Ultra-high-energy cosmic rays from superconducting cosmic strings

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Superconducting cosmic strings may play an important role in the relatively late Universe in formation of structure and in driving highly exoergic processes. With fermionic charge carriers they are expected to eject, in their last stages, high-mass particles which can subsequently decay to produce ultra-high-energy electromagnetic, neutrino, and hadronic radiation. The bosonic cosmic string may undergo a similar saturation behavior. Cosmic-ray physics places significant limits on these scenarios. Furthermore, this provides an example of a fundamental mechanism for the production of the observed ultra-high-energy cosmic rays with some characteristically unusual, perhaps observable, features.

I. INTRODUCTION

Recently Ostriker, Thompson, and Witten[1] (OTW) proposed a dramatic scenario for a highly exoergic late Universe involving the decay of cosmic flux tubes which have superconducting electromagnetic boundary conditions. That such objects might exist in certain grand unified theories (GUT's) was first proposed by Witten.[2] Superconducting cosmic strings rely either upon the existence of superheavy fermions [which have ordinary electric charge and receive a pairing mass from the Higgs boson associated with the breaking of an extra U(1) symmetry and which can become trapped as Jackiw-Rossi zero modes on the string], or upon a bosonic construction, which we shall not consider in full detail in the present paper (some of the estimates we believe will have analogues in the bosonic case). In the fermionic case, massless Jackiw-Rossi zero modes act as carriers of electromagnetic currents and the flux tube becomes superconducting. Thus if one has a closed loop with a primordial threading magnetic field, pairs of zero modes are created on the string as the magnetic field is withdrawn and constitute the induced current. The OTW scenario presupposes the existence of primordial magnetic fields to set up this current.

Once electromagnetic currents are achieved and in the extreme relativistic limit of the string, electromagnetic quadrupole radiation is produced which can drive various effects which may be of importance to form galaxies and large-scale structure in the Universe and accelerate the relaxation of the string. As the loop shrinks the trapped fermion zero modes eventually become degenerate. The upper limit on the Fermi energy is given by the vacuum expectation value (VEV) of the Higgs boson which breaks the extra U(1). Above this energy fermions cease to be trapped on the string and will be ejected into the vacuum.

In the vacuum away from the string the fermions presumably act as superheavy (GUT mass) particles and are expected to decay, probably into three-body final states involving conventional quarks and leptons, or into two-body states with a conventional fermion and some gauge boson or Higgs bosons. This leads to (i) direct neutrinos, (ii) direct electrons and γ's, and (iii) quarks which fragment into hadrons leading to (a) protons, (b) neutrons, (c) neutrinos, and (d) electrons and gammas. At an earthbound detector one records a highly *evolved* spectrum via (a) red-shift of the injection spectrum (b) energy loss and recoil pileup due to collisions with microwave photons (ambient dust, starlight, and ordinary matter are generally negligibly smaller effects), source debris, and in-source magnetic fields, and (c) produced secondaries such as neutrinos and electrons and γ by pion photoproduction in the above collisions. We emphasize that neutrons comprise 50% of the surviving hadronic component because at these energies they can live for >10 Mpc and lose no energy due to Larmor radiation in the source. We do not concern ourselves with the electromagnetic component which involves a more complex evolution study and is more than likely reduced to a degraded thermal spectrum due to the intense B fields in the vicinity of the saturated string (for a discussion see Ref. 3).

In this paper we examine the dynamics of superconducting loops and consider the evolution of the hypothetical unstable fermion pairs they emit. First we show that such loops cannot be supported by degeneracy pressure of fermion zero modes, assuming a perturbative GUT. We then suppose the standard string loop formation distributions and gravitational energy loss as embodied in usual cosmic-string scenarios[4] and, along with very general assumptions about the magnetic field history of loops responsible for establishing the currents, we calculate the density of fermion emitting loops as a function of redshift.

By modeling the fragmentation distribution function of the quarks in a manner consistent with QCD multiplicity expectations and evolving the resultant hadrons through the background radiation, we predict the ultra-high-energy hadronic cosmic-ray spectrum associated with the superconducting loops. The dominant neutrino spectrum is directly obtained from the decays of massive fermions and from pions produced in quark fragmentation. (We can safely neglect the induced neutrino spectrum resulting from the transport of the nucleons through the microwave background as shown below, though results obtained previously[5,6] are readily adaptable.) We find that these scenarios are severely constrained by such limits as the Fly's Eye data on deeply penetrating particles with energies $>10^{17}$ eV. Respecting such limits we find that it is even plausible that the observed ultra-high-energy cosmic rays are generated via the decays of superheavy fermions emitted by saturated superconducting strings.

II. EVOLUTION OF THE SUPERCONDUCTING STRING

A. Do flux tubes have a Chandrasekhar limit?

One important issue is the approach to the extremely degenerate situation. It becomes of interest to see if the loop can be stabilized by fermion degeneracy, i.e., develop a Chandrasekhar limit. Within the assumption of perturbativity we find this does not occur.

We may consider an effective potential for a string of length L:

$$E(L)\approx v^2L+\frac{\pi N^2}{2L},\quad L>L_s\ , \tag{2.1}$$

where v is the Higgs-boson VEV which breaks the U(1) associated with the flux tube, N is the number of fermions of a given chirality plus antifermions of the opposite chirality (thus N has a positive or negative sign associated with the sense of the current and the current is eN/L), L_s is the string length at saturation, and we neglect the energy associated with the self-interaction of the fermions. Note that the mass per unit length, μ, is of order v^2. For typical values expected in grand-unified theories of $v\approx 10^{15}$ GeV we have $\mu\approx 6\times 10^{22}$ g/cm ($G_N\mu\simeq 10^{-6}$).

The number of fermions plus antifermions on the string is related to the Fermi momentum as

$$2L\int_0^{k_F}\frac{dk}{2\pi}=N=\frac{Lk_F}{\pi},\quad E_F=|\,k_F\,|\ , \tag{2.2}$$

and the saturation Fermi energy, E_{Fs}, and saturation current J_s, are given by the Higgs-Yukawa coupling of the heavy fermions to the U(1)$'$-breaking Higgs boson:

$$E_{Fs}\approx gv\ ,$$

hence

$$L_s=\frac{\pi N}{gv}\quad\text{and}\quad J_s=\frac{eM_F}{\pi}=(4\alpha g^2v^2/\pi)^{1/2}\ , \tag{2.3}$$

where g is the Higgs-Yukawa coupling constant, α is the fine-structure constant, and L_s is the saturation length. One can simply view g as the ratio $g=M_F/v$ where M_F is the fermion mass. Since Eq. (2.1) applies only for lengths greater than the saturation length we see that the minimum of $E(L)$ could only be reached if

$$L_s{}^2<\frac{\pi N^2}{2v^2}\quad\text{or}\ 2\pi<g^2\ . \tag{2.4}$$

Such a large Higgs-Yukawa constant is marginally nonperturbative, and, therefore, we presume that there is no stable ground state for the flux tube given by a *Chandrasekhar limit* in the perturbative limit. The present analysis is insufficient to address the issue in the case of a strongly coupled fermion since the free Fermi-Dirac distributions no longer apply and Eq. (2.1) is significantly modified. We have also neglected here the electromagnetic field energy which could alter these conclusions, but probably not significant in the perturbative limit.

