$K \rightarrow \pi\pi\gamma$ in the six-quark model

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We reexamine the decay $K^+ \rightarrow \pi^+\pi^0\gamma$ within the context of the six-quark Kobayashi-Maskawa model of *CP* violation. We estimate the short- and long-distance contributions to the matrix element for direct photon emission. We argue that most of the matrix element is from the long-distance contribution restricted to photons of magnetic polarization and does not contribute to any possible asymmetry. Although this argument fits nicely with the existing experimental data, it implies that any potential asymmetry must be less than $10^{-3}s_2s_3\sin\delta$.

I. INTRODUCTION

The violation of *CP* symmetry is one of the few low-energy observations which has no explanation within the context of the standard model. It is believed that the origin of this phenomenon holds a key to the generation of baryon asymmetry starting with big-bang cosmology. It therefore holds a key to going beyond the standard model.

The original observation of the decay $K_L \rightarrow \pi\pi$ as well as the results of subsequent experimental observations are all consistent with superweak theory in which a $\Delta S = 2$ interaction is responsible for *CP* violation. With present experimental accuracy, many other models mimic the superweak theory. Methods to distinguish these alternative explanations of *CP* violation have attracted much theoretical and experimental investigation.

The most promising candidate for such a search is that for nonvanishing ϵ'. One problem with the ϵ' measurement is that it is suppressed by the $\Delta I = \frac{1}{2}$ rule as

$$\epsilon' = \frac{i}{\sqrt{2}}\mathrm{Im}(A_2/A_0)\exp[i(\delta_2-\delta_0)] \ . \tag{1.1}$$

While the search for ϵ' is in progress, it is worthwhile investigating the possibility of other processes which may shed some light on this puzzle.

Other investigations in *CP*-violating phenomena have led [at least within the context of the Kobayashi-Maskawa (KM) model] to vanishingly small results. Fractional rate differences in hyperon decays were found to be of order 10^{-6} (Refs. 1 and 2). Also muon polarization in $K_L^0 \rightarrow \mu^+\mu^-$ predicts an asymmetry of 10^{-4} or less.[3]

In this paper we reexamine the decay $K^\pm \rightarrow \pi^\pm\pi^0\gamma$. It is hoped that the bremsstrahlung amplitude and internal photon emission amplitude will interfere to produce a difference between $\Gamma(K^+ \rightarrow \pi^+\pi^0\gamma)$ and $\Gamma(K^- \rightarrow \pi^-\pi^0\gamma)$. The asymmetry can be maximized by choosing the kinematical region in which the bremsstrahlung amplitude is comparable to the direct photon emission amplitude. This procedure is helped by the fact that the kinematically enhanced bremsstrahlung amplitude is suppressed by the $\Delta I = \frac{1}{2}$ rule as the *S*-wave $\pi^+\pi^0$ state must have isospin 2. This possibility was emphasized long ago and the decay was a subject of intense research;[4-6] to our knowledge, this possibility has been forgotten since the advent of the standard model.

In Sec. II we repeat the phenomenological analysis given by other authors together with an update of the experimental situation. In Sec. III we present an effective Hamiltonian for $K \rightarrow \pi\pi\gamma$ within the context of the KM model. In Sec. IV we isolate the hadronic matrix elements needed to compute the asymmetry. In Sec. V we give a very rough estimate of the short-distance contribution to the direct $K \rightarrow \pi\pi\gamma$ amplitude and find that the predicted amplitude is too small by an order of magnitude. We then go on to discuss the long-distance contributions. We argue that, unfortunately, most of the long-distance contributions give M_1 transitions which cannot contribute to the asymmetry. In Sec. IV we discuss realistic prospects for observing this decay. In Appendix A we entertain the possibility of observing photon polarization. However unrealistic the measurement of the photon polarization may be, the asymmetry involving the photon polarization is sensitive to the M_1 amplitude and is predicted to be large. There is no kinematical suppression in this asymmetry. For completeness we also discuss the prospects for other electromagnetic decays.

II. PHENOMENOLOGICAL ANALYSIS OF $K^+ \rightarrow \pi^+\pi^0\gamma$

As mentioned above the decay in question proceeds by two possible mechanisms.

(a) Bremsstrahlung emission. First the kaon decays by $K^+ \rightarrow \pi^+\pi^0$, and the two pions are in a state of angular momentum $1\pi\pi = 0$. The final state changes under the interchange of the two pions by a factor $(-1)^{I+l\pi\pi}$; isospin addition alone tells us that the final state must have isospin $I = 1$ or $I = 2$; however, $I = 2$ is picked out as the pions are bosons and this factor must be 1. At that point the photon is emitted from the charged pion and conservation of angular momentum is maintained if the orbital angular momentum of the photon is opposite to its spin. This is an electric-type transition with the matrix element given by

$$A(K \rightarrow \pi + \pi^0)\exp(i\delta_{20})\frac{1}{(q\cdot k)(q\cdot p_+)}F_{\mu\nu}p_+^\mu k^\nu \ , \tag{2.1}$$

where $A(K \rightarrow \pi\pi)$ is the amplitude for $K^+ \rightarrow \pi^+\pi^0$, δ_{20} is the final-state phase shift for $1_{\pi\pi} = 0$ and $I = 2$, k^μ, p_+^μ are the four-momenta of the kaon and charged pion, $F_{\mu\nu} = q_\mu\epsilon_\nu - q_\nu\epsilon_\mu$, and q_μ, ϵ_μ are the four-momenta and polarization of the photon.

(b) Direct emission. The photon is emitted from some intermediate state which subsequently decays into a $\pi^+\pi^0$ final state. For the simplest case of dipole emission $J=1$, where $\mathbf{J}=\mathbf{l}+\mathbf{S}$ is the total angular momentum of the photon, orbital angular momentum plus spin. As $S=1$ we have $J=l+1,l,l-1$ and these are classified as follows: $J=l\pm1$ is the electric transition; $J=l$ is the magnetic transition. Note that for fixed J, electric and magnetic transitions produce photons of opposite parity as $(-1)(-1)^l=(-1)^J$ or $(-1)^{J+1}$ for $J=l\pm1$ or $J=l$.

If J is kept at one we must have $l_{\pi\pi}=1$. As the two pions are bosons we must have $(-1)^I(-1)^{l_{\pi\pi}}=1$; $I=1$ is picked out as $l_{\pi\pi}=1$. The matrix element for this type of transition is given by

$$A(K^+\to\pi^+\pi^0)\frac{e^{i\delta_{11}}}{m_K{}^4}(EF_{\mu\nu}p_+^\mu k^\nu+M\tilde{F}_{\mu\nu}p_+^\mu k^\nu)\ , \tag{2.2}$$

where δ_{11} is the final-state phase shift for $I=1$ and $l_{\pi\pi}=1$, m_K is the mass of the kaon, $\tilde{F}_{\mu\nu}=\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, and E and M are dimensionless form factors for electric- and magnetic-type transitions.

