Conformal extension of gauge theories with spontaneously broken symmetry

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By extending O(n) to the conformal group $CO(n)=O(n)\times R^+$ we show that in an O(n) gauge theory with spontaneously broken symmetry the Higgs scalar field may be regarded as a linear approximation of an internal-space conformal factor in a CO(n) covering theory. The conformal factor enters the theory in such a way that it has a natural physical interpretation as a mass field Ω. In the CO(n) theory the masses of the O(n) and R^+ gauge fields depend on the state of the mass field; however, all mass ratios are constant and the O(n) ratios agree with the mass ratios in the standard theory. The Ω field equation reduces to a constraint equation that determines Ω algebraically in terms of the massive gauge fields, and this constraint may be used to eliminate Ω from the Lagrangian. The resulting CO(n) theory describes the usual number of massive and massless O(n) fields together with a new massive gauge-invariant vector field. The gauge fields have quartic self-coupling terms in the Lagrangian, but the masses are now constants. Solutions of the field equations for the CO(n) gauge fields in turn determine the conformal factor Ω, and therefore the massive vector bosons may be viewed as acting so as to produce "ripples" in the internal conformal geometry.

I. INTRODUCTION

In standard formulations of U(n)-type[1] gauge theories it is tacitly assumed that the internal metric on multiplet space is a nondynamical element of the theory.[2] This assumption is implemented when the constant metric associated with the group is used in writing down the gauge-invariant Lagrangian of the theory. Thus, for example, the Higgs sector of the total Lagrangian of an O(n) gauge theory is typically of the form

$$L_H=\tfrac{1}{2}\delta_{ij}D_\mu\Phi^iD^\mu\Phi^j-V(\delta_{ij}\Phi^i\Phi^j)\ , \tag{1}$$

where $i,j=1,2,\ldots,n$. The constant matrix (δ_{ij}) is the standard metric on the multiplet space R^n that is invariant under the gauge group O(n), and $V(\delta_{ij}\Phi^i\Phi^j)$ is typically an O(n)-invariant quartic potential. In this paper we develop some of the simplest consequences of relaxing this tacit assumption with the eventual goal in mind of ascertaining if it would be of physical interest to drop the assumption altogether and let the internal metric become an additional dynamical element of the theory.[2]

The success of gauge theories that employ the Higgs mechanism and spontaneously broken symmetry is well known. However, there appear to be problems with the standard interpretation of the Higgs particle. The standard interpretation fundamentally requires the presence of at least one massive scalar particle in the theory and, for example, in the Glashow-Weinberg-Salam (GWS) theory[3] the Higgs mechanism introduces a massive scalar particle that has not yet been observed. In addition there appear to be other basic problems that arise due to the presence of the Higgs scalar particle, and these problems have led some researchers to attempt to reformulate theories that employ the Higgs mechanism in such a way that the benefits of the mechanism are retained while the disadvantages are removed.[4] In this paper we study this problem from a geometrical point of view. That is to say, we ask if the Higgs scalar field can be realized as some fundamental feature of the geometry[5] of internal space.

As a first approximation to a theory with a dynamic internal metric we will restrict attention in this paper to the case where the internal metric is allowed to vary over the conformal equivalence class containing the standard metric. This special case leads to a theory with the conformal group $CU(n)=U(n)\times R^+$ as gauge group (R^+ is the set of positive multiples of the identity matrix I). We restrict attention to this special case because there are both physical and mathematical reasons for its consideration, and the generalization is simple enough to allow a rather complete description of its implications.

Consider for the moment the general structure of typical O(n) gauge theories. In the physics literature such theories are standardly formulated in local gauges on the spacetime manifold, as is the Lagrangian given in Eq. (1) above. Such formulations assume the existence of an underlying O(n) principal fiber bundle which is the mathematical arena for the theory. One is thus led to consider the question of the uniqueness of O(n) principal fiber bundles.

A principal fiber bundle B with structure group O(n) may be viewed as a subbundle of a GL(n) principal bundle P. If P is mapped onto itself by a bundle automorphism defined by $u\rightarrow ug$ for all $u\in P$, then B is transformed into a new subbundle $B_1=Bg$ with the gauge group $G_1=g^{-1}O(n)g$. Generally, $G_1\neq O(n)$; however, if g is a conformal transformation matrix then $g=cI$, and $G_1=O(n)$. In this case the new subbundle B_1 is also an O(n) principal subbundle of P. Thus O(n) principal bundles are not unique as subbundles of a GL(n) principal bundle.

It is therefore natural to extend O(n) gauge theories to conformal CO(n)=O(n)$\times R^+$ gauge theories. The corresponding principal fiber bundle CB has the conformal group CO(n) as gauge group, and CB may be considered as a conformal equivalence class of O(n) principal bundles. Each of these O(n) subbundles is associated with an internal metric that is conformally related to the standard metric defined by O(n), and thus conformally related to each other. In local gauges on the spacetime manifold each of these internal metrics is the product of the standard metric with a scalar field, the conformal factor. Now if a CO(n) theory can be formulated in such a way that field equations for the conformal factor are a fundamental part of the theory, then the internal metrical geometry will be a dynamical element of the theory. A conformal factor that is a solution of field equations of this type will determine a specific O(n) subbundle, thus resolving the nonuniqueness problem. In Secs. II and III we will show that an O(n) gauge theory with a Higgs potential can be generalized in a natural way to CO(n) theory in which field equations for the conformal factor are an integral part of the theory. The interesting feature that emerges is that the conformal factor enters the theory in such a way that it has a natural physical interpretation as a mass field. These features of the CO(n) model theory will be seen to depend crucially on the nontrivial ground-state structure of the Higgs potential.

We recall that a fundamental role played by the Higgs Lagrangian in spontaneously broken gauge theories is to provide mass terms for the vector bosons. Moreover, it is well known that conformal invariance can be fundamentally related to notions of mass, although standard arguments that link conformal invariance and mass are usually related to external conformal transformations on the metric tensor of spacetime.[6] As indicated above we are concerned in this paper with internal rather than external conformal invariance. In the case of spontaneously broken gauge theories with one-dimensional vacuum symmetry groups we will show that the conformal factor for the internal metric is related to the single component of the Higgs scalar field that survives the reduction of symmetry. As indicated above we will also show that in spontaneously broken gauge theories the internal conformal factor may be viewed as a mass field, thus giving some concrete realization to one aspect of the well-known proposal of conformally invariant dynamics of all microphysical mass fields as recently reemphasized by Bekenstein and Meisels.[7]

The organization of our paper is as follows. In Sec. II we present a conformal extension of the Higgs and Yang-Mills sectors of a standard O(n) gauge theory with spontaneously broken symmetry, with the Higgs potential chosen so that the symmetry group of the ground state is one dimensional. The structure of the theory may be viewed as an internal space version of classical Weyl geometry on a manifold. The extension of O(n) to CO(n) requires the introduction into the Lagrangian of a new R^+ gauge field C_μ and an extra coupling constant g^*. In order to expose the effects of the generalization the Lagrangian is reformulated so as to conform as closely as possible with the Lagrangian of standard theory. In the reformulation the R^+ gauge field and the conformal factor, both of which transform under CO(n) transformations, are eliminated in favor of two CO(n)-invariant fields, a vector field Q (the internal Weyl vector), and a scalar field Ω.

