On the existence of stable dimesons

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The bound-state problem of two quarks and two antiquarks with coupled channels in color space is studied, using a potential as derived from the MIT bag model. For equal quark and antiquark masses no stable bound state is supported, while for large enough mass ratio a true four-quark bound state is found. The effect of a running coupling constant in the Coulomb part of the potential is studied.

I. INTRODUCTION

The description of the spectrum of $\psi(c\bar{c})$ and $\Upsilon(b\bar{b})$ in terms of the MIT bag model with static, localized quarks as sources of the glue field has been reasonably successful.[1] Therefore, it is attractive to extend such a model to the case of multiquark systems. This was done in a recent paper,[2] to be referred to as I, where the possible existence of stable heavy dimesons was investigated. For this system the potential energy is a two-dimensional matrix in color space. Employing the strict Born-Oppenheimer approximation where it is assumed that (i) only the ground state of this matrix is needed to determine the effective four-quark interaction and (ii) the color wave function varies slowly with the positions of the quarks, it was shown in a variational approach that the interaction is sufficiently attractive to support a bound state of two quarks and two antiquarks of equal masses.

In this paper we study the validity of the strict Born-Oppenheimer approximation by solving (variationally) the coupled-channel Schrödinger equation with trial functions of the exponential type similarly as in I. It is found that we get no binding for the four-quark ground state due to a repulsive contribution from the kinetic energy operator acting on the color wave function. It is of the order of 4 GeV, which renders the dimeson to be unbound.

We have also considered the case of two quarks with mass m and two antiquarks with mass $\overline{m}$, m and $\overline{m}$ being, in general, different. For sufficiently large or small ratio $m/\overline{m}$ the corresponding dimeson becomes bound, in agreement with Refs. 3–6. A problem which arises in allowing the masses to be different is that the size of the four-quark system changes on varying the mass ratio. As a result we need to have a running coupling constant in the interaction to account for the quark loop corrections. This is done in Sec. II which also includes a summary of I. In Sec. III we describe the various variational calculations we have done for both the fixed-coupling and running-coupling case. Some concluding remarks are made in the last section.

II. BAG POTENTIAL WITH RUNNING COUPLING CONSTANT

For not too light quarks it is a reasonable assumption to treat their motion in a nonrelativistic way. The Hamiltonian of such a system is given by

$$H=K+V\ , \tag{1}$$

where $K=\sum_i p_i^2/2m_i$ and V is the potential between the quarks, acting also in color space. Solving the MIT bag model for the glue field due to any number of static, localized quarks and antiquarks gives the potential (2a) below. This is specialized in I to a four-particle system, consisting of two quarks and two antiquarks, where it becomes (2b):

$$V=\alpha_s \sum_{i>j} \frac{F_i \cdot F_j}{r_{ij}} + \frac{k}{\sqrt{2}} \left[\left(\sum_i F_i \mathbf{r}_i \right)^2 \right]^{1/2} , \tag{2a}$$

$$V=W+\frac{kd}{\sqrt{2}} I\ . \tag{2b}$$

The F_i are the color-SU(3) generators of the particles, normalized to $F_i{}^2=\frac{4}{3}$, I is the 2×2 unit matrix, and $d=d_1 d_2/(d_1+d_2)$, where d_i are the color-dipole moments of the system.[2] The string tension k is related to the bag constant B and strong coupling constant α_s through

$$k=\left[\frac{32\pi}{3}\alpha_s B\right]^{1/2} . \tag{3}$$

Insofar as the present paper is concerned the essential point about Eq. (2a) is that it applies to *both* the two-body system and the four-body system. By solving the two-body and the four-body problems we can determine if the dimeson is stable against decay into two mesons. The fact that the potential does not contain an arbitrary additive constant that is permitted to vary from one system to another is clearly essential for this purpose.