Adding the contributions from (a) and (b) gives the total matrix element

$$\begin{aligned}M^+=Ae^{i\delta_{20}}&\frac{1}{(q\cdot p_+)(q\cdot k)}F_{\mu\nu}p_+^\mu k^\nu\\&+A\frac{e^{i\delta_{11}}}{m_K{}^4}(EF_{\mu\nu}p_+^\mu k^\nu+M\tilde{F}_{\mu\nu}p_+^\mu k^\nu)\ .\end{aligned} \tag{2.3}$$

Using CPT invariance one can show that $M^-=M(K^-\to\pi^-\pi^0\gamma)$ is given by

$$\begin{aligned}M^-=A^*e^{i\delta_{20}}&\frac{1}{(q\cdot p_+)(q\cdot k)}F_{\mu\nu}p_+^\mu k^\nu\\&+A^*\frac{e^{i\delta_{11}}}{m_K{}^4}(E^*F_{\mu\nu}p_+^\mu k^\nu-M^*\tilde{F}_{\mu\nu}p_+^\mu k^\nu)\ .\end{aligned} \tag{2.4}$$

We are interested in CP violation in the above decay; this will show up through an asymmetry between the Dalitz plots of $K^+\to\pi^+\pi^0\gamma$ and $K^-\to\pi^-\pi^0\gamma$, that is a nonzero value of

$$a_2=\frac{|M^+|^2-|M^-|^2}{|M^+|^2+|M^-|^2}\ . \tag{2.5}$$

Using the above expressions for M^+ and M^- one obtains the following expression for a_2:

$$a_2=\frac{-4\,|E\,|\sin\phi\sin(\delta_{11}-\delta_{20})}{2x(\,|E\,|^2+|M\,|^2)+2/x+4\,|E\,|\cos\phi\cos(\delta_{11}-\delta_{20})}\,, \tag{2.6}$$

where ϕ is the phase of E and x is a dimensionless kinematic variable given by

$$x=\frac{(q\cdot k)(q\cdot p_+)}{m_K{}^4}=\frac{\omega_\gamma(m_K/2-\omega_0)}{m_K{}^2}\ . \tag{2.7}$$

At this point it is important to note that one advantage this decay has over the $K\to\pi\pi$ decays involved in the measurement of ϵ' is that it is a three-body decay with kinematical degrees of freedom. The interference measurement in the two-pion final state of $K_S^0\to\pi^+\pi^-$ is essentially limited by the fact that the $I=0$ piece of the matrix element dominates over $I=2$. In the $K^+\to\pi^+\pi^0\gamma$ decay however one can choose to look at a region of phase space where the $I=1$ and $I=2$ pieces of matrix element contribute equally thus avoiding the $\frac{1}{20}$ suppression occurring in ϵ' [see Eq. (1.1)].

Indeed one can show that $a_2(x)$ is maximum at $x=(\,|E\,|^2+|M\,|^2)^{-1/2}$ for which the $I=1$ and $I=2$ terms contribute equally in the denominator. The maximum value of the asymmetry over all x is then given by

$$a_{2\max}=\frac{-\sin\phi\sin(\delta_{11}-\delta_{20})}{(1+\gamma^2)^{1/2}+\cos\phi\cos(\delta_{11}-\delta_{20})}\ , \tag{2.8}$$

$$\text{where } \gamma=\frac{|M\,|}{|E\,|}\ .$$

This will provide a useful measure of the magnitude of CP-violating effects parametrized by the values of γ and ϕ.

An experiment due to Abrams *et al.*[7] was performed to measure this asymmetry. Their results can be summarized in the following way:

$$\begin{aligned}&(1)\quad |E\,|^2+|M\,|^2=1000\pm200\sigma\ ,\\&(2)\quad |E\,|\sin\phi\sin(\delta_{11}-\delta_{20})=0.8\pm2.5\sigma\ ,\\&(3)\quad |E\,|\cos\phi\cos(\delta_{11}-\delta_{20})=1.25\pm3.75\sigma\ ,\end{aligned} \tag{2.9}$$

where $\sigma=1$ if we are to use their data to 1 standard deviation, $\sigma=2$ to 2 standard deviations, etc. The data seem to suggest that $\gamma\geq5$ for small ϕ, yet ($\gamma=1$, $\phi=75.5°$, $a_2=0.1$) can be reached with $\sigma=1$, as well as ($\gamma=0$, $\phi=\pi/2$, $a_2=0.173$) for $\sigma=1.51$. In short, although the best fit favors pure magnetic dipole emission, pure electric dipole emission with $\phi=\pi/2$ is less than 2 standard deviations from the most probable value.

III. THE EFFECTIVE HAMILTONIAN FOR $K\to\pi\pi\gamma$ (SHORT DISTANCE)

To evaluate E,M we need a specific model of interaction. We choose the Weinberg-Salam model with six quarks and the standard parametrization of the KM matrix. An analysis with four quarks has been carried out for $K\to\pi\pi\gamma$ by Lucio[8] and Malakyan[9] ($K_L\to\pi^+\pi^-\gamma$ only). We show in Fig. 1 diagrams which are of potential relevance to $K\to\pi\pi\gamma$.

SVZ (Shifman, Vainshtein, and Zakharov) have devised a method to calculate the dependence of any weak amplitude on M_W and on the masses of the quarks to the lowest order in the electromagnetic and weak interactions and includes the strong interactions through the leading-logarithmic approximation.[10] The effective Hamiltonian obtained depends on the scale of the external momenta involved. One never includes fields in the effective Hamiltonian whose mass is greater than the scale at which it is to be used. Rather these fields are integrated out to obtain a new set of vertices involving only the light particles.

We divide the integration regions up into three intervals $[M_W,m_t]$, $[m_t,m_c]$, and $[m_c,m]$, where m is a hadronic mass scale. In the first region 1(b) and 1(c) are zero by Glashow-Iliopoulos-Maiani (GIM) cancellation and the effective Hamiltonian is given by the two operators needed to close 1(a) under renormalization.

In the second region 1(b) and 1(c) would be zero if GIM cancellation were exact in the four-quark model. So their contribution must be proportional to $(a_1+a_2)=s_1c_1c_3(s_2^2+s_2s_3e^{i\delta}c_2/c_1c_3)$. The phase δ will be proportional to CP-violating effects. In order for 1(a) and 1(b) to close under renormalization four more operators must be introduced. This will give the six operators of the SVZ $\Delta S=1$ effective Hamiltonian.[10] In the final region 1(b) and 1(c) are proportional to a_1 so that all operators develop a nonzero coefficient in the limit s_2, s_3 and δ vanish.

