Determination of the Λ magnetic moment by QCD sum rules

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(Received 5 December 1986)

The magnetic moment of the Λ hyperon is calculated using the QCD sum-rule approach of Ioffe and Smilga. It is shown that μ_Λ has the structure $\mu_\Lambda = \frac{2}{3}(e_u + e_d + 4e_s)(e\hbar/2M_\Lambda c)(1+\delta_\Lambda)$, where δ_Λ is small. In deriving the sum rules special attention is paid to the strange-quark mass-dependent terms and to several additional terms not considered in earlier works. These terms are now appropriately incorporated. The sum rule is analyzed using the ratio method. Using the external-field-induced susceptibilities determined earlier, we find that the calculated value of μ_Λ is in agreement with experiment.

I. INTRODUCTION

Associated with the ground states of the $\frac{1}{2}^+$ baryon octet, there are in principle nine magnetic moments including the $\Sigma^0 \to \Lambda$ transition moment. Barring the experimentally inaccessible Σ^0 magnetic moment, the other eight have been measured with considerable accuracy experimentally and have been the object of many theoretical studies. Ioffe and Smilga[1,2] and independently Balitsky and Yung[3,4] have used the QCD sum-rule approach to determine the nucleon and hyperon magnetic moments.

From the point of view of the constituent-quark structure the eight baryons fall into two classes: (i) those of the type (aab) which contain two quarks of the same flavor, viz., p, n, Σ^+, Σ^-, Ξ^-, and Ξ^0; (ii) those of the type (abc) which have all three quarks of different flavors, namely, Λ and Σ^0. In an earlier work[5] it was pointed out that from the QCD point of view, it is natural to write the magnetic moments of the baryons in the former category in the form

$$\mu_B = 4e_a \frac{e\hbar}{2M_B c}(1+\delta_B) , \tag{1}$$

where e_a is the charge of the doubly occurring quark in units of proton charge e, M_B is the baryon mass, and δ_B is a small calculable correction. It is natural to ask whether μ_Λ, the magnetic moment of the Λ, also satisfies an equation similar to Eq. (1). In this work we shall show that μ_Λ can be written in the form

$$\mu_\Lambda = \tfrac{2}{3}(e_u + e_d + 4e_s)\frac{e\hbar}{2M_\Lambda c}(1+\delta_\Lambda) \tag{2}$$

and compute δ_Λ. Using the experimental values of μ_Λ and M_Λ Eq. (2) yields

$$\delta_\Lambda = 0.093 . \tag{3}$$

In our work we shall find that the QCD sum-rule approach yields a value of μ_Λ reasonably close to the experimental number, without involving any *ad hoc* assumptions about constituent-quark mass, quark magnetic moment, configuration mixing, etc.

Our calculational procedure is closely similar to that of Ioffe and Smilga[1,2] and our earlier work.[5] The method basically consists of the evaluation of the current correlation function with quantum numbers of the Λ hyperon in the presence of an external constant magnetic field $F_{\mu\nu}$. As will be seen in Sec. II, the derivation of the sum rule for the Λ hyperon is algebraically more complicated than for the proton case and cannot be obtained from the latter by simple substitution of quark charges. We give some details of the calculations involved, concentrating essentially on the novel aspects for the Λ hyperon as compared to other baryons. Some of the details of the derivations of the strange-quark propagator are relegated to the Appendix. In Sec. III we present an analysis of the sum rule using the ratio method and conclude with a short discussion of the overall agreement between the experimental values of the baryon magnetic moments and the QCD sum-rule predictions.

II. DERIVATION OF THE SUM RULE

We begin by writing the spinor current corresponding to the Λ hyperon:

$$\eta_\Lambda(x) = (\tfrac{2}{3})^{1/2}\epsilon^{abc}[(u^a C\gamma_\mu s^b)\gamma_5\gamma^\mu d^c - (d^a C\gamma_\mu s^b)\gamma_5\gamma^\mu u^c] . \tag{4}$$

Here $u^a(x)$, $d^b(x)$, and $s^c(x)$ refer to the up-, down-, and strange-quark fields, respectively, and a, b, and c are color indices. Following Ioffe and Smilga, we consider the correlation

$$\Pi(p)=i\int d^4x\, e^{ip\cdot x}\langle 0\,|\,T\{\eta_\Lambda(x),\bar{\eta}_\Lambda(0)\}\,|\,0\rangle\ , \qquad (5)$$

in the presence of constant external electromagnetic field $F_{\mu\nu}$. We shall work in the coordinate gauge $x_\mu A^\mu=0$ so that

$$A_\mu=-\tfrac{1}{2}x^\nu F_{\mu\nu}\ . \qquad (6)$$

We shall be using perturbation theory to calculate the Wilson coefficients for the operator product $\eta_\Lambda(x)\bar{\eta}_\Lambda(0)$. Using standard manipulations we find

$$\begin{aligned} i\langle 0\,|\,T\{\eta_\Lambda(x),\bar{\eta}_\Lambda(0)\}\,|\,0\rangle = +i\tfrac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}(&\mathrm{Tr}\{S_u^{aa'}(x)\gamma_\nu C^{-1}[S_s^T(x)]^{bb'}C\gamma_\mu\}\gamma_5\gamma^\mu S_d^{cc'}\gamma^\nu\gamma_5 \\ &+\mathrm{Tr}\{S_d^{cc'}(x)\gamma_\nu C^{-1}[S_s^T(x)]^{bb'}C\gamma_\mu\}\gamma_5\gamma^\mu S_u^{aa'}(x)\gamma^\nu\gamma_5 \\ &-\gamma_5\gamma_\mu S_d^{cc'}(x)\gamma_\nu C^{-1}[S_s^T(x)]^{bb'}C\gamma^\mu S_u^{aa'}(x)\gamma^\nu\gamma_5 \\ &-\gamma_5\gamma_\mu S_u^{aa'}(x)\gamma_\nu C^{-1}[S_s^T(x)]^{bb'}C\gamma^\mu S_d^{cc'}(x)\gamma^\nu\gamma_5)\ . \end{aligned} \qquad (7)$$