This further means that the second term in Eq. (2.1), which is an effective mass density on the string, is never large compared to the first.

B. Energy-loss phases

We wish to obtain a schematic picture of the energy-loss phases of the superconducting string. This is somewhat different than the pure gravitational energy-loss picture of ordinary strings since, as pointed out by OTW, electromagnetic energy loss dominates gravity in the late stages of evolution. In addition, loss of fermions from the top of the Fermi distribution affects the extreme final stage. Our analysis is grossly simplified as we study the static potential of the preceding section which neglects kinetic terms.

We consider a loop of formation size L_f which forms at a time t_f. There may be an initial induced current J_f and thus a fermion number $N_f=J_fL_f/e$ (below we consider the growth of the current as a primordial flux is withdrawn). Initially the string loses energy by gravitational radiation with a power $P_g=\gamma_gG_N\mu^2$, where γ_g is a factor dependent only on the loop's shape and takes values ranging from 50 to 100 (Ref. 7). Thus we have

$$\gamma_gG_N\mu^2=-\left[\mu-\frac{\pi N^2}{2L^2}\right]\dot{L}-\frac{\pi N\dot{N}}{L} \tag{2.5}$$

and as we are far from degeneracy (and things are perturbative) the second and third terms on the right-hand side (RHS) are dropped. The string thus shrinks linearly with time:

$$L(t)\approx L_f-\kappa_1(t-t_f),\quad \kappa_1=\gamma_gG_N\mu\ . \tag{2.6}$$

The rate κ_1 is approximately 3×10^6 cm sec^{-1} for $G_N\mu=10^{-6}$ and $\gamma_g\approx 100$, values which are consistent with the cosmic-string scenario of galaxy formation.[7] We see that loops formed at $z>(1/\kappa_1)^{2/3}\approx 460(100/\gamma_g)^{2/3}(G_N\mu/10^{-6})^{-2/3}$ will have gravitationally evaporated by today.

Assuming with OTW the presence of a *sufficiently coherent primordial magnetic field* there will generally be a buildup of the electromagnetic current as the string shrinks due to the opposition of the withdrawal of magnetic flux. The power loss in electromagnetic radiation is given by $P_e = J^2\gamma_{\rm em} = \gamma_{\rm em}4\pi\alpha N^2/L^2$, where $\gamma_{\rm em}$ is a term analogous to γ_g (we take $\gamma_g \approx \gamma_{\rm em} \approx 100$ in subsequent calculations), and eventually becomes the dominant energy-loss mechanism at a scale $L_{\rm em}{}^2 \le \gamma_{\rm em}4\pi\alpha N^2/(\gamma_g G_N\mu)$. In this regime the loop evolution is described by the equation

$$\gamma_{\rm em}\frac{4\pi\alpha N^2}{L^2} = -\left[\mu - \frac{\pi N^2}{2L^2}\right]\dot{L} - \frac{\pi N\dot{N}}{L} \qquad (2.7)$$

and again neglecting the Fermi energy and taking $\dot{N}=0$, the loop size is now given by

$$\begin{aligned} L(t) &= [L_{\rm em}{}^3 - \bar{\kappa}_2(t-t_{\rm em})]^{1/3} , \\ \bar{\kappa}_2 &= 12\pi\alpha N^2\gamma_{\rm em}/\mu . \end{aligned} \qquad (2.8)$$

For comparison we define the quantity

$$\kappa_2 = \bar{\kappa}_2/L^2 = \frac{12}{\pi}\gamma_{\rm em}\alpha g^2 j^2 , \qquad (2.9)$$

where $j \equiv J/J_s$. We find that $\kappa_2 \approx 3\times 10^{10}(g^2)(j^2)$ cm/sec. Since the OTW scenario relies upon initial values of $P_e/P_g > 10^{-4}$ (assuming $G_N\mu = 10^{-6}$) and since $j \le 1$ we see that the *mathematical* lower limit on the fermion Higgs-Yukawa coupling is $g > 10^{-6}$ and a fermion mass limit of $M_F > 10^9$ GeV, consistent with their scenario. (It should be noted that in the OTW analysis the Higgs-Yukawa coupling is implicitly taken to be of order unity. Thus they conclude that at $j \approx 10^{-2}$ the electromagnetic energy loss dominates gravitational. More generally, this occurs when $\kappa_2/\kappa_1 \approx 1$ or $gj \approx 10^{-4}$ and in most of the expressions in OTW involving j one can substitute gj. The argument that superconducting strings become dominated by electromagnetic energy losses further constrains the fermion mass to $M_F > 10^{11}$ GeV.)

Finally the loop becomes saturated at a length of $L_s = \pi N_s/gv$ and continues to lose energy by emitting fermion pairs as well as electromagnetic radiation. The saturation length must be computed from a knowledge of the magnetic field history experienced by the loop (see Sec. II C). The energy-loss equation below the saturation length becomes

$$\gamma_{\rm em}\frac{4\pi\alpha N(t)^2}{L^2} = -\left[\mu - \frac{\pi N^2}{2L^2}\right]\dot{L} - \frac{\pi N\dot{N}}{L} , \qquad (2.10)$$

where the last term on the RHS reflects the additional energy loss due to the creation of fermion pairs of energy E_F. Again the second and third terms can be neglected on the RHS of Eq. (2.10). Noting that the power on the LHS of Eq. (2.10) involves the ratio $N(t)^2/L(t)^2$ which is a constant when the system is saturated, we find the behavior for $L(t)$ is again linear:

$$\begin{aligned} L(t) &\approx L_s - \kappa_3(t-t_s) , \\ \kappa_3 &= \gamma_{\rm em}J_s{}^2/\mu = \frac{4}{\pi}\alpha\gamma_{\rm em}g^2 , \end{aligned} \qquad (2.11)$$

and the fermion plus antifermion number of the string is just

$$N(t) = \frac{gv}{\pi}L(t), \quad L < L_s . \qquad (2.12)$$

In this phase we note that the particle ejection rate from a saturated string relevant to the cosmic-ray injection spectrum is given by

$$\dot{N}(t) = \frac{g^2}{\pi}(\gamma_{\rm em}J_{\rm sat}{}^2)/M_F = \frac{4}{\pi}\gamma_{\rm em}\alpha g^3 v , \qquad (2.13)$$

where $M_F = gv$ is the fermion mass (note that this is technically the ejection rate of left movers plus anti-right-movers; but the electric charge of the string is zero so $|N_L| = |N_R| = |N|/2$).

