Flux-tube model for ultrarelativistic heavy-ion collisions: Electrohydrodynamics of a quark-gluon plasma

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(Received 17 February 1987)

The mechanism of energy deposition and matter formation in the central rapidity region of ultrarelativistic nucleus-nucleus collisions is studied in terms of the flux-tube model. This model assumes that two Lorentz-contracted nuclei are color charged at the instant of collision by a random color exchange. The strong color-electric field confined between the two capacitor plates will immediately begin to polarize the vacuum making $q\bar{q}$ and gluon pairs and the quanta excited in the system may form a rapidly expanding plasma. We examine the transverse evolution of the plasma within the framework of nonviscous relativistic hydrodynamics, incorporating the matter formation from an expanding background color field and also taking into account the interaction of the plasma with the remaining field. The hydrodynamic equations with a source term for the matter, which is due to the pair creation and Joule heating, are derived from a semiclassical transport equation. We solve these hydrodynamic equations coupled to Abelian equations for the expanding background field, and examine the generation of a transverse flow as well as entropy production at the early stage of the matter evolution. It is shown that only a small portion of the initial field energy can be converted into transverse-collective-flow energy of the plasma fluid and transverse-flow energy never becomes significant in comparison with internal thermal excitation energy of the plasma fluid before the hadronization transition sets in. As expected, most of the deposited energy goes into longitudinal motion.

I. INTRODUCTION

Multiparticle production by ultrarelativistic heavy-ion collisions raises a number of interesting questions concerning the mechanism of formation of matter and its evolution.

It is expected[1] that at sufficiently high beam energies the matter is formed as a dense plasma of nonconfined quarks and gluons which eventually evolves into a large number of ordinary hadrons, leptons, and photons. This theoretical conjecture is partially supported by some spectacular cosmic-ray events which were observed at energies above some tens of GeV per nucleon in the pp center-of-mass frame.[2] According to several theoretical analyses,[3,4] these events in fact indicate that the initial energy density of the matter formed in the central rapidity region must be greater than several GeV/fm^3 which is already one order of magnitude larger than the energy density of protons.

What is the dynamical mechanism of such a large energy deposition? Does the deposited energy immediately get thermalized? If the matter is formed initially as a plasma of nonconfined quarks and gluons, how does it hadronize and what are the characteristic observables which give us the evidence for the plasma formation and the properties of such an extreme form of matter?

It is the purpose of this paper to study the mechanism of the energy deposition and the plasma formation in ultrarelativistic nucleus-nucleus collisions using a dynamical model of particle formation and to examine the initial condition of the evolution of matter in the central rapidity region.

Historically, multiparticle production phenomena observed in cosmic-ray events was first analyzed by thermal[5] and hydrodynamical models.[6] These models assume that in the center-of-mass frame all the kinetic energy of the beam is deposited as heat in a small volume of Lorentz-contracted colliding nuclei. This happens at the very instant of collision, making superdense hadronic matter initially at rest. The fluid mechanics provides a well-defined prescription for the calculation of the initial excitation stage by the shock heating and compression, and the results can be used as initial conditions for the later expansion of the system. These types of models have achieved a certain phenomenological success in reproducing the energy dependence of the total multiplicity and its distribution in phase space.[7] However, the assumption of complete stopping at high-energy collisions and almost instantaneous matter formation and thermalization seems very unrealistic and unacceptable on the basis of the Lorentz time-dilatation effect for the particle formation. Furthermore, the observed leading-particle effect *cannot be explained* by this model.[8]

Recently, a new hydrodynamic model for the nucleus-nucleus collisions was proposed by Bjorken[3] and others.[9,10] In this model the initial conditions for the hydrodynamic evolution of the matter produced in the central rapidity region are imposed to be invariant under the Lorentz boost in the beam direction. This symmetry constraint on the initial conditions has been motivated by the apparent formation of the central rapidity plateau in the particle distribution. This suggests that the space-time evolution of the central rapidity region should look almost the same irrespective of the choice of reference frame, provided that it is somewhere between the target and the projectile rest frame but not too close to either of them. Comparing to the original Landau model, this new hydrodynamic model of ultrarelativistic nucleus-nucleus collision is more consistent with the requirement of the uncertainty principle and special relativity. However, it suffers from the lack of a well-defined prescription for *predicting* the initial conditions for the matter evolution. The best one can do so far is to construct the initial conditions by just *superposing* the assumed space-time picture for the particle formation process in elementary *pp* collisions,[9] or by *integrating the hydrodynamic equations backwards* from the final break-up conditions which are constrained by the actual outcome of nucleus-nucleus collisions.[4] It is desirable to obtain a dynamical description of the plasma formation which tells us how to set up the initial conditions and how to relate them to the beam parameters.

The dynamical model for the energy deposition in nucleus-nucleus collisions which we shall study in this paper is a naive extension of the model of multiparticle production (of jets) by e^+e^- annihilations at high energies. The latter is viewed in its center-of-mass frame as being initiated by the conversion of the e^+e^- pair, through a virtual photon, into a $q\bar{q}$ pair by the electromagnetic interaction. Because of color confinement, these fast $q\bar{q}$ moving oppositely may be connected by a tube of confined color flux. This model explains the multiparticle production as a result of an "inside-outside" cascade of $q\bar{q}$ and gluon pair creation in the tube.[11,12] This successive pair (color-dipole) creation generates the spacelike color current (vacuum-polarization current) in the system which would eventually catch up with the quark and antiquark, moving forward and backwards, respectively, and thus neutralize the external quark colors completely. The pair creation in the color flux tube can be considered as a quantum tunneling of $q\bar{q}$ pair in the strong background color field,[13,14] which is the QCD counterpart of the mechanism first studied by Schwinger in QED.[15] It is very important to note that this model of particle production *predicts* that on the average the process is invariant under the Lorentz transformation along the jets axis as long as the boost velocity does not exceed the velocity of the leading quark or antiquark.

It is not a new idea to apply this picture to the multiparticle production phenomenon in hadronic collisions. The color-flux-tube model of hadronic interaction had been first discussed by Low as a model of the bare-Pomeron exchange[16] in the context of the bag model, and independently by Nussinov.[17] This model assumes the exchange of a single soft gluon when the two hadrons collide. Then the color flux tubes are created in between two receding color-octet hadrons. The pair production inside the tube leads to the multiparticle production exactly as in the case of e^+e^- annihilation. The only qualitative difference from $e^+e^- \rightarrow$jets is that in the hadronic interaction the "jet axis" is already fixed by the initial beam direction. Hence this model asserts that the particle formation process is indeed invariant under a Lorentz boost *along the beam direction* in the central rapidity region.

This picture of multiparticle production in hadronic collisions has recently been extended to hadron-nucleus and nucleus-nucleus collisions.[18–23] In such cases the assumption of a single-gluon exchange seems no longer reasonable since a large number of nucleons participate in the collision simultaneously. One may expect that multiple gluons are exchanged when two nuclei overlap and this leads to the formation of a much stronger color field and hence to the more copious particle production afterwards. A simple but most plausible assumption would be that the average number of gluons exchanged per unit area in central nucleus-nucleus collisions is equal to the number of "binary quark-quark collisions" per unit area which is proportional to the product of the two linear dimensions of the colliding nuclei. If the color orientations among these exchanged gluons are uncorrelated, the average strength of the charge on the "capacitor plates" will increase in proportion to the square root of the number of gluons exchanged just as a result of the random walk in the color space. It has been shown recently that some of the consequences of this simple picture are in an acceptable agreement with the currently existing data of high-energy proton-nucleus and nucleus-nucleus collisions.[22]

When one applies this model to central nucleus-nucleus collisions, it is an almost inescapable consequence that quarks and gluons created from the background color field in the deep interior cannot hadronize immediately but instead form an expanding plasma. In such a plasma two types of interaction will be at work. One is collision among the excited quanta: these collisions maintain the system in local thermodynamic equilibrium during the hydrodynamic expansion. The other is the interaction of the plasma with the background field: since the plasma constituents are color charged, the remnant of the background color-electric field will further accelerate the produced particles and induce a *conductive color current* in the system in addition to the vacuum-polarization current accompanying the particle creation. This conductive current will speed up conversion of the field energy into matter (plasma) energy and as a result heat up the plasma faster; this is an analog of *Joule heating*. Hence in order to describe the nucleus-nucleus collision in terms of the flux-tube model we must also deal with the plasma transport problem in the presence of a strong background color field.

In the next section we shall formulate the problem in terms of semiclassical kinetic theory. We extend the relativistic Boltzmann-Vlasov equation to incorporate particle formation from a background field. We shall construct a particle source term in the kinetic equation from an *expanding* background field, and examine the influence on the field equation. This is an extension of the previous work of Kajantie and one of the present authors[23] which is necessary in order to allow the transverse expansion of the system. In this paper we shall consider an Abelian gauge field as in Ref. 23 to focus on the essential new aspect of the problem related to the transverse evolution of the system. Hence the dynamics of the gauge field is governed by Maxwell's equations, and we neglect some novel features which would appear in the kinetic equation in the case of non-Abelian gauge field theories, such as QCD.[24]

In Sec. III we shall take a hydrodynamic limit and reduce our kinetic equations to hydrodynamic equations which couple to Maxwell's equations. In doing this, we include the dissipative effect due to finite electric condition, but neglect the effect of viscosity. The resultant coupled equations described the hydrodynamic evolution of the plasma produced from a strong electric field by the pair creation, which we call *electrohydrodynamics.* The equations of electrohydrodynamics are then reduced to four coupled differential equations imposing cylindrical symmetry for the charge distribution on the nuclear disks. This implies that we consider only head-on collisions with zero impact parameter and neglect random fluctuations in the charge distribution which one expects in a more realistic situation from the random-walk process. At the end of this section, we present solutions for one-dimensional longitudinal expansion neglecting transverse motion.

Numerical solution for full cylindrically symmetric expansion will be constructed in Sec. IV, and we shall examine how much energy is deposited as transverse collective flow energy of the plasma during the formation stage. In the present work, we calculate only free expansion into the vacuum and leave a proper treatment of the effect of confinement and hadronization for future studies. Hence the boundary conditions at the transverse edge of the system is zero pressure. This causes the edge of the system to expand radially at the velocity of light. It is shown, however, that the transverse expansion is significant only near the transverse edge of the system and only a small part of the original field energy can be converted to the collective tranverse-flow energy of the plasma fluid compared to its internal thermal energy. In Sec. V we summarize our results and conclude with some remarks on the many remaining problems.

Throughout this paper we use the units $\hbar=c=k_B=1$.

II. MODEL KINETIC THEORY

In this section we shall derive the basic equations which determine the dynamics of the plasma of massless charged particles being produced in the (nonstatic) Abelian background gauge field. For this purpose, we start with a semiclassical kinetic equation, known as the Boltzmann-Vlasov equation, and then incorporate particle formation due to pair creation by the expanding background field.

A. Extended Boltzmann-Vlasov equation

The semiclassical kinetic theory assumes that the system is composed of well-separated excitations (quasiparticles) which are on their mass shell $[p_0=(p^2+m^2)^{1/2}]$ and that there are no strong many-body correlations among these quanta ("molecular chaos hypothesis"). Under these assumptions, the space-time evolution of the system is described in terms of the one-particle distribution function $f_i(x,p)$; here $f_i(x,p)$ is a Lorentz-scalar function which is defined so that $f_i(x,p)d^3x\,d^3p$ gives the number of particles of species i (spin, color, flavor, particle-antiparticle) in an infinitesimal volume element $d^3x\,d^3p$ in the one-particle phase space at time t.