Here, $S^u(x)$ is the u-quark Green's function,

$$S_u^{aa'}(x)=\langle 0\,|\,T\{u^a(x),\bar{u}^{a'}(0)\}\,|\,0\rangle\ ,$$

etc., and the superscript T in S^T refers to the transposition in the Dirac space. As usual in the QCD sum-rule approach, although the Wilson coefficients are computed in perturbation theory, one uses, however, phenomenological nonperturbative vacuum expectation values (VEV's) for the local operators involved. With this in mind we write the following expression for the quark Green's function. It is convenient to separate the terms in the absence of the external field $F_{\mu\nu}$, and terms corresponding to interaction with $F_{\mu\nu}$. For the first set we have

$$\begin{aligned} \langle 0\,|\,T\{q_i^a(x),\bar{q}_k^b(0)\}\,|\,0\rangle = &\delta^{ab}(\hat{x})_{ik}\frac{i}{2\pi^2x^4}-\delta^{ab}\delta_{ik}\tfrac{1}{12}\langle\bar{q}q\rangle+\delta^{ab}\delta_{ik}\frac{x^2}{192}\langle\bar{q}g_s\sigma\cdot Gq\rangle-\delta^{ab}\delta_{ik}\frac{m_q}{4\pi^2x^2} \\ &+\delta^{ab}(\hat{x})_{ik}\frac{i}{48}m_q\langle\bar{q}q\rangle-\delta^{ab}(\hat{x})_{ik}x^2\frac{i}{3^2 2^7}m_q\langle\bar{q}g_s\sigma\cdot Gq\rangle \\ &+\left[\frac{\lambda^n}{2}\right]^{ab}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})_{ik}\frac{i}{32\pi^2x^2}g_sG^n_{\alpha\beta} \\ &+\left[\frac{\lambda^n}{2}\right]^{ab}(\sigma^{\alpha\beta})_{ik}\frac{m_q}{32\pi^2}g_sG^n_{\alpha\beta}\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\mathrm{EM}}\right]\ . \end{aligned} \qquad (8)$$

The first term is the usual massless propagator in coordinate space. The second is where the quarks develop a nonzero VEV. The third arises from expanding the quark correlator in a Taylor series to second order in x and replacing ordinary partials by covariant derivatives (which the fixed-point gauge allows us to do). The fourth term is the linear m_q correction to the first term. The fifth term comes from expanding in a Taylor series to first order and using the quark equation of motion. The sixth arises from expanding to third order in x. The seventh is the vertex for a quark interacting with a gluon in fixed point gauge. The eighth term is the linear m_q correction to the seventh term. One computes it in momentum space first keeping all terms linear in m_q. Upon transforming to coordinate space one discovers an infrared divergence which one regulates by replacing, in the denominator, p^2 by $p^2-\Lambda^2$, doing the integration and keeping only terms which are singular in Λ or independent of Λ. The parameter Λ is some cutoff which separates the low-momentum nonperturbative regime from the high-momentum perturbative regime. It should be on the order of a few hundred MeV. For definiteness we choose $\Lambda=500$ MeV. The rationale for this method of regularization comes from the observation[6] that it is only correct to calculate Wilson coefficients by Feynman diagrams when large momenta flow through all internal lines. This means that correlators such as the above come with an understood step function $\theta(-(p^2+\Lambda^2))$ which keeps the momenta large and virtual. Usually the soft momenta present no problem but in the case of an infrared divergence the above replacement in the denominators effectively enforces a cutoff. The constant γ_{EM} is the Euler-Mascheroni constant: $\gamma_{\mathrm{EM}}=0.577\ldots$.

The second set of terms are those of a quark interacting with the external photon field:

$$\langle 0 | T\{q_i^a(x),\bar{q}_k^b(0)\} | 0\rangle_F = \delta^{ab}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})_{ik}\frac{i}{32\pi^2x^2}e_qF_{\alpha\beta}+\delta^{ab}(\sigma^{\alpha\beta})_{ik}\frac{-1}{24}\chi\langle\bar{q}q\rangle e_qF_{\alpha\beta}$$
$$+\delta^{ab}[\sigma^{\alpha\beta}x^2(\kappa+\xi)-2x_\lambda x^\beta\sigma^{\lambda\alpha}(\kappa-\tfrac{1}{2}\xi)]_{ik}\frac{1}{3^2 2^6}\langle\bar{q}q\rangle e_qF_{\alpha\beta}$$
$$+\delta^{ab}(\sigma^{\alpha\beta}x^2-2x_\lambda x^\beta\sigma^{\lambda\alpha})_{ik}\tfrac{1}{288}\langle\bar{q}q\rangle e_qF_{\alpha\beta}$$
$$+\left[\frac{\lambda^n}{2}\right]^{ab}(g_sG^n_{\rho\sigma})(e_qF_{\alpha\beta})\frac{-i}{96\pi^2}g^{\alpha\rho}$$
$$\times\left\{\frac{2}{x^2}x^\beta x^\sigma\hat{x}+[g^{\beta\sigma}\hat{x}-2(\gamma^\beta x^\sigma+\gamma^\sigma x^\beta)]\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\rm EM}\right]\right\}_{ik}$$
$$+\delta^{ab}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})_{ik}\frac{-i}{3^3 2^{10}\pi^2}\langle g_s{}^2G^2\rangle e_qF_{\alpha\beta}\left\{\frac{4}{\Lambda^2}-x^2\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\rm EM}-1\right]\right\}$$
$$+\delta^{ab}(\sigma^{\alpha\beta})_{ik}\frac{1}{32\pi^2}m_qe_qF_{\alpha\beta}\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\rm EM}\right]$$
$$+\delta^{ab}(\hat{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\hat{x})_{ik}\frac{i}{96}m_q\chi\langle\bar{q}q\rangle e_qF^{\alpha\beta}\ . \quad (9)$$