C. Magnetic flux and saturation

We have previously obtained the rate of particle production from a saturated string and we have sketched the time scales from formation to a given epoch at which the string becomes saturated (primarily determined by the gravitational energy loss). To proceed we require the densities of saturated strings at any red-shift. This depends upon the magnetic field history experienced by the loop. We find that a simple parametrization can be given which allows a discussion of the case of OTW, with strong B fields at large z, and more conservative cases in which known B fields are assumed to have formed in the relatively recent past.

The number density of cosmic strings at a given epoch can be estimated from the numerical studies of Albrecht and Turok[4] who find that at a given time t, the probability of having a string at formation, with length of order the horizon size, $H(t)^{-1}$, is approximately one. More precisely, they find that there is one string formed with length

$$L_f \approx 0.87 H_0{}^{-1}(1+z_f)^{-3/2} \qquad (2.14)$$

per horizon volume per Hubble time, where we have assumed that we are interested in the loops formed in a matter-dominated $\Omega = 1$ Universe so that the scale factor evolves as $t^{2/3}$ and the current age of the Universe is just $\frac{2}{3}H_0{}^{-1}$ with H_0 the Hubble constant. This formation rate gives rise to a red-shifted loop length distribution at later times of

$$\frac{dn}{dL_f}(t) = \frac{0.2}{L_f{}^4}\left[\frac{1+z}{1+z_f}\right]^3 = 0.3H_0{}^2\frac{(1+z)^3}{L_f{}^2} , \qquad (2.15)$$

where z_f and z are red-shifts at t_f and t, respectively. The loops are decaying for most of their history by gravitational energy loss and we see using Eq. (2.7), that loops of size $L(z)$ came from loops of size L_f given by

$$L(z) \approx L_f - \frac{2\kappa_1}{3}H_0{}^{-1}[(1+z)^{-3/2} - (1+z_f)^{-3/2}] \qquad (2.16)$$

or

$$L_f \approx \left[L(z) + \left[\frac{2\kappa_1}{3}H_0{}^{-1}\right](1+z)^{-3/2}\right] \qquad (2.17)$$

neglecting κ_1/c relative to unity. Thus we obtain the differential number density of loops with length L at any red-shift:

$$\frac{dn(z)}{dL} = \frac{0.3(1+z)^3 H_0{}^2}{\{L + [2\kappa_1/(3H_0{}^{-1})](1+z)^{-3/2}\}^2} . \qquad (2.18)$$

In the equation above, $[2\kappa_1/(3H_0)](1+z)^{-3/2}$ represents the initial size of loops which have gravitationally evaporated by red-shift z and since we are ultimately interested in small saturated loops with size $L \ll [2\kappa_1/(3H_0)](1+z)^{-3/2}$, we see that the differential distribution of saturated loops is essentially independent of decaying loop length:

$$\frac{dn(z)}{dL} = \frac{0.6H_0^4}{\kappa_1^2}(1+z)^6 \ . \tag{2.19}$$

Thus, if we know the saturation length L_s from the magnetic history of the loop we have the density of saturated loops determined as

$$n(z) = \frac{0.6H_0^4 L_s}{\kappa_1^2}(1+z)^6 \ . \tag{2.20}$$

Using the canonical value of κ_1, Eq. (2.19) represents about $2\times10^8\ L_s H_0$ loops actively decaying within our Hubble volume today.

To calculate the saturation length as a function of red-shift, we must assume a model for the magnetic field. The B field and its correlation length $\lambda(z)$ can be parametrized as

$$B(z) = (1+z)^{-p+3/2}B_0, \quad \lambda(z) = \lambda_0/(1+z) \ , \tag{2.21}$$

where $B_0 \simeq 10^{-9}$ G and $\lambda_0 \simeq 1$ Mpc are the typical values for current epoch intergalactic magnetic fields.[8] With such a parametrization, $p = -\frac{1}{2}$ corresponds to a primordial magnetic field energy density which scales as radiation density (as was assumed by OTW) and $p > -\frac{1}{2}$ would correspond to the fields generated by galactic dynamos in recent epochs.

The saturation length is determined by the history of the magnetic flux crossing the loop during its lifetime and can be parametrized as

$$L_s \simeq \frac{\pi f(L,\lambda,z)\langle\Phi\rangle}{2egv\ \ln(L/2\pi l)} \ , \tag{2.22}$$

where we include the self-inductance of a loop of thickness $l \simeq (gv)^{-1}$. Here $\langle\Phi\rangle$ is taken as the averaged flux given by

$$\langle\Phi\rangle = \frac{1}{z_f - z}\int_z^{z_f} B(z)\lambda(z)^2 dz \tag{2.23}$$

and $f(L,\lambda,z)$ is a factor accounting for $\sqrt{N}$ fluctuations in the B field, given roughly by L/λ for a loop of area L^2. We estimate that loops which produce cosmic rays observable at the current epoch will have both $f(L,\lambda,z) \approx f_\Phi$, and $\ln(L/2\pi l)$ of order 100. One can consider a more detailed model of magnetic field histories but we feel these approximations are sufficient to see the range of possibilities for cosmic-ray production.

The mean flux for loops active at a red-shift of z is

$$\langle\Phi\rangle = \frac{1}{z_f(z)-z}\int_z^{z_f(z)} dz'(1+z')^{-p-1/2}B_0\lambda_0^2 \tag{2.24}$$

$$= \frac{1}{z_f(z)-z}(-p+\tfrac{1}{2})^{-1}B_0\lambda_0^2 \times[(1+z')^{-p+1/2}\,|_z^{z_f(z)}] \ . \tag{2.25}$$

Using $z_f(z) \approx (\kappa_1)^{-1/2}(1+z)$, we find, to a good approximation,

$$\langle\Phi\rangle \approx \frac{B_0\lambda_0^2}{-p+\frac{1}{2}}(1+z)^{-p-1/2}\kappa_1^{(2p+1)/3}, \quad p<\tfrac{1}{2} \tag{2.26}$$

$$\approx \frac{B_0\lambda_0^2}{p-\frac{1}{2}}(1+z)^{-p-1/2}\kappa_1^{2/3}, \quad p>\tfrac{1}{2} \ . \tag{2.27}$$

In particular, for $z=0$ we see that a recent epoch assumption for the B field of $p\simeq1$ gives a mean flux of $\langle\Phi\rangle \approx 10^{37}$ g cm^2 and thus a saturation length $L_s \simeq 10^{18}$ cm$(10^{15}$ GeV$/M_F)$, while the $p=-\frac{1}{2}$ scenario of OTW gives $\langle\Phi\rangle \approx 10^{40}$ G cm^2 and a saturation length $L_s \simeq 10^{21}$ cm$(10^{15}$ GeV$/M_F)$. From Eq. (6) (OTW), we see that the OTW scenario requires roughly 10 saturated loops, with sizes of order 1 kpc, decaying within the present Hubble volume. These loops would carry an energy density of $\approx 10^{-7}$ergs/cm^3, which is roughly ten times the energy density of cosmic rays above 10^{19} eV. With this order-of-magnitude estimate in hand, we proceed to calculate the resultant cosmic-ray spectrum from superconducting strings.