The one-particle distribution function obeys the Boltzmann equation, which in the absence of the pair creation is expressed as

$$\frac{\partial}{\partial t}f_i+\mathbf{v}\cdot\nabla f_i+\mathbf{F}_i\cdot\frac{\partial}{\partial \mathbf{p}}f_i+\mathbf{F}_i\cdot\mathbf{v}\frac{\partial}{\partial p_0}f_i=\left[\frac{\partial f_i}{\partial t}\right]_{\text{coll}}, \tag{2.1}$$

where $\mathbf{v}=\partial p_0/\partial\mathbf{p}=\mathbf{p}/p_0$. The left-hand side of (2.1) represents the temporal change of the distribution function taking into account particle drift in the phase space under the influence of the external force $\mathbf{F}_i$ acting on the particle of species i, while the right-hand side gives the rate of the sudden change of the distribution function due to collisions. In the usual plasma problem where the external force originates from the Abelian gauge field, $\mathbf{F}_i$ is just the Lorentz force:

$$\mathbf{F}_i=g_i(\mathbf{E}+\mathbf{v}\times\mathbf{B}), \tag{2.2}$$

where g_i is the charge of the ith particle, and $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively. When the background field is determined self-consistently by the distribution of the charged plasma constituents, the transport equation (2.1) with (2.2) is usually referred to as the Boltzmann-Vlasov equation.[25]

The manifestly covariant expression of the Boltzmann-Vlasov equation is obtained from Eq. (2.1) by multiplying both sides by the single-particle energy $p_0=(p^2+m^2)^{1/2}$ which yields

$$p^\mu\partial_\mu f_i-g_ip^\mu F_{\mu\nu}\frac{\partial}{\partial p_\nu}f_i=C_i(x,p), \tag{2.3}$$

where $F^{\mu\nu}$ is the antisymmetric field tensor ($E_i=F^{i0}$ and $B_i=\tilde F^{i0}$ where $\tilde F^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$) and $C_i(x,p)=p_0(\partial f_i/\partial t)_{\text{coll}}$ is the scalar collision integral. On the other hand, the self-consistent background field is governed by Maxwell's equations

$$\partial_\mu F^{\mu\nu}(x)=j^\nu_{\text{tot}}(x),\quad \partial_\mu\tilde F^{\mu\nu}(x)=0, \tag{2.4}$$

where the field source is given by the sum of the external current and the current induced in the plasma:

$$j^\mu_{\text{tot}}(x)=j^\mu_{\text{ext}}(x)+j^\mu_{\text{ind}}(x). \tag{2.5}$$

Here the induced current is just the conduction current which arises due to the convective flow of the charged plasma constituents:

$$\begin{aligned} j^\mu_{\text{cond}}(x)&=\sum_i g_i\int\frac{d^3p}{(2\pi)^3p_0}p^\mu f_i(x,p)\\ &\equiv\sum_i g_i\int d\Gamma\,p^\mu f_i(x,p), \end{aligned} \tag{2.6}$$

where we have introduced an abbreviated notation for the phase-space integral.

Equations (2.3)–(2.6) constitute a closed system which may be solved with arbitrary initial conditions for the one-body distribution function $f_i(x,p)$. In the collisionless limit, the above equations describe the self-organized adiabatic motion of the plasma (plasma oscillation) due to the long-range interaction among the plasma constituents, while in the presence of the collision term, they describe the relaxation of the plasma towards complete thermodynamic equilibrium.

To implement the feature that the particles are continuously produced by pair creation from the background field, we add a particle source term on the left-hand side of the Boltzmann-Vlasov equation:[23]

$$p^{\mu}\partial_{\mu}f_i - g_i p^{\mu}F_{\mu\nu}\frac{\partial}{\partial p_{\nu}}f_i = C_i(x,p) + S_i(x,p) \ . \qquad (2.7)$$

The source term $S_i(x,p)$ gives the phase-space distribution of the particles of species i when they are produced. We shall determine the structure of this particle source term according to the WKB formula of the pair-creation rate.

This modification in the Boltzmann-Vlasov equation must also be accompanied by a change in Maxwell's equations. The successive pair creation causes a time-dependent dipole creation which generates a polarization-current flow. We thus expect that the induced current will acquire an additional element

$$j^{\mu}_{\rm ind} = j^{\mu}_{\rm cond} + j^{\mu}_{\rm pol} \ , \qquad (2.8)$$

where the first term on the right-hand side is the usual conductive current generated by the motion of the plasma constituents as given by (2.6), while the second term is the vacuum-polarization current which accompanies the pair creation process. The structure of the latter current is closely related to the source term added into the Boltzmann-Vlasov equation.

B. Particle source term

We now construct the particle source term in the kinetic equation according to the WKB formula for the differential pair-creation rate in a uniform Abelian background field. The pair creation rate for the ith particle (and its antiparticle) which couples to the uniform electric field E with charge g_i may be given by[13–15]

$$p_i = \pm\frac{|g_iE|}{4\pi^2}\int_0^{\infty} dp_T p_T \ln\left[1\pm\exp\left[-\frac{\pi(p_T{}^2+m_i{}^2)}{|g_iE|}\right]\right] , \qquad (2.9)$$

where the upper sign refers to bosons and the lower sign to fermions. The exponent $\pi(p_T{}^2+m_i{}^2)/|g_iE|$ is just the WKB action for the tunneling of a virtual pair with the transverse momentum p_T through the potential barrier

$$V(z) = 2(p_T{}^2+m^2)^{1/2} - |g_iE|z \ .$$

Upon integration over the transverse momentum p_T of the produced particle, this formula gives

$$p_i = \frac{|g_iE|^2}{8\pi^3}\sum_{n=1}^{\infty}\frac{(\mp 1)^{n+1}}{n^2}\exp\left[-\frac{\pi n m_i{}^2}{|g_iE|}\right] . \qquad (2.10)$$

Note that this formula differs from Schwinger's formula[15] by a factor of 2 since the latter contains the sum over the electron's spin.

In the absence of the transverse expansion of the system, the problem can be treated essentially as a one-dimensional problem where only one component of the electric field appears. In this case, one may utilize the above WKB formula for the pair creation rate in a uniform electric field directly in order to determine the particle source term in the kinetic equation. Unfortunately, the formula (2.9) is not complete for our purpose since it still lacks the information about the longitudinal-momentum distribution of the particles. Although it is implicit in the WKB calculation that a tunneling particle possesses zero longitudinal momentum when they become on-shell, we cannot apply this picture to all pairs produced in a certain specific frame: If we were to do so, then we would immediately violate the Lorentz-boost invariance of the original electric field.

To incorporate the Lorentz-boost invariance, we write the particle source term as

$$S_i = \pm|g_iE(\tau)|\ln\left[1\pm\exp\left[-\frac{\pi p_T{}^2}{|g_iE(\tau)|}\right]\right]\delta(\eta-y) \ , \qquad (2.11)$$

where

$$\tau = (t^2-z^2)^{1/2}, \quad \eta = \tfrac{1}{2}\ln\left[\frac{t+z}{t-z}\right] ,$$
$$y = \tfrac{1}{2}\ln\left[\frac{p_0+p_3}{p_0-p_3}\right] . \qquad (2.12)$$

Here the transverse-momentum distribution has been chosen according to the WKB formula (2.9), while the factor $\delta(\eta-y)$ has been introduced to produce a longitudinal-momentum distribution which does not break the Lorentz-boost symmetry. This is an analog to the Thomas-Fermi approximation and is done by assigning a definite longitudinal momentum to each space-time point. Then the Lorentz-boost invariance requires that if a particle is formed at t and z, it must appear with the longitudinal velocity $v_z = z/t$ (Ref. 26). We have determined the normalization of (2.11) so that $\int d\Gamma\, S_i(x,p)$ coincides with the formula (2.10) for the integrated pair-creation rate.

To generalize the above source term for the case of an expanding background field we note that the expanding field can be generated by a space-time-dependent boost of the constant uniform field. In fact, we can express the general form of the Abelian gauge field as

$$F^{\mu\nu} = \mathcal{E}(s^{\mu}t^{\nu}-s^{\nu}t^{\mu}) - \tfrac{1}{2}\mathcal{B}\epsilon^{\mu\nu\alpha\beta}(s_{\alpha}t_{\beta}-s_{\beta}t_{\alpha}) \ , \qquad (2.13)$$

where the spacelike vector $s^{\mu}(x)$ and the timelike vector $t^{\mu}(x)$ are defined so that they always satisfy $s^2=-1$, $t^2=1$, and $s\cdot t=0$. The Lorentz scalar $\mathcal{E}(x)$ and the pseudoscalar $\mathcal{B}(x)$ are related to the two relativistic invariants constructed from the field tensor by

$$F_{\mu\nu}F^{\mu\nu} = -2(\mathbf{E}^2-\mathbf{B}^2) = -2(\mathcal{E}^2-\mathcal{B}^2) \ ,$$
$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\mathbf{E}\cdot\mathbf{B} = -4\mathcal{E}\mathcal{B} \ . \qquad (2.14)$$

The physical meanings of these quantities become clear if one sets $t^\mu(x)=(1,0,0,0)$, and $s^\mu(x)=(0,0,0,1)$. In this case, the field tensor becomes antidiagonal:

$$F^{\mu\nu}=\begin{bmatrix} 0 & 0 & 0 & -\mathcal{E} \\ 0 & 0 & -\mathcal{B} & 0 \\ 0 & \mathcal{B} & 0 & 0 \\ \mathcal{E} & 0 & 0 & 0 \end{bmatrix} . \tag{2.15}$$

Hence we see that $\mathcal{E}(x)$ denotes the proper elastic field strength and $\mathcal{B}(x)$ the proper magnetic field strength at x, while the vectors $s^\mu(x)$ and $t^\mu(x)$ determine the directions of the field orientation (polarization) and the field propagation at this local point.

In the following we set $\mathcal{B}=0$. This would be the only case relevant to our problem since the external current caused by the longitudinal motion of the nuclear capacitor plates does not contain transverse rotational component because of the Lorentz time-dilatation effect. In this case there is a local moving frame at every space-time point x where the local field tensor becomes purely electric, and in such a frame the formula (2.9) for the differential pair creation rate is valid to within the approximation that we may neglect the gradient of the field. Thus the particle source term for the case of an expanding background field would be given by

$$S_i=\pm\,|g_i\mathcal{E}(x)|\ln\left[1\pm\exp\left[-\frac{\pi(\tilde{p}_T{}^2+m_i{}^2)}{|g_i\mathcal{E}(x)|}\right]\right]\delta(\tilde{\eta}-\tilde{y}) , \tag{2.16}$$

where $\tilde{p}_T$ and $\tilde{y}$ are the transverse momentum and the longitudinal rapidity of the particles, measured in the frame where the field becomes purely electric, and thus are related to the particle momentum in the fixed collision frame by

$$\begin{aligned} \tilde{p}_T{}^2+m_i{}^2&=(p^\mu t_\mu)^2-(p^\mu s_\mu)^2 , \\ \tilde{\eta}&=\tfrac{1}{2}\ln\left[\frac{x^\mu t_\mu-x^\mu s_\mu}{x^\mu t_\mu+x^\mu s_\mu}\right] , \\ \tilde{y}&=\tfrac{1}{2}\ln\left[\frac{p^\mu t_\mu-p^\mu s_\mu}{p^\mu t_\mu+p^\mu s_\mu}\right] . \end{aligned} \tag{2.17}$$

C. Vacuum-polarization current

Having seen a covariant expression for the source term, we now seek the corresponding expression for the vacuum-polarization current. Since t^μ and s^μ are the only four-vectors associated with the background field, we may expect that the vacuum-polarization current $j^\mu_{\rm pol}$ is given as a linear combination of these two vectors:

$$j^\mu_{\rm pol}=j^s_{\rm pol}\,s^\mu+j^t_{\rm pol}\,t^\mu , \tag{2.18}$$

where the two unknown coefficients $j^s_{\rm pol}$ and $j^t_{\rm pol}$ will be determined by the energy-momentum-conservation laws in what follows.