The first term in Eq. (9) is the vertex for a quark interacting with a photon in a fixed-point gauge transformed to coordinate space. The second term is the external-field-induced correlation $\langle\bar{q}\sigma_{\alpha\beta}q\rangle_F$. Following Ioffe and Smilga[1] we have defined the susceptibility χ by

$$\langle\bar{q}\sigma_{\alpha\beta}q\rangle_F\equiv e_q\chi\langle\bar{q}q\rangle F_{\alpha\beta}\ . \quad (10)$$

The third term also contains induced operators that arise after expanding to order x^2. The susceptibilities κ and ξ are defined by

$$\langle\bar{q}g_sG_{\alpha\beta}q\rangle_F\equiv e_q\kappa F_{\alpha\beta}\langle\bar{q}q\rangle\ , \quad (11)$$

$$\epsilon_{\alpha\beta\mu\nu}\langle\bar{q}g_sG^{\mu\nu}\gamma_5q\rangle_F\equiv ie_q\xi F_{\alpha\beta}\langle\bar{q}q\rangle\ . \quad (12)$$

The fourth also arises from expanding to order x^2. The fifth and sixth terms arise from the quark interacting with the external electromagnetic field as well as the vacuum gluons. The cutoff momentum Λ is used to separate the high-momentum part of the propagator which should only enter in the calculation of the Wilson coefficients. In the sixth term we have carried out Lorentz and color averaging for the vacuum gluons. A detailed exposition of the derivation of these terms can be found in Ref. 7. The last two terms arise due to finite quark mass m_q and are corrections to the first and second terms, respectively. This derivation can be found in the Appendix.

In addition we shall need the following expression for the vacuum expectation value of the product of the quark and the gluon fields:

$$\langle 0|T\{q_i^a(x),g_sG^n_{\alpha\beta}\bar{q}_k^b(0)\}|0\rangle=\left[\frac{\lambda^n}{2}\right]^{ab}\frac{-1}{3\times2^6}g_s\langle\bar{q}\sigma\cdot Gq\rangle\left[(\sigma_{\alpha\beta})_{ik}-\frac{im_q}{4}(\hat{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\hat{x})_{ik}\right]$$
$$+\left[\frac{\lambda^n}{2}\right]^{ab}\frac{-1}{2^4}\langle\bar{q}q\rangle e_q\left[\delta_{ik}\kappa F_{\alpha\beta}-\frac{i}{4}\xi(\gamma_5)_{ik}\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}\right]\ . \quad (13)$$

The first term is independent of the external electromagnetic field. The second term in large parentheses incorporates the dependence on the (strange-)quark mass and should be retained for a consistent calculation of dimension-six operators in the mass sum rule. (Belyaev and Ioffe[8] do not include this.) The third term arises from external-field-induced VEV and is identical to the expression of Ioffe and Smilga.[1] For a consistent inclusion of all corrections due to quark mass we must also consider terms of the type $\langle\bar{q}G^n_{\alpha\beta}(\lambda^n/2)\hat{\nabla}q\rangle$ which arise in Eq. (13) on expanding

$$q_i^a(x)=q_i^a(0)+x^\mu\nabla_\mu q_i^a(0)+\cdots\ . \quad (14)$$

However such an expression also forces us to introduce new susceptibilities of the type

$$\left\langle\bar{q}\gamma_\alpha G^n_{\beta\rho}\frac{\lambda^n}{2}\nabla^\rho q\right\rangle,\ \ \langle\bar{q}G_{\alpha\rho}\gamma^\rho\nabla_\beta q\rangle\ . \quad (15)$$

On the other hand, from the Borel mass M^2 dependence of the sum rule, these terms contribute to m_sM^2 and are similar to the terms which occur with coefficients $m_sM^2(2\kappa-\xi)$ (cf. below). Since the susceptibilities κ and

ξ are not well determined, no useful purpose is served in adding two more unknown terms. Furthermore, we have checked that inclusion of terms of the type in Eq. (15) coming from the expansion of Eq. (14), do occur with only small coefficients so that there is no danger of the sum rules being overwhelmed by these terms. For this reason we have simply dropped terms of the type Eq. (15) and work with Eq. (13). Following Belyaev and Ioffe,[8] we also define

$$f=\frac{\langle \bar{s}s\rangle}{\langle \bar{u}u\rangle}-1,\quad f_g=\frac{\left\langle \bar{s}\sigma^{\mu\nu}\frac{\lambda^n}{2}G^n_{\mu\nu}s\right\rangle}{\left\langle \bar{u}\sigma^{\mu\nu}\frac{\lambda^n}{2}G^n_{\mu\nu}u\right\rangle}-1\ . \tag{16}$$

Also, we retain only those terms which are linear in $SU(3)_{\text{flavor}}$-breaking parameters. Armed with these details we can now calculate the contributions from various diagrams to the sum rule at the structure $F_{\mu\nu}(\hat{p}\sigma^{\mu\nu}+\sigma^{\mu\nu}\hat{p})$. We indicate the various diagrams generically, it being understood that the summation over appropriate permutations are performed whenever necessary.