III. PRODUCTION AND EVOLUTION OF COSMIC RAYS

Given the rate density for the production of superheavy fermions as described in the preceding section we can estimate the resulting cosmic-ray spectrum. Here we will assume that each heavy fermion undergoes three-body decays into $n \le 3$ quarks (we will not distinguish between quarks and gluons in our fragmentation distributions) and $3-n$ leptons. We begin with a discussion of the fate of the hadronic component.

A. Hadronic component

Quarks or gluons undergo fragmentation into mostly pions and some baryons. The fragmentation distribution cannot be calculated from first principles but Mueller[9] has studied its zeroth moment. Using this and demanding that the first moment be unity (energy conservation) and assuming a convenient $(1-x)^2$ behavior as $x\to1$ [of course in QCD the $x\to1$ behavior is calculable and energy dependent and *not* of the form $(1-x)^2$ but we are not interested in this limit of the spectrum since the observed cosmic rays extend to $\approx10^{11}$ GeV and the characteristic mass scale of the fermions extends to of order $M_F \approx v \approx 10^{15}$ GeV; the low-x limit is more relevant to us]. We can arrive at[10]

$$\frac{dN}{dx} = N(b)\exp[b\sqrt{\ln(1/x)}](1-x)^2 \times[x\sqrt{\ln(1/x)}]^{-1} \ , \tag{3.1}$$

where

$$N(b) = \frac{1}{2}\left[e^{b^2/4}I(b) - \sqrt{2}e^{b^2/8}I(b/\sqrt{2}) + \frac{1}{\sqrt{3}}e^{b^2/12}I(b/\sqrt{3})\right] \tag{3.2}$$

and

$$I(b)=\frac{\sqrt{\pi}}{2}[\mathrm{erf}(b/2)+1], \quad b=4\sqrt{3}/\sqrt{b_0}\ , \qquad b_0=11-\frac{2n_f}{3}\ . \tag{3.3}$$

We have for $n_f=6$ that $b\approx 2.6$ and $N(b)\approx 0.08$. This distribution is engineered so that the zeroth moment obtained in Ref. 9 emerges when Eq. (3.1) is integrated for a jet of energy E from $x=\mu/E$ to $x=1$ and μ is chosen of order 1 GeV. For comparison a simple multiplicity growth of $\sqrt{E}$ follows from the distribution

$$\frac{dN}{dx}=\frac{15}{16}x^{-3/2}(1-x)^2 \tag{3.4}$$

again integrating from $x=\mu/E$ to $x=1$. These distributions are displayed in Fig. 1.

Our hadronic injection spectrum at red-shift z is then

$$\frac{dN}{dE}\propto f_B\left[\left.\frac{dN}{dx}\right|_{x=E/M_f}\right], \tag{3.5}$$

where M_f is the heavy fermion mass. Here f_B is unity for pions and is approximately 0.03 baryons and antibaryons. The pion component induces a neutrino component and we consider this further below. We assume that all baryons ultimately end up as protons though half are initially neutrons which can travel large distances unaffected by magnetic fields at production until they β decay to protons, electrons, and very low-energy neutrinos. Thus, we have a mechanism for injecting extremely energetic nucleons up to energies of order M_f. These will be, in principle, detectable as ultra-high-energy cosmic rays, but must be evolved to $z=0$.

We now must follow the proton cosmic-ray spectrum as it undergoes principally three evolutionary effects: (a) cosmological red-shift; (b) pion photoproduction; (c) Bethe-Heitler processes; all other processes are subleading effects.[5] These have been studied in great detail previously,[5] but we take several justifiable shortcuts in the

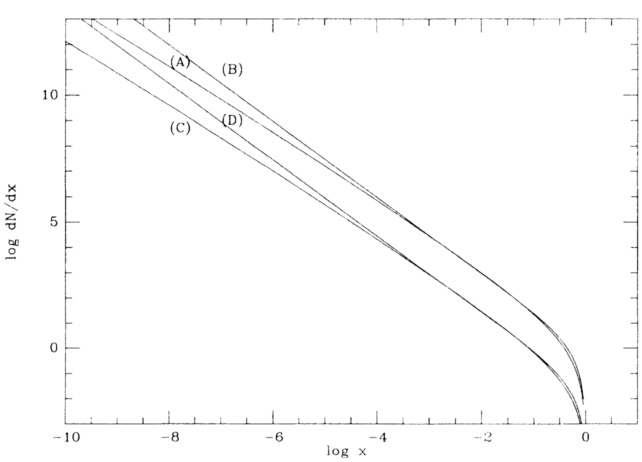

FIG. 1. The x distributions assumed for quark fragmentation (A) Eq. (3.1), (B) Eq. (3.4); and the corresponding nucleon spectra, (C) and (D), assuming 3% nucleons plus antinucleons per total multiplicity.

present analysis. The red-shift effects are straightforwardly included and involve identifying the production energy E_0 at red-shift z_0 with an "observer" energy E' at any other red-shift z', $(1+z_0)E'=(1+z')E_0$ (we will be concerned only with nucleons for which $E>1$ TeV, and hence are ultrarelativistic) and dilution of the number density by $(1+z)^{-3}$. This latter effect is dominant for a spectrum as flat as that produced at injection by Eq. (3.1); we see below that unless the injection spectrum is *increasing* with red-shift faster than $(1+z)^4$ that we are sensitive only to the $z\rightarrow 0$ spectrum. Furthermore, there is a gradual energy loss due to e^+e^- production (Bethe-Heitler) which we can include in principle following Blumenthal[11] and Ref. 5. In practice, however, this effect is negligible on the scale of sensitivity we are presently interested in.

A nucleon colliding with a microwave background photon is above threshold to undergo photoproduction of pions if $E>2m_\pi^2/[(1+z)T_{3\,^\circ\mathrm{K}}]$. The recoil nucleon at extremely high energies is approximately uniformly distributed in energy between incident and threshold energy and will be neglected in the present analysis (see Ref. 5 for a discussion of the relevant corrections). This implies that there will be a Greisen cutoff.[12]

An interesting signature for superconducting strings emerges in this analysis. If there are active strings within a few interactions lengths (one energy-loss length is about 6 Mpc today) then there will be no complete Greisen cutoff, and a dip in the spectrum will occur above the cutoff, extend to very high energies, and super-ultra-high-energy cosmic rays will be seen at energies above the upper limit of the dip.