The final effective Hamiltonian at m can be written in terms of the six operators of SVZ, $O_1,\ldots,O_6$ plus a new operator T explicitly due to the electromagnetic interactions. These are listed below:

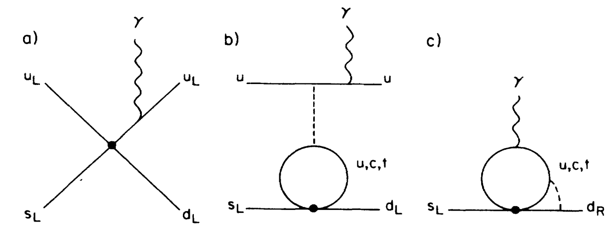

FIG. 1. Contributions to the effective Hamiltonian for $K^+\to\pi^+\pi^0\gamma$. The solid dot denotes the insertion of the operator $G_F/\sqrt{2}\,\bar{q}_L\gamma_\mu q_L\bar{s}_L\gamma_\mu d_L$. The dashed line represents a gluon. (a) Diagram which gives to $LL\to LL$ operators in H_{eff}. (b) Diagram giving penguin operators in H_{eff} with the emission of a photon off any external quark line. (c) Diagram which gives a new operator T solely due to the inclusion of the electromagnetic interaction.

$$
\begin{aligned}
&T=im_s\bar{d}_L\sigma_{\mu\nu}F^{\mu\nu}s_R\ ,\\
&O_1=\bar{d}_L\gamma_\mu s_L\bar{u}_L\gamma_\mu u_L-\bar{d}_L\gamma_\mu u_L\bar{u}_L\gamma_\mu s_L\quad (8_f,\ \Delta I=\tfrac{1}{2})\ ,\\
&O_2=\bar{d}_L\gamma_\mu s_L u_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu u_L u_L\gamma_\mu s_L+2\bar{d}_L\gamma_\mu s_L\bar{d}_L\gamma_\mu d_L+2\bar{d}_L\gamma_\mu s_L\bar{s}_L\gamma_\mu s_L\quad (8_d,\Delta I=\tfrac{1}{2})\ ,\\
&O_3=\bar{d}_L\gamma_\mu s_L\bar{u}_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu u_L\bar{u}_L\gamma_\mu s_L+2\bar{d}_L\gamma_\mu s_L\bar{d}_L\gamma_\mu d_L-3\bar{d}_L\gamma_\mu s_L\bar{s}_L\gamma_\mu s_L\quad (27,\Delta I=\tfrac{1}{2})\ ,\\
&O_4=\bar{d}_L\gamma_\mu s_L\bar{u}_L\gamma_\mu u_L+\bar{d}_L\gamma_\mu u_L\bar{u}_L\gamma_\mu s_L-\bar{d}_L\gamma_\mu s_L\bar{d}_L\gamma_\mu d_L\quad (27,\Delta I=\tfrac{3}{2})\ ,\\
&O_5=\bar{d}_L\gamma_\mu\lambda_a s_L(\bar{u}_R\gamma_\mu\lambda_a u_R+\bar{d}_R\gamma_\mu\lambda_a d_R+\bar{s}_R\gamma_\mu\lambda_a s_R)\quad (8,\Delta I=\tfrac{1}{2})\ ,\\
&O_6=\bar{d}_L\gamma_\mu s_L(\bar{u}_R\gamma_\mu u_R+\bar{d}_R\gamma_\mu d_R+\bar{s}_R\gamma_\mu s_R)\quad (8,\Delta I=\tfrac{1}{2})\ .
\end{aligned}
\tag{3.1}
$$

The λ_a are defined so that the gluon coupling to the fermions is $\psi_i\lambda_a^{ij}A^a\psi_j$. The effective Hamiltonian of SVZ can then be written in the form

$$H=-\sqrt{2}\,G_F\sum_{i=1}^{6}[c_i(a_1+a_2)+d_ia_1]O_i+c_T\sqrt{2}\,G_FT\ . \tag{3.2}$$

These coefficients c_i,d_i can be obtained numerically from the results of Guberina and Peccei[11] who extended the SVZ analysis to the six-quark model. For the choice

$$
\begin{aligned}
&m=0.3\ \text{GeV},\quad m_c=1.5\ \text{GeV}\ ,\\
&m_t=40\ \text{GeV},\quad M_W=80\ \text{GeV}\ ,
\end{aligned}
\tag{3.3}
$$

we obtain the values

$$c_i=\begin{bmatrix}0.0238\\-0.0096\\0\\0\\-0.1102\\-0.0655\end{bmatrix},\quad d_i=\begin{bmatrix}-2.58\\0.1012\\0.0817\\0.4083\\-0.092\\-0.006\end{bmatrix}\ . \tag{3.4}$$

The coefficient c_T of T has been derived in the four-quark model by SVZ (Ref. 10) and in the six-quark model for the first time here. The result is

$$c_T=\tfrac{4}{3}[-0.2545a_1+0.2596\ (a_1+a_2)]\frac{1}{16\pi^2}\ . \tag{3.5}$$

One can see that the numerical coefficients in c_T are small. Indeed the coefficient of a_1 in c_T is only 0.002, a thousand times smaller than d_1, and the coefficient of (a_1+a_2) is also about 0.002, ten times smaller than c_1.

We present the result for c_T only for completeness; it is not expected to be important to this decay unless T has an anomalously large matrix element. It has been shown to be important to other decays, namely, $\Omega^-\to\Xi^-\gamma$ (Refs. 12 and 13), in which the contribution of $O_1,\ldots,O_6$ are inhibited.

IV. MATRIX ELEMENTS

Having obtained the coefficient of each operator above what is left is to calculate the matrix elements of the operators between initial and final states. We have

$$
\begin{aligned}
M^+&=\langle K^+\mid H_{\text{eff}}\mid \pi^+\pi^0\gamma\rangle\\
&=-\sqrt{2}\,G_Fq_ie\langle K^+\mid O_i\mid \pi^+\pi^0\gamma\rangle\\
&\quad+\sqrt{2}\,G_Fc_Te\langle K^+\mid im_s\bar{s}\sigma_{\mu\nu}F^{\mu\nu}d\mid \pi^+\pi^0\rangle\ ,
\end{aligned}
\tag{4.1}
$$

where

$$q_i = s_1 c_1 (d_i c_3 + c_i (c_3 s_2{}^2 + s_2 s_3 c_2 / c_1))$$

and

$$\langle K^+ \mid O_i \mid \pi^+ \pi^0 \gamma \rangle = \int d^4x\, e^{iq\cdot x} \langle K^+ \mid T(O_i j_\mu^{\rm em}(x) \epsilon^\mu) \mid \pi^+ \pi^0 \rangle \ .$$