To do this we calculate the first moment of our model kinetic equation (2.7). The left-hand side is transformed as

$$\sum_i\int d\Gamma\,p^\nu\left[p^\mu\partial_\mu f_i-g_i p^\mu F_{\mu\alpha}\frac{\partial}{\partial p_\alpha}f_i\right] =\partial_\mu T^{\mu\nu}_{\rm kin}-F^\nu{}_\mu j^\mu_{\rm cond} , \tag{2.19}$$

where

$$T^{\mu\nu}_{\rm kin}\equiv\sum_i\int d\Gamma\,p^\nu p^\mu f_i \tag{2.20}$$

is the kinetic energy-momentum tensor of the particles and the second term which involves the conductive current $j^\mu_{\rm cond}$ defined by (2.6) is obtained through integration by parts. The moment of the right-hand side is

$$\Sigma^\nu=\sum_i\int d\Gamma\,p^\nu S_i \tag{2.21}$$

because energy-momentum conservation in the collision terms implies $\sum_i\int d\Gamma\,p^\nu C_i=0$. Hence we find

$$\partial_\mu T^{\mu\nu}_{\rm kin}=F^\nu{}_\mu j^\mu_{\rm cond}+\Sigma^\nu . \tag{2.22}$$

It is clear that Σ^ν is an energy-momentum source for the plasma due to quantal pair creation, while $F^\nu{}_\mu j^\mu_{\rm cond}$ represents an extra source due to Joule heating.

On the other hand, from Maxwell's equation we have

$$\begin{aligned} \partial_\mu T^{\mu\nu}_{\rm field}&=-F^\nu{}_\mu j^\mu_{\rm tot} \\ &=-F^\nu{}_\mu j^\mu_{\rm ext}-F^\nu{}_\mu j^\mu_{\rm cond}-F^\nu{}_\mu j^\mu_{\rm pol} , \end{aligned} \tag{2.23}$$

where

$$T^{\mu\nu}_{\rm field}=F^{\mu\alpha}F_\alpha{}^\nu+\tfrac{1}{4}g^{\mu\nu}F^2 \tag{2.24}$$

is the field energy-momentum tensor. Summing up (2.22) and (2.23), we obtain

$$\partial_\mu(T^{\mu\nu}_{\rm kin}+T^{\mu\nu}_{\rm field})=-F^\nu{}_\mu j^\mu_{\rm ext}-F^\nu{}_\mu j^\mu_{\rm pol}+\Sigma^\nu .$$

Since in our problem the external current vanishes in the region where the induced current is nonvanishing, we demand

$$F^\nu{}_\mu j^\mu_{\rm pol}=\Sigma^\nu \tag{2.25}$$

so that the local energy-momentum-conservation law

$$\partial_\mu(T^{\mu\nu}_{\rm kin}+T^{\mu\nu}_{\rm field})=-F^\nu{}_\mu j^\mu_{\rm ext} \tag{2.26}$$

is satisfied.

Inserting (2.13) (with $\mathcal{B}=0$) and (2.18) into (2.25),

$$\mathcal{E}(j^s_{\rm pol}t^\nu+j^t_{\rm pol}s^\nu)=\Sigma^\nu . \tag{2.27}$$

Since t^μ and s^μ are orthogonal to each other, this implies

$$\begin{aligned} j^s_{\rm pol}&=\mathcal{E}^{-1}\Sigma^\nu t_\nu=\mathcal{E}^{-1}\sum_i\int d\Gamma\,p^\nu t_\nu S_i , \\ j^t_{\rm pol}&=\mathcal{E}^{-1}\Sigma^\nu s_\nu=\mathcal{E}^{-1}\sum_i\int d\Gamma\,p^\nu s_\nu S_i . \end{aligned} \tag{2.28}$$

Using the source term (2.16), this leads to

$$j^\mu_{\rm pol}=\kappa\mathcal{E}^{3/2}(\cosh\tilde{\eta}\,s^\mu+\sinh\tilde{\eta}\,t^\mu) , \tag{2.29}$$

where for massless particles

$$\kappa=\frac{1}{16\pi^3}[\gamma_b g_b{}^{5/2}+(1-2^{-3/2})\gamma_f g_f{}^{5/2}]\zeta(\tfrac{5}{2}) \tag{2.30}$$

with the Riemann ζ function given by $\zeta(\frac{5}{2})=1.341$. Here γ_b (γ_f) and g_b (g_f) stand for the degeneracy factor and the coupling constant of bosons (fermions), respectively. It is interesting to note that the vacuum-polarization current is always spacelike: $j_{\mathrm{pol}}{}^2<0$, corresponding to the assumption that the plasma is neutral.

III. ELECTROHYDRODYNAMICS

The kinetic equation (2.7) and Maxwell's equations (2.4) form a closed set which in principle can be solved numerically for given initial conditions. However, in this section we shall cast these equations into a much simplified form by taking the hydrodynamic limit and imposing cylindrical symmetry.

A. The hydrodynamic limit

When the collision time and the mean free path of the plasma constituents are sufficiently short in comparison with the characteristic time and length scale, the distribution function $f_i(x,p)$ will quickly relax to the local equilibrium distribution function

$$f_{\mathrm{eq}}=\frac{1}{\exp(\beta p^\mu u_\mu)\pm 1} , \tag{3.1}$$

where the parameters $\beta(x)=1/T(x)$ and u^μ are the local temperature and the local flow velocity of the fluid, respectively. In (3.1) the upper sign refers to fermions (quarks) and the lower sign to bosons (gluons). The above expression assumes that the system is in complete local thermodynamic equilibrium and locally neutral with respect to any conserved charges such as baryon number or color charge. If these conditions are satisfied, then the bulk evolution of the system can be described in terms of a few collective variables, namely, $T(x)$ and u^μ, which are determined by solving the hydrodynamic equations.

As usual, the hydrodynamic equations can be obtained from (2.22) by doing a near equilibrium expansion for the energy-momentum tensor $T^{\mu\nu}_{\mathrm{kin}}$ and the conductive current j^μ_{cond}. The leading-order term of the energy-momentum tensor is given by that of a perfect fluid

$$T^{\mu\nu}=-Pg^{\mu\nu}+(\epsilon+P)u^\mu u^\nu , \tag{3.2}$$

where P and ϵ are the local pressure and local energy density, respectively. This result can be derived by inserting (3.1) into the distribution function f_i in (2.20). Note that this derivation also leads to the ideal-gas relations for the pressure and the energy density. For the ideal gas of γ_b massless bosons and γ_f massless fermions,

$$P=\tfrac{1}{3}\epsilon=\frac{\pi^2}{90}(\gamma_b+\tfrac{7}{8}\gamma_f)T^4 . \tag{3.3}$$

On the other hand, the use of (3.1) in the formula (2.6) leads to zero conductive current $j^\mu_{\mathrm{cond}}=0$. In order to obtain a nonzero conductive current we must take into account small deviations of the distribution function from (3.1) due to the finite collision time and the finite mean free path.[24,25] In the single relaxation time approximation to the collision integrals (see the Appendix) we find a covariant form of Ohm's law:

$$j^\mu_{\mathrm{cond}}=\sigma_c F^{\mu\nu}u_\nu . \tag{3.4}$$

In the massless limit, the scalar "color" electric conductivity σ_c is given by

$$\sigma_c=\tfrac{1}{18}(2\gamma_b g_b{}^2+\gamma_f g_f{}^2)\tau_c T^2 , \tag{3.5}$$

where τ_c is the relaxation time.

The deviation of the distribution function from (3.1) causes other nonequilibrium transport effects, such as viscosities.[27-30] In this work, however, we neglect such effects and only take into account the effect of the electric conduction.

Now let us consider the following set of equations:

$$\partial_\mu T^{\mu\nu}=F^\nu{}_\mu j^\mu_{\mathrm{ind}} , \tag{3.6}$$

$$\partial_\mu F^{\mu\nu}=j^\nu_{\mathrm{ext}}+j^\nu_{\mathrm{ind}} , \tag{3.7a}$$

$$\partial_\mu \tilde{F}^{\mu\nu}=0 , \tag{3.7b}$$

where $j^\nu_{\mathrm{ind}}=j^\nu_{\mathrm{cond}}+j^\nu_{\mathrm{pol}}$. If we use the hydrodynamic forms (3.2) and (3.4) for the energy-momentum tensor and the conductive current, and (2.29) for the vacuum-polarization current, (3.6), (3.7a), and (3.7b) form a closed set of equations. Equations (3.6) are the hydrodynamic equations which describe the hydrodynamic evolution of the plasma being produced by the pair creation and the Joule heating, while Eqs. (3.7a) and (3.7b) govern the evolution of the background field which is first created by the external current and attenuates gradually due to the current induced by the pair creation and the conductive current (*electrohydrodynamics*).

It is instructive to decompose the hydrodynamic equations (3.6) into the entropy equation and the acceleration equation. The entropy equation is obtained by projecting (3.6) into the direction of the fluid motion given by u_ν,

$$u^\mu\partial_\mu\epsilon+(\epsilon+P)\partial_\mu u^\mu=u_\mu F^\mu{}_\nu j^\nu_{\mathrm{ind}} , \tag{3.8}$$

and then using the thermodynamic relations (at zero chemical potential),

$$d\epsilon=T\,ds,\quad dP=s\,dT,\quad \epsilon+P=Ts , \tag{3.9}$$

where s is the entropy density. This results in

$$T\partial_\mu(su^\mu)=u_\mu F^\mu{}_\nu j^\nu_{\mathrm{ind}} . \tag{3.10}$$

The acceleration equation is derived by subtracting (3.8) multiplied by u^ν from (3.6):

$$-H^{\mu\nu}\partial_\mu P+(\epsilon+P)u^\mu\partial_\mu u^\nu=H^{\mu\nu}F_{\mu\alpha}j^\alpha_{\mathrm{ind}} , \tag{3.11}$$

where $H^{\mu\nu}=g^{\mu\nu}-u^\mu u^\nu$ is the transverse projection operator with respect to the direction of the fluid motion. Using the thermodynamic relations (3.9) this acceleration equation can be rewritten in terms of the temperature as

$$-H^{\mu\nu}\partial_\mu T+Tu^\mu\partial_\mu u^\nu=s^{-1}H^{\mu\nu}F_{\mu\alpha}j^\alpha_{\mathrm{ind}} . \tag{3.12}$$

We see from the entropy equation (3.10) that $u_\mu F^\mu{}_\nu j^\nu_{\mathrm{ind}}/T$ is the entropy production rate per unit volume. On the other hand, the acceleration equation (3.11) implies that $F_{\mu\alpha}j^{\alpha}_{\rm ind}$ plays the same role as the local pressure gradient $\partial_\mu P$, the driving force of the hydrodynamic expansion. It is not difficult to show that $H^{\nu\mu}F_{\mu\alpha}j^{\alpha}_{\rm ind}=0$ when $u^\mu=t^\mu$. Hence this force arises only when there is a relative motion between the background field and the plasma fluid. This is an analog of the friction force. This force causes further acceleration of the plasma into the transverse radial direction if the background field expands faster than the plasma.