To see this effect (for simplicity we neglect red-shift effects) consider a single cosmic string of total luminosity L_0 at sufficient range to be considered a point source. For definiteness take the particle production rate of the preceding section:

$$L_0=\frac{4}{\pi}\gamma_{\mathrm{em}}\alpha M_F{}^3/v^3\ . \tag{3.6}$$

We see the very strong dependence upon M_F in the activity (there is further dependence through the fragmentation spectrum). The observed differential energy flux at range R for nucleons, including the Greisen cutoff (neglecting red-shift effects) becomes

$$j(E)\approx\frac{f_B n L_0}{4\pi R^2 M_F}\left[\left.\frac{dN}{dx}\right|_{x\rightarrow E/M_f}\right]e^{-R/\lambda(E)}\ . \tag{3.7}$$

Here we neglect pileup effects and a suitable parametrization for $\lambda(E)$ may be taken from Ref. 13. $\lambda(E)$ has weak energy dependence above threshold and is about 6 Mpc over the decade in energies above 10^{20} eV.

In Fig. 2 we give the resulting fluxes at earth for various ranges of the single point source assuming $M_F=10^{15}$ GeV, out to $R\sim 10$ Mpc at which this simple $z\sim 0$ approximation has already broken down. For comparison we plot a $1/E^3$ spectrum with the approximate normalization observed at the Fly's Eye[14] for hadronic ultra-high-energy (UHE) cosmic rays up to 10^{20} eV. We extrapolate the Fly's Eye normalization for a *fixed number of events*

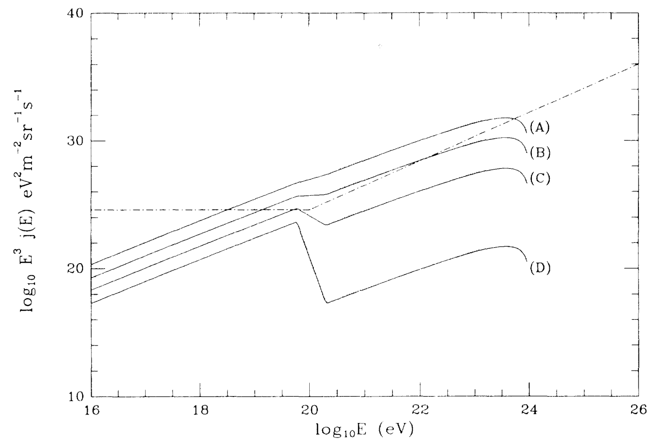

FIG. 2. The UHE nucleon spectrum for a point source at distances (A) 3 Mpc; (B) 10 Mpc; (C) 30 Mpc; (D) 100 Mpc in the approximation of neglecting red-shift effects. We assume $\alpha=\frac{1}{137}$, $\alpha'=\frac{1}{40}$, $\gamma_{\rm em}=100$, and $M_F=10^{15}$ GeV ($g=1$). We have superimposed the "QCD" results with Eq. (3.1) (solid line) upon Eq. (3.4) (dots) to indicate that they are nearly indiscernible. The horizontal line (dot-dashed) represents the Fly's Eye's normalization of the differential spectrum extrapolated up to 10^{20} eV. Above 10^{20} eV we assume a limit determined by a fixed number of events in a given period of time (equivalent to a constant integrated spectrum, independent of energy).

above 10^{20} eV, i.e., a limiting line going as $\approx E^2$.

For the assumed value of the source luminosity we can barely tolerate a single distant point source at range $R \approx 30$ Mpc. Of course, as we see subsequently, by reducing M_F we can have a tolerable contribution to the spectrum. Diffusion in the presumed intergalactic magnetic fields is not a significant effect here. For $B \approx 10^{-9}$ the Larmor radius is of order 100 Mpc for UHE nucleons of 10^{20} eV, larger than the range quoted above. This implies that a point source would be smeared out over a few steradians, i.e., would become a general anisotropy at these energies. By 10^{21} eV the diffusion effects are completely negligible and one would observe a true point source.

Note that the comparison given in Fig. 2 is to the observed diffuse spectrum at these energies since no point sources are seen and the number of events too low to apply meaningful statistical clustering tests. Since at the highest energies we would predict a true point source, the limit we give on range to the source is probably a firm *lower limit,* since we have understated the observed flux by a factor of [(detector angular resolution)$/4\pi]^{-1}$, i.e., we have averaged the predicted flux over 4π steradians. If one does a rudimentary clustering analysis of the arrival directions of the most energetic events a limit on the range of order $R > 100$ Mpc is expected.

We turn now to a more detailed analysis of source versus red-shift distributions. Our procedure for treating the collisional and red-shift effects is much simpler than that employed in Ref. 5 but is reasonably faithful to most of the gross effects. We do not treat the thermal fluctuations in target photon energy and the detailed evolution dynamics have been considerably simplified by approximate recoil distributions.

We will assume the rate densities are determined as in Sec. II C. Thus, the source activity density is given by

$$j(E,z)=\frac{dn(z)}{dL}L_s\dot{N}\frac{1}{M_F}\left[f_B\frac{dN}{dx}\bigg|_{x\to E/M_f}\right] \tag{3.8}$$

and using the result for these quantities as obtained in Sec. II we have

$$j(E,z)=\frac{\gamma_{\rm em}\alpha H_0{}^4}{\kappa_1{}^2}\frac{g}{ev}\left[\frac{f_\Phi\langle\Phi\rangle_z}{\ln(L/2\pi l)}\right](1+z)^6 \times\left[f_B\frac{dN}{dx}\bigg|_{x\to E/M_f}\right] . \tag{3.9}$$

The flux measured at Earth requires integrating over radial cosmic coordinates $r(z)$ and is given by

$$J(E,z)=\int_0^\infty \frac{j(E/(1+z),z(r))R(t_r)^3t^2dr}{R(t_0)^2r^2(1-kr^2)^{1/2}}e^{-r/\Lambda'[E/(1+z)]} \tag{3.10}$$

$$=\int_0^\infty \frac{j(E/(1+z),z)(1+z)^{-4}dz}{(1+2q_0z)^{1/2}H_0}e^{-r(z)/\Lambda'[E/(1+z)]}, \tag{3.11}$$

where $\Lambda'(E)$ depends upon red-shifted energy. We assume heretofore that the deceleration parameter is $q_0=\frac{1}{2}$. We find then using the approximate expressions for the flux as given in Eq. (2.26) that the spectrum for $p<\frac{1}{2}$ is

$$J(E,z)\approx\frac{2\gamma_{\rm em}\alpha g^2H_0{}^4f_Bf_\Phi B_0\lambda_0{}^2}{eM_F\kappa_1{}^2(1-2p)\ln(L/2\pi l)}\left[\frac{\kappa_1}{c}\right]^{(1+2p)/3}\int_0^\infty dz(1+z)^{1-p}\theta(E_c/(1+z)-E)\left[\frac{dN}{dx}\bigg|_{x\to E/(1+z)M_f}\right], \tag{3.12}$$

where $\theta(E_c/(1+z)-E)$ [$\theta(x)$ is the ordinary step function] approximates the large red-shift cutoff effects.[5] There is an analogous expression corresponding to the magnetic evolution $p>\frac{1}{2}$ using Eq. (2.27). In practice, we must evaluate this numerically. We find no significant differences between the simple distribution, Eq. (3.4) and that of Eq. (3.1).