In the single-singlet, octet-octet representation with basis states

$$\Psi_1 = |[(1\,2)^1\ (3\,4)^1]^1\rangle ,$$
$$\Psi_8 = |[(1\,2)^8\ (3\,4)^8]^1\rangle , \tag{4}$$

where particles 1 and 3 are quarks and 2 and 4 are antiquarks, W can be written as

$$W=\frac{1}{3}\begin{bmatrix} 4y_{12} & \sqrt{2}(y_{13}-y_{14}) \\ \sqrt{2}(y_{13}-y_{14}) & -\frac{1}{2}y_{12}+y_{13}+\frac{7}{2}y_{14}\end{bmatrix} , \tag{5}$$

where $y_{ij}=w_{ij}+w_{kl}$ with all four indices different and

$$w_{ij}=\frac{k}{\sqrt{2}}\frac{r_{ij}{}^2}{d_1+d_2}-\frac{\alpha_s}{r_{ij}} . \tag{6}$$

Equation (2a) is derived under the assumption that the bag has a spherical shape together with the use of the dipole approximation for the homogeneous part of the potential. As long as the distances between the quarks are small as compared to the bag size it is expected to be a reliable description of the four-quark system. Since the color-dipole moments d_i depend on the positions of all four particles, the interaction (2b) cannot be written as a sum of pair potentials and therefore constitutes a genuine *many-body* potential.

The quark-antiquark potential obtained from the *same* bag model is given by

$$V=-\frac{4}{3}\frac{\alpha_s}{r}+(\tfrac{2}{3})^{1/2}kr . \tag{7}$$

This potential also contains no arbitrary additive constant, and is capable of providing a good fit to the $c\bar{c}$ and $b\bar{b}$ spectra.

In I a slightly modified form of W has also been studied by changing W for large separations of the two quark-antiquark pairs in order that W goes into a sum of pair potentials of the form (7). Here we shall neglect this complication and take the unmodified form of W as potential. Hyperfine interactions, which are not considered in this paper, can be treated as a perturbation.

The above potentials can be used as input for the study of the possible existence of stable $q^2\bar{Q}^2$ states, with masses m and $\bar{m}$ for the quark and antiquark, respectively, in general chosen differently. Since the coupling constant α_s runs with the size of the system, it was taken in I to be smaller for heavier mass systems. Here we follow a different procedure. A regularized form for α_s that satisfies the renormalization-group analysis of QCD in the one-loop approximation is

$$\alpha_s=\alpha_0\frac{\ln\gamma}{\ln\left[\frac{1}{(\Lambda r)^2}+\gamma\right]} . \tag{8}$$

The scale constant Λ is chosen to be 1 fm^{-1}, which is in the range of values extracted from high-energy experiments. Recent lattice gauge simulations[7,8] of the heavy-quark potential indicate that the r dependence of α_s is *not* seen for $r>0.15$ fm, so it is reasonable to take $\gamma \gtrsim 100$. For $\gamma=\infty$ a simultaneous fit can be made to the ground and first excited s states of both the $c\bar{c}$ and $b\bar{b}$ systems, by varying the two parameters α_0 and B and the quark masses m_c and m_b. A good fit is found for $\alpha_0=0.37$ and $B^{1/4}=0.245$ GeV (see Fig. 4 in I). A question which we would like to address is the sensitivity of the binding energy of the four-quark system to the specific choice of γ. Keeping the values of α_0 and B fixed, the quark masses were changed accordingly for γ in the range of 50—1000 to yield the correct ground-state levels of $c\bar{c}$ and $b\bar{b}$. Since the correct asymptotic freedom behavior can be satisfied by having the coupling constant running only in the Coulomb part of the potential, we have replaced α_s by α_0 in the constant k [see Eq. (3)], which essentially determines the potential at large separations. The calculated $2S$-$1S$ splittings are still reproduced nicely for these values of γ. The results are shown in Table I. We see that over this wide range of γ the splitting changes by at most 15%. In Fig. 1 are plotted the resulting $Q\bar{Q}$ potentials for two values of γ. They are very similar to previous potentials that describe the spectra of the $c\bar{c}$ and $b\bar{b}$ systems, differing approximately by an additive constant.[1]

The existence of stable heavy dimesons has been explored in I for the case of equal-mass quarks and antiquarks in the strict Born-Oppenheimer approximation. The wave function is expressed in terms of the eigenfunctions $\chi_\alpha(\mathbf{r}_i)$ of the potential matrix V:

$$\psi_0(\mathbf{r}_i)=\sum_\alpha \phi_\alpha(\mathbf{r}_i)\chi_\alpha(\mathbf{r}_i) . \tag{9}$$