$j_\mu^{\rm em}(x)$ is the electromagnetic current. We will neglect the contribution of T due to the small numerical value of c_T and set

$$\langle K^+ \mid O_i \mid \pi^+ \pi^0 \gamma \rangle = m_\pi{}^{-1} (E_i F_{\mu\nu} p_+^\mu k^\nu + M_i \tilde{F}_{\mu\nu} p_+^\mu k^\nu) \ , \tag{4.2}$$

where we have used Lorentz and gauge invariance to express the matrix elements of O_i in terms of the dimensionless form factors E_i and M_i. Our immediate goal is to express the form factors E and M of Sec. II in terms of E_i and M_i. From the expression

$$M^+ = \frac{eAe^{i\delta_{20}}}{(q\cdot p_+)(q\cdot k)} F_{\mu\nu} p_+^\mu k^\nu + ee^{i\delta_{11}} (-\sqrt{2}\, G_F m_\pi{}^{-1} q_i)(E_i F_{\mu\nu} + M_i \tilde{F}_{\mu\nu}) p_+^\mu k^\nu \tag{4.3}$$

we obtain

$$E = -\sqrt{2}\, G_F m_\pi{}^{-1} m_K{}^4 A^{-1} (q_i E_i) \ . \tag{4.4}$$

We need now an expression for the amplitude $A = A(K^+ \to \pi^+ \pi^0)$. In the transition $K^+ \to \pi^+ \pi^0$ the two pions must be in a $I=2$ state. The process involves a $\Delta I = \frac{3}{2}$ transition hence the operator O_4. Note that q_4 is real so we can take the whole amplitude to be real. The magnitude of A can be inferred from experimental data and is given by SVZ as[10]

$$A = 1/(1.6)(c_4 G_F s_1 c_1 c_3 m_K{}^2 f_\pi) \ , \tag{4.5}$$

where we have normalized the above to the result obtained using the vacuum-saturation approximation[10] contained in the second set of parentheses. Giving us

$$E = [-\sqrt{2}(1.6) m_K{}^2 f_\pi{}^{-1} m_\pi{}^{-1}] \times E_i \left[d_i + c_i \left[s_2{}^2 + s_2 s_3 \frac{c_2}{c_3 c_1} e^{i\delta} \right] \right] \ . \tag{4.6}$$

As c_i and d_i were determined from the short-distance analysis of Sec. III, the unknowns are the matrix elements E_i and M_i. One might not think that we have gained anything by expressing the unknowns E and M with the 12 unknowns E_i and M_i, but these matrix elements are, in principle, calculable within various approximations and the coefficients c_i and d_i give the weights with which they come in. We give an estimate in Sec. V using the vacuum-saturation approximation.

We would like to give a rough estimate of the E_i and M_i consistent with experiment and use these values to get an idea of the $a_{2\max}$ one would expect. There is a $\Delta I = \frac{1}{2}$ enhancement in the decay $K^0 \to \pi^+ \pi^-$ and this is thought to be due to a large enhancement in the matrix elements of the penguin operators O_5 and O_6 which contain right-handed currents and only contribute to $\Delta I = \frac{1}{2}$. An analysis using the vacuum-saturation approximation and soft-pion theorems has been carried out for $K \to \pi\pi\gamma$ in which no such enhancement was found.[8] Nevertheless we will keep E_5 and E_6 as free variables and see how the asymmetry $a_{2\max}$ depends on them.

We find that we can parametrize the asymmetry by $r = E_5/E_1$ and $\gamma = |M| / |E|$. The results are shown in Fig. 2. In summary we can say that if we assume the bound by Abrams *et al.* for $|E| \cos\phi$ is only good to 4σ, taking the smallest value of γ we possibly can and assuming E_5 and E_6 are strongly enhanced we get an asymmetry as high as $0.04\, s_2 s_3 \sin\delta$. One loses about a factor of 10 if E_5 and E_6 are not enhanced and at least a factor of 4 if the data of Abrams *et al.* for $|E| \cos\phi$ to 1σ is used instead, forcing us to use a larger value of γ.

V. LONG-DISTANCE CONTRIBUTION TO THE MATRIX ELEMENT

For the amplitude

$$\langle K \mid H \mid \pi\pi\gamma \rangle = \int dx\, e^{iq\cdot x} \langle K \mid T(H j_\mu^{\rm em}(x)) \mid \pi\pi \rangle \epsilon^\mu$$

we shall distinguish between a short- and a long-distance region. Short-distance strong-interaction effects can be calculated using perturbation theory as was done in Sec. III. In doing so one integrates a renormalization-group equation (RGE) from a scale M_W down to some scale μ usually taken to be about 1 GeV, beyond this perturbation theory is thought to break down as the strong force becomes confining. Following Ref. 9 we shall call the region satisfying $|x| \geq \mu^{-1}$ long-distance regions. The short-distance effective Hamiltonian we derived in Sec. III will not be valid in this region.

The short-distance contribution to $K \to \pi\pi\gamma$ can be calculated from the effective Hamiltonian of Sec. III using the vacuum-saturation approximation and soft-pion theorems as was done in Ref. 9. This leads to a value of the branching ratio given by

$$B_{\rm SD} = 4 \times 10^{-7} \tag{5.1}$$

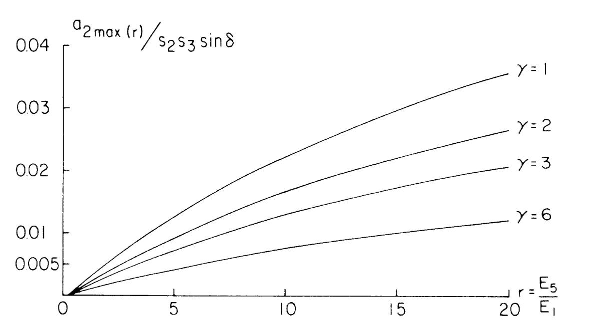

FIG. 2. Plot of the maximum value of the asymmetry across a two-dimensional Dalitz plot as a function of the ratio $r = E_5/E_1$ of matrix elements of penguin operators over ordinary four-fermion operators [see Eqs. (4.3)–(4.6)] for various values of $\gamma = |E| / |M|$.

while the experimental result due to Abrams gives

$$B_{\text{expt}} = 1.6 \times 10^{-5} \ . \tag{5.2}$$

Although it must be emphasized that the short-distance calculation represents only a very crude estimate, this indicates the long-distance effects are important. Indeed in a similar decay $K_L \to \gamma\gamma$ it has been shown that the long-distance contribution is dominant. In order to calculate such contributions we shall assume that the process takes place to leading order through one-particle intermediate states. Here several possibilities arise. There are pole diagrams $K \to P \to \pi\pi\gamma$ and vector-meson pole diagrams $K \to \pi \to \rho\omega \to (\pi\pi)\gamma$ (see Fig. 3). Contributions of the form $K^{\pm} \to K^0\pi^{\pm}\gamma \to \pi^0\pi^{\pm}\gamma$ are suppressed by a factor $m_\pi{}^2/\mu^2$ according to Refs. 14 and 15.