In Fig. 3 we present the results for the evolved injection spectra following various assumptions about the primordial B field as parametrized by p and a fermion mass

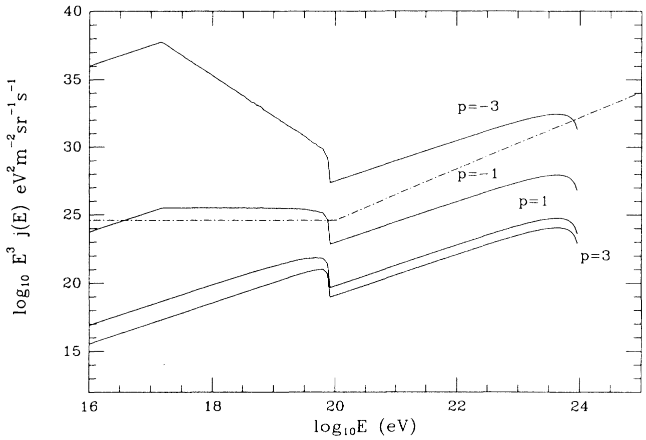

FIG. 3. The UHE nucleon spectrum integrated over red-shift for $M_F = 10^{15}$ GeV and all other parameters as in Fig. 2.

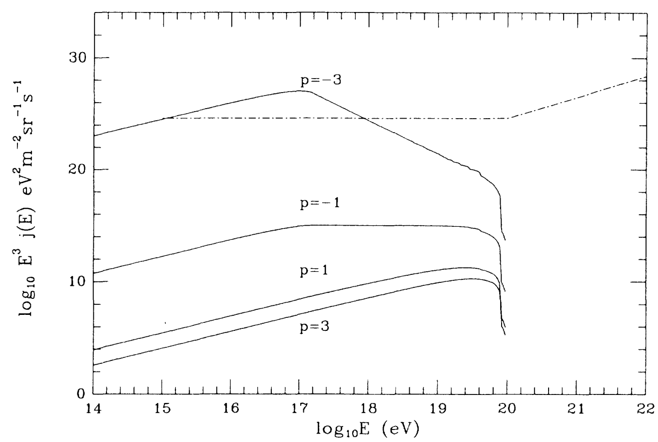

FIG. 5. The UHE nucleon spectrum integrated over red-shift for the indicated magnetic-flux history parametrization, p. We assume here the minimal $M_F = 10^{11}$ GeV and all other parameters as in Fig. 2.

equal to $v = 10^{15}$ GeV. We see that negative p can be ruled out, and the possibility of a dip extending to very high energies. Here the dashed line is defined by the Fly's Eye normalization of the UHE spectrum,[14] which has been extrapolated above 10^{20} eV by assuming a constant (with energy) integrated spectrum, with normalization fixed by the limit at 10^{20} eV, produces the same likelihood of a recorded event per unit time.

In Figs. 4 and 5 we consider the light fermion masses of 10^{13} and 10^{11} GeV, respectively. We see that a light fermion of 10^{11} GeV is minimally compatible with OTW and cannot be ruled out. These cases *may in fact account in part for the observed UHE cosmic rays.*

B. The neutrino spectrum

Presently we consider the neutrinos which may emerge from (a) direct decay products of the massive fermion ejected from the string, (b) fragmentation products from the decays of pions in the quark jets, and (c) induced neutrinos from the proton collisions with microwave photons

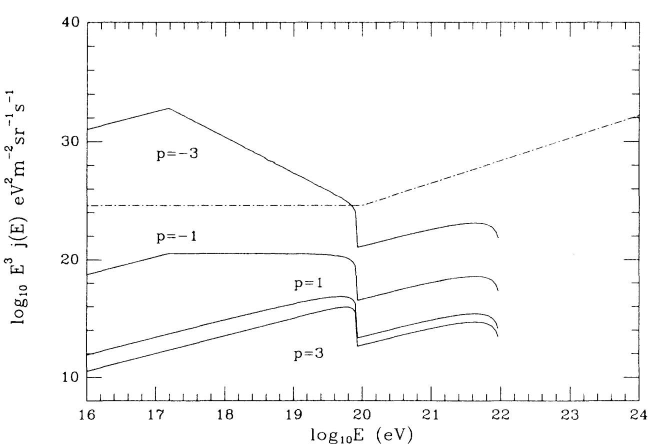

FIG. 4. The UHE nucleon spectrum integrated over red-shift for $M_F = 10^{13}$ GeV and all other parameters as in Fig. 2.

in transit to detector.[5,6]

Neutrinos produced by mechanism (a) constitute approximately a flat energy distribution at injection (boosted β-decay distribution):

$$\frac{dN_v}{dE} \simeq \frac{3}{M_F}\theta(M_F/3 - E(1+z)) \ , \tag{3.13}$$

where E is the energy at detection. In mechanism (b) each muon neutrino produced in charged pion decay has a flat energy (x) distribution up to the pion energy. Upon convoluting the quark fragmentation distribution with this flat spectrum we obtain

$$\begin{aligned}\frac{dN_\nu}{dE_\nu} &= \int \theta(x - x_\nu)\frac{1}{x}\frac{dN_\pi}{dx}dx \bigg|_{x \to E/M_f} \\ &= \frac{15}{16}\left[\frac{16}{3} - \frac{6x_v{}^2 + 12x_v - 2}{3x_v{}^{3/2}}\right]\bigg|_{x \to E/M_f}\end{aligned} \tag{3.14}$$

and the pions, for simplicity, have been described by the quark fragmentation distribution of Eq. (3.4) Finally (c), the neutrinos produced by the decays of secondary pions in $N + \gamma \to \pi + N'$ are determined by the nucleon spectrum above the Greisen cutoff energy. Let $x = 1$ correspond to the maximum energy and x_c the Greisen cutoff energy. Thus it has essentially the form of Eq. (3.14) for $x > x_c$ and is flat for $x < x_c$. In normalizing this contribution we must include the multiplier effect due to the fact that the recoil nucleon N' does not drop immediately below threshold and the average number of successive photoproduction reactions per nucleon is roughly $0.3\ln(x/x_e)$.

