[19] at multi-TeV colliders. If one imagines that the pion had been discovered before the proton or neutron or any other hadron and that it had been recognized as a Goldstone boson, then we can formulate the analogous problem: given only $F_\pi = 93$ MeV, what is the energy scale at which strong interactions set in and hadron resonances occur? Naive extrapolation of the low-energy theorem for the $I = J = 0$ partial-wave amplitude

$$a_{00}(\pi\pi) = \frac{s}{16\pi F_\pi{}^2} \tag{5.1}$$

suggests a scale of $4\sqrt{\pi}F_\pi \sim 700$ MeV whereas the $I = J = 1$ amplitude would suggest a scale that is larger by $\sqrt{3}$ or 1100 MeV. These values are the order of magnitude of typical low-lying hadron masses and, not coincidentally, of the energy scale at which pion-pion scattering saturates unitarity. It is also, again not coincidentally, the scale set by one-loop corrections in low-energy chiral Lagrangians.[10]

If the electroweak $SU(2)\times U(1)$ is broken by new strong-interaction dynamics, then the W_L, Z_L low-energy theorems suggest a scale of $4\sqrt{\pi}v \simeq 1.8$ TeV for the onset of strong interactions and the emergence of resonances. The experimental implications of this expectation have been explored in preliminary fashion for proton-proton colliders[3,4] with the conclusion that a collider with the SSC design parameters, $\sqrt{s} = 40$ TeV and $\mathcal{L} = 10^{33}$ cm^{-2}sec^{-1}, is near the minimal configuration needed to be sure of seeing the strong-interaction signal from WW fusion. More detailed studies, corresponding more closely to experimentally implementable signals, are presently in progress.[20] A similar investigation for electron-positron colliders concluded that a collider of $\sqrt{s} = 3-5$ TeV and $\mathcal{L} = (1-2.5)\times 10^{33}$ cm^{-2}sec^{-1} is needed to be able to observe the generic strong-interaction signal.[21]

A collider with the capability to see the strong-interaction signal puts us in the enviable position of being able to learn from either the absence or presence of the signal. If the signal is absent, we learn that the quanta of the symmetry-breaking sector are much lighter than 1 TeV. In that case if the new quanta in the $J = 0$ channel are near or above the WW threshold, they would be copiously produced at the multi-TeV collider. If the strong-interaction signal is seen, it means that the new quanta are at the TeV scale or above. Then the most likely possibility is that the masses of the lightest states correspond to the TeV scale of the onset of strong interactions, and that indications of resonance structure will emerge not long after the first observations of the strong-interaction signal.

ACKNOWLEDGMENTS

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-82-15249.

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