(1) Coefficient of $F_{\mu\nu}$, given in Fig. 1:

$$\Pi(x)\,|_{\text{Fig. 1}}=\frac{e_u+e_d+4e_s}{4\pi^6x^8}(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ . \tag{17}$$

(2) Coefficient of $F_{\mu\nu}G^{\alpha\beta}G_{\alpha\beta}$, given in Fig. 2:

$$\Pi(x)\,|_{\text{Fig. 2}}=\frac{\langle g_s{}^2G^2\rangle}{x^4\pi^6}\left\{\frac{1}{9\times 192}(e_u+e_d+\tfrac{7}{4}e_s)+\frac{1}{27\times 512}(5e_u+5e_d+2e_s)\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\text{EM}}\right]\right.$$
$$\left.+\frac{1}{27\times 128}(e_u+e_d+4e_s)\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\text{EM}}-1-\frac{4}{x^2\Lambda^2}\right]\right\}\ . \tag{18}$$

The calculation of the diagrams in Fig. 2 is fairly complicated and the details can be found in Ref. 7.

(3) Coefficient of $m_s\langle\bar{q}\sigma_{\mu\nu}q\rangle_F$, given in Fig. 3:

$$\Pi(x)\,|_{\text{Fig. 3}}=-\frac{m_s}{6\pi^4x^6}(e_u+e_d-2e_s\phi)\chi\langle\bar{u}u\rangle(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ . \tag{19}$$

(4) Coefficient of $m_sF_{\mu\nu}\langle\bar{q}q\rangle$, given in Fig. 4. The contribution of Figs. 4(a), 4(b), and 4(c) is

$$m_s\frac{19}{288}\frac{e_u+e_d}{\pi^4x^4}\langle\bar{u}u\rangle(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ , \tag{20}$$

while the contribution of Fig. 4(d) is

$$\frac{m_se_s}{12\pi^4x^4}\langle\bar{u}u\rangle(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\text{EM}}\right]\ . \tag{21}$$

(5) Coefficient of $m_s\langle\bar{q}(\lambda^n/2)G^n_{\mu\nu}q\rangle$ and $m_s\langle\bar{q}\gamma_5(\lambda^n/2)G^n_{\mu\nu}q\rangle$, is given in Fig. 5. The contributions of Figs. 5(a) and 5(b) are given by

$$-\frac{m_s}{144}\frac{e_u+e_d}{\pi^4x^4}(2\kappa-\xi)\langle\bar{u}u\rangle(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ , \tag{22}$$

while the contribution of Fig. 5(c) is

$$-\frac{m_s\kappa(e_u+e_d)}{48\pi^4x^4}\langle\bar{u}u\rangle(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\left[\ln\left[\frac{-x^2\Lambda^2}{4}\right]+2\gamma_{\text{EM}}\right]\ . \tag{23}$$

(6) Coefficient of $F_{\mu\nu}\langle\bar{q}q\rangle^2$, given in Fig. 6:

$$\Pi(x)\,|_{\text{Fig. 6}}=\frac{\langle\bar{u}u\rangle^2}{27\times 16\pi^2x^2}[e_s+7(e_u+e_d)+4f(e_s+2e_u+2e_d)](\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ . \tag{24}$$

(7) Coefficient of $\langle\bar{q}q\rangle\langle\bar{q}\sigma_{\mu\nu}q\rangle$, given in Fig. 7:

$$\Pi(x)\,|_{\text{Fig. 7}}=-\frac{\langle\bar{u}u\rangle^2}{36\pi^2x^2}[\chi(e_u+e_d)(1+2f)+4\chi_se_s](\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ . \tag{25}$$

(8) Coefficient of $\langle\bar{q}\sigma_{\mu\nu}q\rangle\langle\bar{q}\sigma_{\mu\nu}(\lambda^n/2)G^{n\mu\nu}q\rangle$, given in Fig. 8:

$$\Pi(x)\,|_{\text{Fig. 8}}=-\frac{m_0{}^2\langle\bar{u}u\rangle}{9\times 128\pi^2x^2}[\chi(e_u+e_d)+4\chi_se_s+2f_g\chi(e_u+e_d)](\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)\ . \tag{26}$$

(9) Coefficients of $\langle\bar{q}q\rangle\langle\bar{q}(\lambda^n/2)G^n_{\mu\nu}q\rangle$ and $\langle\bar{q}q\rangle\langle\bar{q}\gamma_5(\lambda^n2)\tilde{G}^n_{\mu\nu}q\rangle$, given in Fig. 9 [the correct combination that

occurs is $(2\kappa-\xi)$ and not $(\kappa-2\xi)$ as reported in previous works; see Refs. 1 and 5]

$$\Pi(x)\,|_{\text{Fig. 9}}=-\frac{\langle\bar{u}u\rangle^2}{27\times 32\pi^2x^2}(\sigma\cdot F\hat{x}+\hat{x}\sigma\cdot F)[(2\kappa-\xi)(e_u+e_d)+4(2\kappa_s-\xi_s)e_s+2f(2\kappa-\xi)(e_u+e_d)]\ . \tag{27}$$