Assuming that only the ground state χ_1 needs to be taken into account, and that it varies slowly with the $\mathbf{r}_i$, the Schrödinger equation becomes a one-channel equation:

$$\left[\sum_i \frac{p_i{}^2}{2m_i}+v_1(\mathbf{r}_i)\right]\phi_1(\mathbf{r}_i)=E_4\phi_1(\mathbf{r}_i) , \tag{10}$$

where $v_1(\mathbf{r}_i)$ is the lower eigenvalue of V. Equation (10) has been solved variationally in I, using exponential trial wave functions and found to support a four-particle bound state with binding energies of typically 200 MeV. For one case where there is an exact Monte Carlo Green's-function calculation of E_4 in Eq. (10), the results agree very well.[9]

TABLE I. The mass difference between the $2S$ and $1S$ levels of the $c\bar{c}$ and $b\bar{b}$ systems at a fixed bag constant $B^{1/4}=0.245$ GeV and coupling parameter $\alpha_0=0.37$ for various values of the cutoff parameter γ in Eq. (8). Masses are in GeV. The experimental value of the energy difference is the same as for $\gamma=\infty$. The masses of the c and b quarks are fitted to reproduce the ground state of $c\bar{c}$ and $b\bar{b}$ to be 3.095 GeV, respectively, 9.460 GeV.

γ	m_c	$M(2S)$-$M(1S)$	m_b	$M(2S)$-$M(1S)$
∞	1.364	0.591	4.781	0.550
1000	1.366	0.590	4.771	0.539
500	1.363	0.588	4.765	0.531
100	1.356	0.585	4.737	0.499
50	1.349	0.580	4.717	0.476

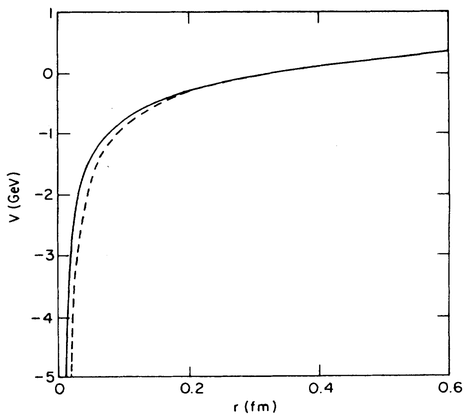

FIG. 1. The $Q\bar{Q}$ potentials of our model with $\alpha_0=0.37$ and $B^{1/4}=0.245$ GeV for two cases of the cutoff parameter $\gamma=100$ and 1000.

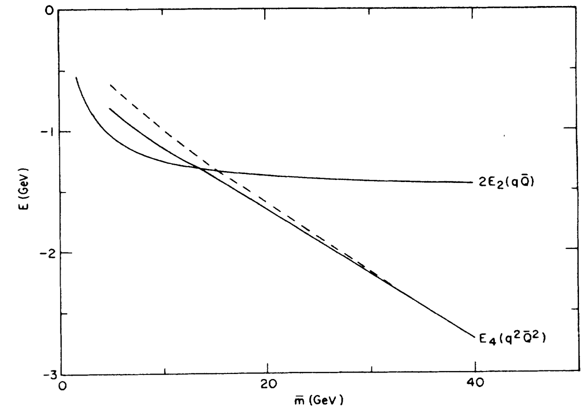

FIG. 2. Coupled-channel results for the bound-state energy E_4 of the $q^2\bar{Q}^2$ system in the case of the nonrunning coupling constant α_s as a function of the antiquark mass $\bar{m}$. The other parameters of the bag model are taken from Ref. 1, corresponding to the c-quark case: $\alpha_s=0.747$, $B^{1/4}=0.145$ GeV, and $m=1.685$ GeV. Also are shown two times the bound-state energy E_2 of the $q\bar{Q}$ system and the results of a one-channel calculation where only the $3\bar{3}$ component of the wave function is kept (dashed curve).