In Sec. III we discuss the physical implications of the theory. The CO(n) theory predicts the usual number of massive and massless O(n) vector bosons, but now the masses depend on the state of the Ω field. However, the ratios of the masses are constants and agree with the ratios of the masses in the standard theory. The new gauge-invariant vector boson Q is also massive, with the mass of Q also involving a factor of Ω. We argue that the properties of the gauge-invariant scalar field Ω suggest that identification of Ω with a particle field is incorrect, and we propose to interpret Ω as a mass field. By a proper choice of the gauge we show that Ω may be identified with the internal conformal factor, and thus the Ω field equations are equations that determine the mathematical arena for the O(n) theory. Moreover, we show that a certain low-energy limit of the CO(n) theory is a pure O(n) theory with the nonvanishing field Ω playing the role of the Higgs scalar field. When Ω is approximated to first order the theory reduces to the standard O(n) model.

In Sec. IV we examine the dynamics of the mass field Ω. By defining a new variable ($S_\mu \equiv Q_\mu - \partial_\mu \ln\Omega$) we show that the Ω field equation reduces to an algebraic equation[8] for Ω in terms of scalar invariants formed from the gauge fields. Treating the Ω field equation as a constraint and substituting it into the Lagrangian completely eliminates the Ω field from the dynamical equations. The resulting Lagrangian describes only vector bosons with quartic self-coupling terms, and the masses of the vector bosons are constants. We show that if the algebraic variable Ω is calculated after having solved the field equations, then the action of the massive vector bosons is to produce "ripples" in the internal metrical geometry. We also discuss in Sec. IV a simple application of the CO(n) theory in which constant classical solutions for Ω induce a classical mass spectrum for the particles in the theory.

In Sec. V we sketch a conformal extension of the GWS gauge theory, and conclusions and suggestions for further work are given in Sec. VI.

II. A MODEL CO(n) GAUGE THEORY

Our objective in this section is set up to a CO(n) conformal extension of a standard O(n) gauge theory, and then to recast the generalized Lagrangian into a form that clearly shows the effects of the generalization. Consider first the Higgs and Yang-Mills sectors of the total Lagrangian for an O(n) gauge theory:

$$L_0 = -\tfrac{1}{4}\,|F_A|^2 + \tfrac{1}{2}\delta_{ij} D_\mu \Phi^i D^\mu \Phi^j - V(\delta_{ij}\Phi^i\Phi^j)\ . \qquad (2)$$

Here $A = A_\mu{}^M (dx^\mu \otimes T_M)$ is the gauge field, $F_A = DA$ is its curvature, Φ^j is the R^n-valued Higgs field, and $D_\mu \Phi^j = \partial_\mu \Phi^j + g A_\mu{}^M (T_M)^j{}_i \Phi^i$. In these expressions greek suffixes refer to flat-spacetime coordinates and the internal-index ranges are $M = 1,2,\ldots,n(n-1)/2$, $i,j = 1,2,\ldots,n$. The T_M are the matrices for the standard basis of the Lie algebra o(n), and g is a coupling constant.

We generalize to a conformal CO(n) gauge theory by introducing the following CO(n)-invariant Lagrangian:

$$L_{CO}=-\tfrac{1}{4}(\,|F_A|^2+|F_C|^2)+\tfrac{1}{2}\sigma_{ij}D_\mu\Phi^i D^\mu\Phi^j-V(\sigma_{ij}\Phi^i\Phi^j)\ . \tag{3}$$

All quantities are now subject to CO(n) gauge transformations, and as we are considering an internal metric (σ_{ij}) on R^n that is conformally related to the standard metric we assume

$$\sigma_{ij}(x)=\beta^{-2}(x)\delta_{ij}\ . \tag{4}$$

The coefficient $\beta^{-2}(x)$ is the conformal factor, where β is a positive-valued function. The extra R^+ degree of freedom requires us to introduce a new gauge field component $C=C_\mu dx^\mu$ and a new coupling constant g^*. In the Lagrangian in (3) above $F_C=dC$ is the curvature component associated with C, and the covariant derivative of the Higgs field Φ^j now takes the form

$$D_\mu\Phi^j=\partial_\mu\Phi^j+gA_\mu{}^M(T_M)^j{}_i\Phi^i+g^*C_\mu\Phi^j\ . \tag{5}$$

Under a CO(n) gauge transformation $x\rightarrow g(x)=[b(x)I]h(x)$ with $b(x)I\in R^+$ and $h(x)=\exp[\xi^M(x)T_M]\in O(n)$, the fields Φ^j, σ_{ij}, $A_\mu{}^M$, and C_μ transform[9] as

$$\Phi\rightarrow\bar{\Phi}\equiv b^{-1}h^{-1}\Phi\ , \tag{6a}$$

$$\sigma\rightarrow\bar{\sigma}\equiv b^2\sigma\ , \tag{6b}$$

$$A\rightarrow\bar{A}\equiv h^{-1}Ah+(1/g)h^{-1}dh\ , \tag{6c}$$

$$C\rightarrow\bar{C}\equiv C+(1/g^*)d[\ln(b)]\ . \tag{6d}$$

Since $\sigma_{ij}=(\beta)^{-2}\delta_{ij}$, Eq. (6b) implies that β transforms according to the rule

$$\beta\rightarrow\bar{\beta}\equiv b^{-1}\beta\ . \tag{7}$$

An important consequence of Eqs. (6) and (7) is that the covariant derivative of the internal metric σ_{ij} can be expressed as

$$D_\mu\sigma_{ij}=(\partial_\mu\beta^{-2}-2g^*C_\mu\beta^{-2})\delta_{ij}\equiv 2Q_\mu\sigma_{ij}\ . \tag{8}$$

The covector field $Q=Q_\mu dx^\mu$ thus has the defining formula

$$Q\equiv -d(\ln\beta)-g^*C\ . \tag{9}$$

It is not difficult to show that Q is invariant under the full conformal group CO(n) and therefore may be considered as a covariant vector field on spacetime. Because of the structure of Eq. (8) we refer to Q as the (generalized) Weyl vector for the internal geometry.[10]

Counting independent degrees of freedom we have $1+n(n-1)/2$ vector fields A^M and C, together with $(n+1)$ scalar fields Φ^j and β. The question of the physical identification of these fields will be dealt with in the next section.

