Delayed thresholds and heavy-flavor production in the dual parton model

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It is shown that the two-chain structure of the cut Pomeron in the dual parton model for low- p_T multiparticle production provides a natural explanation for the phenomenon of delayed thresholds for heavy-flavor production in proton-proton collisions.

INTRODUCTION

Many recent papers have convincingly demonstrated that the dual parton model^{1,2} provides a rather complete quantitative description of soft $(low-p_T)$ multiparticle production in high-energy hadronic collisions. This partonic approach is systematically based on the dual-topologicalunitarization (DTU) scheme,³ which advocates the dominance at high energies of those diagrams which have the simplest topology. The most important contribution to particle production consists of a two-chain mechanism¹ corresponding to a unitarity cut of the cylindrical dual Pomeron. This is essentially the only contributing diagram for c.m. energies $\sqrt{s} < 60$ GeV. For higher collision energies, topologically more complicated multichain^{4,5} diagrams (coming from unitarity corrections) give noticeable effects. The model has been successfully used to quantitatively describe inclusive single-particle rapidity distribu-tions,^{4,5} rising rapidity plateaus,^{1,4,5} charged-particle mul-tiplicity moments and violations of Koba-Nielsen-Olesen scaling,^{4,5} charge distributions in rapidity,⁶ correlations between $\langle p_T \rangle$ and multiplicity,⁷ long-range rapidity corre-lations,⁸ antiproton-proton annihilation,⁹ diffraction dissociation,¹⁰ two-particle inclusive reactions,¹¹ and hadron-nucleus,¹² and nucleus-nucleus interactions.¹³

In previous work using the dual parton model, inclusive distributions into pions and kaons were mainly considered. In this article we focus on baryon, antibaryon, and heavy-meson production in hadronic collisions. In proton-proton collisions, the so-called *delayed threshold*¹⁴ behavior has long been an outstanding problem. For instance, although the kinematic threshold for producing a nucleon-antinucleon pair in *pp* collisions is $s \approx 15 \text{ GeV}^2$, the production rate for antiprotons remains negligible up to $s \approx 150 \text{ GeV}^2$ and then increases rapidly by a factor of more than 5 from s = 150 to $s = 3000 \text{ GeV}^2$. In contrast, the pion production rate varies smoothly over a much

larger energy range. Similarly, even though the effective charm-quark mass is of the order of 1.5 GeV, the cross section for charm production in *pp* collisions is hardly measurable until $s \approx 600 \text{ GeV}^2$ and then shows a rapid rise with energy.¹⁵ Also, it has been observed that inclusive distributions into charmed particles (e.g., $pp \rightarrow \Lambda_{c^+} + X$) are relatively flat.¹⁶ This behavior cannot be understood in perturbative QCD calculations involving gluon-gluon fusion, which give predominantly central production.¹⁷ Other approaches such as single-chain fragmentation¹⁸ or the assumption of "intrinsic charm" in protons¹⁹ also cannot quantitatively account for the observed delayed-threshold phenomenon for heavy-flavor production.

This peculiar phenomenon can be understood qualitatively in those models where particles are produced with a p_T cutoff and with short-range ordering in rapidity. Since, on the average, half of the initial energy is retained by the incoming protons, a longitudinal-phase-space suppression mechanism then sets in. However, for models of single-chain type, quantitative agreement cannot be achieved without introducing additional *ad hoc* rapidity suppression factors.¹⁴ Furthermore, average considerations are by themselves not enough in the context of heavy-flavor production, since there is none in an average event. A more unified as well as detailed model is necessary for quantitative calculations.

We shall show that the experimental observations described above can be naturally accommodated in a dual parton approach.²⁰ Care must be taken in handling the single-chain threshold region. By first examining pion and kaon production, we note that the two-chain dual parton model, with a smooth extrapolation down to low subenergies, already becomes meaningful for $\sqrt{s} > 15$ GeV. To properly describe baryon and heavy-flavor production and its threshold behavior, one must pay attention to mass effects and extrapolate the concept of frag-