III. COUPLED-CHANNEL ANALYSIS

We now study the effect of solving the full two-channel problem. It is convenient to use instead of the representation (4) the **3**-$\bar{\mathbf{3}}$, **6**-$\bar{\mathbf{6}}$ representation given by the basis states

$$\Psi_3=|[(1\ 3)^{\bar{3}}\ (2\ 4)^3]^1\rangle\ ,$$
$$\Psi_6=|[(1\ 3)^6\ (2\ 4)^{\bar{6}}]^1\rangle\ , \qquad (11)$$

with Ψ_3 antisymmetric under interchange of either pair of identical particles, and Ψ_6 symmetric. The relation with Eq. (4) is simply given by a unitary transformation

$$\Psi_3=-(\tfrac{1}{3})^{1/2}\Psi_1-(\tfrac{2}{3})^{1/2}\Psi_8\ ,$$
$$\Psi_6=(\tfrac{2}{3})^{1/2}\Psi_1-(\tfrac{1}{3})^{1/2}\Psi_8\ . \qquad (12)$$

An arbitrary function of the color and spatial coordinates can be written as

$$\psi(\mathbf{r}_i)=\phi_3(\mathbf{r}_i)\Psi_3+\phi_6(\mathbf{r}_i)\Psi_6\ . \qquad (13)$$

The possible solutions to the coupled-channel equations can be classified according to their spatial symmetry character as being of a T(true) and M (mock) type.[5] For the T type the (**3**-$\bar{\mathbf{3}}$) and (**6**-$\bar{\mathbf{6}}$) components are symmetric and antisymmetric, respectively, in the spatial coordinates under interchange of the quarks or of the antiquarks, whereas for the M type the (**3**-$\bar{\mathbf{3}}$) and (**6**-$\bar{\mathbf{6}}$) components are antisymmetric and symmetric, respectively. The Pauli principle is then satisfied by taking the proper spin combinations.

Let us consider the case of T-type wave functions. Similarly, as in I, we may study variationally the coupled-channel problem with trial functions of the form

$$\phi_n(\mathbf{r}_i)=N_n[1-(-1)^nP_{13}]$$
$$\times\exp\{-[\alpha_{12}^{(n)}(r_{12}+r_{34})+\alpha_{12,34}^{(n)}R_{12,34}]\} \qquad (14)$$

with

$$\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2,\ \ \mathbf{r}_{34}=\mathbf{r}_3-\mathbf{r}_4\ ,$$
$$\mathbf{R}_{12,34}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_1+m_2}-\frac{m_3\mathbf{r}_3+m_4\mathbf{r}_4}{m_3+m_4}\ ,$$

and N_n and the $\alpha^{(n)}$'s are the variational parameters. P_{13} is the permutation operator for the coordinates of particles 1 and 3. Equation (14) has been chosen such that the components of the trial function satisfy the proper symmetries under exchange of $\mathbf{r}_1$ with $\mathbf{r}_3$ or of $\mathbf{r}_2$ with $\mathbf{r}_4$. The potential employed in these calculations is the one from I with $\alpha_0=0.747$, $\gamma=\infty$, and $B^{1/4}=0.145$ GeV, corresponding to the $c\bar{c}$ system. We now also allow the quark mass to be different from that of the antiquark. In Fig. 2 the results for the bound-state energy E_4 of the dimeson and $2E_2$, E_2 being the ground-state energy of $q\bar{Q}$, are shown as a function of $\bar{m}$ with m taken to be the c-quark mass $m=1.685$ GeV. From this figure we see for sufficiently large mass $\bar{m}$ a stable bound state exists. A bound state is considered stable if it cannot dissociate into a pair of $q\bar{Q}$ mesons. No stable bound state is supported for the equal-mass case $m=\bar{m}$. Similar results are obtained if we take the quark mass to be that of a b instead of a c quark. Also shown in Fig. 2 are the results if we do a variational calculation keeping only the **3**-$\bar{\mathbf{3}}$ component of the wave function. The results are close to the coupled-channel calculation, indicating that the coupling of the $\mathbf{3}\bar{\mathbf{3}}$ state to the $\mathbf{6}\bar{\mathbf{6}}$ state is suppressed because of the p-wave nature of the $\mathbf{6}\bar{\mathbf{6}}$ state.