Now the Higgs potential function[11] V supplies the mechanism for the breaking of symmetry, and as in standard theories we take it to be of the form

$$V(z^2)=-\mu^2z^2+\lambda z^4\ . \tag{10}$$

Computing $\partial V/\partial\Phi^j=0$ from this equation with $z^2=\beta^{-2}\delta_{ij}\Phi^i\Phi^j$ we obtain the nontrivial ground-state condition

$$\beta^{-2}(x)\delta_{ij}\Phi^i(x)\Phi^j(x)=\mu^2/2\lambda\equiv v^2/2\ . \tag{11}$$

The conformal factor $\beta^{-2}(x)$ and a field $(\Phi_0)^j$ that minimizes V are thus related by (11). Clearly we may rewrite (11) as

$$\delta_{ij}(\beta^{-1}\Phi_0)^i(\beta^{-1}\Phi_0)^j=v^2/2 \tag{12}$$

and by an O(n) gauge transformation rotate $(\beta^{-1}\Phi_0)^j$ to the form $(0,0,\dots,0,v/\sqrt{2})$. Thus a general vector $(\Phi_0)^j$ can be parametrized as

$$(\Phi_0)^j(x)=\beta(x)\exp[\xi^m(x)T_m]\begin{bmatrix}0\\0\\\vdots\\0\\v/\sqrt{2}\end{bmatrix}\ , \tag{13}$$

where the T_m are the generators of O(n) that do not annihilate the vector $(0,0,\dots,0,v/\sqrt{2})$.

In a standard interpretation the Higgs field Φ^j in the Lagrangian would be a general R^n-valued field, and would be parametrized by a scalar field $\eta(x)$ as

$$\Phi^j(x)=\beta(x)\exp[\xi^m(x)T_m]\begin{bmatrix}0\\0\\\vdots\\0\\\dfrac{v+\eta(x)}{\sqrt{2}}\end{bmatrix}\ . \tag{14}$$

Clearly this parametrized field need not be everywhere nonzero. In the following we will use instead of (14) the parametrization

$$\Phi^j(x)=\tau(x)\exp[\xi^m(x)T_m]\begin{bmatrix}0\\0\\\vdots\\0\\v/\sqrt{2}\end{bmatrix}\ , \tag{15}$$

where we assume that the real-valued function $\tau(x)$ is everywhere positive. We are thus assuming that the Higgs field Φ^j never vanishes, and in particular that any state of this field is CO(n) related to the ground-state vector $(0,0,\dots,0,v/\sqrt{2})$. This assumption will play an important role in our interpretation[12] of the fields in Sec. III.

Working with a Higgs field parametrized in this way we decompose it as $\Phi^j(x)=L(x)\chi^j(x)$ where $L(x)\equiv(\delta_{ij}\Phi^i\Phi^j)^{1/2}\neq 0$ and $\chi^j(x)\equiv\Phi^j(x)/L(x)$. The scalar field $L(x)$ is invariant under O(n) transformations and transforms as

$$L\rightarrow\bar{L}=b^{-1}L \tag{16}$$

under the pure conformal transformation $x\rightarrow b(x)I$, while the δ unit vector χ^j is invariant under pure conformal transformations. The covariant derivative $D\Phi^j$ can now be reexpressed as

$$D_\mu \Phi^j = L D_\mu \chi^j + (D_\mu L)\chi^j$$
$$= L[D_\mu \chi^j + L^{-1}(D_\mu L)\chi^j] \ . \qquad (17)$$

Since χ^j is invariant under pure conformal transformations the covariant derivative $D_\mu \chi^j$ will involve only the O(n) gauge fields.

Equation (17) can be used to decompose the kinetic term for the Higgs field in (3) as

$$\tfrac{1}{2}\sigma_{ij} D_\mu \Phi^i D^\mu \Phi^j = \tfrac{1}{2}(\beta^{-2}) L^2 \delta_{ij}[D_\mu \chi^i + (D_\mu L/L)\chi^i][D^\mu \chi^j + (D^\mu L/L)\chi^j]$$
$$= \tfrac{1}{2}(\beta^{-2}) L^2 [\delta_{ij} D_\mu \chi^i D^\mu \chi^j + L^{-2}(D_\mu L)(D^\mu L)] \ . \qquad (18)$$

In the calculation we have used the fact that $\delta_{ij}\chi^i D_\mu \chi^j = 0$. We introduce the following notation for the quantity $\beta^{-1}L$ that appears in this equation:

$$\Omega \equiv (\beta^{-1} L)/\sqrt{2} \ . \qquad (19)$$

Note that Ω is a positive-valued function. Using Eqs. (7) and (16) it is easy to show that the function Ω, like the Weyl vector Q, is invariant under the full group CO(n) and therefore may be considered as a function on space-time. Ω^2 is in fact one-half the square of the σ length of Φ: $\Omega^2 = \tfrac{1}{2}\sigma(\Phi,\Phi)$.

To simplify further we observe that Eq. (16) implies that $(DL)/L = d(\ln L) + g^* C$. Using Eq. (9) to eliminate C we have the formula

$$(DL)/L = d(\ln \beta^{-1} L) - Q = d(\ln \Omega) - Q \ . \qquad (20)$$

Rewriting Eq. (18) using (19) and (20) and inserting the result into (3) we get the following decomposition of our Lagrangian:

$$L_{\rm CO} = (-\tfrac{1}{4}|F_A|^2 + \Omega^2 \delta_{ij} D_\mu \chi^i D^\mu \chi^j)$$
$$+ \left[-\frac{1}{4g^{*2}} |dQ|^2 + \Omega^2 Q^2 \right]$$
$$+ [(d\Omega)^2 - V(2\Omega^2)] - Q^\mu \partial_\mu (\Omega^2) \ . \qquad (21)$$

In the decomposition we have also rewritten the R^+ curvature component as $F_C = -(1/g^*) dQ$ using Eq. (9).

