PHYSICAL REVIEW D VOLUME 36, NUMBER 4 15 AUGUST 1987

Cosmic balls of trapped neutrinos

Bob Holdom
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
(Received 29 December 1986)

Fermions trapped inside a closed domain wall may cool to degeneracy and form a long-lived structure. In the context of spontaneous left-right-symmetry breaking, we show that trapped right-handed neutrinos cool due to annihilations to electron-positron pairs if the initial temperature is less than $0.21m_e$. The surface tension of the wall must be less than $(1.93\ \text{TeV})^3$. The lifetime of the neutrino ball (NB) is determined by neutrino annihilation to three photons and may be comparable to the age of the Universe. These NB's are in the 10^4-10^7 solar mass range and radiate γ rays in the few hundred keV range at a rate of $10^{40}-10^{44}$ erg/sec range. NB's die in a $10^{56}-10^{59}$ erg electron-positron burst.

INTRODUCTION

In the early Universe domain walls may have existed such that degenerate vacuum states on opposite sides of a wall are related by a discrete symmetry.[1] Particles on opposite sides which are related to each other by the discrete symmetry operation have the same mass. But it is possible that a particle has a mass which changes across the wall. Such a particle with energy small compared to the mass difference will not penetrate the wall. A wall enclosing a volume will then trap such particles. For decreasing volume the pressure of the gas grows much more quickly ($\propto V^{-4/3}$ for adiabatic compression of a relativistic gas) than the pressure exerted on the gas by the surface tension ($\propto V^{-1/3}$). The structure is stabilized about some volume; we wish to study the properties and evolution of these structures.

Domain walls arise from a spontaneous breakdown of a discrete symmetry, and we will base our analysis on an explicit example of this phenomenon. We will treat the case of spontaneous parity (and charge-conjugation)[2] breakdown in a left-right-symmetric theory. We take the order parameter for this breakdown to be a large Majorana mass for right-handed (RH) neutrinos, $\langle \nu_R \nu_R \rangle \neq 0$. (The reader may assume that this order parameter develops either due to appropriate Higgs structure or due to appropriate strong gauge dynamics in the absence of Higgs fields.) In another region of space it may be the LH neutrino which develops a large mass, and this region will be separated from our region by a domain wall. RH neutrinos on the other side of the wall from us are light (with masses equal to our LH neutrino masses). If their energies are small compared to the large RH neutrino mass on our side then they will not be able to pass through the wall to us. All other fermions have equal mass on either side. We refer to balls of trapped light RH neutrinos as neutrino balls (NB's).

Domain walls in the early Universe must not dominate the energy density and influence the evolution of the Universe in an unfavorable way.[1] But causality implies that there is at least one wall stretched across each horizon volume. Then, if the surface tension of the wall is σ, the time at which the one wall dominates the total energy contained in the horizon volume is[1]

$$\hat{t} \approx 0.03/(G\sigma) \ . \tag{1}$$

[We have used $\rho_b = 3/(32\pi G t^2) \approx 0.03/(Gt^2)$ for the background radiation energy density of the Universe.] There are two mechanisms by which these "infinite" walls may disappear before $\hat{t}$. One involves cosmic strings which form earlier than the walls and which become boundaries of walls when walls form. The system of hole-dominated walls and walls bounded by strings eventually decays away before $\hat{t}$ (Ref. 3). Note that the spontaneous creation of a hole in a wall has an exponentially suppressed tunneling probability and that this phenomena plays no role in the infinite-wall removal.[3] Rather, a piece of wall bounded by string striking an infinite wall suffices to puncture the latter. The evolution and disappearance of the wall-string system has been discussed for the case of interest here, that of a left-right-symmetric theory emerging from SO(10) (Ref. 3). The other mechanism involves a slight explicit breaking of the discrete symmetry.[1,4] This slightly favors one vacuum over the other and eventually the walls get pushed everywhere toward the side of higher vacuum energy. This mechanism creates an additional complication to the following analysis which we will treat. But for the most part we will not be concerned with the actual mechanism for wall removal. We only assume that some NB's remain after the infinite walls (and strings) have disappeared.

We will be interested in the possibility of long-lived NB's. We will find that NB's may undergo a cooling and become long-lived if $\sigma^{1/3} \lesssim 1.93$ TeV. It might be expected that $\sigma^{1/3}$ is of the order of the scale of left-right-symmetry breaking, in which case we are identifying consequences of the latter occurring at a relatively low energy. But inside the wall thickness both $\langle \nu_L \nu_L \rangle$ and $\langle \nu_R \nu_R \rangle$ will be nonzero and thus σ is not directly related to the potential-energy difference between the symmetric phase [when both vacuum expectation values (VEV's) are zero] and the broken phase (when one VEV is nonzero). Thus for suitable values of parameters in a Higgs potential, for example, $\sigma^{1/3}$ could be arbitrarily small compared to the left-right-breaking scale. We will say no more about the physics of left-right-symmetry breaking. To give the reader some idea of the time scales of interest we note that $\hat{t} \approx 3000$ sec for $\sigma \approx (1\ \text{TeV})^3$. The temperature of the Universe at this time is about 20 keV, well below the electron mass.

We start by assuming that the NB wall is completely opaque to neutrinos. We will discuss initial NB properties, the cooling phenomena, and the NB lifetime before returning to the question of possible leakage through the NB wall. We then treat the case of a slightly preferred vacuum. In the concluding section we describe the properties and evolution of NB's living much longer than the cooling time.