The rest of the calculation proceeds in the standard manner.[1,5] We first Fourier transform the various terms listed above and then Borel transform them to the variable M^2 by

$$\hat{B}f(p^2)=\frac{1}{\pi}\int_0^\infty ds\,\mathrm{Im}f(s)e^{-s/M^2}\ . \tag{28}$$

To obtain a sum rule for $\Pi(p)$ in Eq. (5) at the structure $(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$, we write a dispersion relation and compute the absorptive part using physical intermediate states. The sum rule reads as

$$\begin{aligned}
-(e_u+e_d+4e_s)\frac{M^4}{12L^{4/9}}&+\frac{a^2L^{4/9}}{108M^2}[2(7e_u+7e_d+e_s)+8f(2e_u+2e_d+e_s)]\\
&+\frac{b}{288L^{4/9}}\left[-8(e_u+e_d+\tfrac{7}{4}e_s)+(5e_u+5e_d+2e_s)\tfrac{2}{3}[\ln(M^2/\Lambda^2)-1-\gamma_{\mathrm{EM}}]\right.\\
&\qquad\left.+(e_u+e_d+4e_s)\tfrac{8}{3}\left[\ln(M^2/\Lambda^2)-\gamma_{\mathrm{EM}}-\frac{M^2}{2\Lambda^2}\right]\right]\\
&+\frac{\chi a^2}{54L^{4/27}}\left[1-\frac{m_0{}^2}{8M^2L^{4/9}}\right][3(e_u+e_d)(1+2f)+12e_s\phi]+(e_u+e_d-2e_s\phi)\frac{m_sa\chi M^2}{6L^{28/27}}\\
&-\frac{a^2}{108M^2}(2\kappa-\xi)[(1+2f)(e_u+e_d)+4e_s\phi]+(e_u+e_d)\frac{m_sa}{6}\kappa[\ln(M^2/\Lambda^2)-1-\gamma_{\mathrm{EM}}]\\
&+(e_u+e_d)\frac{19}{36}\frac{m_sa}{L^{4/9}}-(e_u+e_d)\frac{m_sa}{18}(2\kappa-\xi)\\
&+\frac{2}{3}\frac{m_sae_s}{L^{4/9}}[1+\gamma_{\mathrm{EM}}-\ln(M^2/\Lambda^2)]=\beta_\Lambda{}^2\left[\frac{F_1+F_2}{M^2}+A'\right]e^{-M_\Lambda{}^2/M^2}+\text{e.s.c.}\ ,
\end{aligned} \tag{29}$$

where e.s.c. stands for excited-state contributions. In computing the first term on the right-hand side we have used the definitions

$$\langle 0\,|\,\eta_\Lambda(0)\,|\,\Lambda(p)\rangle=\lambda_\Lambda u(p),\quad \beta_\Lambda{}^2=\frac{\lambda_\Lambda{}^2(2\pi)^4}{4}\ , \tag{30}$$

$$\langle\Lambda(p_2)\,|\,J_\mu^{\mathrm{EM}}(0)\,|\,\Lambda(p_1)\rangle=\bar{u}(p_2)\left[F_1\gamma_\mu+F_2\frac{i\sigma_{\mu\nu}k^\nu}{2M_\Lambda}\right]u(p_1)\ , \tag{31}$$

$$k^\mu=p_2^\mu-p_1^\mu,\quad F_1(0)=1\ ,$$

and the total magnetic moment is given by

$$\mu_\Lambda=(F_1+F_2)\frac{e\hbar}{2M_\Lambda c}\ . \tag{32}$$

The coefficient A' on the right-hand side arises from the nondiagonal transitions induced by the external field between the ground-state Λ and excited states.

On the left-hand side we have incorporated the anomalous dimensions of the various operators where

$$L=\ln(M^2/\Lambda_{\mathrm{QCD}}{}^2)/\ln(\mu^2/\Lambda_{\mathrm{QCD}}{}^2),\quad \mu\approx 500\ \mathrm{MeV},\quad \Lambda_{\mathrm{QCD}}\approx 100\ \mathrm{MeV}\ . \tag{33}$$

The factor ϕ measures the difference in the susceptibilities between the strange quark and the up or down quarks. It is defined by

$$\chi_s/\chi=\phi\ . \tag{34}$$

Simple pole dominance estimates of the susceptibilities suggest that ϕ should be approximately the same for the various susceptibility ratios and we set uniformly $\phi=m_\rho{}^2/m_\phi{}^2\approx 0.6$. For the analysis given in the next section we also need the mass sum rule again at the odd structure $\hat{p}$. This has been worked out by Belyaev and Ioffe.[8] We have redone their calculations incorporating the additional m_s-dependent terms given in Eqs. (8), (9), and (13), and the final result is

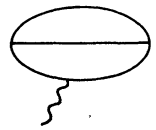

FIG. 1. Contribution of the identity operator.