In particular the second diagram in Fig. 3 can be calculated as the effective coupling of $g_{\rho\omega\pi}$ and $f_{\rho\pi\pi}$ are known.[16] Also $\langle K \mid H_W \mid \pi \rangle$ is known.[17] We calculated this diagram and showed that it led to a branching ratio

$$B_{\text{LD}} = 2.7 \times 10^{-5} \ . \tag{5.3}$$

This is the same order of magnitude as the experimental measurement. More interestingly, this diagram led to a contribution which was entirely magnetic. This was dictated by the coupling of ρ and ω to π which is given by $g_{\rho\omega\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\rho_\alpha\partial_\nu\omega_\beta\pi$.

Owing to the parity conservation of the low-energy effective vertices parity violation must reside in $\langle K \mid H_W \mid \pi \rangle$. As the K and π are pseudoscalers this tells us nothing about the tensor structure. This is dictated by $\langle \pi \mid H_{\text{eff LD}} \mid \pi\pi\gamma \rangle$. As $H_{\text{eff LD}}$ must conserve parity we have $(-1)(-1)(-1)(-1)^{l\pi\pi}(-1)^{J+\sigma} = 1$ where $\sigma = 1,0$ for magnetic, electric transition. For decay from a scaler or pseudoscaler $l\pi\pi = J$ and $\sigma = 1$, only a magnetic transition is possible.

To modify this conclusion we must flip the parity at the level of $\langle K \mid H_W \mid S \rangle$ with S an intermediate scaler. As it must be charged the only possibility is the δ resonance. Owing to the large mass of δ and the fact that δ^0 does not contribute very much to a $K^0 \to \bar{K}^0$ transition matrix element we feel that the contribution of δ to $K \to \pi\pi\gamma$ should be small. In principle its effect could be estimated in the chiral-Lagrangian framework.[18] Then if the long-distance contribution does indeed dominate this would suggest that the decay proceeds through a predominantly magnetic transition. Indeed if we take the magnitude for electric transition from short-distance direct emission and compare it to the magnitude of the long-distance magnetic

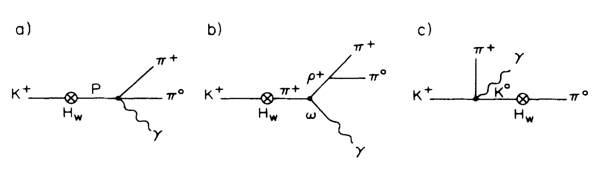

FIG. 3. Long-distance contributions to the decay $K^+ \to \pi^+\pi^0\gamma$. The circled $\times$ denotes the insertion of the effective weak Hamiltonian. The solid dot is an effective coupling valid at long distances. P denotes an arbitrary intermediate state.

transition, we find roughly

$$|E| / |M| \approx |E_{\text{SD}}| / |M_{\text{LD}}| \approx \tfrac{1}{7} \ . \tag{5.4}$$

VI. PROSPECTS FOR MEASURING THE ASYMMETRY

Let us first compare the predicted asymmetry $a_{2\max}$ with the asymmetry parameters ϵ and ϵ'. Under the most favorable circumstances $\gamma = 0$ and $r = 20$ $a_{2\max} = 0.04$ $s_2s_3\sin\delta$. Whereas various theoretical estimates based on the KM model give a range for

$$|\epsilon'| \approx (4.9 \times 10^{-3}) - (1.8 \times 10^{-2}) s_2s_3\sin\delta \ ,$$

$$|\epsilon| \approx (1.03 - 19.1) s_2s_3\sin\delta \ ,$$

and

$$|\epsilon'/\epsilon| \approx (1.1 \times 10^{-3}) - (6.0 \times 10^{-3})$$

(see Refs. 11 and 19). For $\gamma = 0$ and $r = 20$ we see that $a_{2\max}$ is somewhat larger than ϵ'. A more conservative guess $\gamma = 7$ and $r = 1$ yields $a_{2\max} = 6.8 \times 10^{-4} s_2s_3\sin\delta$ about an order of magnitude less than ϵ'. If photon polarization could be measured one could increase this by about a factor of 4 (see Appendix A).

Experimentally the limit on $a_{2\max}$ is about $(2.5 \times 10^{-2} \pm 8.3 \times 10^{-2})$. How far away is the measurement from the predicted value? From above $a_{2\max}(\gamma = 0, r = 20) = 0.04 s_2s_3\sin\delta$. To proceed further we need to use an acceptable range for the KM angles. For the experimental result for $|\epsilon| = (2.274 \pm 0.022) \times 10^{-3}$ to fall within the estimates $(1.03 - 19.1)$ $s_2s_3\sin\delta$ we should take $s_2s_3\sin\delta \approx (2.2 \times 10^{-3}) - (1.2 \times 10^{-4})$. So that

$$a_{2\max}(\gamma = 0, r = 20) = 0.04 s_2s_3\sin\delta$$
$$\approx (8.8 \times 10^{-5}) - (4.8 \times 10^{-6}) \ .$$

This is 3 orders of magnitude smaller than the experimental measurement. Further experiments with higher statistics could hope to improve this measurement by an order of magnitude but the measured upper limit will stay above the predicted value for some time.

VII. CONCLUSIONS

One primary motivation in studying $K^+ \to \pi^+\pi^0\gamma$ is the enhancement of the direct emission amplitude according to the $\Delta I = \frac{1}{2}$ rule. This would be very interesting as the $\Delta I = \frac{1}{2}$ rule is thought to be due to a large matrix element of the penguin operators which have an appreciable imaginary coefficient. This would lead to a large asymmetry in the decay although not outside experimental bounds (Fig. 2). To determine whether there is a $\Delta I = \frac{1}{2}$ enhancement in $K^+ \to \pi^+\pi^0\gamma$ one must look at a Dalitz plot for this decay and determine if it has an appreciable direct transition. The present experiments indicate that there is an appreciable direct transition but only of the magnetic type. Consequently there is only evidence for $\Delta I = \frac{1}{2}$ enhancement in the magnetic channel. The interference term involves only electric transitions and has not been observed—consequently, neither has any asymmetry.