INITIAL PROPERTIES

Inside a NB we have a relativistic gas of n_ν flavors of light stable RH neutrinos. We assume the gas starts with total neutrino number zero and with zero chemical potential for each flavor. The number of neutrinos of one flavor will always equal the number of antineutrinos of that flavor.

First consider a static spherical NB. The internal neutrino pressure, always $\frac{1}{3}$ of the neutrino energy density ρ, must balance the pressure exerted on the neutrinos by the wall tension. We ignore here the external LH neutrino pressure which eventually becomes negligible due to the expansion of the Universe. The wall tension pressure is $2\sigma/r$ if the radius of the NB is r. Thus

$$\rho = 6\sigma/r \ . \tag{2}$$

We then find the total mass of the NB ignoring gravitational effects:

$$M = 4\pi r^2 \sigma + \tfrac{4}{3}\pi r^3 \rho = 12\pi r^2 \sigma \ . \tag{3}$$

Relations (2) and (3) rely only on the neutrinos being relativistic and will thus be true throughout the evolution of the NB.

For an initial temperature T_i the initial energy density is $\rho_i = (7\pi^2 n_\nu/120) T_i^4$ and the initial number density of neutrinos is $n_i = 0.182\,69 n_\nu T_i^3$. Inserting ρ_i into (2) gives a relation between T_i and r_i. Thus for a given total number of trapped neutrinos $N = n_i V_i$, the initial T_i and r_i are determined.

We note that there is an upper limit on the initial mass M_i since if M_i is too large the NB becomes a black hole. Requiring $2GM_i < r_i$ and using (3) gives

$$M_i < (48\pi G^2 \sigma)^{-1}$$
$$\approx 1.3 \times 10^8 / \sigma(\text{TeV}^3) \text{ solar masses} \ . \tag{4}$$

(This bound is approximate since we have not incorporated gravitational effects in the determination of the mass.) We also note that a NB which is not close to being a black hole is then automatically smaller than the horizon size at time $\hat{t}$. This allows NB's to form before the infinite walls must disappear.

In general, NB's will be formed with a nonspherical shape and with moving walls. In a vacuum the result would be NB's oscillating around the equilibrium configuration. But the background density of LH neutrinos which are reflected at the NB surface causes a damping of these oscillations. The typical damping time is

$$\tau_d \approx \sigma/\zeta\rho_b \approx t^2/\zeta\hat{t} \ , \tag{5}$$

where ζ is the neutrino fraction of the total background energy density ρ_b. For example, if $n_\nu = 2$ and if there are no other massless particles besides photons then ζ tends toward ≈ 0.3 as the electrons and positrons annihilate. NB's formed before $\zeta\hat{t}$ will have their oscillations damped within one expansion time to end up as static, spherical NB's. NB's formed after $\zeta\hat{t}$ will not have their oscillations damped in this way, but kinetic energy will still be lost through gravitational radiation. The radiation loss for large oscillations may be estimated (Vilenkin[3]) using the quadrupole formula, noting that the typical frequency of oscillation will be $\omega \approx r^{-1}$:

$$\dot{M} \approx -GM^2 r^4 \omega^6 \approx -G\sigma M \ . \tag{6}$$

Thus in a time $\approx \hat{t}$ the large oscillations of these NB's will also be damped. We therefore continue our analysis of static NB's or NB's with an amplitude of oscillation small compared to the NB radius.

COOLING

Inside the NB the RH neutrinos have the same weak interactions (only parity flipped) as the LH neutrinos outside the NB. Thus trapped neutrinos and antineutrinos may annihilate to electron-positron pairs. The produced electrons and positrons are able to drift outside the NB. If $T_i \gtrsim m_e$ these annihilations produce a short and uninteresting lifetime for the NB. If T_i is sufficiently below m_e then a more interesting phenomena can take place. Only those neutrinos on the high-energy tail of the thermal distribution have enough energy to annihilate. As this process continues the average energy of the remaining neutrinos drops. Thus the neutrinos cool and a nonzero chemical potential develops, the same for neutrinos and antineutrinos of each flavor. The decreasing average energy makes it more and more difficult to produce electron-positron pairs through annihilation and the process slows. We are left with a ball of relativistic neutrinos slowly approaching degeneracy.

We note that the neutrinos originally decoupled from the electrons when the temperature of the Universe was $\approx 2m_e$. The NB's of interest are formed well below this temperature so that the trapped neutrinos are decoupled from the background radiation and matter in the Universe. It is easy to check that they remain so over the lifetime of the NB. We also note that some electrons and positrons emitted by the NB itself may remain gravitationally bound to the NB. But rapid annihilations to photons keep this electron-positron cloud from building up to levels significant enough to affect the NB.

We wish to derive the critical initial temperature below which the cooling phenomena occurs. We first find the relation between the change in the total mass M of the NB and the change in the total number N of trapped neutrinos, due to annihilations to e^+e^-. A neutrino and antineutrino each having energy $\approx m_e$ may annihilate to e^+e^-, implying $dM \approx m_e dN$. We may write the result of integrating over neutrino energies and phase space as $dM = \chi m_e dN$. Since we are supposing that the temperature is well below m_e the Boltzmann factor ensures that most of the contribution arises from neutrinos with energies $\approx m_e$. Then χ is a number only slightly larger than one. χ also has a slight time dependence, approaching unity even closer as the NB cools. For simplicity we set $\chi = 1$. But we then anticipate a small correction to our result for the critical temperature.