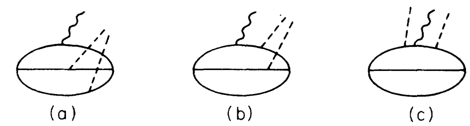

FIG. 2. Contribution of the gluon condensate $\langle g_s{}^2G^2\rangle$. Diagrams (b) and (c) include infrared divergences which are regularized by a cutoff Λ.

$$\frac{M^6}{L^{4/9}}+\frac{bM^2}{4L^{4/9}}+\frac{4}{3}a^2L^{4/9}(1+\tfrac{4}{3}f)-\frac{a^2m_0{}^2}{3M^2}[1+\tfrac{2}{3}(f+f_g)]+\tfrac{2}{3}am_s\frac{M^2}{L^{4/9}}-\frac{4}{3}m_sam_0{}^2=\beta_\Lambda{}^2e^{-M_\Lambda{}^2/M^2}+\text{e.s.c.} \quad (35)$$

III. ANALYSIS

After dividing Eq. (30) through by $-\frac{2}{3}(e_u+e_d+4e_s)/M^2$ we can write

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left[2+\tfrac{2}{3}[\ln(M^2/\Lambda^2)-1-\gamma_{\rm EM}]-\tfrac{8}{3}\left[\ln(M^2/\Lambda^2)-\gamma_{\rm EM}-\frac{M^2}{2\Lambda^2}\right]\right]$$
$$+\frac{a^2L^{4/9}}{72}(4+\tfrac{8}{3}f)+\frac{\chi a^2}{36L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right](1+2f-4\phi)+\frac{m_sa\chi M^4}{4L^{28/27}}$$
$$-\frac{a^2}{216}(2\kappa-\xi)(1+2f-4\phi)+\frac{m_saM^2}{12L^{4/9}}[\ln(M^2/\Lambda^2)-1-\gamma_{\rm EM}]-\frac{m_saM^2}{36}\left[2\kappa-\xi-\frac{19}{2L^{4/9}}\right]$$
$$=\beta_\Lambda{}^2(1+\delta_\Lambda+AM^2)e^{-M_\Lambda{}^2/M^2}+\text{e.s.c.}\ , \quad (36)$$

where we have defined

$$F_1+F_2=\tfrac{2}{3}(e_u+e_d+4e_s)(1+\delta_\Lambda)\ , \quad (37)$$

or, equivalently,

$$\mu_\Lambda=\tfrac{2}{3}(e_u+e_d+4e_s)\frac{e\hbar}{2M_\Lambda c}(1+\delta_\Lambda)\ . \quad (38)$$

The close similarity between Eq. (37) and the mass sum rule Eq. (35) at the structure $\hat{p}$ is now evident. We now take the ratio of the left-hand sides of Eq. (37) and (35) to define

$$R(M^2)=\left[\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2}{6}L^{4/9}(1+\frac{4}{3}f)-\frac{a^2m_0{}^2}{24M^2}[1+\frac{2}{3}(f+f_g)]+\frac{m_saM^2}{12L^{4/9}}-\frac{1}{24}m_sam_0{}^2\right]^{-1}$$
$$\times\left[\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left\{2+\frac{2}{3}\left[\ln\left[\frac{M^2}{\Lambda^2}\right]-1-\gamma_{\rm EM}\right]-\frac{8}{3}\left[\ln\left[\frac{M^2}{\Lambda^2}\right]-\gamma_{\rm EM}-\frac{M^2}{2\Lambda^2}\right]\right\}\right.$$
$$\times\frac{a^2L^{4/9}}{72}(4+\frac{8}{3}f)+\frac{\chi a^2}{36L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right](1+2f-4\phi)+\frac{a}{12}m_s\chi(1+2\phi)M^4L^{-28/27}$$
$$-\frac{a^2}{216}(2\kappa-\xi)(1+2f-4\phi)+\frac{a}{12}m_sM^2\kappa\left[\ln\left[\frac{M^2}{\Lambda^2}\right]-1-\gamma_{\rm EM}\right]-\frac{a}{36}m_sM^2\left[2\kappa-\xi-\frac{19}{2L^{4/9}}\right]$$
$$\left.+\frac{am_sM^2}{3L^{4/9}}\left[\ln\left[\frac{M^2}{\Lambda^2}\right]-1-\gamma_{\rm EM}\right]\right]\ . \quad (39)$$

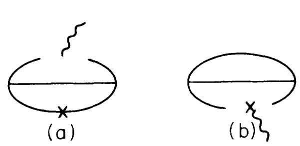

FIG. 3. Contribution of the $m_x\chi$ operator. Diagram (b) arises from expanding the quark correlator to first order in x and using its equation of motion.

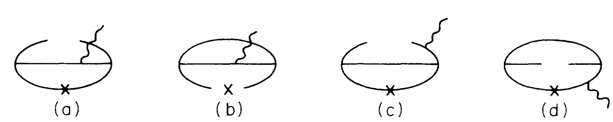

FIG. 4. Contribution of the m_sa operator. Diagram (d) has an infrared divergence which is regularized by a cutoff Λ.

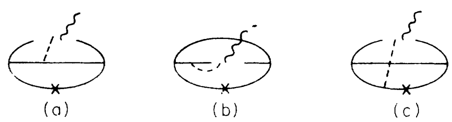

FIG. 5. Contributions of the operators $m_s a\kappa$ and $m_s a\xi$. Diagram (c) contributes only to the κ term and has an infrared divergence which is regularized by a cutoff Λ.

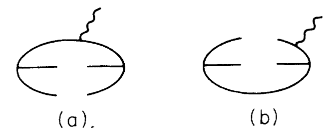

FIG. 6. Contribution of the a^2 operator.