These three contributions to the neutrino spectrum are shown in Fig. 6. By far, the multiplicity is dominated by the pion decays in a quark jet. In Fig. 7 we present the results for the evolved neutrino spectra following various assumptions about the primordial B field as parametrized by p. We assume here that the fermion mass is equal to v, or $g = 1$ and $v = 10^{15}$ GeV. We see

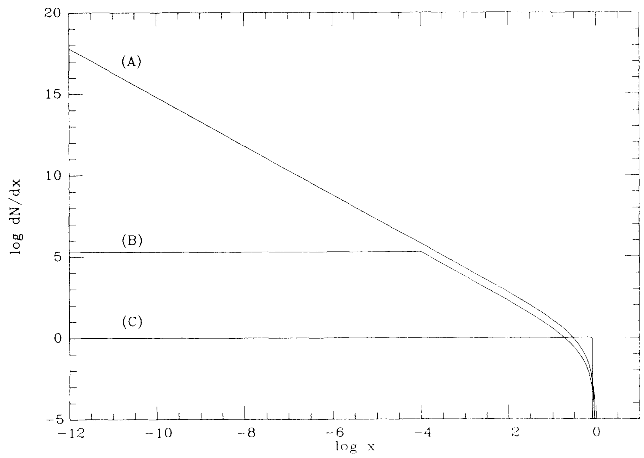

FIG. 6. The x distribution of produced neutrinos due to (A) quark fragmentation pion decays; (B) secondary pion decays in $N+\gamma(3\ ^\circ\mathrm{K})\rightarrow\pi+N'$ assuming $x_c=10^{-4}$; (C) direct neutrinos from heavy fermion decay. Note that $x=1$ correspond to maximum energy $\approx M_F/3$ in three-body decays.

that all $p<-1$ can be ruled out. Here the dashed line is defined by the Fly's Eye limit on deeply penetrating particles[14] using total neutrino cross sections of Ref. 15, and which has been extrapolated above 10^{20} eV by assuming a constant (with energy) integrated spectrum with normalization fixed by the limit at 10^{20} eV produces the same likelihood of a recorded event per unit time. Remarkably, the limit on the neutrino spectrum is somewhat more constraining than that of the nucleon spectrum, a consequence of the larger number of neutrinos per heavy fermion decay than baryons and the lack of a Greisen cutoff.

In Figs. 8 and 9 we consider the light fermion masses of 10^{13} and 10^{11} GeV, respectively. We see that a light fermion of 10^{11} GeV is minimally compatible with OTW and cannot be ruled out.

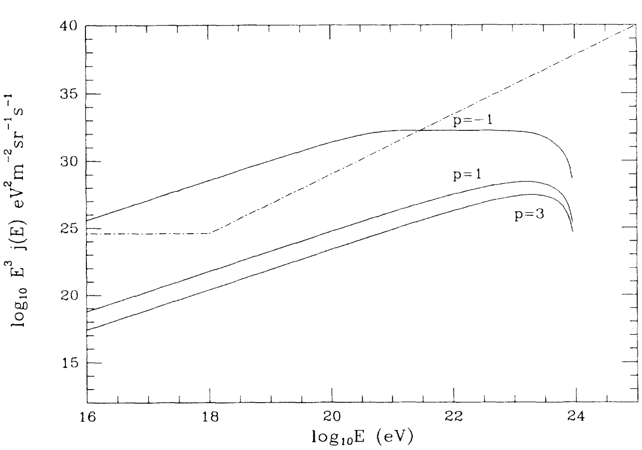

FIG. 7. The UHE neutrino spectrum integrated over red-shift for $M_F=10^{15}$ GeV and all other parameters as in Fig. 2.

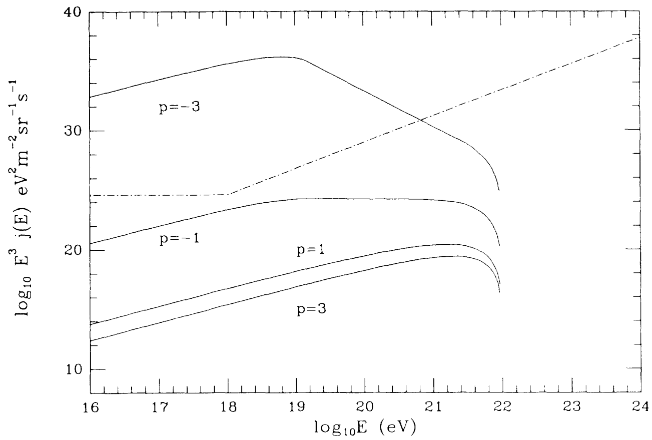

FIG. 8. The UHE neutrino spectrum integrated over red-shift for $M_F=10^{13}$ GeV and all other parameters as in Fig. 2.

IV. SUMMARY AND CONCLUSIONS

A. General conclusions

In this work we have discussed the production of ultra-high-energy cosmic rays by superconducting loops of cosmic string. We assume that the superconducting cosmic strings have size distributions as a function of red-shift and mass per unit length so that they are consistent with galaxy formation scenarios. A general parametrization of the magnetic flux history of these loops then allows one to calculate the production of massive fermion pairs from saturated loops and to follow the evolution of the high-energy cosmic rays produced by their decay. Our calculations may be summarized as follows

(1) An active loop of superconducting cosmic string within 30 Mpc, which decays into fermions with masses of the order of the VEV of the broken U(1) which produced the string ($\approx 10^{15}$ GeV), would generate observable

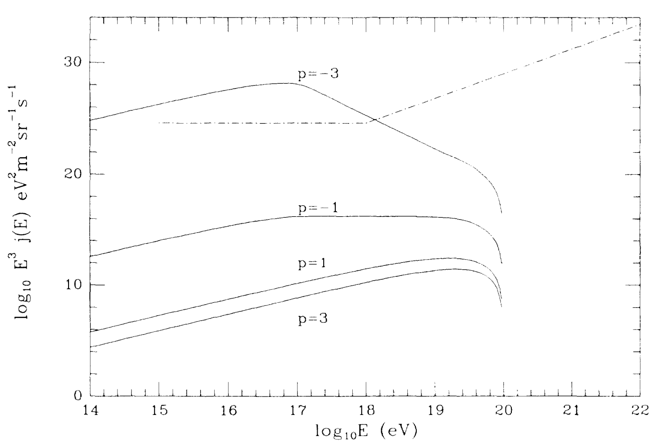

FIG. 9. The UHE neutrino spectrum integrated over red-shift for the indicated magnetic-flux history parametrization, p. We assume here the minimal $M_F=10^{11}$ GeV and all other parameters as in Fig. 2.

cosmic rays above the Greisen cutoff ($\approx 10^{20}$ eV). While this prediction is independent of the magnetic field history of the loop (although the density of such objects in the Universe is), the critical range outside which such a loop could not be "seen" can be tuned by an appropriate choice of fermion mass.