Theoretically we find evidence for $\Delta I=\frac{1}{2}$ enhancement only in the long-distance component which is primarily magnetic. This seems to support the experimental result $\gamma=|M|/|E|\geq 5.5$. The interfering electric transition is present in the short-distance component to the decay. It occurs in the standard model through the graphs of Fig. 1. We estimated the magnitude of this electric transition and found it to be about a 12% effect compared to the other pieces of the matrix element in a particular region of the two-dimensional Dalitz plot. It occurs at a level just below present experimental sensitivity and would be discernible to an experiment with higher statistics. *CP* violation arises for $K^+\to\pi^+\pi^0\gamma$ in the standard model through the penguin graph [Fig. 1(b)]. It is a one-loop effect and comes in with a small coefficient of order $1/\pi^2$. Unless the matrix elements of the penguin operators are large the predicted asymmetry will be small of order 0.003 $s_2s_3\sin\delta$ (see Fig. 2). Also if $\gamma=|M|/|E|$ is large, the asymmetry will be suppressed by a factor $1/\gamma$, a photon polarization measurement would then be necessary to observe an asymmetry (see Appendix A). A better measurement of γ, similar to what was done for $\pi^-\to\mu^-\nu\gamma$ (Ref. 20), would decide if this is necessary.

Note added in proof. After submitting this paper we learned that the long-distance contribution to $K^+\to\pi^+\pi^0\gamma$ had previously been computed by Moshe and Singer.[23]

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APPENDIX A: PHOTON POLARIZATION

One of the troublesome aspects of the above analysis is that

$$|E|\cos\phi\leq 5\sigma,\quad |E|^2+|M|^2=1000\pm 220\sigma\ , \tag{A1}$$

implies $\gamma/\cos\phi\geq 5.5$ which for small ϕ would favor purely magnetic dipole emission. However if the polarization is summed over, the magnetic term will not interfere with the other terms in the amplitude, electric dipole, and bremsstrahlung, although it contributes to the total decay rate. To observe the interference with the magnetic term it is necessary to fix the photon polarization.[21] Then a potential asymmetry could develop unrestricted by a large value of γ.

The interference between the magnetic and other terms will be proportional to

$$\tilde{F}_{\mu\nu}p_+^{\mu}k^{\nu}F_{\alpha\beta}p_+^{\alpha}k^{\beta}=E_K{}^2[\boldsymbol{\epsilon}\cdot(\mathbf{p}_+\times\mathbf{k})](\mathbf{q}\cdot\mathbf{p}_+\epsilon_0-q_0\mathbf{p}_+\cdot\boldsymbol{\epsilon})\ , \tag{A2}$$

where ϵ_μ is the photon polarization. Set $\epsilon_\mu=(0,\boldsymbol{\epsilon})$ and $\mathbf{q}\cdot\boldsymbol{\epsilon}=0$ and expand $\boldsymbol{\epsilon}$ in the basis suggested by Christ:[5]

$$\boldsymbol{\epsilon}=\epsilon_e[\mathbf{p}_+/|p_+|-(\mathbf{p}_+\cdot\mathbf{q})\mathbf{q}/(|p_+|\,|q|^2)]+\epsilon_m(\mathbf{p}_+\times\mathbf{q})/(|p_+||q|)\ . \tag{A3}$$

Inserting this form of $\boldsymbol{\epsilon}$ into the above gives

$$\tilde{F}_{\mu\nu}p_+^{\mu}k^{\nu}F_{\alpha\beta}p_+^{\alpha}k^{\beta}=\epsilon_e\epsilon_m\,|p_+|^2|q|^2E_K{}^2\sin^4\theta\ , \tag{A4}$$

where $\cos\theta=(\mathbf{p}_+\cdot\mathbf{q})/(|p_+||q|)$. Similarly one has

$$F_{\mu\nu}p_+^{\mu}k^{\nu}F_{\alpha\beta}p_+^{\alpha}k^{\beta}=\epsilon_e{}^2|p_+|^2|q|^2E_K{}^2\sin^4\theta\ . \tag{A5}$$

Formulas can now be given for the asymmetry as a function of $\boldsymbol{\epsilon}$, they are similar to those of Sec. II except we have an interference term between the magnetic and bremsstrahlung portions of the matrix element and the presence of the additional polarization variables. Actually one would only push to observe such a term if the interference between the electric and bremsstrahlung portions of the matrix element turned out to be very small, so from here on we shall set $|E|=0$. We then get

$$a_2(\epsilon_e,\epsilon_m,x)=\frac{|M^+|^2-|M^-|^2}{|M^+|^2+|M^-|^2}\ \text{(polarization unsummed)}=\frac{-2\epsilon_e\epsilon_m\sin\phi\sin(\delta_{11}-\delta_{20})\,|M|}{x\,|M|^2\epsilon_m{}^2+1/x+2\epsilon_e\epsilon_m\cos\phi\cos(\delta_{11}-\delta_{20})} \tag{A6}$$

and x is defined in Sec. II. Defining $y=(\epsilon_m/\epsilon_e)\,x$ we obtain

$$a_2(y)=\frac{-2\,|M|\sin\phi\sin(\delta_{11}-\delta_{20})}{y\,|M|^2+1/y+2\,|M|\cos\phi\cos(\delta_{11}-\delta_{20})}\ . \tag{A7}$$

This formula is zero if either ϵ_m or ϵ_e vanish, i.e., if y is 0 or ∞, so that we need look for photons of mixed polarizations. Note also that this formula is maximum at $y=1/|M|$ for which it has the value

$$a_{2\max}=\frac{-1\sin\phi\sin(\delta_{11}-\delta_{20})}{1+\cos\phi\cos(\delta_{11}-\delta_{20})}\ . \tag{A8}$$

One amusing fact is that $a_{2\max}$ does not depend on the unknown coefficient $|M|$ although its position in phase space does. By not summing over the polarization we have avoided the suppression from the factor $1/\gamma$. In light of the fact that the asymmetry is zero whenever ϵ_m or ϵ_e vanish one should not look for photons with a polarization lying along $\mathbf{p}_+$ or along $\mathbf{p}_+\times\mathbf{q}$ but only along a linear combination of them. In particular for $\epsilon_e=\epsilon_m$ we would look for photons with polarization proportional to $\mathbf{p}_+|q|-(\mathbf{p}_+\cdot\mathbf{q})\mathbf{q}/|q|+\mathbf{p}_+\times\mathbf{q}$.

APPENDIX B: OTHER ELECTROMAGNETIC DECAYS

To extend the analysis to other decays let us briefly note that $K\to\pi\pi\pi\gamma$ can proceed via a $\Delta I=\frac{1}{2}$ enhanced $K\to\pi\pi\pi$ followed by bremsstrahlung emission. This is similar to $K_S^0\to\pi^+\pi^-\gamma$ for which a measurement of direct emission has yet to be made owing to the heavy bremsstrahlung background. Such bremsstrahlung dominance would make it difficult to observe an asymmetry in $K\to\pi\pi\pi\gamma$.