Table II gives the calculated values of the variational parameters for the coupled-channel calculations, together with the probability of finding the four-particle system in the $\mathbf{3}\bar{\mathbf{3}}$ state. It is seen that with increasing mass $\bar{m}$ the system tends to go into a pure $\mathbf{3}\bar{\mathbf{3}}$ state. This is just what one would expect since the two very heavy antiquarks are

TABLE II. The variational parameters in the trial functions (13) and (14) obtained by minimizing the $q^2\overline{Q}^2$ bound-state energy as given in Fig. 2. The mass of the quark is held fixed at $m=1.685$ GeV and $\overline{m}$ is the mass of the antiquark. The bag-model parameters are $\alpha_s=0.747$ and $B^{1/4}=0.145$ GeV. The $\alpha^{(n)}$'s are in fm^{-1}. Also shown is the probability $P_{3\overline{3}}$ to find the $q^2\overline{Q}^2$ state in the $\mathbf{3}\overline{\mathbf{3}}$ color configuration.

$\overline{m}$ (GeV)	$\alpha_{12}^{(3)}$	$\alpha_{12,34}^{(3)}$	$\alpha_{12}^{(6)}$	$\alpha_{12,34}^{(6)}$	$P_{3\overline{3}}$
5	6.1	5.6	6.3	4.7	0.863
10	6.2	11	7.2	8.5	0.988
20	6.3	23	9.0	15	0.999
30	6.4	35	9.5	20	1.000
40	6.4	46	9.5	25	1.000

able to take advantage of their Coulomb attraction in the color 3 state. To demonstrate that this is indeed the case we choose a different trial function

$$\phi_n(\mathbf{r}_i)=\exp[-(\alpha_{13}^{(n)}r_{13}+\alpha_{24}^{(n)}r_{24}+\alpha_{13,24}^{(n)}R_{13,24})] , \qquad (15)$$

expressed in terms of the relative coordinate $\mathbf{r}_{24}$ of the two antiquarks, $\mathbf{r}_{13}$, and $\mathbf{R}_{13,24}$. [These are defined in analogy with the expressions below Eq. (14).] If the Coulomb attraction of the two antiquarks in the color 3 state is dominant, then the variational parameter $\alpha_{24}^{(3)}$ should take on the hydrogenic value corresponding to a reduced mass of $\overline{m}/2$ and a coupling constant $\frac{2}{3}\alpha_s$:

$$\alpha_{24}^{(3)}(\text{hydrogenic})=\frac{\overline{m}\alpha_s}{3} .$$

The results of these calculations, which do not include any coupling between the two color states, are shown in Fig. 3 and Table III. For the M-type wave functions ($n=6$) no stable bound-state solutions are found for the range of mass $\overline{m}$ considered. The T-type solutions ($n=3$) are qualitatively in agreement with the results shown in Fig. 2. The binding energies obtained with Eq. (15) are, in general, about 20-MeV larger than the corresponding one-channel results with trial functions (14). It is seen in Table III that the variational parameter $\alpha_{24}^{(3)}$ approaches the hydrogenic value as $\overline{m}$ increases, as expected.

We now turn to the case using the four-particle potential with *running coupling constant*. Let us confine ourselves to the T-type solutions. Since the coupling of the $\mathbf{3}\overline{\mathbf{3}}$ state to the $\mathbf{6}\overline{\mathbf{6}}$ state is small we have carried out the variational calculations using only the $\mathbf{3}\overline{\mathbf{3}}$ channel and adopting the trial functions (15). Figure 4 shows the dependence of E_4 and $2E_2(q\overline{Q})$ as a function of the mass of the antiquarks for the case that we have c quarks with $m=1.356$ GeV and $\gamma=100$. We see that there occurs a stable four-particle bound state if the mass $\overline{m}>$ 12 GeV. The magnitude of the binding energies are considerably smaller in this case due to the effect of the weakening of the $1/r$ attraction at small distances; however, the critical value for the mass $\overline{m}$ to support a four-particle bound state is similar to that obtained for a fixed-coupling constant. Within the model considered here the $c^2\overline{t}^2$ system would form a stable dimeson, but $c^2\overline{b}^2$ would not. Figure 5 shows the dependence on γ for the case of $c^2\overline{t}^2$. In the considered range of γ the results are not very sensitive to this cutoff parameter.