We may consider this Lagrangian to be composed of the sum of four terms:

$$L_{\rm CO} = L_A + L_Q + L_\Omega + L_{\rm int} \ ,$$

where

$$L_A \equiv -\tfrac{1}{4}|F_A|^2 + \Omega^2 \delta_{ij} D_\mu \chi^i D^\mu \chi^j \ , \qquad (22a)$$

$$L_Q \equiv -\frac{1}{4g^{*2}} |dQ|^2 + \Omega^2 Q^2 \ , \qquad (22b)$$

$$L_\Omega \equiv (d\Omega)^2 - V(2\Omega^2) \ , \qquad (22c)$$

$$L_{\rm int} \equiv -Q^\mu \partial_\mu (\Omega^2) \ . \qquad (22c)$$

The original CO(n)-invariant Lagrangian given in (3) has thus been split into a Lagrangian L_A for the $n(n-1)/2$ O(n) gauge fields, Lagrangians L_Q and L_Ω for the CO(n)-invariant fields Q and Ω, respectively, and an interaction Lagrangian $L_{\rm int}$ that couples the Q and Ω fields. It should be noted that each of these four Lagrangians is individually CO(n) invariant, and that no CO(n) gauge conditions have yet to be imposed.

III. INTERPRETATION OF THE MODEL

We are now in a position to examine the implications and particle content of the theory. The Lagrangian L_A describes the $n(n-1)/2$ O(n) vector bosons, where the second term in (22a) is the mass term for these bosons. L_A is identical with the Lagrangian of standard O(n) theory except for the factor Ω^2 that multiplies the mass term. Thus by an O(n) transformation we may rotate the vector χ^j to the ground-state vector $(0,0,\ldots,0,v/\sqrt{2})$ and conclude that the theory describes $n(n-1)/2-1$ massive vector bosons and 1 massless vector boson. We observe that the *ratios* of the masses of these bosons are constants, and will be identical in both this CO(n) theory and in standard O(n) theory, but in the CO(n) theory the masses will depend on the state of the Ω field.

The remaining three terms in (22) describe the dynamics of the two gauge-invariant fields Q and Ω. Since the interaction Lagrangian $L_{\rm int}$ is composed of CO(n)-invariant quantities it cannot be removed by a gauge transformation, and without further assumptions it must be considered as an essential coupling of these two fields. Before examining the effects of this coupling term in detail we consider a special case that allows the interaction Lagrangian $L_{\rm int}$ to be discarded.

Specifically, suppose that the Weyl vector satisfies the divergence-free condition $\partial_\mu Q^\mu = 0$. Then

$$L_{\rm int} = -Q^\mu \partial_\mu (\Omega^2) = -\partial_\mu (\Omega^2 Q^\mu) + \Omega^2 \partial_\mu Q^\mu$$
$$= -\partial_\mu (\Omega^2 Q^\mu) \ . \qquad (23)$$

In this case $L_{\rm int}$ is a pure divergence and may be discarded in the variational principle. The Lagrangian L_Q now describes a massive gauge-invariant vector boson Q whose mass $\sqrt{2} g^* \Omega$ also depends on the state of the Ω field. Note, however, that the ratios of the mass of the Q field with the masses of the O(n) bosons are independent of Ω. Even when the divergence-free condition is not satisfied it is clear that the mass of Q will involve Ω as a multiplicative factor and that the mass ratios will still be independent of Ω.

Returning to the general case we consider the structure of the Lagrangian L_Ω. Standard interpretation of this Lagrangian would suggest that the Ω field describes a massive scalar particle. However, there are two reasons that suggest that an alternative interpretation of this field is needed. The first is the fact that by construction the Ω field never vanishes, and, as is well known, most physical particle fields do not enjoy this property. Moreover, as we have seen above the Ω field occurs as a multiplicative factor in the masses of the Q and A^M fields. For these reasons we propose to interpret Ω as a *mass field*. It can be linked to the geometrical structure of the theory as follows.

Recall that by definition $\Omega(x)=\beta^{-1}(x)L(x)/\sqrt{2}$. Now although $\Omega(x)$ itself is CO(n) invariant, the factors $\beta^{-1}(x)$ and $L(x)$ are not. Under the pure conformal transformation $x\rightarrow b(x)I\in R^+$ we have $\beta\rightarrow b^{-1}\beta$ and $L\rightarrow b^{-1}L$, so that the product $\beta^{-1}L$ remains unchanged. Since $L(x)\neq 0$ by assumption, we may perform the R^+ gauge transformation $x\rightarrow L(x)I$, and in the new gauge we find

$$\Omega(x)=(\bar{\beta})^{-1}(x)/\sqrt{2}\ .$$

We may thus interpret solutions of the Ω-field equations as defining the conformal factor for the internal metric. As alluded to in the Introduction we have the result that the Ω field, which in standard theory would represent the Higgs scalar particle, in this CO(n) theory serves to define the mathematical arena[13] for the O(n) theory. The internal conformal geometry is thus now a dynamical element of the theory. The physical implications are that the masses of the physical vector bosons in the theory depend on the state of the internal geometry, while the ratios of the masses are constants that are independent of the state of Ω. This theory is a concrete example of a theory that relates a mass field to conformal geometry, but with the new feature that the conformal geometry is on internal space and is not related to spacetime geometry.[14]

We would expect that standard O(n) theory should in some way be a low-energy limit of this CO(n) theory, and by considering a limiting form of the Q field equations we can show this. If standard O(n) theory is to be a low-energy limit of the CO(n) theory, then we would need $m_Q\gg m_A$, or equivalently, $g^*\gg g_M$. Let us consider the case that g^* is so large and Q is sufficiently slowly varying that we can ignore the first term $(-1/4g^{*2})\,|\,dQ\,|^2$ in (22b). The Lagrangian for Q then reduces to $L_Q=\Omega^2Q^2$. The only terms in (22) that involve Q are L_Q and L_{int}, and variation of the sum of these two Lagrangians with respect to Q^μ leads to the field equations

$$2\Omega^2Q_\mu-\partial_\mu(\Omega^2)=0\ . \tag{24}$$

Solving (24) for Q_μ we obtain

$$Q_\mu=\partial_\mu(\ln\Omega)\ . \tag{25}$$

In this limiting case the Weyl vector is a gradient, and by Eq. (9) this condition implies that the R^+ component C_μ of the full CO(n) gauge field is also a gradient, and is thus trivial. When this is the case the CO(n) geometry reduces to O(n) geometry,[10] and thus this limit of the CO(n) theory leads to an O(n) gauge theory.