mentation functions down to low energies. Qualitatively, in a dual parton approach, the phenomenon of delayed thresholds for pp collisions is a consequence of the fact that only a (small) fraction of the initial collision energy is actually available for particle production. In a protonproton collision particle production comes from two quark-diquark chains. Since the c.m. of each chain does not coincide with the overall pp c.m., a substantial part of the initial energy goes into the c.m. motion of the chains and furthermore the rest is shared by the two quarkdiquark chains. These two effects are responsible for delayed thresholds. (The mean available energy in each chain is very roughly $\sqrt{s_1} \approx [\bar{x}(1-\bar{x})s]^{1/2}$, where $\bar{x} =$ mean momentum fraction of the "held-back" valence quark ≈ 0.05 (Ref. 1).) Moreover, if one is looking at unfavored fragmentation products (e.g., antibaryons), additional particles (e.g., baryons) must also necessarily be produced-which further displaces the production threshold to higher energies. The dual parton model makes specific predictions for the presence or absence of delayed thresholds in other hadronic reactions. For example, no delayed thresholds are expected in $\overline{p}p$ collisions—a prediction which is borne out by data. The reason for this is the following. In pp collisions, the two chains are $\bar{q}q$ and $\overline{q} \, \overline{q}$ -qq, and their c.m., on the average, coincides with the overall $\overline{p}p$ c.m. (Ref. 1). The $\overline{q}q$ chain is usually short and has a typical energy $\sqrt{s_1} \approx x\sqrt{s}$ whereas the $\bar{q} \bar{q}$ -qq chain is long and has energy $\sqrt{s_2} \approx (1-x)\sqrt{s}$, which is almost the entire available incident energy. Thus, no delayed threshold is expected.

We shall now briefly review the main ideas of the dual parton model and then proceed to make quantitative calculations of heavy-flavor production.

REVIEW OF THE DUAL PARTON MODEL

When two hadrons collide, the resulting reaction is assumed to be a two-step process: color separation and subsequent fragmentation. For example, in a proton-proton collision, the simplest topological diagram comes from each proton splitting into a valence quark and a diquark, giving rise to two quark-diquark chains (see Fig. 1).

In order to calculate the contribution of this diagram, it is necessary to specify the probability that the interaction separates the protons into two quarks with momentum fractions x_1 and x_2 and two diquarks with the remaining

$$dN/dy^{pp\to h}(s,y) = \int \int dx_1 dx_2 \rho(x_1,x_2) [(dN/dy) |_1 (y - \Delta_1,s_1) + (dN/dy) |_2 (y - \Delta_2,s_2)],$$

where $dN/dy \mid_{1,2}$ are the contributions from chains 1 and 2, which are given by quark and diquark fragmentation functions into the detected hadron *h*. Thus, the only inputs are these fragmentation functions. At energies $\sqrt{s} > 60$ GeV, multichain contributions become important. Relevant formulas are given in Ref. 4.

FRAGMENTATION FUNCTIONS

Since there are, as yet, no clean measurements of quark and diquark fragmentation functions into baryons and FIG. 1. The two-chain diagram for proton-proton collisions.

momentum fractions $(1-x_1)$ and $(1-x_2)$. This probability $\rho(x_1,x_2)$ is given in terms of Regge intercepts^{1,4,12(a)} in a separable form, $\rho(x_1,x_2) = \rho(x_1)\rho(x_2)$, with

$$\rho(x) = c \, (1 - x)^{1.5} x^{-1/2} \tag{1}$$

and $0 \le x \le 1$. The coefficient *c* is determined by normalizing the probability ρ to unity. Since the quark structure functions are peaked near x = 0 $[\rho(x) \sim x^{-1/2}]$, the interaction usually results in two "held-back" quarks near $x_1, x_2 = 0$. Typically, $x_1 = x_2 \approx 0.05$ (Ref. 1). The total energy \sqrt{s} in the overall pp c.m. frame is shared between the two chains labeled 1 and 2 in Fig. 1:

$$s_1 \approx sx_2(1-x_1), \quad s_2 \approx sx_1(1-x_2)$$
, (2)

where $\sqrt{s_1}$ ($\sqrt{s_2}$) is the energy of chain 1 (2) in its own c.m. frame. The rapidity shift Δ_1 (Δ_2) necessary to go from the overall *pp* c.m. frame to the c.m. of chain 1 (2) is $\Delta = \frac{1}{2} \ln[(1+\beta)/(1-\beta)]$, where β is the corresponding Lorentz boost.