To understand why the strict Born-Oppenheimer approximation breaks down we remove the approximations made in Eq. (10). For this purpose we again work with the eigenstates of the potential matrix V. Let

$$\sum_m V_{nm}(\mathbf{r}_i)\chi_{m\alpha}(\mathbf{r}_i)=v_\alpha(\mathbf{r}_i)\chi_{n\alpha}(\mathbf{r}_i), \quad \alpha=1,2 \qquad (16)$$

with normalized eigenfunctions χ_α with components $\chi_{m\alpha}$ $(m=1,2)$ with respect to a fixed color basis Ψ_m.

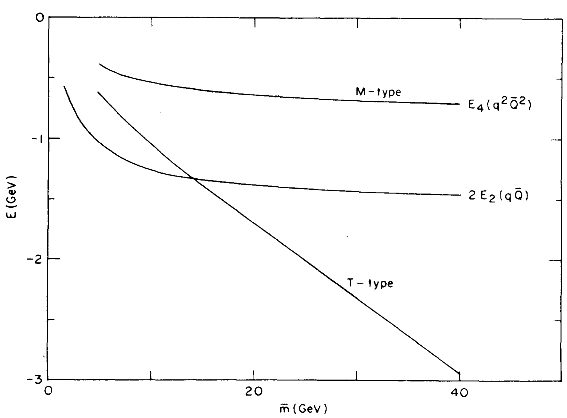

FIG. 3. One-channel results for the T- and M-type $q^2\overline{Q}^2$ ground state using the trial function (15). See caption of Fig. 2 for the meaning of the various curves and the bag-model parameters used.

TABLE III. The variational parameters (fm^{-1}) in the trial function (15), taken to be a pure 3-$\overline{3}$ color state, obtained by minimizing the $q^2\overline{Q}^2$ bound-state energy as shown in Fig. 3. $\overline{m}$ is the mass of the antiquarks. See the caption to Table II for the values of the input parameters. The final column shows the theoretical value of the parameter $\alpha_{24}^{(3)}$ for a pure hydrogenic relative wave function of the two antiquarks.

$\overline{m}$ (GeV)	$\alpha_{13}^{(3)}$	$\alpha_{24}^{(3)}$	$\alpha_{13,24}^{(3)}$	$\frac{\overline{m}\alpha_s}{3}$
5	4.0	7.3	5.7	6.3
10	4.4	14	6.8	13
20	4.5	26	7.2	25
30	4.5	38	7.2	38
40	4.6	51	7.3	51

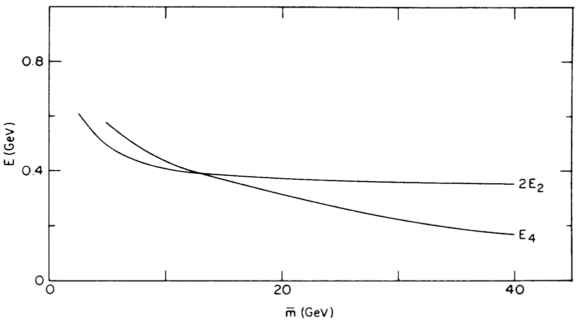

FIG. 4. Dependence of the ground-state energy of the T-type $q^2\overline{Q}^2$ state on the antiquark mass $\overline{m}$ using the trial function (15) for the case of the running coupling constant α_s given by Eq. (8) with $\gamma=100$ and $\alpha_0=0.37$. The quark mass is taken to be that of the c quark $m=1.356$ GeV, while the bag constant is $B^{1/4}=0.245$ GeV.

Writing the normalized ground-state wave function as

$$\psi_0(\mathbf{r}_i)=\sum_{n,\alpha}\chi_{n\alpha}(\mathbf{r}_i)\phi_\alpha(\mathbf{r}_i)\Psi_n \ , \qquad (17)$$

the kinetic energy operator K produces three terms. The strict Born-Oppenheimer approximation corresponds to keeping only the one term in which K acts on ϕ_1 (the lower eigenvector), and taking $\phi_2(\mathbf{r}_i)=0$. In first-order perturbation theory we find, for the energy shift,

$$\delta E_4=\sum_n \int \phi_1(\mathbf{r}_i)\chi_{n1}(K\chi_{n1})\phi_1(\mathbf{r}_i)\prod d\mathbf{r}_i \ , \qquad (18)$$

which is repulsive. Numerical evaluation of Eq. (18) for the case of $c^2\overline{c}^2$ gives an additional repulsion of 4 GeV, which is consistent with the fact that no binding is found in the coupled-channel calculation. Apart from the diagonal matrix elements ($\alpha=\beta$) of the kinetic energy operator, there are also nondiagonal ones describing the coupling between the ground state and excited state. These terms give rise to an attractive contribution to E_4. A rough estimate shows that they are small as compared to the diagonal contribution.