More explicitly, we show that when Eq. (25) holds there exists an internal metric π_{ij} such that $D\pi_{ij}=0$. Specifically define π_{ij} by

$$\pi_{ij}\equiv\Omega^{-2}\sigma_{ij}\ . \tag{26}$$

We recall that $\Omega(x)$ is a CO(n)-invariant function so that $D\Omega=d\Omega$. Thus

$$\begin{aligned}D\pi_{ij}&=\Omega^{-2}D\sigma_{ij}+\sigma_{ij}d\Omega^{-2}\\&=(2\Omega^{-2}Q+d\Omega^{-2})\sigma_{ij}\ ,\end{aligned} \tag{27}$$

where we have used Eq. (8) to substitute for $D\sigma_{ij}$. Combining Eqs. (25) and (27) we obtain

$$D\pi_{ij}=0\ . \tag{28}$$

This condition implies that the CO(n) connection reduces to the O(n) subbundle[15] defined by π_{ij}.

Finally, we show that on this O(n) subbundle the linear approximation of the field Ω reduces the Lagrangian L_Ω to the standard Lagrangian[16] for the Higgs scalar field $\eta(x)$. We parametrize the positive-valued function Ω as

$$\Omega(x)=(v/\sqrt{2})\exp[\eta(x)/v]\ .$$

Expanding the exponential to first order we obtain

$$\Omega(x)\approx[v+\eta(x)]/\sqrt{2}\ , \tag{29}$$

and substitution of this approximation into the Lagrangian L_Ω leads to

$$L_\Omega=(\partial\eta)^2-V([v+\eta(x)]^2)\ .$$

This Lagrangian is the standard form of the Lagrangian for the scalar Higgs field.[16] We have therefore shown that the Higgs scalar field may be interpreted as a remnant of a "mass-field–conformal factor" in a CO(n) covering theory.

IV. THE Q AND Ω EQUATIONS

By examining the Q and Ω field equations in the CO(n) model we can show that Ω is an algebraic variable[8] in the theory. This result, together with the identification of the linear approximation of Ω with the Higgs scalar field in the O(n) limit, lends further support to our interpretation of the Higgs scalar field as a remnant of a mass field as opposed to a physical particle field.

From the Lagrangian L_{CO} given in Eq. (22) we derive the following field equations for Ω and Q:

$$\Box\Omega+\tfrac{1}{2}\partial V/\partial\Omega=\Omega[Q^2+\partial_\mu Q^\mu+(D\chi)^2]\ , \tag{30}$$

$$\partial_\mu\partial^{[\mu}Q^{\lambda]}=-4g^{*2}\Omega^2(Q^\lambda-\partial^\lambda\ln\Omega)\ . \tag{31}$$

These coupled equations may be simplified by using the identity

$$\partial_\mu[\Omega^2(Q^\mu-\partial^\mu\ln\Omega)]=0 \tag{32}$$

that is implied by Eq. (31). Expansion of (32) and substitution of the result into (30) leads to

$$(\tfrac{1}{2}\Omega)\partial V/\partial\Omega=(Q^\mu-\partial^\mu\ln\Omega)^2+(D\chi)^2\ ,$$

$$\partial_\mu\partial^{[\mu}Q^{\lambda]}=-(4g^{*2}\Omega^2)(Q^\lambda-\partial^\lambda\ln\Omega)\ .$$

If we make the definition $S_\mu\equiv Q_\mu-\partial_\mu\ln\Omega=-(D_\mu L)/L$ [see Eq. (20)] then these last two equations may be rewritten as

$$(\tfrac{1}{2}\Omega)\partial V/\partial\Omega=S^2+(D\chi)^2\ , \tag{33}$$

$$\partial_\mu \partial^{[\mu} S^{\lambda]} = (-4g^{*2})\Omega^2 S^\lambda \ . \tag{34}$$

The form of the field equations shows that the field Ω is an algebraic variable in the theory, since it appears in the field equations undifferentiated [Ω will also enter the remaining O(n) field equations algebraically].

We can solve (33) formally for Ω^2 as

$$\Omega^2 = f(S^2 + (D\chi)^2) \tag{35}$$

for some function f that depends on the explicit form of V

This formula can then be used to eliminate Ω^2 from the remaining field equations, and specific solutions for A^M and S would then in turn determine Ω^2 through Eq. (35).

Let (A^M, S, Ω^2) be a solution of the field equations, with Ω^2 calculated from (35). In the last section we have argued that by a choice of gauge we can make the identification $\Omega^2 = \beta^{-2}/2$. A solution of the field equations thus leads to the formula

$$\sigma_{ij} = 2f(S^2 + (D\chi)^2)\delta_{ij} \ . \tag{36}$$

Recall that when $D\chi$ is evaluated at the vector $(0,0,\ldots,v/\sqrt{2})$, then $(D\chi)^2 = M_{BC} A^B{}_\mu A^{C\mu}$ where M_{BC} is the usual mass matrix with rank $(n-1)$. Equation (36) shows that the massive vector bosons A^M ($M = 1,2,\ldots,n-1$) and S act so as to produce "ripples" in the metric geometry of internal space. For the trivial solutions $S^2 = (D\chi)^2 = 0$ we have $f(S^2 + (D\chi)^2) = \text{const} \neq 0$ and the metric geometry on internal space is everywhere the same.

Now since $S_\mu = Q_\mu - \partial_\mu \ln\Omega$, the limiting case $S = 0$ leads to the reduction of the CO(n) theory to the O(n) theory as discussed in the last section. In this O(n) limit $\sigma_{ij} = 2f((D\chi)^2)\delta_{ij}$ by Eq. (36), and we thus have the result that the internal metric σ_{ij} on internal space is determined by the massive O(n) bosons.