As we check below, the cooling process is slow enough so that the NB continues to satisfy (2) and (3) as it shrinks. Then M is a factor of $\frac{3}{2}$ times the energy in neutrinos at all times. Combining $M = \frac{3}{2}(\rho/n)N$ with $dM = m_e dN$ gives

$$dM/M = (2m_e n/3\rho)dN/N \ . \tag{7}$$

Equations (2) and (3) also imply that

$$dM/M = 2dr/r = -2d\rho/\rho \ . \tag{8}$$

Then since $n = N/(\frac{4}{3}\pi r^3)$,

$$\begin{aligned} dn/n &= dN/N - 3dr/r \\ &= [-(3\rho/m_e n) + 3]d\rho/\rho \ . \end{aligned} \tag{9}$$

For decreasing M, ρ and n are increasing. But the average energy per neutrino, ρ/n, will decrease if $3-(3\rho/m_e n) > 1$ or

$$\rho/n < \tfrac{2}{3} m_e \ . \tag{10}$$

Thus ρ/n will continue to decrease after starting to decrease. By inserting the initial values ρ_i and n_i into (10) we find that the initial temperature must satisfy

$$T_i < T_c \quad \text{where } T_c \equiv 0.211\,55 m_e \ . \tag{11}$$

Henceforth we use $y \equiv T_i/T_c$ in place of T_i.

The initial NB mass is

$$M_i = (6\,220\,800\sigma^3/49\pi^3 n_\nu^2 T_c^8)y^{-8} \equiv M_c y^{-8} \ . \tag{12}$$

The upper bound $y < 1$ means that M_c is a lower bound on M_i:

$$M_c \approx 2\times 10^5[\sigma(\text{TeV}^3)]^3/n_\nu^2 \text{ solar masses} \ . \tag{13}$$

Combining this lower bound on M_i with the upper bound in (4) we find that for NB's to exist at all we must have

$$\sigma^{1/3} \lesssim 1.93 \text{ TeV} \ . \tag{14}$$

We now discuss the cooling phenomenon in more detail in order to find the NB mass after cooling. We may rewrite (9) as

$$dn/d\rho - 3n/\rho + 3/m_e = 0 \ , \tag{15}$$

which may be solved to give

$$n(\rho) = 3\rho/2m_e + C(T_i)\rho^3 \ . \tag{16}$$

$C(T_i)$ is determined by the initial n_i and ρ_i. Defining $x \equiv \rho/\rho_i$ and $w \equiv n/n_i$ we find that (16) is equivalent to

$$w(x) = xy + x^3(1-y) \ . \tag{17}$$

In the limit of complete neutrino degeneracy we have $n_d = n_\nu \mu_d^3/3\pi^2$ and $\rho_d = n_\nu \mu_d^4/4\pi^2$ for some chemical potential μ_d. The relation between n_d and ρ_d may be written in terms of $x_d \equiv \rho_d/\rho_i$ and $w_d \equiv n_d/n_i$:

$$w_d(x_d) = 1.9244 x_d^{3/4} \ . \tag{18}$$

Thus a NB will tend toward a case of complete degeneracy if the functions in (17) and (18) intersect. A solution to $w(x_{\rm ac}) = w_d(x_{\rm ac})$ for $x_{\rm ac} \neq 0$ turns out to exist for all initial temperatures $y < 1.000\,231$. The fact that y may be slightly larger than unity implies that degeneracy may be achieved even though ρ/n increases slightly.

$x_{\rm ac}(y)$ is the ratio of energy density after cooling to the initial energy density: $x_{\rm ac}(y) = \rho_{\rm ac}/\rho_i = r_i/r_{\rm ac}$. We plot $x_{\rm ac}(y)$ in Fig. 1 [where $x_{\rm ac}(1.000\,231) = 21.968$]. From this we may obtain the limiting mass after cooling:

$$M_{\rm ac} = x_{\rm ac}(y)^{-2} y^{-8} M_c \ . \tag{19}$$

We also note that the chemical potential after cooling $\mu_{\rm ac}(y)$ has the maximum value $\mu_{\rm ac}(1.000\,231) = m_e$.

To find the cooling time we consider the evolution of the total number of trapped neutrinos $N(t)$:

$$\begin{aligned} (dN(t)/dt)/N(t) &= V(t)(dn/dt\,|_{\rm ann})/N(t) \\ &= (dn/dt\,|_{\rm ann})/n(t) \ . \end{aligned} \tag{20}$$

$(dn/dt\,|_{\rm ann})$ denotes the change in the number density due to annihilations [and not due to the changing total volume $V(t)$]. We find the time τ_c it takes for annihilations to significantly change N by inserting initial values of quantities into (20):

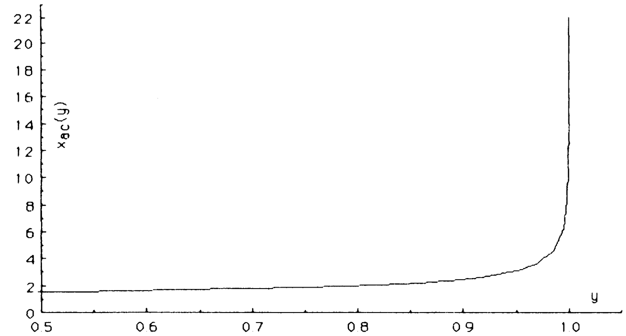

FIG. 1. $x_{\rm ac} \equiv \rho_{\rm ac}/\rho_i = r_i/r_{\rm ac}$ (ac denotes "after cooling") as a function of $y \equiv T_i/T_c$.