Computing this ratio in terms of physical intermediate states we can write

$$R(M^2)\,|_{\rm RHS}=\frac{(1+\delta_\Lambda+A_\Lambda M^2)+\sum_{i\neq\Lambda}^{\infty}\frac{\beta_i^2}{\beta_\Lambda^2}(1+\delta_i+A_iM^2)e^{-(M_i^2-M_\Lambda^2)/M^2}}{1+\sum_{i\neq\Lambda}^{\infty}\frac{\beta_i^2}{\beta_\Lambda^2}e^{-(M_i^2-M_\Lambda^2)/M^2}}\;. \tag{40}$$

We again use the ansatz form introduced in Ref. 5, viz.,

$$R(M^2)\,|_{\rm RHS}=1+\delta_\Lambda+A_\Lambda M^2+[\rho+\sigma(W^2-M_\Lambda^2+M^2)]e^{-(W^2-M_\Lambda^2)/M^2}\;. \tag{41}$$

As explained in Ref. 5, matching the asymptotic behavior of $R(M^2)$ and $R(M^2)\,|_{\rm RHS}$ we get

$$1+\delta_\Lambda+\rho=1 \tag{42}$$

and

$$\sigma+A_\Lambda=0\;. \tag{43}$$

We start with an initial value of $\sigma=0$ and an arbitrary value of ρ and compute

$$R(M^2)-\rho\exp\left[-\frac{W^2-M_\Lambda^2}{M^2}\right]=F(M^2)\;. \tag{44}$$

The function $F(M^2)$ is fitted by

$$F(M^2)=\gamma+\delta M^2\;, \tag{45}$$

in the fiducial region

$$1.2\leq M^2\leq 1.4\ {\rm GeV}^2\;. \tag{46}$$

If the fitted value of γ does not satisfy the condition $\gamma=1+\delta_\Lambda=1-\rho$, then a new value $\rho'=(\gamma+\rho)/2$ is chosen. Next the left-hand side of Eq. (44) is reevaluated and a new fitted value γ' is obtained. This process is iterated. After the ith iteration, the acquired value of γ is given by

$$\rho_{i+1}=(\gamma_i+\rho_i)/2\;. \tag{47}$$

The convergence of this iteration for the quantity $1+\delta_i=1-\rho_i$ is shown in Fig. 10(a). We note that the iteration in ρ converges rapidly, and more importantly the final value of ρ is independent of its initial value. However, when we try to satisfy Eq. (43) by iterating σ, a small nonzero value of $A+\sigma$ persists. We chose $\sigma=0$ and let the constraint of Eq. (43) be mildly violated in the large-M^2 region. Figure 10(b) shows the match between the function $R(M^2)$ and our ansatz Eq. (41). It is seen that our failure to match Eq. (43) has little effect in the mass region of interest, Eq. (46). We take our final value of μ_Λ to be the limit to which ρ converges. For the numerical analysis we need to supply the values of the susceptibilities χ, κ, and ξ. In Ref. 5 the values of these quantities were estimated using the procedure of Belyaev and Kogan.[9] It has been kindly pointed out to us by Ioffe and Smilga[10] that the use of unsubtracted dispersion relations is not quite justified for estimating the values of κ and ξ. Here in this work (cf. also Ref. 11), we have treated χ and κ as free parameters. (We retain however the feature $\xi=-2\kappa$ of the estimates provided in Ref. 5 using single- and double-pole approximations.)

IV. RESULTS

In Ref. 11 we have considered all the baryons together and searched for the values of χ and $2\kappa-\xi$ which give the best agreement for all the baryons. We find that close agreement with experimental values is possible if

$$\chi=-3\ {\rm GeV}^{-2},\quad 2\kappa-\xi=3,\quad \xi=-2\kappa\;. \tag{48}$$

For the Λ magnetic moment calculation, we note that at this stage the contribution of the $m_s a\kappa$ and $m_s a\xi$ operators is necessarily incomplete. We need the expan-

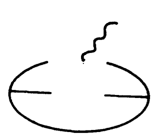

FIG. 7. Contribution of the χa^2 operator.

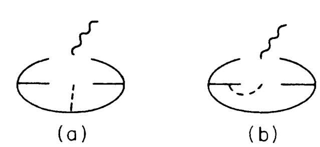

FIG. 8. Contribution of the $\chi a^2 m_0^2$ operator.

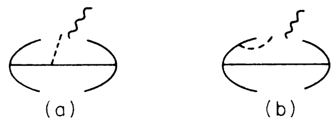

FIG. 9. Contribution of the linear combination of operators $(2\kappa-\xi)a^2$.

sion of $\langle q_i^a(x)G_{\alpha\beta}\bar{q}_k^b(0)\rangle$ to first order in x. This, however, leads to new and unknown susceptbilities. We have taken the following stand. The terms which we have calculated are of the same dimension as the omitted terms. As we are not sure of the value of $2\kappa-\xi$, there is little sense in introducing more unknown terms in the sum rule. Hopefully our search for $2\kappa-\xi$ compensates for this lacuna in our calculation.

To investigate the effect of these terms, we have done the calculation with and without the operators $m_s a\kappa$ and $m_s a\xi$. Let us call the calculation including these terms result 1, and the calculation omitting them result 2. Then for the values of the susceptibilities given in Eq. (48) we obtain

$$\text{Result 1} \quad \mu_\Lambda = -0.50 \ , \tag{49}$$

$$\text{Result 2} \quad \mu_\Lambda = -0.54 \ , \tag{50}$$

$$\text{Experiment} \quad \mu_\Lambda = -0.61 \ . \tag{51}$$

Keeping only the $m_s(2\kappa-\xi)$ contribution gives a value in between results 1 and 2. We have also varied the value of the infrared cutoff Λ from 500 to 600 MeV and found only small variation in the output values of μ_Λ.