The single-particle inclusive cross section for $pp \rightarrow h + X$ is given by the superposition of chains 1 and 2:

charmed mesons in high-energy processes, we have resorted to guidance from dimensional counting rules²¹ and the recursive cascade model^{22,23} for hadronization. Our choices for fragmentation functions into $\bar{p}, \bar{\Lambda}, \Lambda, \Lambda_c +, D^0$ are given in Table I. In most instances, we have just chosen a simple power of (1-x) for $\bar{x}D(x)$ $=(x^2 + 4\mu^2/s)^{1/2}D(x)$, where *m* is the transverse mass of the detected hadron. For favored diquark fragmentation into baryons (e.g., $ud \rightarrow \Lambda, \Lambda_c$ +), we have taken a favored first breakup followed by subsequent unfavored fragmen-



Detected hadron h	m _h (GeV)	Minimal additional system s	<i>m</i> ₃ (GeV)	Fragmentation functions $\tilde{x}D_h(x)$			
				и	d	ии	ud
\overline{p}	0.94	рр	2.0	$c_{\bar{p}}(1-x)^4$	$c_{\bar{n}}(1-x)^4$	$c_{\bar{p}}(1-x)^5$	$c_{\overline{p}}(1-x)$
$\overline{\Lambda}$	1.12	Λp	2.1	$c_{\overline{\Lambda}}^{r}(1-x)^{4}$	$c_{\overline{\Lambda}}^{r}(1-x)^{4}$	$c_{\bar{\Lambda}}^{\prime}(1-x)^{5}$	$c_{\overline{\Lambda}}(1-x)^5$
Λ	1.12	ĸ	0.5	$c_{\Lambda}(1-x)^2$	$c_{\Lambda}(1-x)^2$	$c_{\Lambda}(1-x)^3$	$c_{\Lambda}[60x^{1.5}(1-x)^{1.5}+(1-x)^{4.5}]$
Λ_{c^+}	2.1	D_0	1.9	$c_{\Lambda_c}(1-x)^2$	$c_{\Lambda_c}(1-x)^2$	$c_{\Lambda_c}(1-x)^3$	$c_{\Lambda_c}[4x(1-x)^{0.2}+(1-x)^4]$
<i>D</i>	1.9	Λ_{c}^{+}	2.1	$c_D(1-x)$	$c_D(1-x)$	$c_D(1-x)^2$	$c_D(1-x)^2$

TABLE I. Quark and diquark fragmentation functions into various detected hadrons: $c_{\bar{p}} = 0.084$, $c_{\bar{\Lambda}} = 0.028$, $c_{\Lambda} = 0.020$, $c_{\Lambda_c} = 0.036$, and $c_D = 0.036$.

tation.²³ For any detected hadron h, the normalization constant c_h is a parameter to be determined. These constants, along with the powers in favored diquark fragmentation, can be fixed by looking at available inclusive distributions at any one energy. For example, the powers involved in $D_{ud \to \Lambda}$ have been chosen by fitting the $d\sigma/dy$ distributions of $pp \to \Lambda + X$ at $p_L = 405$ GeV/c (see Fig. 2).

EXTRAPOLATION TO THRESHOLD

For quarks, diquarks, or any colored system, a description of hadronization in terms of jets and fragmentation makes good sense only when several particles are produced. However, as mentioned in the Introduction, we would like to take the liberty of extrapolating down in energy to near the threshold where only two or three particles are produced. Intuitively, one expects that in the threshold region the rapidity plateau should develop gradually as the energy increases at a rate determined by the scale of masses involved. Whereas the onset of scaling can be fairly rapid for pions and kaons, the mass effect is important for heavy-flavor productions. To be specific, we appeal to the fact that hadron production tends to be



FIG. 2. Rapidity distributions for Λ and $\overline{\Lambda}$ production in *pp* collisions at $p_L = 405$ GeV/c. Data from Ref. 24.