The fact that the conformally invariant field Ω is an algebraic variable in the theory tends to support our interpretation of Ω as a "mass-field–conformal-factor" that does not introduce more scalar particles into the theory. We can show this explicitly by reformulating the original Lagrangian L_{CO} using the field equation (33) as a constraint. When the potential V is the Higgs quartic potential given in (10) the solution of (33) is

$$\Omega^2 = [2\mu^2 + S^2 + (D\chi)^2]/8\lambda \ . \tag{37}$$

When this expression is substituted into L_{CO} given in (22), and Q_μ and Ω are eliminated in favor of $S_\mu \equiv Q_\mu - \partial_\mu \ln\Omega$, we find that the Lagrangian takes the form

$$L_{\text{CO}} = -\tfrac{1}{4}\,|F_A|^2 + (-1/4g^{*2})\,|dS|^2 + [2\mu^2 + S^2 + (D\chi)^2]^2/16\lambda \ . \tag{38}$$

This Lagrangian describes 1 massless and $(n-1)$ massive O(n) vector bosons together with a new massive vector boson S. These bosons have quartic self-coupling terms plus an interaction term that is proportional to $S^2(D\chi)^2$, and the masses are now constants and may be read off the quadratic terms in (38). The Ω field has completely disappeared from the Lagrangian at the price of the introduction of a new massive vector boson S. We can now regard the CO(n) model theory as being described by the Lagrangian (38), together with the formula (37) for computing the conformal factor Ω^2.

As a simple application of the conformal theory we briefly indicate one way in which a classical mass spectrum may be included in a CO(n) gauge theory. We seek classical solutions of Eqs. (33) and (34) for which Ω is constant.[17] When Ω is constant Eq. (33) implies that the quantity $q^2 \equiv S^2 + (D\chi)^2$ is also constant. We now rewrite Eq. (33) as

$$\partial V^{\#}/\partial\Omega = 0, \quad V^{\#} \equiv V - q^2\Omega^2 \ . \tag{39}$$

The constant Ω solutions of (33) are thus given by the extreme values of the effective potential $V^{\#}$.

Up to now we have assumed that the potential V represents the quartic Higgs potential given in Eq. (10). However, V may be more general[11] provided that it defines a nonzero ground state with a one-dimensional symmetry group. Now if we take for V the quartic Higgs potential then (39) has the single solution ($\Omega > 0$) given in Eq. (37). Suppose, however, that V is a potential function such that $V^{\#}$ has the general form shown schematically in Fig. 1.

There will be in this case a finite sequence of solutions of (39). Discarding the unstable equilibrium solutions, we denote the remaining stable equilibrium solutions by Ω_i, $i = 0,1,2,\ldots,N$.

In this multilevel equilibrium situation these constant solutions Ω_i provide a discrete mass spectrum for the vector bosons in the theory. The O(n) vector bosons A^M ($M = 1,2,\ldots,n-1$) are described by mass parameters $(m_M)_i = \Omega_i g$, while the Q boson has mass parameters $(m_Q)_i = \Omega_i g^*$. If other particle fields were included in the theory in an appropriate conformally invariant way, then these constant solutions for Ω would also induce a mass spectrum for these particles.

V. CONFORMAL EXTENSION OF THE GWS THEORY

The model theory described in Sec. II can be modified easily to a conformal extension of the GWS theory of the electroweak interaction. The symmetry group is now

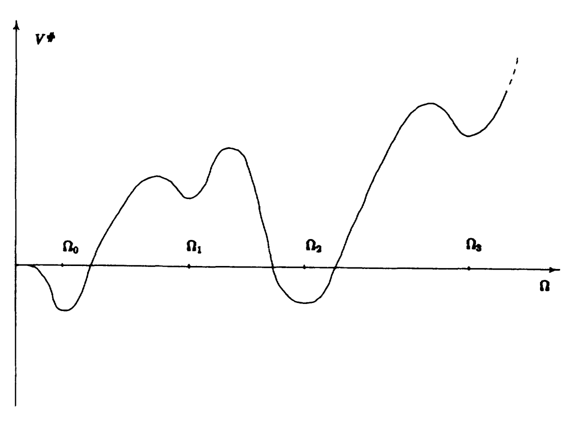

FIG. 1. A prototypical effective potential $V^{\#}$.

$CU(2)=U(2)\times R^+=(SU(2)\times U(1))\times R^+$, and the Lagrangian for the Higgs and Yang-Mills sectors of the total Lagrangian has the form

$$L=-\tfrac{1}{4}\,|F_A|^2+-\tfrac{1}{4}\,|F_C|^2 +\tfrac{1}{2}\sigma_{ij}(D_\mu\Phi^i)(\overline{D^\mu\Phi^j})-V(\sigma_{ij}\Phi^i\overline{\Phi}^j)\ . \tag{40}$$

The notation for the various quantities is parallel to the notation of Sec. II. The nonzero field Φ has values in $\mathbb{C}^2$, and σ is an internal metric on $\mathbb{C}^2$ that is conformally related to the usual metric (δ_{ij}). Thus

$$\sigma_{ij}=\beta^{-2}\delta_{ij} \tag{41}$$

and $\sigma(X,Y)\equiv\beta^{-2}\delta_{ij}X^i\overline{Y}^j$.

The decomposition of the Lagrangian in (40) along the lines described in Sec. II leads to $L=L_A+L_Q+L_\Omega+L_{\rm int}$ where

$$L_A=-\tfrac{1}{4}\,|F_A|^2+\Omega^2\delta_{ij}D_\mu X^i\overline{D^\mu X^j}\ , \tag{42a}$$

$$L_Q=-\frac{1}{4g^{*2}}\,|dQ|^2+\Omega^2Q^2\ , \tag{42b}$$

$$L_\Omega=(d\Omega)^2-V(2\Omega^2)\ , \tag{42c}$$

$$L_{\rm int}=-Q^\mu\partial_\mu(\Omega^2)\ . \tag{42d}$$

The Lagrangian L_A for the $SU(2)\times U(1)$ bosons is identical to the corresponding Lagrangian in GWS theory except for the factor of Ω^2 in the mass term. Analogous to our discussion in Sec. III of the model $CO(n)$ theory we conclude that in this conformal extension of the GWS theory we have (i) three massive vector bosons W^+, W^-, and Z^0, (ii) one massless vector boson A, (iii) a new massive, uncharged vector boson Q^0, and (iv) a mass field Ω.

The ratios of the masses of the $SU(2)\times U(1)$ bosons of the GWS theory will be identical with the ratio of the masses of the corresponding bosons in this extended theory. The new massive vector boson Q^0 is uncharged since it is gauge invariant and therefore does not couple to the $SU(2)\times U(1)$ bosons via a $U(1)$ subgroup of $CU(2)$.

The low-energy limit of this theory will be a pure $SU(2)\times U(1)$ theory that agrees with the standard GWS theory except that the field Ω never vanishes. If the theory is specialized further by approximating Ω to first order as in Eqs. (29), then the resulting theory reproduces the standard GWS theory.