B. Cylindrically symmetric expansion

To proceed with the calculation further we assume that the external current is created by the left-moving disk with the surface charge density $\sigma(\rho)$,

$$j^\mu_L=\sigma(\rho)\delta(t+z)(1,0,0,-1)\ , \tag{3.13}$$

and by the right-moving oppositely charged disk,

$$j^\mu_R=-\sigma(\rho)\delta(t-z)(1,0,0,1)\ . \tag{3.14}$$

We have assumed that these disks are moving with the velocity of light and hence infinitesimally thin due to the Lorentz contraction. This choice of the external current ensures that the solution will be invariant to a Lorentz boost in the longitudinal direction and to rotation around the collision axis. We have ignored possible fluctuations in the charge density and thus the expansion is radial and is not accompanied by rotational motion. These are the idealizations of a head-on collision at ultrarelativistic energies.

For scalar quantities, such as the local temperature $T(x)$ and the local proper field strength $\mathcal{E}(x)$, these conditions imply that they are the functions only of the proper time $\tau=(t^2-z^2)^{1/2}$ and the radial coordinate $\rho=(x^2+y^2)^{1/2}$ and do not depend on $\eta=\frac{1}{2}\ln[(t+z)/(t-z)]$ or the azimuthal angle ϕ ($y/x=\tan\phi$).

For the four-fluid velocity we take

$$u^\mu=(\cosh\alpha(\tau,\rho)\cosh\eta,\sinh\alpha(\tau,\rho)\cos\phi\ ,\quad \sinh\alpha(\tau,\rho)\sin\phi,\cosh\alpha(\tau,\rho)\sinh\eta)\ . \tag{3.15}$$

This form gives the scaling relation $v_z=z/t$ for the longitudinal fluid velocity, no rotational flow $v_\phi=0$, and $v_T=\tanh\alpha$ for the transverse radial velocity at $z=0$. We call $\alpha=\alpha(\tau,\rho)$ the transverse rapidity of the plasma fluid. We may take a similar form for t^μ,

$$t^\mu=(\cosh\beta(\tau,\rho)\cosh\eta,\sinh\beta(\tau,\rho)\cos\phi\ ,\quad \sinh\beta(\tau,\rho)\sin\phi,\cosh\beta(\tau,\rho)\sinh\eta)\ , \tag{3.16}$$

with

$$s^\mu=(\sinh\eta,0,0,\cosh\eta)\ , \tag{3.17}$$

which expresses a radially expanding field. Indeed, using (2.13) one finds $E_z=\mathcal{E}\cosh\beta$, $B_\phi=-\mathcal{E}\sinh\beta$, $E_\rho=E_\phi=B_\rho=B_z=0$ at $z=0$. We may call $\beta=\beta(\tau,\rho)$ the transverse rapidity of the expanding background field in the sense that if one observes the field at $z=0$ on the frame moving with the tranverse radial boost velocity $v_B=\tanh\beta$ it looks purely electric. These four-vectors satisfy the following useful relations:

$$\partial_\mu u^\mu=\frac{1}{\tau\rho}\left[\frac{\partial}{\partial\tau}(\tau\rho\cosh\alpha)+\frac{\partial}{\partial\rho}(\tau\rho\sinh\alpha)\right]\ , \tag{3.18a}$$

$$u^\mu\partial_\mu=\cosh\alpha\frac{\partial}{\partial\tau}+\sinh\alpha\frac{\partial}{\partial\rho}\ , \tag{3.18b}$$

$$\partial_\mu t^\mu=\frac{1}{\tau\rho}\left[\frac{\partial}{\partial\tau}(\tau\rho\cos\beta)+\frac{\partial}{\partial\rho}(\tau\rho\sinh\beta)\right]\ , \tag{3.18c}$$

$$t^\mu\partial_\mu=\cosh\beta\frac{\partial}{\partial\tau}+\sinh\beta\frac{\partial}{\partial\rho}\ , \tag{3.18d}$$

$$\partial_\mu s^\mu=0\ , \tag{3.18e}$$

$$s^\mu\partial_\mu=\frac{1}{\tau}\frac{\partial}{\partial\eta}\ , \tag{3.18f}$$

$$x^\mu s_\mu=0\ , \tag{3.18g}$$

where in the last equality

$$x^\mu=(\tau\cosh\eta,\rho\cos\phi,\rho\sin\phi,\tau\sinh\eta)\ .$$

With these symmetry constraints on the solutions, the equations of electrohydrodynamics are greatly simplified. We first note that (3.18g) gives $\tilde{\eta}=0$ so that the vacuum-polarization current (2.29) becomes proportional to s^μ:

$$j^\mu_{\rm pol}=\kappa\mathcal{E}^{3/2}s^\mu\ . \tag{3.19}$$

Also using (3.15)–(3.17) one can show

$$j^\mu_{\rm cond}\equiv\sigma_c F^{\mu\nu}u_\nu=\sigma_c\mathcal{E}\cosh(\alpha-\beta)s^\mu\ . \tag{3.20}$$

Hence the total induced current is written in the form of

$$j^\mu_{\rm ind}=J_{\rm ind}s^\mu\ , \tag{3.21}$$

where

$$J_{\rm ind}=\kappa\mathcal{E}^{3/2}+\sigma_c\mathcal{E}\cosh(\alpha-\beta) \tag{3.22}$$

and the constants κ and σ_c are given by (2.30) and (3.5), respectively.

Using (3.18a) and (3.18b), the entropy equation (3.10) and the temperature equation (3.12) are reduced to

$$\frac{\partial}{\partial\tau}(\tau\rho s\cosh\alpha)+\frac{\partial}{\partial\rho}(\tau\rho s\sinh\alpha)=\frac{\tau\rho\mathcal{E}J_{\rm ind}\cosh(\beta-\alpha)}{T}\ , \tag{3.23}$$

$$\frac{\partial}{\partial\tau}(T\sinh\alpha)+\frac{\partial}{\partial\rho}(T\cosh\alpha)=\frac{\mathcal{E}J_{\rm ind}\sinh(\beta-\alpha)}{s}\ . \tag{3.24}$$

Similarly, Maxwell's equations (3.7) are reduced to two independent equations:

$$\frac{\partial}{\partial\tau}(\mathcal{E}\cosh\beta)+\frac{\partial}{\partial\rho}(\mathcal{E}\sinh\beta)+\frac{1}{\rho}\mathcal{E}\sinh\beta=-J_{\rm ext}-J_{\rm ind}\ , \tag{3.25}$$

$$\frac{\partial}{\partial\tau}(\mathcal{E}\sinh\beta)+\frac{\partial}{\partial\rho}(\mathcal{E}\cosh\beta)+\frac{1}{\tau}\mathcal{E}\sinh\beta=0\ , \tag{3.26}$$

where

$$J_{\rm ext} \equiv -s_\mu j^\mu_{\rm ext} = -\sigma(\rho)\delta(\tau) \ . \tag{3.27}$$

To obtain Eq. (3.25) we have taken a contraction of Eq. (3.7a) with s^μ and then used (3.18c)–(3.18f). Equation (3.26) can be derived from Eq. (3.7b). Recall that $E_z = \mathcal{E}\cosh\beta$ and $E_x = E_y = 0$, $B_\phi = -\mathcal{E}\sinh\beta$ at $z=0$. Hence Eq. (3.25) is just the z component of

$$-\partial\mathbf{E}/\partial t + \nabla\times\mathbf{B} = \mathbf{j}_{\rm ext} + \mathbf{j}_{\rm ind}$$

while Eq. (3.26) is the ϕ component of Faraday's induction law:

$$\nabla\times\mathbf{E} + \partial\mathbf{B}/\partial t = 0 \ .$$

The other two equations in Maxwell's equations are automatically satisfied owing to the symmetry in the solution.

The singular external current (3.27) can be eliminated from the right-hand side of (3.25) by replacing $\mathcal{E}(\tau,\rho)$ with $\mathcal{E}(\tau,\rho)\theta(\tau)$ and then setting the initial condition as $\mathcal{E}(0,\rho) = \sigma(\rho)$ and $\beta(\tau,\rho) = 0$ at $\tau = 0$.

Before presenting the numerical solutions for full cylindrical expansion we shall first examine a one-dimensional longitudinal expansion neglecting the transverse expansion. In this case a simple analytic expression for the solution exists.

C. A one-dimensional expansion

In the absence of the transverse expansion, T and $\mathcal{E}$ depend only on τ and $\alpha = \beta = 0$ for all τ and ρ. In this case Eqs. (3.23)–(3.26) are further reduced to

$$\frac{d}{d\tau}(\tau s) = \frac{\tau\mathcal{E}J_{\rm ind}}{T} \ , \tag{3.28}$$

$$\frac{d}{d\tau}\mathcal{E} = -J_{\rm ind} \ , \tag{3.29}$$

and the induced current becomes

$$J_{\rm ind} = \kappa\mathcal{E}^{3/2} + \sigma_c\mathcal{E} \ . \tag{3.30}$$

If we neglect the electric conduction in the plasma fluid by taking the short collision time limit $\sigma_c \propto \tau_c T^2 \to 0$, the solution becomes particularly simple. In this case the field equation (3.29) decouples from the hydrodynamic equation (3.28) and hence can be integrated first. For the initial condition $\mathcal{E}(0) = E_0$, the solution is given by

$$\mathcal{E} = E_0 f(\tau/\tau_0) \ , \tag{3.31}$$

where $f(x) = (1+x)^{-2}$ and

$$\tau_0 = \frac{2}{\kappa E_0^{1/2}} \tag{3.32}$$

sets the time scale for the attenuation of the field.

The hydrodynamic equation (3.28) can now be integrated using the equation of state:

$$c_s^2 = \frac{dP}{d\epsilon} = \frac{s\,dT}{T\,ds} = \frac{d(\ln T)}{d(\ln s)} \tag{3.33}$$

which gives, for a constant sound velocity c_s,

$$s = a(1+c_s^2)T^{1/c_s^2}, \quad \epsilon = aT^{1+c_s^{-2}} + b \ , \tag{3.34}$$

where a and b are the integration constants. For a massless ideal gas, $c_s^2 = \frac{1}{3}$ and

$$a = \frac{\pi^2}{30}(\gamma_b + \tfrac{7}{8}\gamma_f) \ . \tag{3.35}$$

In this case, with the initial condition $T(\tau=0) = 0$, we find, for the energy density with $b = 0$,

$$\epsilon = \epsilon_0 g(\tau/\tau_0) \ , \tag{3.36}$$

where $\epsilon_0 = \frac{1}{2}E_0^2$ is the initial field energy density and the dimensionless function $g(x)$ is

$$g(x) \equiv 4\int_0^x dy \left[\frac{y}{x}\right]^{4/3} [f(y)]^{5/2} \ . \tag{3.37}$$

This function possesses the following asymptotic behavior:

$$g(x) \sim \begin{cases} \frac{12}{7}x & \text{for } x \ll 1 \ , \\ 4\dfrac{\Gamma(\frac{7}{3})\Gamma(\frac{8}{3})}{\Gamma(5)}x^{-4/3} \sim 0.30x^{-4/3} & \text{for } x \gg 1 \end{cases}$$

and reaches its maximum at $x \sim 0.5$ as shown in Fig. 1. Note that the maximum value of the plasma energy density is only $\sim\frac{1}{5}$ of the initial field energy density. The rest of the energy goes to the longitudinal collective flow energy. On the other hand, the entropy density becomes

$$s(\tau) = a^{1/4}E_0^{3/2}h(\tau/\tau_0) \ , \tag{3.38}$$

where $h(x) = (2^{5/4}/3)[g(x)]^{3/4}$. Since asymptotically $h(x) \propto 1/x$ the entropy per unit rapidity $dS/dy \propto \tau s$ approaches to a constant value.