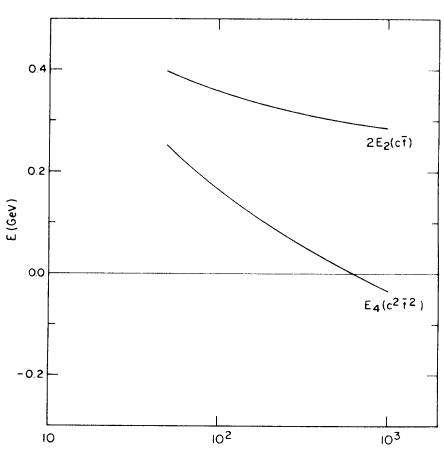

FIG. 5. Dependence of the ground-state energy of the T-type $q^2\overline{Q}^2$ state on the cutoff parameter γ using the trial function (15) for the antiquark mas $\overline{m}=40$ GeV. The other parameters are the same as for Fig. 4.

IV. SUMMARY

We have calculated the ground-state energy of a system of two quarks and two antiquarks, treating it as a two-channel problem in color space. Using the potential energy derived from the MIT bag model, we find no stable bound state if all four quarks have the same mass.

We now understand why the use of the strict Born-Oppenheimer approximation in I is not valid. If the two-dimensional potential matrix is diagonalized for every set of positions of the quarks, and only the lower eigenstate kept, it turns out that the corresponding eigenvector varies sufficiently rapidly with quark position to make a large contribution to the kinetic energy. It was the neglect of this energy in I that led us to conjecture, incorrectly, that the equal-mass case would support a bound state.

Returning to the full two-channel problem, and varying the antiquark mass, we have found that for large enough $\overline{m}$ a stable bound state is supported. This result can rigorously be shown for $\overline{m}\rightarrow\infty$ (Ref. 10). The physical reason for the existence of a bound state is that the color Coulomb interaction within the antiquark pair is attractive when the pair is in the $\overline{3}$ representation and becomes dominant for $\overline{m}\rightarrow\infty$. Because of this attraction the antiquark pair tends in this limit to sit on top of each other, whereas the quarks will stay at a finite distance from each other and the pair. As a result the $3\overline{3}$ configuration of the $q^2\overline{Q}^2$ system is predominantly favored, yielding a binding energy for the ground state which increases linearly with $\overline{m}$ for large $\overline{m}$. This result is weakened only logarithmically by the variation in the coupling constant. Since the $q\overline{Q}$ system has a ground state with a finite binding energy for $\overline{m}\rightarrow\infty$, we see that indeed the $q^2\overline{Q}^2$ system supports a stable bound state for large enough $\overline{m}$ and which is of the T-type. This argument is far more general than the specific dynamical structure used in this paper, and applies to systems such as $u^2\overline{b}^2$ even though the light quarks are relativistic.[10]

Although our numerical results are variational upper-bounds, in view of the fact that the application of the trial function used here for the one-channel problem gave reliable answers, we may hope that this also is the case in the coupled-channel problem. To be sure a more systematic analysis is needed such as the use of hyperspherical methods. In any event, improved wave functions can only produce more binding.

Since on varying the masses of the quarks the system size changes, we have to account also for the running coupling constant in the interaction between the quarks. This aspect is also considered in this paper. As a result we find that for a given mass ratio this decreases the binding energy considerably as compared to a fixed coupling constant, but still yielding the presence of a stable ground state for a mass ratio greater than of the order of 10. Therefore narrow dimeson states of type $u^2\overline{b}^2$ $s^2\overline{b}^2$ and $c^2\overline{t}^2$ are expected to exist in nature.

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