(2) Since the cosmic-ray "acceleration" mechanism envisioned here involves a fundamental process rather than a complicated dynamolike mechanism we expect in general (i) a preponderance of neutrinos over nucleons of order $\sim 30\times$ and (ii) *no heavy nuclei at the highest energies,* $\gg 10^{18}$ eV. This latter point is extremely important and may be subject to definitive test in the next few years. If the composition above 10^{19} eV is significantly iron rich, it is impossible to have a fundamental process as the origin of UHE cosmic rays and much of the transport analysis as in Ref. 5 also becomes irrelevant.

(3) The evolved cosmic-ray spectrum which results from the contributions of large-red-shift active superconducting cosmic strings can be made consistent with the observed ultra-high-energy cosmic rays by tuning the fermion mass and/or the evolution of magnetic fields. In fact, if we assume a magnetic field energy density which scales as the radiation density to a value of $\approx 10^{-9}$ G today, then our calculations indicate that the cosmic rays produced by evaporating superconducting cosmic strings could be responsible for all of the observed ultra-high-energy cosmic rays.

In both cases, decreasing the fermion mass or requiring that intergalactic magnetic fields are produced during a recent epoch will reduce the resultant flux of cosmic rays. It is striking that UHE cosmic-ray physics places such limits upon a hypothetical microphysical process and conversely that such fundamental processes may be observable by large-scale cosmic-ray detectors.

It should be noted that it is difficult at best to produce extremely energetic cosmic rays by conventional mechanisms, i.e., by "weather." Also, our present proposal is generic to mechanisms involving the decay of topological defects of cosmic origin, such as a monopole-antimonopole bound state,[10] and may also include collapsing or annihilating domain walls, and false-vacuum bubbles, as well as bosonic cosmic strings. All of these are likely to produce effects similar to those discussed herein, and it is certainly to be hoped that we might have observational access to such phenomena.

B. Additional remarks

Subsequent to the initial release of this work we have enjoyed many useful discussions with various colleagues and find that additional issues may be considered in the context of this problem. We wish to argue presently that the main results presented here are essentially unaffected by many additional considerations, though, under certain circumstances, they may be significantly modified.

An important point raised by Spergel[16] is the effect of cusps in the string on this analysis. Cusps may exist and become singular points on the string which can attain extremely large relativistic γ factors, $\sim 10^7$. Fermions ejected from these points may therefore have large relativistic energies prior to their decay and our estimate is therefore significantly modified. Though we agree that this should be further scrutinized, it is not clear to us that this is a real or significant effect on the charge carriers. The actual curvature at a cusp depends upon the detailed microphysics of the vortex solution and may not be large enough to significantly alter the rate at which charge carriers are ejected; ejection in the vicinity of the cusp is probably given more or less by Eq. (2.13) as $\dot{N}/L$ *in the rest frame of the cusp* and the external observer therefore sees, by time dilation, a deficiency of particles coming from the highly relativistic segments of string. Also, one must carefully distinguish between the current motion and the vortex motion, or world-sheet motion. Even though the latter becomes relativistic the current may remain small. Nonetheless, a further analysis of this potentially important point goes beyond the present paper.

An extremely important criticism was raised by Thompson and Ostriker[16] which contends that if the charge carriers are ejected from the string and immediately decay, the magnetic fields are so large near the string that Larmor radiation is instantaneous and one rapidly produces plasma of photons and electrons. In fact, quarks will preferentially Larmor radiate gluons which produce quark-antiquark pairs, so the likely outcome could be a thermal quark-gluon-lepton-boson plasma.

However, it is not unreasonable that the current carriers have lifetimes sufficiently long that they drift into the weak-field region and decay without significant Larmor radiation (this can presumably only happen for the fermionic string). We might expect that the lifetime is determined by a unification scale above M_X, e.g., by M_{Planck}, and the corresponding lifetime of the charge carrier is of order $\tau \approx (M_{\text{Planck}})^4/\alpha^2 M_X{}^5$. Assuming ejection with $\beta \approx 0.3$ and the γ factor for a decay product quark of order M_F/Λ_{QCD}, one easily sees that the fermion can drift into a region of sufficiently weak field that Larmor radiation is of no consequence (we use the criterion that the energy loss per turn in a uniform field of strength equal to the string field at r be small compared to the energy; then the particle will not even be trapped since the Larmor radius is of order r/α). This is a reasonable *underestimate* of the lifetime since we have supplied no Cabibbo-angle suppression to the decay rate. Nonetheless, the point of Thompson and Ostriker suggests that one should analyze carefully the plasma possibility. This is not unrelated to the following issue.

Recently Amsterdamski[16] has argued that large currents do not occur because of pair production in the vacuum in the presence of the large inhomogeneous magnetic field. This argument is simply incorrect. First, our current carriers have unspecified quantum numbers other than electric charge—they could carry baryon number, or something exotic, and that could be strictly conserved quantum number. Then pair creation of e^+e^- cannot neutralize the current. Even if the latter occurred and somehow collapsed the field, it would have no effect upon the arguments given here (and would only help in answering the previously discussed point of Thompson and Ostriker). As the string shrinks our current increases, eventually saturating, independent of vacuum-polarization effects. On the other hand, if this effect is relevant it *would* have drastic consequences for the OTW scenario.

One may further argue, however, that Amsterdamski's basic conclusion is wrong. The vacuum no doubt is rearranged by strong inhomogeneous B fields, but how? Simply arguing the effective potential of the free theory in the presence of such a field develops an imaginary part, Amsterdamski's analysis, does not answer this question. One might expect that when the Dirac operator is separated into positive and negative energy quasi-Landau levels, that one gets a stable Dirac sea in these states which are Bogoliubov combinations of positive- and negative-energy plane waves. Some particle production occurs while the field is "turned on" as the vacuum rearranges itself into negative-energy Landau levels, but then we have Fermi degeneracy of some of the positive-energy levels in the plane-wave basis which stabilizes the system against further decay. Amsterdamski's calculation is done in a negative-energy plane-wave Dirac sea (a state devoid of positive-energy plane waves) and he is therefore insensitive to the Fermi degeneracy which will zero his imaginary part; he is really only probing the onset of the vacuum rearrangement in his calculation and not the final state. Note too that the magnetic field will have to rearrange itself in this process.

Finally, some have remarked[16] on the possibility of internal annihilation of the particle right movers and antiparticle left movers which constitute the current. Any such effect must show up in the appropriate axial-vector current divergence which controls the phenomenon (see Hill and Widrow[17]). Certainly the anomaly itself contributes; this is just the normal radiation produced by the accelerated string. Other terms could occur which are suppressed by large mass scales and do not occur in simple models. Again, the right and left movers are best viewed as independent flavor species and need not communicate at scales below M_X.

ACKNOWLEDGMENTS

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