The simple E_0 dependence which appears in Eqs. (3.31), (3.37), and (3.38) is just the consequence of the fact that E_0 is the only parameter which has a dimension [of (energy)^2 and (length)$^{-2}$].[22] The inclusion of the finite electric conductivity of the plasma fluid may at first glance seem to break this simple behavior since $\sigma_c \propto \tau_c T^2$ introduces another scale into the problem. However, the collision time τ_c may depend on the temperature as $\tau_c \propto T^{-1} \propto \epsilon^{-1/4}$, hence it is ultimately determined by E_0 and the overall E_0 dependence should be preserved. Hence the maximum energy density will grow in proportion to the initial field energy density ϵ_0 and the entropy per unit rapidity in proportion to the initial field intensity E_0. The inclusion of the electric conduction changes the functional form of $f(x)$, $g(x)$, and $h(x)$. In Fig. 1 we plot the numerical result of $f(x)$ and $g(x)$ for several different values of the new dimensionless parameter:

$$\zeta = \frac{\sigma_c}{\kappa\epsilon^{1/4}} \ . \tag{3.39}$$

It is seen that finite electric conduction accelerates the heating process (Joule heating) and the plasma acquires higher energy densities at earlier times. At large x, $f(x)$ attenuates exponentially and $g(x)$ approaches the hydrodynamic behavior faster. Hence the plasma cools down faster due to the longitudinal expansion. Figure 2 shows that $s\tau$ (the entropy per unit rapidity) saturates faster at lower values as the electric conductivity increases.

IV. NUMERICAL SOLUTIONS

We now construct the cylindrically symmetric solution allowing transverse expansion of both the plasma fluid and the background field. The numerical calculation can be most conveniently performed by casting (3.23)–(3.26) in characteristic form:

$$\left[\frac{\partial}{\partial\tau}+\tanh(\alpha\pm y_s)\frac{\partial}{\partial\rho}\right]a_{\pm}=I^{a}_{\pm}(a_{\pm},b_{\pm}) , \tag{4.1}$$

$$\left[\frac{\partial}{\partial\tau}\pm\frac{\partial}{\partial\rho}\right]b_{\pm}=I^{b}_{\pm}(a_{\pm},b_{\pm}) , \tag{4.2}$$

where T, α, $\mathcal{E}$, and β have been transformed to a set of new dimensionless variables, $a_{\pm}$ and $b_{\pm}$, according to

$$a_{\pm}=\left[\frac{T}{T_0}\right]^{1+c_s^{-2}}e^{\pm(c_s+c_s^{-1})\alpha}$$
$$=\left[\frac{s}{s_0}\right]^{1+c_s^{2}}e^{\pm(c_s+c_s^{-1})\alpha} , \tag{4.3}$$

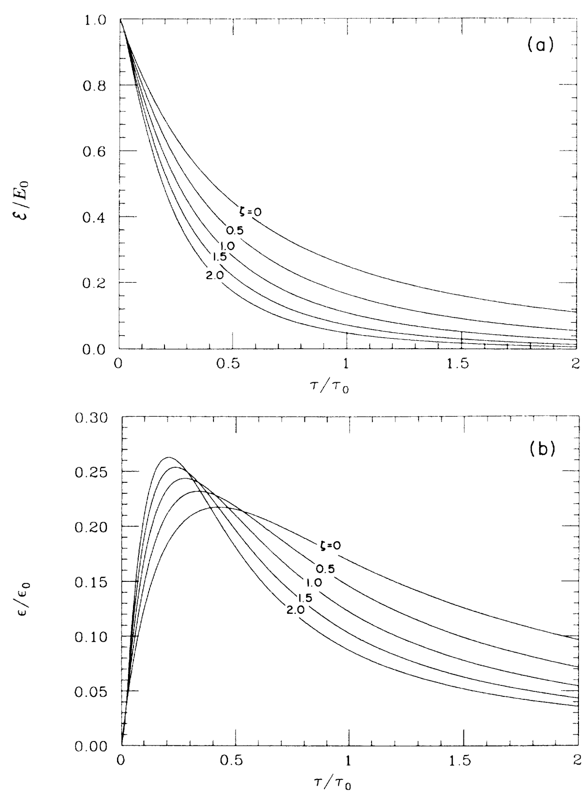

FIG. 1. The time evolution of (a) the field intensity and (b) the proper energy density of the plasma in the one-dimensional longitudinal expansion. The field intensity and the energy density of plasma are measured in the units, the field intensity E_0 and the initial field energy density $\epsilon_0=\frac{1}{2}E_0{}^2$, respectively. Each curve is labeled by the value of ζ defined by Eq. (3.38) which parametrizes the significance of the electric conduction in the plasma.

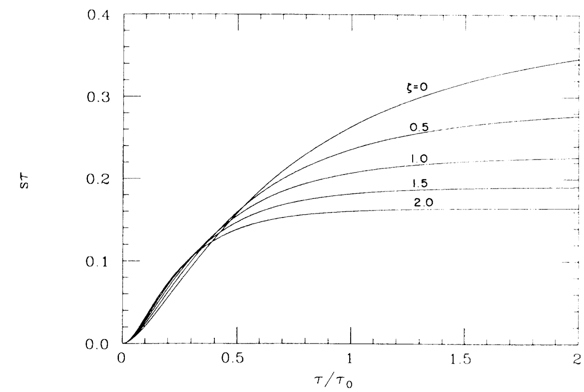

FIG. 2. The time evolution of the entropy density multiplied by the proper times $s\tau$. This quantity is proportional to the entropy per unit rapidity dS/dy. We have set the scale by the unit of $(2^{5/4}/3)a^{1/4}E_0{}^{3/2}\tau_0$. The labeling is the same as in Fig. 1.

$$b_{\pm}=\left[\frac{\mathcal{E}}{E_0}\right]e^{\pm\beta} , \tag{4.4}$$

where T_0, s_0, and E_0 are some arbitrary constants which set the scale for the temperature, the entropy density, and the field strength, and we have used the thermodynamic relation (3.34) for a constant sound velocity $c_s=\tanh y_s$.

The inhomogeneous terms of the characteristic equations are given by

$$I^{a}_{\pm}=-\left[\frac{\sinh\alpha}{\rho}+\frac{\cosh\alpha}{\tau}\right]\left[c_s+\frac{1}{c_s}\right]\frac{\sinh y_s}{\cosh(\alpha\pm y_s)}a_{\pm}$$
$$\pm\frac{J_{\mathrm{ind}}\mathcal{E}}{Ts}\left[c_s+\frac{1}{c_s}\right]\frac{\sinh(\beta-\alpha\pm y_s)}{\cosh(\alpha\pm y_s)}a_{\pm} , \tag{4.5}$$

$$I^{b}_{\pm}=-\frac{1}{2}\left[\frac{1}{\rho}\pm\frac{1}{\tau}\right](b_{+}-b_{-})-\frac{J_{\mathrm{ind}}}{\mathcal{E}}(b_{+}b_{-})^{1/2} , \tag{4.6}$$

respectively, where the first terms have arisen due to the cylindrical geometry of the expansion and the second terms are due to the source in the hydrodynamic equations and the sink in Maxwell's equations. These inhomogeneous terms give the changes of $a_{\pm}$ and $b_{\pm}$ along the characteristic lines $\rho^{a}_{\pm}(\tau)$ and $\rho^{b}_{\pm}(\tau)$ defined by

$$\frac{d\rho^{a}_{\pm}}{d\tau}=\tanh(\alpha\pm y_s) , \tag{4.7}$$

$$\frac{d\rho^{b}_{\pm}}{d\tau}=\pm 1 . \tag{4.8}$$

Clearly, the characteristic lines of the field equations are just straight lines propagating in $\pm\rho$ directions with the velocity of light. The characteristic lines for the fluid motion, however, must be determined by integrating (4.7) self-consistently with (4.1) and (4.2).

In the actual calculation, we have used the variables $a(\tau,\rho)$ and $b(\tau,\rho)$, which are defined on the extended range $-\infty<\rho<\infty$ by

$$a(\tau,\rho)=a_{+}(\tau,\rho) \quad\text{and}\quad b(\tau,\rho)=b_{+}(\tau,\rho) \quad\text{for } 0<\rho ,$$

$$a(\tau,\rho)=a_-(\tau,-\rho) \quad \text{and} \quad b(\tau,\rho)=b_-(\tau,-\rho) \quad \text{for } \rho<0 ,$$

and obey the characteristic equations for a_+ and b_+, respectively, in the whole extended region $-\infty<\rho<\infty$. This procedure automatically incorporates the boundary conditions $\alpha=\beta=0$ at $\rho=0$.

We set the initial conditions at $\tau=0$ as

$$a(0,\rho)=0 \quad \text{and} \quad b(0,\rho)=[1-(\rho/R)^2]^{1/2} , \tag{4.9}$$

so that

$$T(0,\rho)=s(0,\rho)=0, \quad \alpha(0,\rho)=0 , \tag{4.10}$$

and

$$\begin{aligned} &\mathcal{E}(0,\rho)=\sigma(\rho)=E_0[1-(\rho/R)^2]^{1/2} , \\ &\beta(0,\rho)=0 , \end{aligned} \tag{4.11}$$

where R is the radius of the nuclei in a head-on collision. Here we have chosen a smooth initial condition (4.11) for the field, assuming a smooth charge distribution on the disks with the ρ dependence suggested by geometry along with the random walk ansatz for the color charging mechanism which we have discussed in the Introduction. Again we have neglected possible fluctuations in $\sigma(\rho)$. We have set E_0 as the initial field strength at the center. Note that the second condition of (4.11) has been required by the structure of our external source current. On the other hand, the condition $\alpha(0,\rho)=0$ follows from the limiting behavior of the characteristic equations: if one sets $T_0 s_0=E_0{}^2$, then one finds

$$a(\tau,\rho)=\frac{2(c_s{}^2+1)}{c_s{}^2+2}\frac{\tau}{\tau_0} \quad \text{for } \tau\ll\tau_0 . \tag{4.12}$$

We use this behavior to initiate the numerical integration.

Unlike the one-dimensional expansion, solutions now depend also on the ratio $R/\tau_0 \propto A^{1/2}$. Here we present the solution only for a nonconductive ideal-gas plasma ($\zeta=0$ and $c_s{}^2=\frac{1}{3}$) with $R/\tau_0=7$, which corresponds to central U + U collisions if $\tau_0=1$ fm for this case. Owing to the scale invariance of the electrohydrodynamic equations, the same solution, however, also represents, say, a Ca + Ca collision with $\tau_0=0.5$ fm.