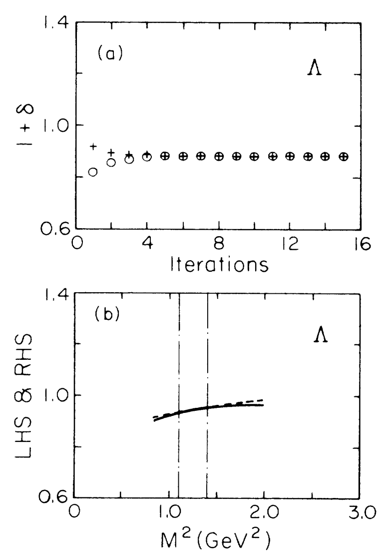

FIG. 10. (a) shows the convergence of the iterative solution for δ. Two seed values were picked and both converge to the same answer. (b) shows the agreement between the ratio and fitted solution. The ratio is the solid curve and the fitted solution is the dashed curve.

FIG. 11. m_s corrections to quark propagators. (a) is the m_s correction to the quark-photon vertex and leads to an infrared divergence in coordinate space. (b) is an m_s correction to the χ susceptibility which is calculated by expanding the quark correlator to first order in x.

To summarize then, we have established that from QCD sum rules one can write

$$\mu_\Lambda = \frac{2}{3}(e_u+e_d+4e_s)\frac{e\hbar}{2M_\Lambda c}(1+\delta_\Lambda) \ , \tag{52}$$

and can compute δ_Λ. Given the approximations involved in the use of QCD sum rules, we consider our numerical estimate to be in satisfactory accord with experiment.

APPENDIX: MASS CORRECTIONS TO THE QUARK PROPAGATOR

In this appendix we want to evaluate mass corrections to the quark Green's function as indicated in Fig. 11, where there are a- and b-type corrections. Diagram (a) has the form

$$\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \int \frac{d^4p}{(2\pi)^4} \frac{i}{\hat{k}+\hat{p}-m_q} \gamma^\mu \frac{i}{e_q} F_{\mu\nu} \times \int d^4y\, e^{ip\cdot y} y^\nu \frac{i}{\hat{k}-m_q} \ , \tag{53}$$

where we have used the fixed-point gauge. Retaining only those terms which are $O(m_q)$, we get

$$\frac{ie}{2} m_q F_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \frac{\sigma^{\mu\nu}}{(k^2+i\epsilon)^2} \ , \tag{54}$$

which is divergent in the infrared region. As explained in the text, to keep only the high-momentum components we replace $k^2+i\epsilon \rightarrow k^2-\Lambda^2+i\epsilon$. Using this we can write, for Eq. (54),

$$\frac{m_q}{32\pi^2}\sigma_{\mu\nu}e_q F^{\mu\nu}\left[\ln\left[-\frac{x^2\Lambda^2}{4}\right]+2\gamma_{\rm EM}\right]+O(\Lambda^2) \ . \tag{55}$$

This is the term that enters in Eq. (9) of the text.

Diagram (b) indicates the m_q correction to the correlator $\langle \bar{q}\sigma_{\alpha\beta}q\rangle_F$. This term arises from expanding $\langle 0 \,|\, T\{q_i^a(x), q_k^b(0)\} \,|\, 0\rangle_F$ in x. The first term in the expansion gives $x_\mu \langle 0 \,|\, \nabla^\mu q_i^a(0), \bar{q}_k^b(0) \,|\, 0\rangle_F$ which is sufficient to give a linear term in the mass. Next we must take its vector and axial-vector components only and then antisymmetrize the Lorentz indices in order to get a field-induced nonzero VEV:

$$x^{\mu}\langle 0\,|\,\nabla_{\mu}q_i^a,\bar{q}_k^b\,|\,0\rangle_F=\frac{\delta^{ab}}{3}x^{\mu}[-\tfrac{1}{8}(\gamma^{\nu})_{ik}\langle 0\,|\,\bar{q}(\gamma_{\mu}-\gamma_{\mu}\nabla_{\nu})q\,\,|\,0\rangle_F+\tfrac{1}{8}(\gamma^{\nu}\gamma_5)_{ik}\langle 0\,|\,\bar{q}(\gamma_{\nu}\nabla_{\mu}-\gamma_{\mu}\nabla_{\nu})\gamma_5 q\,\,|\,0\rangle_F]\ . \quad (56)$$

But the vector component will not contribute due to a charge-conjugation argument. Using the relations

$$\gamma^{\sigma}\gamma_5=\tfrac{1}{12}\epsilon^{\alpha\beta\lambda\sigma}(\gamma_{\lambda}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\gamma_{\lambda})\ , \quad (57)$$

$$\tfrac{1}{2}\gamma_5(\gamma_{\nu}\nabla_{\mu}-\gamma_{\mu}\nabla_{\nu})=\tfrac{1}{2}\epsilon_{\alpha\beta\mu\nu}\epsilon^{\alpha\beta\lambda\sigma}\gamma_5\gamma_{\lambda}\nabla_{\sigma}\ , \quad (58)$$

we get

$$x^{\mu}\langle 0\,|\,\nabla_{\mu}q_i^a,\bar{q}_k^b\,|\,0\rangle_F=\delta^{ab}\frac{i}{96}m_q(\hat{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\hat{x})\langle\bar{q}\sigma^{\alpha\beta}q\rangle_F\ . \quad (59)$$

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