$$\tau_c^{-1} \approx 2(5G_F^2 m_e^2/3\pi)[(m_e^4/4\pi^4)T_i^2 \exp(-2m_e/T_i)](0.1827T_i^3)^{-1} \ . \tag{21}$$

Two neutrinos are lost per annihilation. The next factor in (21) is the weak-interaction annihilation cross section $\sigma(\nu_e\bar{\nu}_e \Rightarrow e^+e^-)$ for two neutrinos of energy $\approx m_e$, neglecting phase-space factors. The next factor is the squared number density of neutrinos of one flavor with energies $\gtrsim m_e$. And the last is $1/n$ (except that the $1/n_\nu$ factor has canceled). Thus the cooling time is

$$\tau_c \approx 2\times 10^3 y \exp(9.454/y) \text{ sec} . \tag{22}$$

$y=1$ gives the lower limit $\tau_c \gtrsim 1$ yr. (Neglected phase-space factors should increase τ_c no more than a factor of 10.)

The corresponding time scale for neutrino-(anti)neutrino scattering, which is not Boltzmann suppressed and which maintains thermal equilibrium, is of the order $10^4/y^3$ sec. Also, the process $\nu_i\bar{\nu}_i \rightarrow \nu_j\bar{\nu}_j$ maintains equal numbers of different flavors.

To study the time evolution in more detail we express the RH side of (20) in terms of $N(t)$ and then solve for $N(t)$. Suppressing the time dependence of quantities we write $N=Vn$, $V=V_i/x^3$, and using (17) we find

$$\tilde{N} \equiv N/\xi = y^3[y/x^2+(1-y)] \quad \text{where } \xi \equiv 0.1827 n_\nu V_i T_c{}^3 . \tag{23}$$

Then

$$1/x^3=[\tilde{N}/y^4+(y-1)/y]^{3/2} . \tag{24}$$

Thus

$$(d\tilde{N}/dt)/\tilde{N}=(1/n)(dn/dt\,|_{\text{ann}})=(1/\tilde{N})(V_i/\xi)(1/x^3)(dn/dt\,|_{\text{ann}})$$
$$\approx -(2\times 10^3 \text{ sec})^{-1}(1/\tilde{N})[\tilde{N}/y^4+(y-1)/y]^{3/2} f(\tilde{N}) . \tag{25}$$

It remains to determine the factor

$$f(\tilde{N}) \equiv [T(\tilde{N})/T_c]^2 \exp\{-2[m_e-\mu(\tilde{N})]/T(\tilde{N})\} \tag{26}$$

which cuts off the annihilations when $T(\tilde{N})$ becomes small. We do this by numerically inverting the relations

$$n=(n_\nu/\pi^2)\int_0^\infty d\epsilon\,\epsilon^2/\{\exp[(\epsilon-\mu)/T]+1\}, \quad \rho=(n_\nu/\pi^2)\int_0^\infty d\epsilon\,\epsilon^3/\{\exp[(\epsilon-\mu)/T]+1\} , \tag{27}$$

to express T and μ in terms of ρ and n which in turn are functions of $\tilde{N}$ through (24) and (17). Equation (25) may then be numerically integrated. A plot of the result for $y=\frac{3}{4}$ is shown in Fig. 2 where we have used $\tilde{N}(t)$ to obtain $M(t)/M_i$.

The above analysis has neglected kinetic energy of the wall as the NB shrinks. We may check this assumption by using (23) and the fact that $x=r_i/r$ to find the velocity of the wall:

$$dr/dt=(r_i/t)\{(\tilde{N}/2y^4)[\tilde{N}/y^4+(y-1)/y]^{-1/2}(d\ln\tilde{N}/d\ln t)\} . \tag{28}$$

Since $r_i/t \ll 1$ at the time of cooling we may conclude that the wall velocity is negligible (at least for y not extremely close to 1.000 231).

LIFETIME

We now treat the effect of neutrino annihilations to photons. We consider the diagram in Fig. 3 with some number of photons attached. With external momenta below the electron mass we need only consider an electron loop. We first show that the two-photon amplitude is negligible. The weak $\nu\bar{\nu}e^+e^-$ vertex in this diagram may be Fierz transformed to the form

$$\bar{\nu}_R\gamma^\mu\nu_R\bar{e}\gamma_\mu(a+b\gamma_5)e .$$

Thus the amplitude for this diagram takes the form

$$\bar{\nu}_R\gamma^\mu\nu_R T_{\mu\alpha\beta}(k_1,k_2)\epsilon_1^\alpha\epsilon_2^\beta , \tag{29}$$

where we let p_i^μ, k_i^μ, and ϵ_i^μ $(i=1,2)$ be the incoming neutrino momenta, the outgoing photon momenta, and photon polarizations, respectively. $T_{\mu\alpha\beta}(k_1,k_2)$ is the triangle diagram amplitude for one axial-vector current

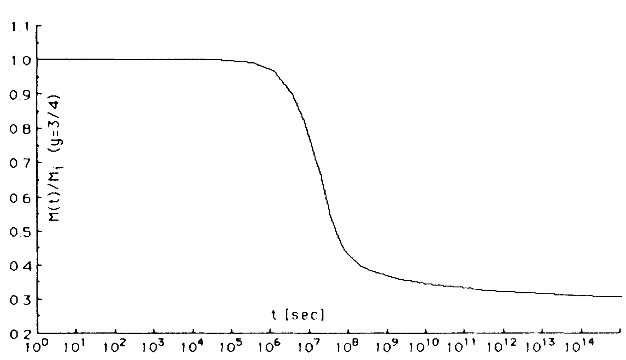

FIG. 2. The time evolution of the NB mass due to the cooling process for $y=\frac{3}{4}$.