VI. CONCLUSIONS

In this paper we have shown how to set up a conformal extension of $U(n)$-type gauge theories. This generalization of nondynamical internal metric geometry was motivated by the nonuniqueness of $O(n)$ principal bundles, and by the desire to geometrize the Higgs mechanism and to eliminate the Higgs scalar particle from the theory. The fact that the role of the Higgs mechanism is to provide masses for the vector bosons, together with the well-known association of conformal geometry and notions of mass, supplied additional motivation for the consideration of a $CO(n)$ theory.

As shown in Sec. II the Lagrangian L_{CO} of a $CO(n)$ theory, initially containing $n(n-1)/2+1$ gauge fields and $n+1$ scalar fields, decomposes naturally into a sum of four $CO(n)$-invariant Lagrangians $L_{CO}=L_A+L_Q+L_\Omega+L_{\rm int}$. In the reformulation the R^+ gauge fields C_μ and the conformal factor $\beta(x)$, both of which transform under R^+ transformations, were eliminated in favor of two gauge-invariant fields, a vector field Q (the internal Weyl vector), and a nonvanishing scalar field Ω. L_Q and L_Ω are Lagrangians for the Q and Ω fields, respectively, while $L_{\rm int}$ in an interaction Lagrangian for these two fields.

L_A is the Lagrangian for the $n(n-1)/2$ $O(n)$ bosons, and is identical with the Lagrangian of standard theory except that the mass term is now multiplied by the factor $\Omega^2(x)$, so that the masses of these bosons depend on the state of the scalar field $\Omega(x)$. The new gauge-invariant vector boson Q with Lagrangian L_Q also has a factor of Ω^2 in its mass term. However, it was pointed out in Sec. III that the ratios of the masses of all the vector bosons are constants, independent of Ω. For the $O(n)$ bosons the mass ratios in the $CO(n)$ theory are identical with the mass ratios in the standard $O(n)$ theory. These facts, together with the nonvanishing nature of the field $\Omega(x)$, led us to propose that Ω be interpreted as a mass field.

In Sec. III we showed that in a low-energy limit the $CO(n)$ theory reduces to a pure $O(n)$ theory. If the theory is further specialized by approximating the positive-valued function Ω to first order as $\Omega(x)\approx[v+\eta(x)]/\sqrt{2}$, then the theory goes over to the standard $O(n)$ theory upon identifying $\eta(x)$ with the Higgs scalar field. The $O(n)$ theory therefore may be regarded as a low-energy limit of the $CO(n)$ theory, and the scalar Higgs field may be regarded as a remnant of a mass field in a $CO(n)$ covering theory. We also showed that by a proper choice of gauge in the $CO(n)$ theory the Ω field can be identified with the internal conformal factor. Each solution of the Ω-field equations may be viewed as selecting a unique $O(n)$ subbundle, thus providing a resolution of the nonuniqueness problem.

It is well known that in the context of gravitational theory pure conformal structure corresponds to a strictly local pure mass field. Following others, Bekenstein and Meisels[7] have concluded that microphysics should be conformally invariant, and that all rest masses "define a mass field with important physical implications." Bekenstein and Meisels[7] refer to *external* conformal invariance and a corresponding mass field in spacetime. These external concepts can be linked in various ways to certain scalar-tensor gravitational theories and extended gravitational theories with conformal structure.[18] The mass field that we have introduced in the specific theoretical model above relates to *internal* conformal geometry; however it shares some of the properties of external mass fields (e.g., a functional scale transformation law for the masses). This special case might be viewed as providing a deeper explanation of the Bekenstein-Meisels functional scale transformation law for the masses in all conformally invariant microphysical theories. However, we are not proposing here to link the external and internal conformal transformations.[14] The internal conformal formulation discussed above suggests that the geometrized Higgs field is more appropriately thought of as a nonvanishing local mass field that interacts with the other particles in the theory, but which does not actually produce another particle in the theory. However, in the nontrivial case the CO(n) dynamics also produces a new massive vector particle that is naturally coupled to the other particles in the theory by the Higgs-boson mass field.

In Sec. IV we considered the dynamics of the new fields Q and Ω associated with the R^+ gauge freedom. Using a differential identity supplied by the Q-field equation we showed that the Ω-field equation reduced to an algebraic constraint equation that determines the conformal factor in terms of scalar invariants formed from the massive gauge fields. The massive vector bosons in the theory therefore may be considered to act so as to produce "ripples" in the internal conformal geometry. Substitution of the constraint into the Lagrangian eliminated Ω from the dynamical equations, and the resulting Lagrangian described only vector bosons with quartic self-coupling terms and constant mass parameters.

In Sec. V we sketched a conformal extension of the Higgs and Yang-Mills sectors of the GWS theory of the electroweak interaction. The general features of the theory are the same as those of the model CO(n) theory described above. The theory describes the usual number of massive and massless SU(2)$\times$U(1) vector bosons, together with an electrically neutral massive vector boson Q and a mass field Ω. The GWS theory was also shown to be a low-energy limit of the CU(2) theory.

It should be observed that in CU(2) extension of the GWS theory presented in Sec. V, the new vector particle Q couples to the other vector particles only through the mass field Ω. In a more complete CU(2) theory that also includes Lagrangian terms for the leptons, Q would couple to the leptons through the CU(2)-covariant derivatives. We hope to examine this feature of the CU(2) theory in future work.

We have remarked that because of the presence of the mass field in the CU(2) extension of the GWS theory, the masses of the particles depend on the state of the field Ω. If leptons were included in such a theory, then presumably the masses of the leptons would also depend on the state of the mass field Ω. At the end of Sec. IV we described a simple example of a classical mechanism for inducing a mass spectrum for particles in a gauge theory. It is interesting to speculate that this feature of the conformal theory might be related to the similarity of the electron and muon. As is well known these two leptons appear to be identical in all respects except mass. We end with a question. Could it be that the muon is really an electron whose mass is measured in a configuration in which the mass field is not in its ground state?

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[1]By a U(n)-type gauge theory we mean a gauge theory whose group is that subgroup U(n) of GL(n,K) which leaves some positive-definite metric σ on K^n invariant. Here K may be either the real or complex scalar field.

[2]Clearly U(n)-type gauge theories implicitly define a trivial metrical substructure on their internal spaces. L. O'Raifeartaigh, Rep. Prog. Phys. **42**, 159 (1979), pointed out that, up until 1978, gauge theories other than gravitational theories were presumed to have no nontrivial metrical substructure. However, since 1978 a number of authors have considered generalizations. See, for example, K. Cahill, Phys. Rev. D **18**, 2930 (1978); **26**, 1916 (1982); J. Math. Phys. **21**, 2676 (1980); J. E. Kim and A. Zee, Phys. Rev. D **21**, 1939 (1980); R. O. Fulp and L. K. Norris, J. Math Phys. **24**, 1871 (1983); J. Dell, J. L. deLyra, and L. Smolin, Phys. Rev. D **34**, 3012 (1986).