The decay $K^{\pm} \to \pi^{\pm} e^{+} e^{-}$ can also have possible charge asymmetry. Figure 4 represents contributions to this decay. In Fig. 4(a) there is no strong-interaction phase shift; however 4(b), 4(c), and 4(d) contain a phase shift associated with the ρ meson: $\exp(i\Gamma_\rho/2m_\rho)$. This is obvious for 4(b) and can be seen for 4(c) and 4(d) by relating the matrix elements $\langle K^{+} \mid O_i \mid \pi^{+} e^{+} e^{-} \rangle$ to electromagnetic form factors of the kaon and pion as was done in Ref. 22. However the contribution of 4(a) to the matrix element is only about 0.1 of the others. This is because it arises from a one-loop electromagnetic penguin which is down by a factor $1/\pi^2$. This gives an asymmetry which is roughly

$$0.1 \sin(\Gamma_\rho/2m_\rho) s_2 s_3 \sin\delta \tag{B1}$$

less than $10^{-2} s_2 s_3 \sin\delta$. As the branching ratio for $K^{+} \to \pi^{+} e^{+} e^{-}$ is only 10^{-7} it would be difficult to see such a small asymmetry.

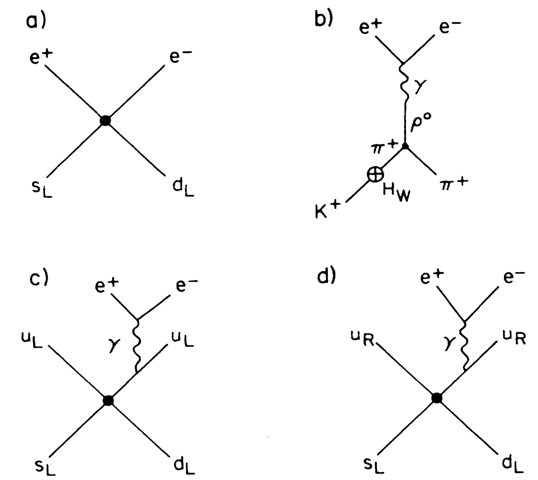

FIG. 4. Contributions to the effective Hamiltonian for $K^{+} \to \pi^{+} e^{+} e^{-}$. (a) represents the insertion of $O_7 = e^2/4\pi \bar{s}\gamma_\mu(1-\gamma_5)d\bar{e}\gamma_\mu e$ at the vertex, (b) represents a long-distance contribution to the decay, (c) the insertion of $O_1, \ldots, O_4$ which gives rise to left-left transitions only, (d) the insertion of O_5, O_6 which can give rise to left-right transitions.

APPENDIX C: TRANSFORMATION PROPERTIES OF MULTIPOLE EXPANSION

Here we discuss certain facts which were largely covered in Ref. 5 but are included here to make our treatment complete.

A. Effect of *CPT*, *CP*, and *T* on the matrix element

Under charge conjugation we have, using the conventions of Sec. II,

$$M^{\pm} = \langle \pi^{\pm}(p_c)\pi^0(p_0)\gamma(q,\epsilon)^{\text{out}} \mid H \mid K^{\pm}(k) \rangle = \langle \pi^{\mp}(p_c)\pi^0(p_0)\gamma(q,-\epsilon)^{\text{out}} \mid C^{-1}HC \mid K^{\mp}(k) \rangle \ . \tag{C1}$$

Now operating with a parity transformation

$$\begin{aligned} CP\colon\ M^{\pm} &= -\langle \pi^{\mp}(p_{cT})\pi^0(p_{0T})\gamma(q_T,-\epsilon_T)^{\text{out}} \mid (CP)^{-1}H(CP) \mid K^{\mp}(k_T) \rangle \\ &= \langle \pi^{\mp}(p_{cT})\pi^0(p_{0T})\gamma(q_T,\epsilon_T)^{\text{out}} \mid (CP)^{-1}H(CP) \mid K^{\mp}(k_T) \rangle \ , \end{aligned} \tag{C2}$$

where $p_{cT} = (-\mathbf{p}_c, p_{c0})$ in accordance with Christ.[5] Now subject the above to a time-reversal transformation keeping in mind that under such a transformation both H and the external states get transformed into their complex conjugates, so the whole matrix element is transformed into its complex conjugate. Also (in) and (out) states are interchanged:

$$CPT\colon\ M^{\pm} = \langle \pi^{\mp}(p_c)\pi^0(p_0)\gamma(q,\epsilon)^{\text{in}} \mid (CPT)^{-1}H(CPT) \mid K^{\mp}(k) \rangle^{*} \tag{C3}$$

going back now if we apply a time-reversal transformation to the states untransformed by CP we find

$$T\colon\ M^{\pm} = \langle \pi^{\pm}(p_{cT})\pi^0(p_{0T})\gamma(q_T,\epsilon_T)^{\text{in}} \mid T^{-1}HT \mid K^{\pm}(k_T) \rangle \ . \tag{C4}$$

B. Multipole expansion of the matrix element

The photon has spin one and as it is massless it has $m_s = \pm 1$. If we choose the quantization axis in the direction of the initial kaon beam then $\mathbf{k}$ is proportional to $\hat{\mathbf{z}}$. If we take the center-of-momentum frame of the two pions then $\mathbf{q} \propto \hat{\mathbf{z}}$, in that case the orbital angular momentum of the photon in the z direction is zero and $M = m_s = \pm 1$ where M is the total angular momentum of the photon in the z direction. As the decay proceeds from a spinless kaon the same statements are true for the orbital angular momentum of the two pions. Note that the final state is not necessarily an eigenstate of J^2. It can however be expressed as a superposition of states with arbitrary J and $M = \pm 1$, as these form a complete set.

The three-body decay $K \to \pi\pi\gamma$ has two independent variables which we take to be $\cos\theta = \hat{\mathbf{p}}_c \cdot \hat{\mathbf{q}}$ and E_c in the c.m. frame of the two pions. The matrix element is a function of $\cos\theta$, E_c, and the photon polarization ϵ_μ. As $M(E_c, \cos\theta, \epsilon) = (\Psi_f(E_c, \cos\theta, \epsilon), H\Psi_i)$. Note that the $\cos\theta$ dependence resides in the final state which can be expanded in terms of

$$P_l^{\pm 1}(\cos\theta) = \sin\theta P_l'(\cos\theta) \tag{C5}$$

which is an eigenstate of l with $m_l = \pm 1$, l the angular momentum of the two pions. This will give us the expansion

$$M(E_c, \cos\theta, \epsilon) = \sum_{l=1}^{\infty} A_l(E_c, \epsilon) P_l^{\pm 1}(\cos\theta) \ . \tag{C6}$$