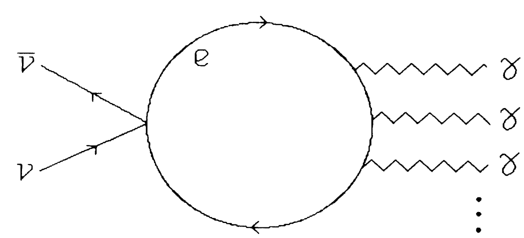

FIG. 3. Neutrino-antineutrino annihilation to photons.

and two vector currents. Gauge invariance ($k_1^\alpha T_{\mu\alpha\beta}=k_2^\beta T_{\mu\alpha\beta}=0$), Bose symmetry ($k_1,\alpha\leftrightarrow k_2,\beta$), and on-shell photons allow only the following two combinations in $T_{\mu\alpha\beta}(k_1,k_2)$:

(1) $(k_1+k_2)_\mu \epsilon_{\alpha\beta\sigma\rho} k_1^\sigma k_2^\rho$,

(2) $\epsilon_{\mu\alpha\beta\sigma}(k_1-k_2)^\sigma(k_1\cdot k_2)-\epsilon_{\mu\beta\sigma\rho}k_1^\sigma k_2^\rho k_{2\alpha}-\epsilon_{\alpha\mu\sigma\rho}k_1^\sigma k_2^\rho k_{1\beta}$.

We choose $\epsilon_1\cdot k_1=\epsilon_2\cdot k_2=0$ and $\epsilon_1\cdot(k_1+k_2)=0$. Thus our amplitude takes the form

$$\bar{\nu}_R\gamma^\mu\nu_R[A\,(k_1+k_2)_\mu\epsilon_{\alpha\beta\sigma\rho}k_1^\sigma k_2^\rho + B\epsilon_{\mu\alpha\beta\sigma}(k_1-k_2)^\sigma(k_1\cdot k_2)]\epsilon_1^\alpha\epsilon_2^\beta\ . \quad (30)$$

We need not determine A or B since both terms inside the brackets are proportional to $(k_1+k_2)_\mu=(p_1+p_2)_\mu$ (for the B term this is most easily seen in the center-of-mass frame). We therefore find that our amplitude is proportional to $\bar{\nu}_R(\not p_1+\not p_2)\nu_R$, implying a neutrino mass factor.

The vanishing of the $\bar{\nu}\nu\Longrightarrow\gamma\gamma$ amplitude to first order in G_F and for zero neutrino mass was first discussed by Gell-Mann.[5] A nonvanishing contribution appears at order $G_F{}^2$ (Ref. 6). This corresponds to the appearance of a dimension-8 effective operator involving two factors of $F_{\mu\nu}$, the factor $\bar{\nu}_R\gamma^\mu\nu_R$, and one derivative, appearing in the theory below the weak scale.

With the cosmological mass limit on light stable neutrinos in mind we conclude that the NB lifetime is determined by neutrino annihilations to three photons, since the amplitude for Fig. 1 with three photons is not proportional to the neutrino mass. Below the electron mass scale this amplitude corresponds to a dimension-10 effective operator involving three factors of $F_{\mu\nu}$, the factor $\bar{\nu}_R\gamma^\mu\nu_R$, and one derivative. Thus a $G_F m_e{}^{-4}$ factor appears in the amplitude. To estimate the numerical factors we note that the cross section $\sigma(\gamma\gamma\Longrightarrow\gamma\bar{\nu}\nu)$ has been calculated.[7] If $R\equiv\sigma(\bar{\nu}\nu\Longrightarrow\gamma\gamma\gamma)/\sigma(\gamma\gamma\Longrightarrow\gamma\bar{\nu}\nu)$ then the characteristic NB lifetime τ is

$$1/\tau\approx(n/2n_\nu)\sigma(\bar{\nu}\nu\Longrightarrow\gamma\gamma\gamma)$$
$$\approx(\mu_{\rm ac}{}^3/6\pi^2)4.5\times10^{-5}R\alpha^3G_F{}^2E_\nu{}^{10}/m_e{}^8\ . \quad (31)$$

We have expressed n in terms of the chemical potential after cooling, $\mu_{\rm ac}$. If we also set $E_\nu=\mu_{\rm ac}$ (we will improve this approximation below) then we have

$$\tau\approx5\times10^{14}R(m_e/\mu_{\rm ac})^{13}\ {\rm sec}\ . \quad (32)$$

This estimate assumes that $(m_e/\mu_{\rm ac})^{13}\gg1$. We see that the lifetime τ is much greater than the cooling time τ_c [at least for y not too small, see (22)]. The ratio $m_e/\mu_{\rm ac}$ may be obtained as a function of y:

$$m_e/\mu_{\rm ac}=2.165x_{\rm ac}(y)^{-1/4}/y\ . \quad (33)$$

We plot the factor $(m_e/\mu_{\rm ac})^{13}$ appearing in the lifetime versus y in Fig. 4.

