Expansion of the early Universe and the equation of state

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(Received 26 February 1987)

New advances in high-energy physics concerning the equation of state and the phase transition from quark-gluon plasma to hadronic matter are used to investigate the hydrodynamical expansion of the Universe by solving the equations of hydrodynamics and the Einstein gravitational equations.

The expansion of the Universe after the big bang is described by the equations of hydrodynamics

$$\tilde{T}_{ik}^{;k}=0 \ , \tag{1}$$

the Einstein gravitational equation

$$\tilde{R}_{ik}-\tfrac{1}{2}\tilde{R}g_{ik}=-\kappa\left[T_{ik}+\frac{\Lambda}{\kappa}g_{ik}\right] , \tag{2}$$

and the equation of state

$$p=p(\epsilon) , \tag{3}$$

where $\tilde{T}_{ik}$ is the energy-momentum tensor, $\tilde{R}_{ik}$ the Ricci tensor, and R the curvature scalar defined by $R=R_{ik}g^{ik}$. κ and Λ are the curvature constant and the cosmological constant, respectively; g_{ik} is the space-time metric and p and ϵ are the pressure and energy density.

A fundamental assumption of the cosmological theory is the "cosmological principle" stating that the Universe is homogenous and isotropic which implies that the space-time metrics can be parametrized by the Robertson-Walker formula

$$ds^2=dt^2-R(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta+r^2\sin\theta\, d\theta\right] . \tag{4}$$

By using this metric (1) and (2) can be reduced to

$$R^2-\frac{\kappa\epsilon R^2}{3}+k-\frac{\Lambda}{3}R^2=0 , \tag{5}$$

$$\dot{\epsilon}+3(\epsilon+p)\dot{R}/R=0 . \tag{6}$$

In the early Universe ($\epsilon\sim 1$ GeV/fm^3) Λ and k can be neglected. Combining (5) and (6) we get

$$-\frac{d\epsilon}{3\sqrt{\epsilon}(\epsilon+p)}=\left[\frac{\kappa}{3}\right]^{1/2}dt . \tag{7}$$

In the region of interest where $\epsilon\sim 1$ GeV/fm^3 we can now calculate the time scale which is of the order of $t\sim 10^{-6}$ sec.

Equation (3) reflects at large ϵ microscopic properties of matter and the possible existence of a phase transition from quark-gluon plasma to hadronic matter. So far the influence of this phase transition on the expansion of the Universe has been studied under the following assumptions: (a) The phase transition is of first order; (b) the two phases are described by two different equations of state: the quark-gluon plasma by

$$p_p=\tfrac{1}{3}(\epsilon-4B)=g_p\frac{\pi^2}{90}T^4-B \tag{8}$$

and the hadronic phase by the equation

$$p_h=\tfrac{1}{3}\epsilon=g_h\frac{\pi^2}{90}T^4 . \tag{9}$$

g_p and g_h are statistical factors and B the bag-model constant. Calculations for cases (a) and (b) were done in Refs. 1 and 2.

However assumption (a) has to be qualified. Indeed there are indications[3] from lattice QCD calculations that when fermions are included the phase transition might be of higher (second?) order. Furthermore a more satisfactory theoretical description of the two phases is expected to be given by a single equation of state which contains the phase transition as an output and not as an input as in (b).

Lattice QCD for gluonic systems[4] or a phenomenological approach with a density-dependent quark mass[5] provide such an equation of state:

$$p=c^2(\epsilon)\epsilon , \tag{10}$$

where $c(\epsilon)$ is an effective velocity of sound related to the true velocity of sound c_0 through the equation

$$c_0{}^2=\left.\frac{dp}{d\epsilon}\right|_s=c^2+\epsilon\frac{dc^2}{d\epsilon} , \tag{11}$$

where S is the entropy. As a consequence of Eq. (11) the relation between energy density ϵ and temperature T is not

$$T=T_0\epsilon^{c_0{}^2/(1+c_0{}^2)}$$

but

$$T=T_0\epsilon[1+c^2(\epsilon)]\exp\left[-\int_{\epsilon_0}^{\epsilon}\frac{d\epsilon}{\epsilon[1+c^2(\epsilon)]}\right] . \tag{12}$$

The purpose of this paper is to investigate the expansion of the early Universe under assumptions more general than (a) and (b), in particular, by using an equation of state which at high ϵ describes a quark-gluon phase and at low ϵ a hadronic phase and where the phase transition can be either of first or higher order. An equation of state or a $c_0{}^2(\epsilon)$ which satisfies these requirements is

TABLE I. The parameters for the function $c_0{}^2$ [Eq. (12)].