In Fig. 3 we plot the profiles of $\mathcal{E}(\tau,\rho)$, $\tanh\beta(\tau,\rho)$, $\epsilon(\tau,\rho)$, and $\tanh\alpha(\tau,\rho)$ at several different early times. As shown in Fig. 3(a), the background field attenuates rapidly in the interior where the initial field strength is high and therefore the shape of the field intensity distribution is being flattened. Although the field lines are initially at rest, the gradient in the field intensity causes a fast expansion

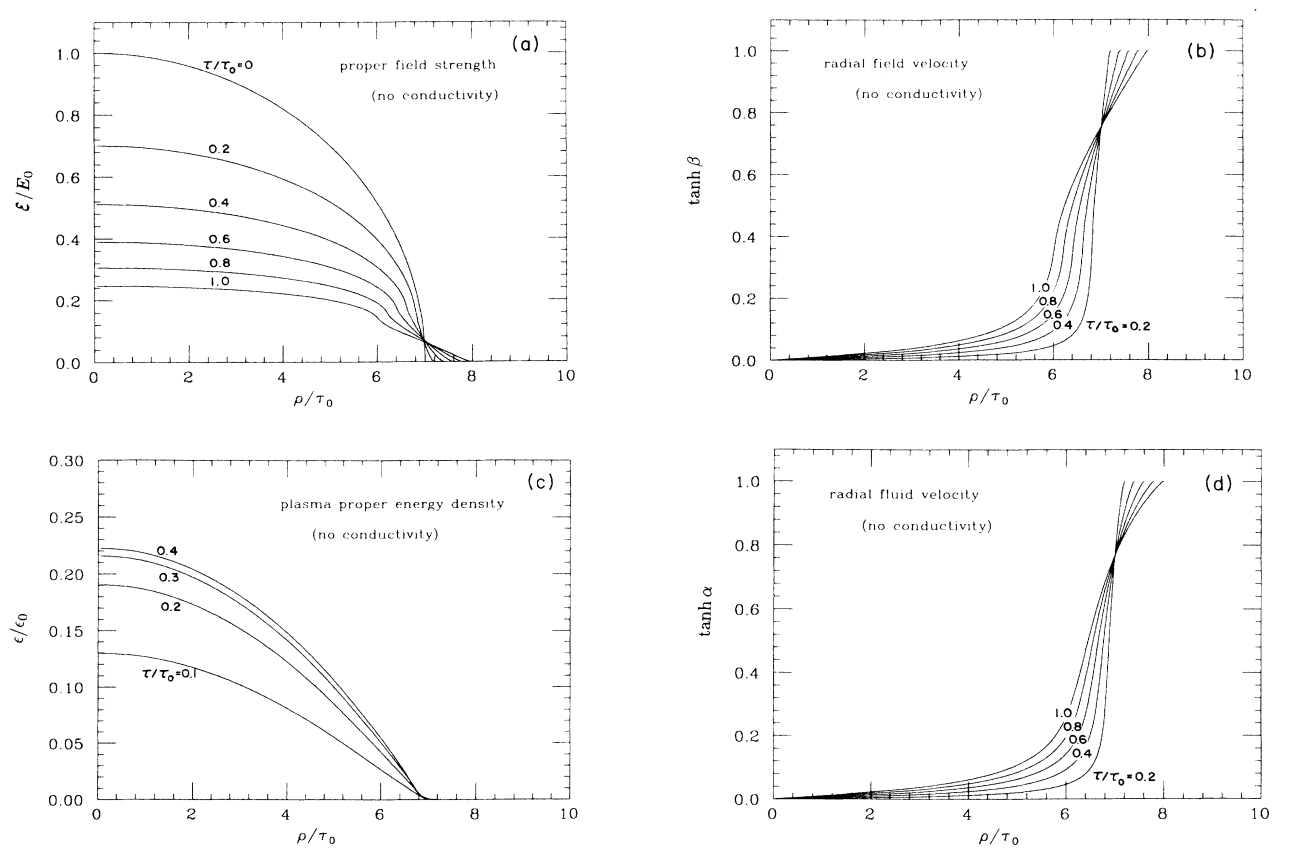

FIG. 3. Profiles of the radial distribution of (a) the proper field strength, (b) the plasma proper energy density, (c) the radial field velocity, and (d) the radial plasma fluid velocity at early times. The number of each curve indicates the proper time τ/τ_0 corresponding to the distribution. The field strength and the plasma energy density are measured in the units of the initial field strength E_0 at $\rho=0$ and the initial field energy density $\epsilon_0=\frac{1}{2}E_0{}^2$ at $\rho=0$, respectively.

of the field near the transverse edge of the cylinder as seen in Fig. 3(b). It is seen that the derivative singularity in the initial field-strength distribution at $\rho=R$ propagates inward at the velocity of light, while the field at the transverse edge expands radially, also at the velocity of light. This behavior, however, may be significantly changed if the surface boundary conditions are properly imposed incorporating the effects of confinement and hadronization which are absent in the present treatment.

In the meantime, the plasma energy density grows rapidly in the interior and reaches its maximum at $\tau=0.4\tau_0$ [see Fig. 3(c)], and then decreases monotonically mainly due to the rapid longitudinal expansion. As seen in Fig. 3(d), the plasma collective transverse flow is gradually built up at the transverse periphery. This is caused partially by the hydrodynamic expansion of the plasma already produced, but also a part of the plasma flow results from the fact that new elements of the plasma are continually being produced from the expanding field. From a comparison of Figs. 3(b) and 3(d), one can see that the field expands only slightly faster than the plasma fluid. This implies that the effect of any "frictional" force between the plasma and the background field is probably not significant.

The growth of the transverse expansion of the system can be better illustrated by plotting the magnetic field strength $|B_\phi|=\mathcal{E}\sinh\beta$, and the radial energy flux of the plasma fluid $T^{0\rho}=(\epsilon+P)\sinh\alpha\cosh\alpha$. This is done in Figs. 4 and 5. It is seen that the magnetic field strength increases only up to one-tenth of the magnitude of the initial electric field strength before it fades away due to pair creation, indicating that field expansion does not play a major role. Similarly, the radial energy flux which the plasma gains during its formation stage is about one order of magnitude smaller than its internal excitation energy. This implies that transverse expansion of the plasma fluid will be mainly generated at a later stage of the hydrodynamic evolution.

To show the large time scale evolution of the plasma fluid, we plot the contour maps of $\epsilon(\tau,\rho)$ and $\tanh\alpha(\tau,\rho)$ on the τ-ρ plane in Figs. 6 and 7, respectively. One can read from this plot, for instance, that the plasma element with $\epsilon>0.1\epsilon_0$ exists only until $\tau/\tau_0=1.8$ in a limited space-time region bounded by the curve marked by 0.1, and there is no significant transverse-flow effect in this region. The region where the fluid has gained radial velocity greater than $0.4c$ exists only inside the very low energy density contour corresponding to $\epsilon=0.02\epsilon_0$.

These contour maps in the τ-ρ plane can be converted to snapshots of the matter profile in the x-z plane at given time t by making use of the relation $\tau=(t^2-z^2)^{1/2}$. Figures 8 and 9 are the resultant snapshots of contour maps for the plasma proper energy density and the radial flow velocity, respectively, at several sequential times. Note that the pattern change of the energy density contours in Fig. 8 is caused mostly by the longitudinal expansion and is not due to the transverse expansion except near the transverse edge. The propagation of the transverse rarefraction wave is more clearly seen in Fig. 9.

We would like to extract quantitative information about the significance of the tranverse expansion. For this purpose we calculate the total amount of energy which has been converted to the collective tranverse flow energy before the fluid elements are diluted below a certain critical energy density ϵ_c. This energy density ϵ_c may be considered as a threshold for the onset of the hadronization transition beyond which our present description of the

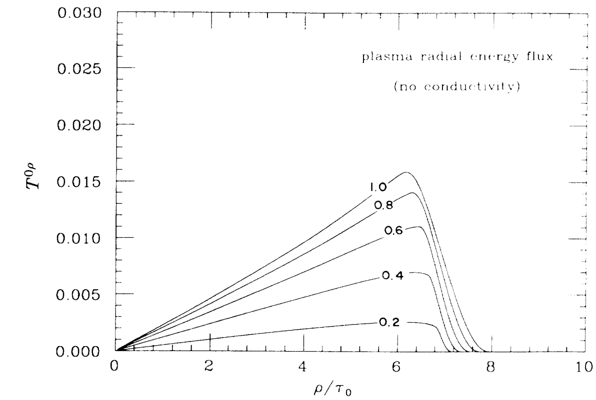

FIG. 5. Profiles of the radial energy flux $T^{0\rho}$ measured in the units of the initial field energy density at $\rho=0$.

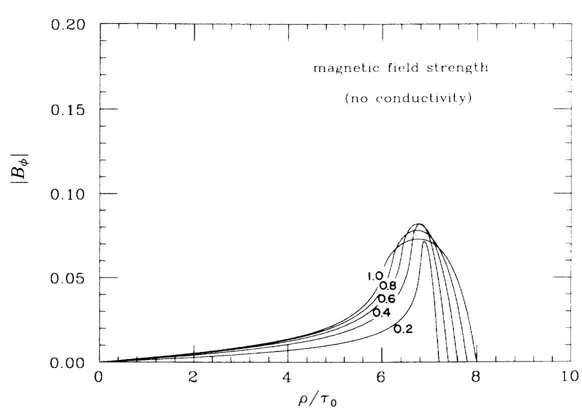

FIG. 4. Profiles of the magnetic field strength $|B_\phi|$ measured in the units of the initial field intensity at $\rho=0$.

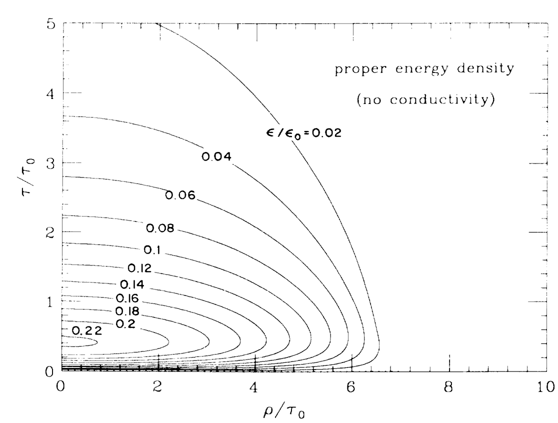

FIG. 6. Isotherms in the τ-ρ plane. Each curve is marked by the corresponding value of ϵ/ϵ_0.

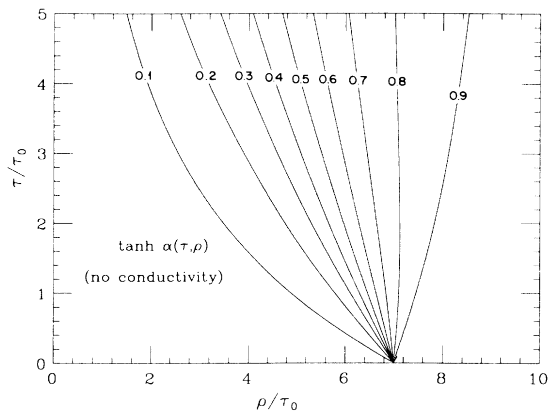

FIG. 7. A contour map of the radial velocity distribution of the plasma in the τ-ρ plane. Each curve is marked by the corresponding value of tanhα.

matter evolution is no longer adequate. The points at which the plasma elements reach this critical energy density form a three-dimensional hypersurface in the Minkowski space given by $\epsilon(x)=\epsilon_c$.