[3]S. Weinberg, Rev. Mod. Phys. **52**, 515 (1980); A. Salam, *ibid.* **52**, 525 (1980); S. L. Glashow, *ibid.* **52**, 539 (1980).

[4]A number of authors have either implicitly or explicitly supported the idea that it would be desirable to eliminate the Higgs field from present theory provided one could retain its effects. In particular, see V. Weisskopf, Phys. Today **34** (No. 11), 69 (1981); J. D. Bekenstein, Found. Phys. **16**, 409 (1986); R. Jackiw, in *Current Algebra and Anomalies*, edited by S. B. Treiman, R. Jackiw, B. Zumino, and E. Witten (Princeton University Press, Princeton, NJ, 1985), p. 236.

[5]This idea of absorbing the Higgs mechanism into the geometrical substructure is similar to the ideas of Bekenstein (see Ref. 4) who linked the Higgs mechanism to the exterior geometry of spacetime rather than to internal geometry as we propose here.

[6]A relationship between conformal structure and mass can be illustrated in a very simple way. In Riemannian geometry a conformal transformation of the metric tensor does not change the conformal curvature tensor, but it does alter the Ricci part of the curvature tensor and hence it alters the Einstein tensor as well. Since mass-energy builds the source terms in Einstein's equations, and since the Einstein tensor is altered by a conformal transformation, we have a fundamental link between spacetime conformal structure and mass.

[7]J. D. Bekenstein and A. Meisels, Phys. Rev. D **22**, 1313 (1980).

[8]In Sec. IV we show that the Ω-field equation reduces to an equation [see Eq. (33)] in which Ω appears undifferentiated. When the potential for the Higgs scalar field is a polynomial then the Ω-field equation turns out to be a polynomial equation for Ω, and in this sense Ω is an algebraic variable in the theory. We will also refer to Ω as an algebraic variable if Ω enters the field equations undifferentiated even when the potential is not a polynomial.

[9]Since $bh=hb$ for all $bI\in R^+$ it follows that the transformation of the pair (A^M,C) to the pair $(\overline{A}^M,\overline{C})$ due to the gauge transformation $x\rightarrow b(x)h(x)$ decouples so that the $\overline{A}^M$ do not depend on b and $\overline{C}$ does not depend on h.

[10]R. O. Fulp and L. K. Norris, J. Math. Phys. **24**, 1871 (1983).

[11]For the purposes of the present paper the potential V may be quite general. We need only that it supply a mechanism for breaking the symmetry to a nontrivial ground state which retains a one-dimensional symmetry group.

[12]We show in Sec. III that our assumption that Φ never vanishes allows us to interpret solutions for the field Ω [defined in Eq. (19)] as solutions for the nonvanishing conformal factor.

[13]The CO(n) theory we have been discussing can be formulated on a subbundle P of the bundle of all frames of internal space. Given the standard inner product δ_p on the vector space of internal state vectors at p, the fiber bundle P consists of all pairs (p,e_j) where p is in Minkowski space and

$\{e_j\}$ is a $\beta^{-2}\delta_P$ orthonormal frame in internal space at p for some β. If we work in a gauge in which $\Omega=\beta^{-1}/\sqrt{2}$ then the corresponding metric is $\sigma=\beta^{-2}\delta=2\Omega^2\delta$. The field equations then determine Ω and the corresponding metric $2\Omega^2\delta$; this metric defines the O(n) arena of our theory: namely, it selects the O(n) sub-bundle of P which consists of $2\Omega^2\delta$ orthonormal frames.

[14]The fact that external conformal geometry has also been utilized in an effort to eliminate the Higgs scalar field (see Bekenstein, Ref. 4) opens the intriguing possibility of developing a theory in which conformally scaled internal metrics are coupled via the conformal factor to conformally scaled external metrics. We defer investigation of such coupled theories to future work.

[15]This is a well-known fact from modern differential geometry. See, for example, Ref. 10, and S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry* (Interscience, New York and London, 1963), Vol. I.

[16]E. S. Abers and B. W. Lee, Phys. Rep. **9C**, 10 (1973).

[17]If Ω were interpreted as a particle field then certain of the constant solutions would have infinite energy and would clearly be unphysical. However, our interpretation of Ω as a mass-field–conformal-factor on internal space does not rule out the constant solutions. In fact, setting "mass field" = const corresponds to selecting "particle units" in Bekenstein and Meisel's work (Ref. 7).

[18]Contrary to the works of Dirac (Ref. 19) and Canuto, Adams, Hsieh, and Tsiang (Ref. 19), which may be linked with certain forms of the Brans-Dicke (Ref. 19) and Jordan (Ref. 19) theories, Bekenstein and Meisels (Ref. 7) have a way of looking at the dynamics of the mass field of conformally invariant microphysics in the context of ordinary general relativity that precludes the notion of a variable gravitational constant. In any case all of these theories can be embraced in extended gravitational theories with a type of torsion field that can be viewed as further geometrizing the mass field, and which also include the notion of a variable gravitational constant (see, for example, S. Hojman, M. Rosenbaum, and M. P. Ryan, Jr., in *Relativity and Gravitation,* Proceedings of the Third Latin American Symposium, edited by S. Hojman, M. Rosenbaum, and M. P. Ryan, Jr. (Universidad Nacional Autonoma de Mexico, 1982). Davis, Baker, and Green (Ref. 20) discuss simple U(4) gravitational theories with a mass field of this type that exhibit coupled conformal-projective invariance.

[19]P. A. M. Dirac, Proc. R. Soc. London **A333**, 403 (1973); **A333**, 439 (1973); V. M. Canuto, P. J. Adams, S. H. Hsieh, and E. Tsiang, Phys. Rev. D **16**, 1643 (1977); C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961); R. H. Dicke, *ibid.* **125**, 2163 (1962); P. Jordan, *Schwerkraft und Weltall* (Vieweg und Sohn, Brunschweig, 1955).

[20]W. R. Davis, W. M. Baker, and L. H. Green, in *Proceeding of the Sir Arthur Eddington Centenary Symposium on Relativity Theory,* edited by Y. Choquet-Bruhat and T. M. Karade (World Scientific, Singapore, 1985), Vol. 2.