Model	α	β	γ	δ	Γ
I	$\frac{1}{3}$	0	0	0	0
II	0.295	0.035	0.31	0	0
III	$\frac{5}{21}$	$\frac{2}{21}$	0.31	0	1
IV	$\frac{5}{21}$	$\frac{2}{21}$	0.31	0	0

$$c_0{}^2(\epsilon)=\left[\alpha+\beta\tanh\left[\gamma\ln\frac{\epsilon}{\epsilon_c}+\delta\right]\right]\times\left[1-\frac{\Gamma^2}{\ln^2\frac{\epsilon}{\epsilon_c}+\Gamma^2}\right], \tag{13}$$

where $\alpha,\beta,\gamma,\delta,\Gamma$ are constants defined below. ϵ_c is the critical density (Fig. 1). (This is a generalization of the equation of state considered in Ref. 6.)

By an appropriate choice of the constants $\alpha,\beta,\gamma,\delta,\Gamma$ we are able to study via Eq. (13) the following scenarios (Table I): (I) a radiation-dominated universe (no phase transition); (II) a first-order phase transition which fits the lattice QCD calculations leading to $c^2=\frac{1}{5}$; (III) a first-order phase transition which leads to a hadronic phase with $^5c^2=\frac{1}{6}$; (IV) a second- or higher-order phase transition which leads to a hadronic phase with $c^2=\frac{1}{6}$.

The system of Eqs. (3) and (6) with the equations of states (I), (II), (III), (IV) was solved numerically. The results are compared with those of assumption (b).

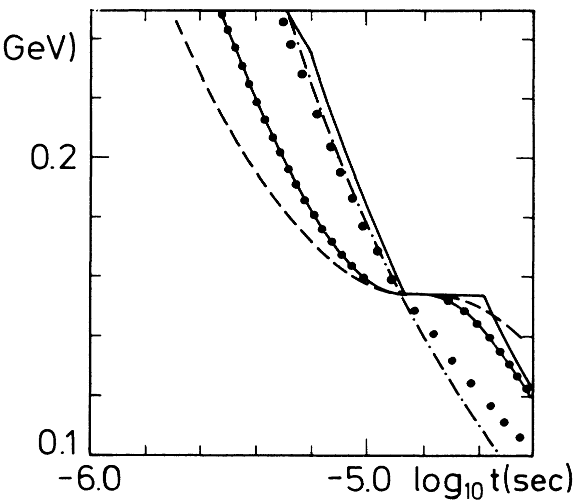

FIG. 2. Temperature vs time for models (I)–(IV). For comparison the results of the bag model [case (b)] are included.

RESULTS

As can be seen in Fig. 2 the time evolution of the temperature is smooth in all cases (II)–(IV) while this does not happen under assumption (b).

If we go back in time the temperature increases slower in cases (II) and (III) than in (IV) and in the bag model. Furthermore in (II) and (III) the time spent in the mixed

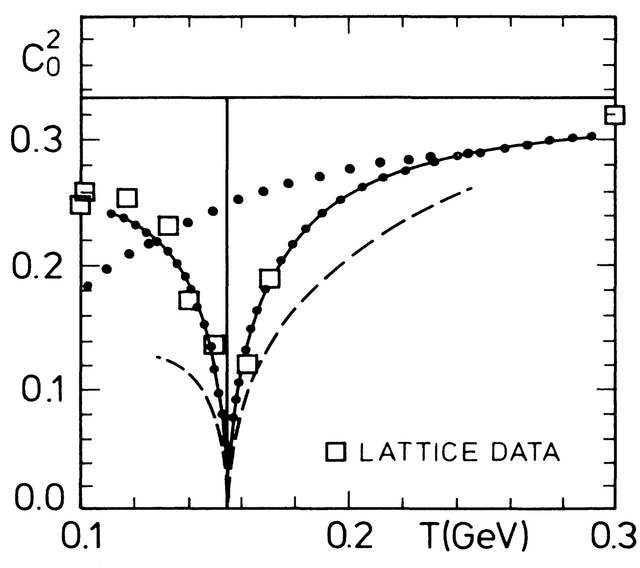

FIG. 1. Input sound velocity vs temperature. The data from QCD lattice calculations are from Ref. 4. For an explanation of symbols see Fig. 3.