The total energy on this hypersurface is given by

$$E_c=\int_{\epsilon(x)=\epsilon_c} T^{0\mu} d\sigma_\mu \ , \tag{4.13}$$

where $d\sigma_\mu$ is the infinitesimal surface element on the hypersurface. This surface integral can be converted to the four-volume integral by making use of the identity

$$\int_{\epsilon(x)=\epsilon_c} A(x)d\sigma_\mu = \int d^4x \ A(x)\partial_\mu \epsilon(x)\delta(\epsilon(x)-\epsilon_c) \ , \tag{4.14}$$

where $A(x)$ is a function of space and time. Using the cylindrical coordinate system whose four-volume element is given by $d^4x = d\tau\, d\eta\, d\rho\, d\phi\, \tau\rho$, we find that

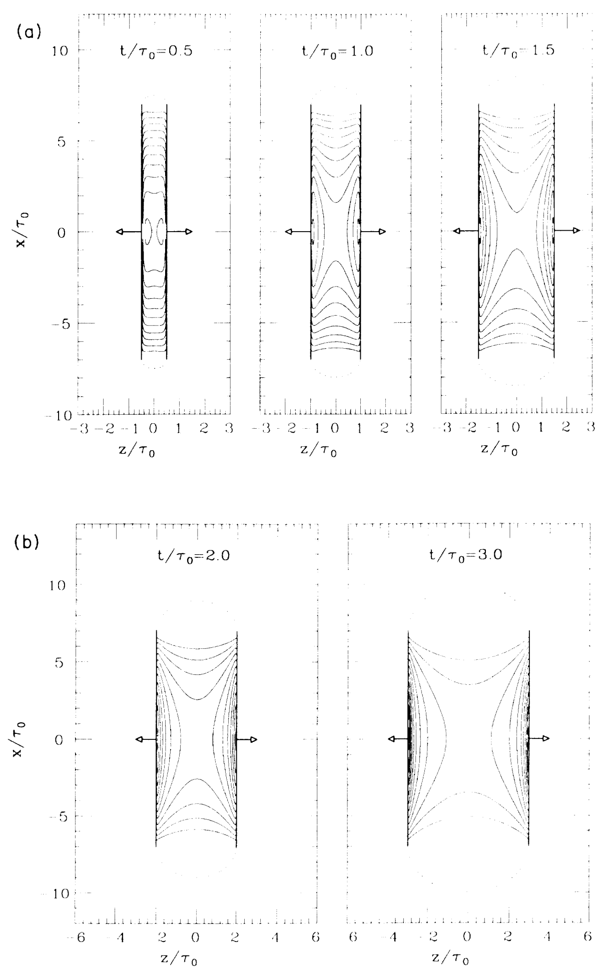

FIG. 8. Snapshots of the isotherms in x-z plane at $y=0$ at several different times. The edge of the plasma where energy density vanishes is drawn by a dotted line. The energy density corresponding to each solid curve increases by $0.02\epsilon_0$ in each step as one goes from the edge to the interior.

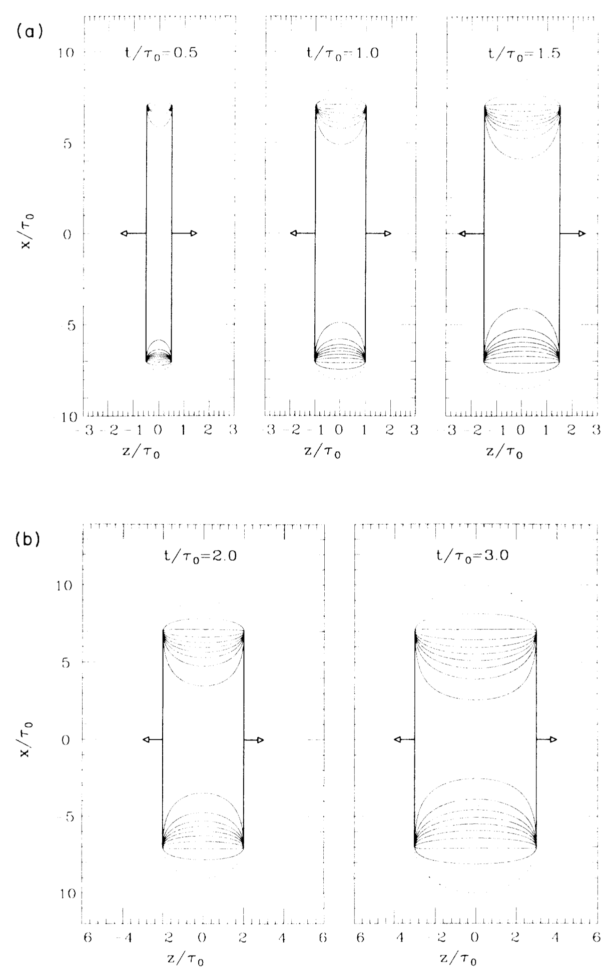

FIG. 9. Snapshots of the radial velocity distribution of the plasma in the x-z plane at $y=0$ at several different times. At the edge drawn by the dotted line the plasma velocity is equal to the velocity of light. The value of tanhα for each curve decreases by 0.1 as one goes from the edge to the interior.

$$E_c = 2\pi \int d\eta [\,(\epsilon \cosh^2\alpha + P \sinh^2\alpha) d\rho + (\epsilon + P)\cosh\alpha \sinh\alpha \, d\tau] \tau\rho \cosh\eta \; . \tag{4.15}$$

Since the longitudinal fluid rapidity y is equal to $\eta = \frac{1}{2}\ln[(t+z)/(t-z)]$, (4.15) implies that the energy per unit fluid rapidity at $y=\eta=0$ is given by

$$\left[\frac{dE_c}{dy}\right]_{y=0} = 2\pi \int d\rho\, \rho\tau(\rho)[\epsilon_c \cosh^2\alpha(\tau(\rho),\rho) + P_c \sinh^2\alpha(\tau(\rho),\rho))] + 2\pi(\epsilon_c + P_c) \int d\tau\, \tau\rho(\tau)\cosh\alpha(\tau,\rho(\tau))\sinh\alpha(\tau,\rho(\tau)) \; , \tag{4.16}$$

where τ and ρ are related by $\epsilon(\tau,\rho)=\epsilon_c$ and P_c is the pressure at $\epsilon=\epsilon_c$. Similarly, the total entropy which comes out of the same hypersurface can be calculated by

$$S_c = \int_{\epsilon(x)=\epsilon_c} s^\mu d\sigma_\mu \tag{4.17}$$

and a straightforward calculation yields

$$\left[\frac{dS_c}{dy}\right]_{y=0} = 2\pi s_c \left[\int d\rho\, \tau(\rho)\rho \cosh\alpha(\tau(\rho),\rho) + \int d\tau\, \tau\rho(\tau)\sinh\alpha(\tau,\rho(\tau)) \right] , \tag{4.18}$$

where s_c is the entropy density at $\epsilon=\epsilon_c$. We define the quantity

$$\mathcal{R} = \frac{(1+c_s{}^2)(dE_c/dy)_{y=0}}{T_c(dS_c/dy)_{y=0}} \; . \tag{4.19}$$

This ratio is 1 in the absence of transverse collective flow, namely, $\alpha=0$, since $\epsilon+P=(1+c_s{}^2)\epsilon=Ts$. Hence the deviation of $\mathcal{R}$ from 1 measures the significance of the collective tranverse flow energy in comparison with the thermal energy of the fluid.

In Fig. 10 we plot $\mathcal{R}$ as a function of the ratio ϵ_0/ϵ_c for several different values of R/τ_0 with no electric conduction ($\zeta=0$). It is seen that the amount of energy converted to transverse-collective-flow energy increases as R/τ_0 decreases or ϵ_0/ϵ_c increases, but it never becomes significant even for a very strong initial field. This implies that most of the deposited energy is converted to the kinetic energy of longitudinal motion of the plasma by the fast scaling expansion. The same plot is made in Fig. 11 for fixed R/τ_0 with varying ζ. The inclusion of the finite conductive current further suppresses the generation of transverse flow. This is so because the electric conduction shortens the plasma formation time which results in the enhancement of the cooling rate due to longitudinal hydrodynamic expansion.

Although we still lack a description of the hadronization and freeze-out stage of the matter evolution, this result has a significant implication for the observations. We note that dS/dy is always a monotonically increasing function of time and will eventually be reflected in the multiplicity of secondary hadrons, mostly pions and kaons: If we use the massless ideal Bose gas formulas to relate the particle multiplicity to the fluid entropy when the number of particles in the fluid is frozen, then

$$\left[\frac{dS_c}{dy}\right]_{y=0} \le \left[\frac{dS_f}{dy}\right]_{y=0} \sim 4\frac{dN}{dy} \; . \tag{4.20}$$

On the other hand, dE/dy will most likely be a monotonically decreasing function of time since the change of dE/dy is caused by the work done by fluid pressure while

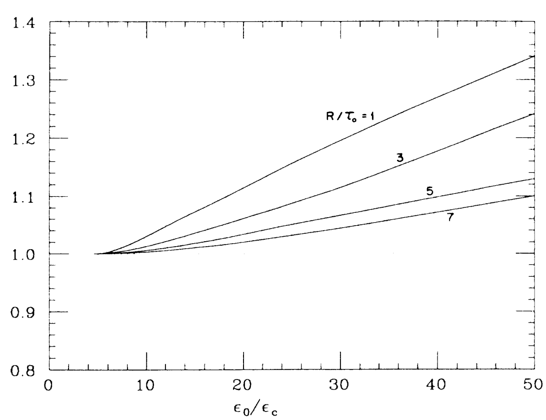

FIG. 10. The ratio $\mathcal{R}$ defined by Eq. (4.19) as a function of ϵ/ϵ_0 for a nonconductive plasma ($\zeta=0$). Each curve is marked by the corresponding value of R/τ_0.

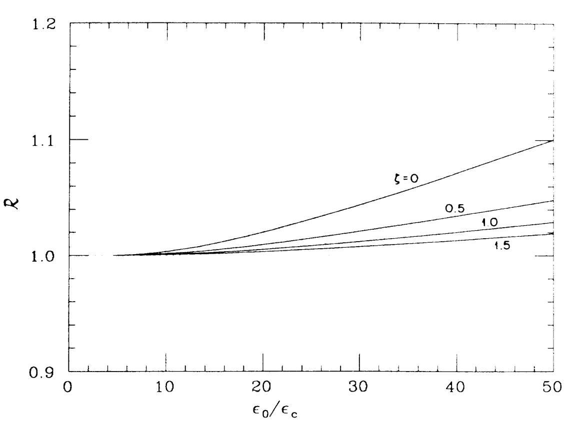

FIG. 11. The ratio $\mathcal{R}$ defined by Eq. (4.19) as a function of ϵ/ϵ_0 for $R/\tau_0=7$. Each curve is marked by the corresponding value of ζ defined by (3.39).

the system undergoes longitudinal hydrodynamic expansion and it increases only when the fluid pressure becomes negative by a significant supercooling:

$$\left[\frac{dE_c}{dy}\right]_{y=0} \geq \left[\frac{dE_f}{dy}\right]_{y=0} . \tag{4.21}$$

It then follows that the above ratio $\mathcal{R}$ gives the upper bound for the average transverse energy of hadrons created in the central rapidity region:

$$\begin{aligned}\langle E_T \rangle \equiv \frac{(dE_f/dy)_{y=0}}{(dN/dy)_{y=0}} &\leq 4\frac{(dE_c/dy)_{y=0}}{(dS_c/dy)_{y=0}} \\ &= \frac{4T_c}{1+c_s{}^2}\mathcal{R} = 600\mathcal{R}\ \mathrm{MeV} ,\end{aligned} \tag{4.22}$$

where in the last equality we have set $T_c = 200$ MeV.

V. CONCLUSION

In this paper we have presented a model for energy deposition and plasma formation in ultrarelativistic nucleus-nucleus collisions. We have assumed that at relativistic collider energies nuclear collisions produce an intermediate giant color flux tube by random color exchange between the colliding nuclei. The strong color-electric field midway between the two receding nuclear "capacitor" plates will immediately begin to polarize the vacuum creating $q\bar{q}$ and gg pairs, and thus the energy of the field will be deposited as a hot quark-gluon plasma. In this paper we have elaborated on the basic formulation to deal with the dynamics of the plasma produced in such a way and have shown how one can set up the initial conditions for the hydrodynamic expansion of the plasma. For this purpose we have started with the semiclassical kinetic theory and derived a set of coupled differential equations which self-consistently determine the hydrodynamic motion of the plasma fluid produced in an expanding electric field. These equations have been solved for the case of cylindrically symmetric expansions.