As a single photon is emitted the $A_l(E_c, \epsilon)$ must be proportional to ϵ_μ. By Lorentz and gauge invariance it must be proportional to $\epsilon_\mu \epsilon^{\sigma\mu}$ where ϵ^σ are defined by

$$\epsilon_\mu \epsilon^{+\mu} = \frac{1}{(k\cdot q)m_K} F_{\mu\nu} p_+^\mu k^\nu$$

and

$$\epsilon_\mu \epsilon^{-\mu} = \frac{1}{m_K{}^3} \tilde{F}_{\mu\nu} p_+^\mu k^\nu \; .$$

This gives us $A_l(E_c,\epsilon) = \alpha_l^\sigma(E_c)\epsilon_\mu \epsilon^{\sigma\mu} e^{i\delta(l)}$ where we have explicitly pulled out the phase shift δ_l. Finally we have

$$M(E_c,\cos\theta,\epsilon) = \sum_{l=1}^{\infty} \alpha_l^\sigma(E_c)\epsilon_\mu \epsilon^{\sigma\mu} P_l^{\pm 1}(\cos\theta) \; . \tag{C7}$$

This expression can be inverted to give $\alpha_l^\sigma(E_c)$ in terms of the matrix element. To do this one has to choose a basis in which the final-state polarization can be expanded, that is, $\epsilon_\mu = a^\sigma \epsilon_{\sigma\mu}$. If we define $N_{\alpha\sigma} = \epsilon_\alpha^\mu \epsilon_\mu^\sigma$ then we have

$$\alpha_l^\sigma(E_c) = (2l+1)/[2l(l+1)] \int_{-1}^{1} d(\cos\theta)(N^{-1})_{\sigma\alpha}(\cos\theta) M(E_c,\cos\theta,\epsilon_\alpha) P_l^{\pm 1}(\cos\theta) e^{-i\delta_l} \; . \tag{C8}$$

For the choice $\epsilon_+ = (\hat{\mathbf{p}}_c - \hat{\mathbf{q}}(\hat{\mathbf{p}}_c\cdot\hat{\mathbf{q}}),0)$ and $\epsilon_- = (\hat{\mathbf{p}}_c \times \hat{\mathbf{q}},0)$ we have $N_{\alpha\sigma} = \delta_{\alpha\sigma}\sin^2\theta/N_\sigma$ where $N_+ = m_K/|p_c|$ and $N_- = m_K{}^3/(2E_c q\,|p_c|)$ in which case

$$\alpha_l^\sigma(E_c) = (2l+1)/[2l(l+1)] N_\sigma \int_{-1}^{1} d(\cos\theta)\sin^{-2}(\theta) M(E_c,\cos\theta,\epsilon_\sigma) P_l^{\pm 1}(\cos\theta) e^{-i\delta_l} \; . \tag{C9}$$

C. Implications of *CP*, *CPT*, and *T* for the multipole moments

We have the expansions

(1) $\langle \pi^\pm(p_c)\pi^0(p_0)\gamma(q,\epsilon)^{\text{out}} | H | K^\pm(k)\rangle$

$$= \sum_{l=1}^{\infty} \alpha_{l\sigma}^{\pm} \epsilon_\mu \epsilon^{\sigma\mu} e^{i\delta_l} P_l^{\pm 1}(\cos\theta) \; , \tag{C10}$$

(2) $\langle \pi^\pm(p_c)\pi^0(p_0)\gamma(q,\epsilon)^{\text{in}} | H | K^\pm(k)\rangle$

$$= \sum_{l=1}^{\infty} \beta_{l\sigma}^{\pm} \epsilon_\mu \epsilon^{\sigma\mu} P_l^{\pm 1}(\cos\theta) e^{-i\delta_l} \; .$$

Note that the relevant S matrix in (1) is $\exp(i\delta_l) = S_l$ while in (2) it is $\exp(-i\delta_l) = S_l{}^{-1}$. Under *CP* we have, using the results of subsection A,

$$CP\!: \; M^{\mp}(p_{cT},p_{0T},q_T,\epsilon_T) = \sum_{l=1}^{\infty} \alpha_{l\sigma}^{\mp} \epsilon_{\mu T} \epsilon^{\mu\sigma T} e^{i\delta_l} P_l^{\pm 1}(\cos\theta) \; , \tag{C11}$$

for $\sigma = +$

$$\begin{aligned} \epsilon_{\mu T}\epsilon^{+\mu T} &= -\boldsymbol{\epsilon}\cdot[-\mathbf{p}_c + \mathbf{k}(p_c\cdot q)/(k\cdot q)] \\ &\quad -\epsilon_0[p_{c0} - k_0(p_c\cdot q)/(k\cdot q)] \\ &= \epsilon_\mu \epsilon^{+\mu} \; , \end{aligned} \tag{C12}$$

for $\sigma = -$

$$\begin{aligned} \epsilon_{\mu T}\epsilon^{-\mu T} &= -\epsilon_i(\epsilon_{i\mu\nu\lambda} p_c^\mu q^\nu k^\lambda) \\ &\quad -\epsilon_0(-\epsilon_{0ijk} p_c^i q^j k^k) = -\epsilon_\mu \epsilon^{-\mu} \; . \end{aligned} \tag{C13}$$

So *CP* invariance implies

$$CP\!: \; \alpha_{l\sigma}^{\pm} = \alpha_{l\sigma}^{\mp} s_\sigma \; ,$$

where $s_\sigma = 1$ for $\sigma = +$ and -1 for $\sigma = -$. (C14)

Likewise for *CPT* invariance we have

$$CPT\!: \; \sum_{l=1}^{\infty} \alpha_{l\sigma}^{\pm} \epsilon_\mu \epsilon^{\sigma\mu} e^{i\delta_l} P_l^{\pm 1}(\cos\theta) = \sum_{l=1}^{\infty} [\beta_{l\sigma}^{\mp} \epsilon_\mu \epsilon^{\sigma\mu} e^{-i\delta_l} P_l^{\pm 1}(\cos\theta)]^* \; ,$$

$$CPT\!: \; \alpha_{l\sigma}^{\pm} = \beta_{l\sigma}^{\mp *} \; , \tag{C15}$$

and, for *T* invariance,

$$T\!: \; \sum_{l=1}^{\infty} \alpha_{l\sigma}^{\pm} \epsilon_\mu \epsilon^{\sigma\mu} e^{i\delta_l} P_l^{\pm 1}(\cos\theta) = \sum_{l=1}^{\infty} [\beta_{l\sigma}^{\pm} \epsilon_{T\mu} \epsilon^{\sigma T\mu} e^{-i\delta_l} P_l^{\pm 1}(\cos\theta)]^* \; ,$$

$$T\!: \; \alpha_{l\sigma}^{\pm} = \beta_{l\sigma}^{\pm *} s_\sigma \; . \tag{C16}$$

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