We have obtained cosmologically interesting lifetimes which are independent of σ and thus independent of the underlying particle physics giving rise to walls (assuming that σ is less than our upper limit).

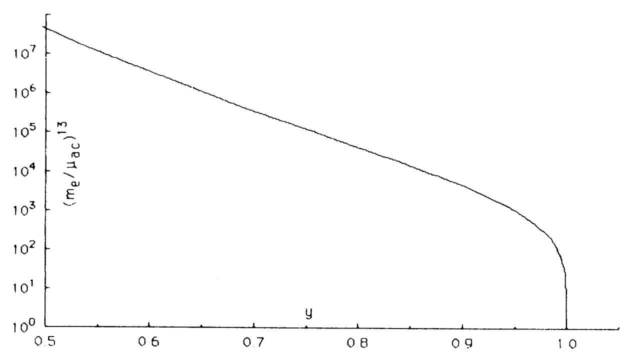

FIG. 4. The factor $(m_e/\mu_{\rm ac})^{13}$ appearing in the NB lifetime as a function of y.

LEAKAGE

We now consider possible leakage through the wall. If a Dirac condensate $\langle\bar{\nu}_R\nu_L\rangle$ develops inside the wall thickness of the same order of magnitude as the Majorana condensates then the neutrino mass eigenstates are completely different inside and outside the wall thickness. In this case it is possible that a ν_R colliding with the wall could convert and emerge as a ν_L on the outside. For the one-dimensional problem of a plane wave incident normally to the wall, the probability for transmission is $\approx l^2/\lambda^2$. l is the thickness of the wall and λ the wavelength of the incident plane wave. (This is the probability found for transmission of a boson through a domain wall not arising from a discrete symmetry.[8])

But the formation of a nonzero $\langle\bar{\nu}_R\nu_L\rangle$ breaks discrete symmetries. If the operations $\nu_L\Longrightarrow-\nu_L$ and $\nu_R\Longrightarrow-\nu_R$ are independent symmetries everywhere in space then $\langle\bar{\nu}_R\nu_L\rangle=0$ even in the wall. In this case the mass eigenstates do not change inside the wall (even though the masses do) and there is no way for a ν_R to convert to a ν_L. The ν_R would always be reflected. On the other hand, we should still contemplate a Dirac neutrino mass of order $\sqrt{m_Rm_L}$ or less generated by physics on lower mass scales (having nothing to do with the wall). Then the mass eigenstates will change slightly in the wall, and the transmission probability is now $\lesssim(l^2/\lambda^2)(m_L/m_R)$ (where these are neutrino masses on our side of the wall). In this case leakage of neutrinos is a negligible effect.

But even if the ν_R is always reflected, another possible effect of the collision with the wall could be to cause a virtual excitation of the wall to radiate a pair of neutrinos to the outside. Then energy is lost even though neutrinos do not escape. This is just another form of cooling. As energy is radiated and the radius r decreases, ρ increases with r^{-1} while n increases with r^{-3}. Thus $n\propto\rho^3$ and the neutrino gas is driven closer to degeneracy. But as degeneracy is approached further shrinkage due to energy loss stops; a completely degenerate ball of neutrinos can radiate no more.

We estimate the time scale for this form of cooling. The probability per collision for this process should be less than $\approx l^2/\lambda^2\approx E_\nu{}^2/\sigma^{2/3}$. We consider the total rate of collisions with the wall, $dN/dt\,|_{\rm wall}$, and note that

$$(dN/dt\,|_{\rm wall})/N \approx nA/nV = 3/r = (n_\nu/8\pi^2)(\mu_{\rm ac}{}^4/\sigma) \ . \quad (34)$$

We then find the time τ' at which this energy loss becomes significant:

$$1/\tau' \lesssim (dN/dt\,|_{\rm wall})N^{-1}(E_\nu{}^2/\sigma^{2/3}) \ ,$$
$$\tau' \gtrsim 10^{12}(m_e/\mu_{\rm ac})^6(\sigma^{5/3}/{\rm TeV}^5) \ {\rm sec} \ . \quad (35)$$

Thus τ' is far larger than the cooling time τ_c found above.

Only if a large $\langle \bar{\nu}_R \nu_L \rangle$ forms inside the wall thickness could the NB lifetime τ be affected. Henceforth we will assume that this is not the case (e.g., for an appropriate range of parameters in a Higgs potential).

A SLIGHTLY PREFERRED VACUUM

Thus far our analysis would apply to the case that strings were responsible for wall removal. We now instead suppose that a slight breaking of the vacuum degeneracy was responsible for wall removal. If the difference in vacuum energies on either side of a wall is ϵ then there is an extra pressure, also given by ϵ, exerted toward the side of higher vacuum energy. But before the preferred vacuum starts to dominate the volume of the Universe, two constraints must be satisfied. First, ϵ must exceed the radiation pressure of the neutrinos or otherwise this radiation pressure on both sides of walls will maintain equal volumes of the two vacua. Thus $\epsilon \gtrsim \tau\rho_b/3 \approx 0.01\zeta/(Gt^2)$ (ζ is the fraction of the background energy density in neutrinos). Second, ϵ must exceed typical forces due to surface tension before ϵ can influence the motion of walls. Since typical radii of curvature $\approx$ horizon size we have $\epsilon \gtrsim \sigma/t$.