Although we are not yet at the stage of making a definite prediction for any directly observable quantity, the results of our calculation are very suggestive. It is shown that only a tiny portion of the original field energy can be converted into collective-transverse-flow energy of the plasma even if we start with a significantly large value for the initial field strength. We have shown that the inclusion of the Joule heating process works to suppress both the transverse expansion and the entropy production per unit rapidity. In either case the rest of the energy is transmitted to longitudinal motion by the fast scaling expansion. Hence in this model we cannot expect a large enhancement of the transverse momentum of the secondaries although the multiplicity grows rather rapidly in proportion to the initial field strength. This may account for the observed feature of the central rapidity region of high-energy pA collisions where the transverse energy of the secondaries depends very weakly on the target size,[31] while the increase in multiplicity is reasonably well reproduced by the flux-tube model.[22]

In any case, a number of refinements still need to be made before we can make quantitative predictions from this model. Probably the most urgent one is to incorporate the effects of confinement and hadronization. In this calculation we have not taken into account either of these important physical effects. The transverse evolution may be very sensitive to these effects as the previous studies[32-35] suggest. It is also interesting to extend the present formalism to describe the fragmentation regions, taking into account the finite longitudinal extension of the nuclei, and to study the slowing down mechanism of the color charged nuclear capacitor plates.[36] At a more formal level, it is important to reformulate the problem with a non-Abelian color charge. This may result in some fundamental modifications in the dynamics of the plasma formation as well as the background-field evolution.[18,37] In particular, color charge fluctuations on the nuclear plates may cause a nontrivial behavior in color field evolution due to the nonlinearity of the Yang-Mills equations which is absent in the present linearized (Abelian) treatment.

ACKNOWLEDGMENT

This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under Contract No. DE-AC02-76ER03069.

APPENDIX: ELECTRIC CONDUCTIVITY OF A RELATIVISTIC PLASMA

Here we shall present a derivation of Ohm's law (3.4) in the relaxation time approximation.[38]

In the single-relaxation approximation, the collision terms are written as

$$C_i = -\frac{p\cdot u}{\tau_c}(f_i - f_{\mathrm{eq}}) , \tag{A1}$$

where τ_c gives the time scale for the relaxation of the distribution function f_i to the local equilibrium function f_{eq}:

$$f_{\mathrm{eq}} = \frac{1}{\exp(\beta p^\mu u_\mu) \pm 1} . \tag{A2}$$

Then, from the kinetic equation (2.7), we find

$$\begin{aligned}\delta f_i &\equiv f_i - f_{\mathrm{eq}} \\ &= \frac{\tau_c}{(p\cdot u)}\left[p^\mu \partial_\mu f_i - g_i p^\mu F_{\mu\nu}\frac{\partial}{\partial p_\nu} f_i - S_i(x,p)\right] .\end{aligned} \tag{A3}$$

Since $\sum_i g_i \int d\Gamma\, p^\mu f_{\mathrm{eq}} = 0$ which implies that the system is locally neutral in equilibrium with respect to the charge g_i, the conductive current (2.6) can be rewritten as

$$j^\mu_{\mathrm{cond}}(x) = \sum_i g_i \int d\Gamma\, p^\mu [f_i(x,p) - f_{\mathrm{eq}}(x,p)] , \tag{A4}$$

which, upon the insertion of (A3), becomes

$$j^\mu_{\mathrm{cond}}(x) = -F_{\alpha\nu}\tau_c \sum_i g_i{}^2 \int d\Gamma \frac{p^\mu p^\alpha}{(p\cdot u)}\frac{\partial}{\partial p_\nu} f_i . \tag{A5}$$

Now we make a near equilibrium expansion of the right-hand side of (A5). The leading-order term is obtained by replacing f_i with the local equilibrium distribution (A2):

$$j^{\mu}_{\rm cond}(x)=F_{\alpha\nu}\tau_c \sum_i g_i{}^2 \int d\Gamma \frac{p^{\mu}p^{\alpha}}{(p\cdot u)}\beta u^{\nu} f_{\rm eq}(1\mp f_{\rm eq}) \ . \tag{A6}$$

Since u^{μ} is the only four-vector which survives after the integral, the phase-space integral which appears on the right-hand side can be written as

$$\int d\Gamma \frac{p^{\mu}p^{\alpha}}{(p\cdot u)} f_{\rm eq}(1\mp f_{\rm eq})=g^{\mu\alpha}c_0+u^{\mu}u^{\alpha}c_1 \ , \tag{A7}$$

where c_0 and c_1 are the scalars. Inserting (A7) into (A6), we find Ohm's law:

$$\begin{aligned} j^{\mu}_{\rm cond}(x)&=F_{\alpha\nu}\tau_c \sum_i g_i{}^2\beta u^{\nu}(g^{\mu\alpha}c_0+u^{\mu}u^{\alpha}c_1) \\ &=\tau_c \sum_i g_i{}^2\beta c_0 F^{\mu\nu}u_{\nu} \ . \end{aligned} \tag{A8}$$

The electric conductivity σ_c is given by

$$\sigma_c=\tau_c\beta\sum_i g_i{}^2 c_0 \ . \tag{A9}$$

The constant c_0 can be calculated from (A6) by going over to the fluid comoving frame, where $u^{\mu}=(1,0,0,0)$ and comparing the $\mu=\nu=1$ component of both sides of (A6). This yields

$$c_0=\frac{1}{3}\int \frac{d^3p}{(2\pi)^3}\left[\frac{\mathbf{p}}{p_0}\right]^2 \frac{\exp(\beta p_0)}{[\exp(\beta p_0)\pm 1]^2} \ . \tag{A10}$$

In the massless particle limit, this integral can be carried out analytically and we find

$$\begin{aligned} c_0&=\tfrac{1}{18}\beta^{-3} \quad \text{for bosons} \\ &=\tfrac{1}{36}\beta^{-3} \quad \text{for fermions} \ . \end{aligned} \tag{A11}$$

Inserting this into (A9) we obtain (3.5).

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[1]For a recent review, see *Quark Matter '84*, proceedings of the Fourth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, edited by K. Kajantie (Lecture Notes in Physics, Vol. 221) (Springer, New York, 1985).

[2]T. Burnett *et al.*, Phys. Rev. Lett. **50**, 2062 (1983).

[3]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[4]M. Gyulassy and T. Matsui, Phys. Rev. D **29**, 419 (1984).

[5]E. Fermi, Prog. Theor. Phys. **5**, 570 (1950).

[6]L. D. Landau, Izv. Akad. Nauk SSSR **17**, 51 (1953); S. Z. Belen'kii and L. D. Landau, Usp. Fiz. Nauk **56**, 309 (1955), reprinted in *Collected Papers of L. D. Landau,* edited by D. ter Haar (Gordon and Breach, New York, 1965), pp. 569 and 665.

[7]P. Carruthers, Ann. N.Y. Acad. Sci. **229**, 91 (1974).

[8]Several variants of the original Landau model have been proposed to incoporate the leading-particle effect; see, for example, S. Pokorski and L. Van Hove, Acta Phys. Pol. **B5**, 229 (1974); E. V. Shuryak, Phys. Rep. **61**, 71 (1979); P. Carruthers and Minh Duong-Van, Phys. Rev. D **28**, 130 (1983).

[9]K. Kajantie and L. D. McLerran, Phys. Lett. **119B**, 203 (1982); Nucl. Phys. **B214**, 261 (1983); K. Kajantie, R. Raitio, and P. V. Ruuskanen, *ibid.* **B222**, 152 (1983).

[10]G. Baym, B. L. Friman, J. P. Blaizot, M. Soyeur, and W. Czyż, Nucl. Phys. **A407**, 541 (1983).

[11]J. Bjorken, in *Current Induced Reactions,* proceedings of the International Summer Institute on Theoretical Particle Physics, Hamburg, 1975, edited by J. G. Körner, G. Kramer, and D. Schildnecht (Lecture Notes in Physics, Vol. 56) (Springer, New York, 1976), p. 93.

[12]A. Casher, J. Kogut, and L. Susskind, Phys. Rev. D **10**, 732 (1974).

[13]A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D **20**, 179 (1979).

[14]E. G. Gurvich, Phys. Lett. **87B**, 386 (1979); B. Andersson, G. Gustafson, and T. Sjostrand, Z. Phys. C **97**, 235 (1980); N. K. Glendenning and T. Matsui, Phys. Rev. D **28**, 2890 (1983).

[15]J. Schwinger, Phys. Rev. **82**, 664 (1951); E. Brezin and C. Itzykson, Phys. Rev. D **2**, 1191 (1970).

[16]F. Low, Phys. Rev. D **12**, 163 (1975).

[17]S. Nussinov, Phys. Rev. Lett. **34**, 1286 (1975).

[18]H. Ehtamo, J. Lindfors, and L. McLerran, Z. Phys. C **18**, 341 (1981).

[19]T. S. Biro, H. B. Nielsen, and J. Knoll, Nucl. Phys. **B245**, 449 (1984).

[20]A. Białas and W. Czyż, Phys. Rev. D **31**, 198 (1984).

[21]M. Gyulassy and A. Iwazaki, Phys. Lett. **165B**, 157 (1985).

[22]A. Kerman, T. Matsui, and B. Svetitsky, Phys. Rev. Lett. **56**, 219 (1986).

[23]K. Kajantie and T. Matsui, Phys. Lett. **164B**, 373 (1985).

[24]U. Heinz, Phys. Rev. Lett. **51**, 351 (1983); Ann. Phys. (N.Y.) **161**, 48 (1985).

[25]S. R. de Groot, W. A. van Leeuwen, and Ch. G. van Weert, *Relativistic Kinetic Theory* (North-Holland, Amsterdam, 1980).

[26]The constraint of the Lorentz-boost symmetry alone does not single out this δ-function form for the longitudinal-momentum distribution, and any function of $\eta-y$ would be allowed. To eliminate this uncertainty in determining the source term, the proper procedure would be to go beyond the static uniform field assumption which gives the formulas (2.9) and (2.10) for the pair-creation rate and to perform a quantum-mechanical calculation of the pair-creation process in the field produced by expanding capacitor plates.

[27]A. Hosoya and K. Kajantie, Nucl. Phys. **B250**, 666 (1985).

[28]P. Danielowicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985).

[29]S. Gavin, Nucl. Phys. **A435**, 826 (1985).

[30]M. C. Chu, Phys. Rev. D **34**, 2764 (1986).

[31]C. De Marzo *et al.*, Phys. Rev. D **29**, 363 (1985).

[32]N. Glendenning and T. Matsui, Phys. Lett. **141B**, 149 (1984).

[33]B. Friman, Phys. Lett. **159B**, 369 (1985).

[34]H. von Gersdorff, L. McLerran, M. Kataya, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986); M. Kataja, P. V. Ruuskanen, L. McLerran, and H. von Gersdorff, *ibid.* **34**, 2755 (1986).

[35]J.-P. Blaizot and J. Y. Ollitrault, Nucl. Phys. **A458**, 745 (1986).

[36]Some interesting results in the direction have already been reported: M. Gyulassy and L. Csernai, Report No. LBL-20610, 1986 (unpublished).

[37]This problem has been recently studied in the collisionless limit based on the framework of Ref. 24; A. Białas and W. Czyż, Acta Phys. Pol. **B17**, 635 (1986).

[38]A more elaborate calculation of the electric conductivity of a relativistic plasma by the Chapman-Enskog method can be found in Ref. 25. For a calculation of the color-electric conductivity in a non-Abelian field theory, see U. Heinz, Ann. Phys. (N.Y.) **168**, 148 (1986).