peripheral (or multiperipheral), leading naturally to limited p_T distributions. For two-body reactions, the p_T suppression is explicitly accomplished by a cutoff in the four-momentum transfer squared. For unequal masses, this in turn leads to the t_{min} -suppression effect which can be dominant when large mass differences are involved. We therefore introduce into each fragmentation function a factor

$$F = e^{2\alpha t_{\min}} . ag{3}$$

We shall treat α as a mass-independent constant which, from two-body phenomenology, can take on values from 0 to 5 GeV². For a reaction $a + b \rightarrow h + d$ at c.m. energy \sqrt{s} , t_{\min} is given by the familiar formula

$$t_{\min} = m_a^2 + m_h^2 - [(s + m_a^2 - m_b^2)(s + m_h^2 - m_d^2) - \Lambda^{1/2}(s, m_a^2, m_b^2) \times \Lambda^{1/2}(s, m_h^2, m_d^2)]/2s , \quad (4)$$

where $\Lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. Note that when $m_a \approx m_b \approx 0$, $t_{\min} \approx -m_h^2 m_d^2/s$. As a specific example, consider $pp \rightarrow \Lambda + X$, in which one has two quarkdiquark chains. The minimal additional hadronic system d, which must be present at threshold is (Λp) (see Table I). Thus, with $m_a = m_q \approx 0.3$ GeV, $m_b = m_{qq} \approx 0.6$ GeV, $m_h = m_{\overline{\Lambda}} = 1.12$ GeV, and $m_d = m_{(\Lambda p)} \approx 2.1$ GeV we have at $\sqrt{s} \approx 4$ GeV a value of $t_{\min} = -0.64$ GeV².



FIG. 3. Anti- Λ and antiproton production in proton-proton collisions. Theoretical curves (solid line for $\overline{\Lambda}$ and dashed line for \overline{p}) are calculated within the multichain dual parton model with $\alpha = 3 \text{ GeV}^{-2}$. The dotted curve: for $\overline{\Lambda}$ production is obtained with $\alpha = 0$. Data from a compilation are given in Ref. 25.



FIG. 4. Inclusive total cross sections for $\overline{\Lambda}/\Lambda$ production in *pp* and $\overline{p}p$ collisions calculated within the dual parton model with $\alpha = 3$ GeV⁻². Data from a compilation given in Ref. 26.

RESULTS AND DISCUSSIONS

The average number of \overline{p} and $\overline{\Lambda}$ produced in *pp* collisions as a function of energy \sqrt{s} is shown in Fig. 3. These reactions only involve unfavored fragmentation functions. The dual parton model curves with $\alpha = 0$ [no t_{\min} suppression factor *F* from Eq. (3)] and $\alpha = 3$ GeV⁻² are shown. As expected, a t_{\min} factor helps to steepen the rise above threshold, but its effect is not dramatic.

Figure 4 is a plot of the cross sections for Λ or $\overline{\Lambda}$ production in pp and $\overline{p}p$ collisions. The fragmentation functions involved in both processes are the same, but as discussed before, pp and $\overline{p}p$ have chains of different lengths which lead to a delayed threshold in pp but not in $\overline{p}p$. As a consequence, the pp cross section is smaller than the $\overline{p}p$ one at finite energies and their difference vanishes asymptotically. Clearly, the dual parton model calculations are in good qualitative agreement with the data.

We have also applied our approach to study the production of charm in hadronic collisions. Using the quark and diquark fragmentation into Λ_{c^+} and D^0 given in Table I, we get the curves shown in Fig. 5. A rapid rise from threshold is present both in the theoretical curves and in the data. The normalization for fragmentation functions into charmed particles comes from $d\sigma/dx$ inclusive distributions.¹⁶ Once this normalization is fixed, the *s* dependence is fully calculable and in reasonable accord with experiment. Note that our nonperturbative fragmentation approach is not incompatible with a certain amount of intrinsic charm in the initial hadrons,¹⁹ but



FIG. 5. (a) Inclusive cross sections for $D \cdot \overline{D}$ pair production in *pp* collision (Ref. 27). The solid (dotted) curve is obtained within the dual parton model with $\alpha = 3 \text{ GeV}^{-2}$ ($\alpha = 0$). (b) same as in (a) for Λ_c production (Ref. 27).

such a component is not necessary.

The dual parton model allows us to make qualitative estimates for the production of other heavy flavors. A quick calculation yields the thresholds for the production of heavy quarks of mass m_q :

$$\begin{split} (s_{\rm th}^{pp})^{1/2} &\approx [x\,(1-x)]^{-1/2}(2m_q+m_p) \ , \\ (s_{\rm th}^{pp})^{1/2} &\approx (2m_q+m_p) \ , \end{split}$$

with a typical average value of $\bar{x} \approx 0.05$ (Ref. 1). This shows a definite advantage of using an antiproton beam for heavy-flavor production. (This is an important point in favor of the antiproton-beam option which is being considered for the proposed Superconducting Super Collider.)

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