At a time $\tilde{t}$ when both conditions become satisfied the vacuum with lower energy begins to dominate the volume of the Universe. If ϵ is such that $\tilde{t} \lesssim \zeta\hat{t}/3$ [$\hat{t}$ is defined in (1)] then one may show that the horizon size $\tilde{t}$ exceeds σ/ϵ. In this case, NB's may form both larger and smaller than σ/ϵ. The larger NB's have pressure exerted on the inside neutrinos due mostly to the ϵ pressure rather than the wall surface tension. On the other hand, if $\tilde{t} > \zeta\hat{t}/3$ then NB's are always smaller than σ/ϵ and the surface tension dominates. The evolution of NB's smaller than σ/ϵ would be as outlined above.

It is amusing to consider the NB's larger than σ/ϵ. The inside neutrino pressure, $\rho/3$, must balance ϵ, thus fixing the neutrino energy density ρ independent of radius. Then the mass of the NB is proportional to r^3. With this difference we can carry out the same analysis which led to (17). The corresponding expression is

$$w(x) = y + (1-y)x^3 \ , \quad (36)$$

where $w \equiv n/n_i$ and $x \equiv r_i/r (\neq \rho/\rho_i)$. y is defined as before and is now fixed from $\rho/3 = \epsilon$ rather than being related to the size of the NB. For $y<1$, ρ/n is now decreasing faster than previously for the same change in radius. This produces more efficient cooling. Eventually the radius becomes less than σ/ϵ and $w(x)$ switches back to the behavior in (17). The result would be a NB approaching degeneracy with a lower $\mu_{\rm ac}$ and hence a somewhat longer lifetime than the corresponding NB in the previous analysis.

LONG-LIVED NEUTRINO-BALLS

Here we are interested in NB's surviving until recent epochs and beyond. We therefore treat the case that the factor $(m_e/\mu_{\rm ac})^{13}$ appearing in the lifetime (32) is large. But from Fig. 4 we find only a weak constraint on y; interesting lifetimes occur for y close to unity. This in turn places a weak additional constraint on σ due to the black-hole upper limit on the initial NB mass. By comparing (4), (12), (13), and (14) we find

$$\sigma^{1/3} \lesssim 1.93 y^{2/3} \ {\rm TeV} \ . \quad (37)$$

We may work out the time evolution of a NB after it has cooled and as it gradually shrinks and dies due to neutrino annihilations to photons. The tendency for these annihilations to move the trapped neutrinos away from degeneracy competes with the opposite tendency arising from possible annihilations to e^+e^-. Since the latter cross section is so much larger (for neutrinos of sufficient energy), the trapped neutrinos remain close to degeneracy. We can use this fact to find the relative frequency of annihilations producing photons compared to those producing e^+e^-.

First we find the constraint on the average energy lost per neutrino per annihilation. If we denote this by $\bar{E}$ then we may repeat the arguments which led to (9) (by replacing m_e by $\bar{E}$). We have

$$dn/n = [-(3\rho/\bar{E}n) + 3]d\rho/\rho \ . \quad (38)$$

But if the NB remains close to degeneracy then from (18),

$$dn/n = \tfrac{3}{4} d\rho/\rho \ . \quad (39)$$

We conclude that $\bar{E} = \frac{3}{4}\rho/n = \mu$. (The time dependence of quantities is suppressed.) If p is the probability that an annihilation produces photons then

$$\mu = \bar{E} = E_\nu(\gamma\gamma\gamma)p + E_\nu(e^+e^-)(1-p) \ . \quad (40)$$

$E_\nu(\gamma\gamma\gamma)$ and $E_\nu(e^+e^-)$ are the average energies per neutrino per the respective annihilation mode. $E_\nu(\gamma\gamma\gamma)$ will be some significant fraction η of $\mu, E_\nu(\gamma\gamma\gamma) = \eta\mu$. (We will not attempt to determine η, but a guess would be $\eta \approx 0.9$.) And $E_\nu(e^+e^-) = \chi m_e$ where, as before, we lose little by setting $\chi = 1$.

We now solve for p to obtain

$$p = (m_e - \mu)/(m_e - \eta\mu) \ . \quad (41)$$

As expected, $p \Longrightarrow 0$ as $m_e/\mu \Longrightarrow 1$. And the two annihilation modes have equal probability for $m_e/\mu = 2-\eta$. Thus for $m_e/\mu(t)$ sufficiently greater than $2-\eta$, $p \approx 1$, and the decrease in the total number of trapped neutrinos is due mostly to annihilations to photons.