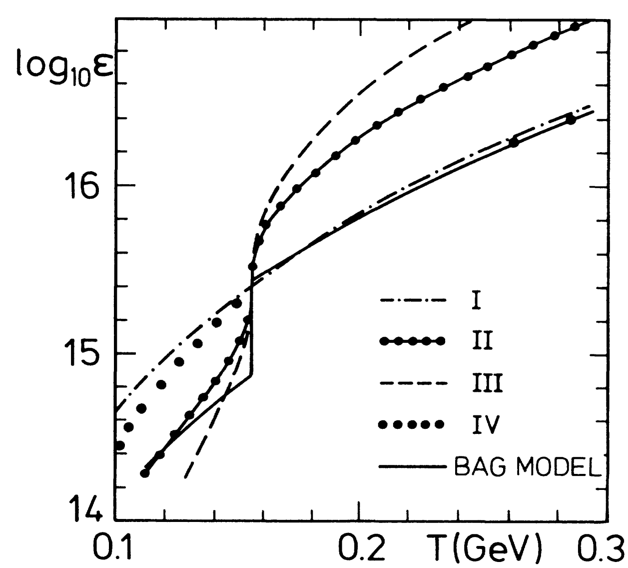

FIG. 3. Energy density in g/cm^3 vs temperature for models (I)–(IV).

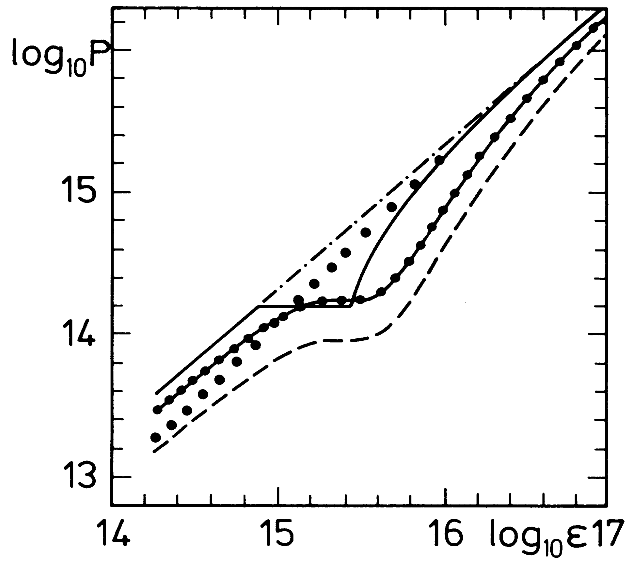

FIG. 4. Pressure in g/cm sec^2 vs energy density. For model (III) the pressure is lower by a factor of 0.5.

phase (quark-gluon plasma and hadrons) is shorter than in the bag model by a factor of ~ 0.58. If structures developed during this period, as assumed by some authors[7,1] (black-hole formation, lumps of quark matter, etc.), then their dimension would be reduced approximately by this factor which was calculated here under more realistic assumptions.

The jump in energy density as a function of temperature (Fig. 3) is in cases (II) and (III) larger by a factor 1.5–2 than in case (b).

A higher-order phase transition (IV) considered here for the first time would be hardly distinguishable from a situation in which no phase transition would have taken place (radiation-dominated universe).

In case (III), where we have a first-order phase transition and a low sound velocity ($c_0{}^2=\frac{1}{6}$) in the hadronic phase, we get a decrease of pressure in the mixed phase by a factor of ~ 0.5 (Fig. 4).

These results are summarized in Table II.

TABLE II. Some results for the models with a first-order phase transition. τ is the time the system remains in the mixed phase, $P(T_c)$ the pressure at the critical temperature T_c.

	II	III	Bag model
$\tau(\mu\text{sec})$	7.2	9.6	12.4
$P(T_c)$ (MeV/fm^3)	157	84	140
ϵ^h/ϵ^p	0.4	0.29	0.19

ACKNOWLEDGMENT

R. W. acknowledges instructive discussions with A. Albrecht and D. Schramm. This work was supported in part by Deutsche Forschungsgemeinschaft.

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