The shrinking NB has $n(t) \propto \mu(t)^3$ and $V(t) \propto 1/\rho(t)^3 \propto 1/\mu(t)^{12}$. Thus we may define $\hat{N}(t) \equiv N(t)/\xi'$

$=[m_e/\mu(t)]^9$ where $\xi' \equiv n_\nu V_{ac}\mu_{ac}{}^{12}/(3\pi^2 m_e{}^9)$. We have $(d\hat{N}/dt)/\hat{N} \approx (n/2n_\nu)\sigma(\bar{\nu}\nu \Longrightarrow \gamma\gamma\gamma)$ and thus similar to the derivation of (32) we find

$$(d\hat{N}/dt)/\hat{N} \approx -\hat{N}^{-13/9}/T \ , \tag{42}$$

where $T=5\times 10^{14}\eta^{-10}R$ sec. We solve (42) and obtain $\mu(t)$:

$$m_e/\mu(t) \approx [13(\tau-t)/(9T)]^{1/13} \quad (t<\tau) \ . \tag{43}$$

This expression holds until $m_e/\mu(t)$ gets too close to $2-\eta$. For $(m_e/\mu_{ac})^{13} \gg 1$ the lifetime is $\approx \tau$, determined from (43) by letting $t \Longrightarrow 0$ and $\mu(t) \Longrightarrow \mu_{ac}$. The τ so determined improves our previous estimate (32) by a factor $\frac{9}{13}\eta^{-10}$.

From the evolution of $\mu(t)$ we may obtain the evolution of the NB mass and radius after cooling. Equation (43) may be inserted into

$$M(t) \approx 410[m_e/\mu(t)]^8[\sigma(\text{TeV}^3)]^3/n_\nu{}^2 \text{ solar masses} \ , \tag{44}$$

$$r(t) \approx 2.3[m_e/\mu(t)]^4[\sigma(\text{TeV}^3)]/n_\nu \text{ light seconds} \ . \tag{45}$$

We also find the power radiated:

$$-dM(t)/dt \approx 1.3\times 10^{42}[m_e/\mu(t)]^{-5}[\sigma(\text{TeV}^3)]^3/(R\eta^{-10}n_\nu{}^2) \text{ erg/sec} \ . \tag{46}$$

Note that the power output increases as the NB decreases in size [and $\mu(t)$ increases]. The upper cutoff on the primary γ-ray spectrum is of order $\mu(t)$. The fraction of power appearing as primary electrons and positrons is $(1-\eta)m_e/[m_e-\eta\mu(t)]$. When $m_e/\mu(t)$ reaches $\approx 2-\eta$ the NB dies in a burst of primary electrons and positrons of total energy $\approx 7.3\times 10^{56}(2-\eta)^8[\sigma(\text{TeV}^3)]^3/n_\nu{}^2$ erg.

We conclude with some remarks on remaining issues. We have made no attempt to study the initial NB production to find the expected number NB's as a function of mass. This would depend on details of the infinite-wall removal mechanism. Most crucial is the time at which the infinite walls disappear. This is a function of the mass density of the string in the string scenario or the vacuum energy splitting in the preferred vacuum scenario. It is also very sensitive to any previous supercooling.[3] We note as well that large NB's may form through collisions of smaller NB's even after the infinite walls have disappeared. In any case for long-lived NB's never to dominate the energy density of the Universe (for example) would require that no more than one NB with $y \lesssim 1$ be formed for every $\approx 10^7\sigma^3/(1.93\ \text{TeV})^9$ horizon volumes when the temperature of the Universe was $\approx T_c$. Thus the efficiency for production of NB's above the critical size must be small.

Gravitational effects have not been included; they will somewhat alter the properties of NB's near the top end of the mass range. Also, two numbers introduced in our analysis remain to be calculated, R and η. They both appear in the power output (46).

And finally, the astrophysical implications of NB's remain to be explored. To which of the known energetic sources in the sky could NB's correspond? The initial number of NB's is expected to be a rapidly falling function of their mass. Thus the population of long-lived NB's is expected to be bunched near the shoulder of the lifetime curve in Fig. 4, implying that many expire at a similar time.

Topics of astrophysical interest include the possibility of oscillating NB's giving rise to periodic bursts. Another would be the question of how NB properties change when they accrete matter. If a NB accretes enough matter a supermassive black hole could result. In any case the mass range of NB's makes them of interest for seeding galaxy formation. These possibilities will be pursued elsewhere.

ACKNOWLEDGMENTS

I thank A. Hadeed for collaborating on some preliminary work and R. Bond, B. Campbell, and R. Mann for discussions. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys. JETP **40**, 1 (1975)].

[2]For our discussion we may assume that CP is not spontaneously broken. Alternatively, it could be that both P and CP are explicitly broken (e.g., by nonzero vacuum angles). In this case our discussion would apply to spontaneously broken C instead of P.

[3]A. E. Everett and A. Vilenkin, Nucl. Phys. **B207**, 43 (1982); T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. D **26**, 435 (1982); A. Vilenkin, Phys. Rep. **121**, 263 (1985).

[4]T. W. B. Kibble, J. Phys. A **9**, 1387 (1976); Phys. Rep. **67**, 183 (1980); in *Quantum Structure of Space-Time,* edited by M. J. Duff and C. J. Isham (Cambridge University Press, Cambridge, England, 1982); A. Vilenkin, Phys. Rev. D **23**, 852 (1981); B. Holdom, *ibid.* **27**, 332 (1983); **28**, 1419 (1983).

[5]M. Gell-Mann, Phys. Rev. Lett. **6**, 70 (1961).

[6]M. J. Levine, Nuovo Cimento **48**, 67 (1967).

[7]N. Van Hieu and E. P. Shabalin, Zh. Eksp. Teor. Fiz. **44**, 1396 (1963) [Sov. Phys. JETP **17**, 681 (1963)].

[8]A. E. Everett, Phys. Rev. D **10**, 3161 (1974).