Electromagnetic mass differences of $\frac{1}{2}^+$ baryons in quark and Skyrme models

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We study electromagnetic mass differences of $\frac{1}{2}^+$ baryons in the constituent-quark model and the Skyrme model. We find that the mass differences can be explained better in the presence of a photon-cloud contribution.

I. INTRODUCTION

Since the beginning of hadron spectroscopy, electromagnetic (em) mass differences have posed a problem in their understanding. The proton-neutron mass difference is probably the oldest puzzle in hadron physics.[1] In the quark models, this problem has only shifted to understanding the up-down quark mass difference. The em mass differences among hyperons have also been studied via different approaches. Recently, with the appearance of the Skyrme model[2] as a promising candidate to explain hadron phenomenology, these mass differences have again attracted the attention of some authors.[3-5]

For a long time, strong interactions were supposed to be isospin symmetric and the observed isospin violations were attributed to em interaction. In order to solve the problems faced by this idea, the tadpole mechanism was proposed. This mechanism, associating the bulk of em self-energy with an SU(3) octet, led to the celebrated Coleman-Glashow sum rule.[6] Then other em mass sum rules were derived in SU(3) and higher symmetries.[7] In quark models, these mass differences were thought to arise from up-down quark mass differences and two quark interactions:[8] i.e., Coulombic and magnetic interactions.[9] Quantum chromodynamics has provided significant insight into hadron masses.[10] The idea that hyperfine splitting in the mass formula arises from single-gluon exchange between quarks led to the realization that such an exchange would also contribute to em mass differences.[11] But this development only led to partial success.

In the past few years, the Skyrmion, a soliton solution of an effective field theory of mesons, has been used to reproduce nucleon behavior.[12] It has been extended to the SU(3) sector and many of the baryonic properties have been studied.[13] The agreement with experiment, however, is within 30%, indicating that new features may be required. The em mass differences of the baryons have also been calculated in this model.[3-5] But these show large discrepancies with observed values.

A probable reason for the discrepancy found in the study of the em mass differences may be the neglect of a cloud of virtual photons surrounding the baryon state. The order of magnitude of its contribution to the baryon mass is similar to isospin breaking due to $m_u \neq m_d$ (Ref. 14). Normally one assumes that the cloud contribution might have been included in the quark mass terms,[15] but explicitly calculating, this does not seem to be the case.[14] Therefore one should consider these, while comparing theory with experiment.

We, in this paper, study the em mass differences among the $\frac{1}{2}^+$ baryons in the constituent-quark model and the Skyrme model. We find that the agreement improves in the presence of the self-energy contribution due to the photon cloud.

II. ELECTROMAGNETIC MASS DIFFERENCE IN CONSTITUENT-QUARK MODEL

A. Without photon-cloud energy

In the constituent-quark models, the s-wave baryon mass formula can be written[10,16] as

$$M=\sum_i m_i+\sum_{i>j}\frac{b}{m_i m_j}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) , \tag{2.1}$$

where

$$b=\frac{\pi}{6}(\tfrac{2}{3}\alpha_s-\alpha Q_i Q_j)\langle\psi_0\,|\,\delta^3(\mathbf{r}_{ij})\,|\,\psi_0\rangle . \tag{2.2}$$

m_i and Q_i denote mass and charge of ith quark. The em mass difference, therefore, gets effective contributions from the constituent-quark mass difference and hyperfine splitting due to spin-spin interaction. Other flavor-dependent effects arising from the difference in kinetic energies of the quarks and the Coulombic interactions,[10] etc., have been absorbed in renormalizing the quark mass terms in (2.1). In our analysis, we ignore $\alpha Q_i Q_j$, i.e., the photon-exchange term in (2.2), since this is very small in comparison to the $\alpha_s \sim 0.5$ required for hadron masses.[10]

The mass expressions for different $\frac{1}{2}^+$ baryons are given in columns 2 and 3 of Table I. The Coleman-Glashow formula (particle symbol denotes its mass)

$$\begin{array}{cc}(p-n)=&(\Sigma^+-\Sigma^-)+(\Xi^- -\Xi^0)\\ (-1.3\text{ MeV})&(-1.6\pm0.7\text{ MeV})\end{array} \tag{2.3}$$

and the isospin mass formula

TABLE I. Mass contributions to $\frac{1}{2}^+$ baryons.

Baryon	Quark constituent mass	Hyperfine splitting $\times b/m_u^2$ ($x=m_u/m_d$, $y=m_u/m_s$)	Isospin breaking ($\Delta I=1$) in the Skyrme model
p	$2m_u+m_d$	$1-4x$	$-\mu/10$
n	m_u+2m_d	x^2-4x	$+\mu/10$
Σ^+	$2m_u+m_s$	$1-4y$	$-\mu/2$
Σ^0	$m_u+m_d+m_s$	$\frac{1+x^2}{2}-2y-2xy$	0
Σ^-	$2m_d+m_s$	x^2-4xy	$+\mu/2$
Ξ^0	m_u+2m_s	y^2-4y	$-2\mu/5$
Ξ^-	m_d+2m_s	y^2-4xy	$+2\mu/5$

$$\Delta\Sigma=\Sigma^+ +\Sigma^- -2\Sigma^0=0$$

$$(1.78\pm0.19 \text{ MeV}) \tag{2.4}$$

follow. Determining $m_0=(m_u+m_d)/2\sim 340$ MeV, $m_0/m_s\simeq\frac{2}{3}$ from the average isomultiplet mass and $\delta m=m_d-m_u$ from the n-p mass difference, the em mass differences can be obtained as given in column 2 of Table II. The disagreement with experiment is clear.

B. With photon-cloud energy

In the presence of em interaction, the hadron emits and absorbs photons. The cloud of virtual photons would then contribute to the mass of the state. This is expected to be about the fine-structure constant α multiplied with the scale of the strong interactions. To lowest order the effective Lagrangian describing this contribution[14] is

$$\mathscr{L}_{\rm em}=-\tfrac{1}{2}e^2\int d^4y\, D(x-y)TJ^\mu(x)J_\mu(y)\ , \tag{2.5}$$

where D is the photon propagator and $J_\mu(x)$ is the em current. Renormalization of the Lagrangian (2.5) would introduce some terms which effectively can be absorbed in the quark mass differences.[14] The photon-cloud energy can be calculated either within the quark model[11,15] or through the electron scattering data.[17] Both of these estimates are in good agreement.[14] We use the latest estimate given in column 3 of Table II, as made in Ref. 14, from the Born term in the electron scattering. The inelastic contribution introduces only an uncertainty of ±0.30 MeV in the estimates.[14]

It may be noted that $\Delta\Sigma=\Sigma^+-\Sigma^--2\Sigma^0$, which was predicted to be zero earlier, is found to be 1.78 ± 0.14 MeV, due to the photon-cloud effect, in nice agreement with the experimental value of 1.79 ± 0.19 MeV. All the em mass differences calculated in column 4 of Table II have improved in the presence of the photon-cloud contribution.

III. ELECTROMAGNETIC MASS DIFFERENCE IN SKYRME MODEL

A. Without photon-cloud energy

A promising soliton picture for the baryon has been provided by the Skyrme model.[2–5,12,13] In this model isospin breaking ($\Delta I=1$) can be introduced through the meson masses[3] as

$$\Delta H^3=\frac{m_{K^0}{}^2-m_{K^+}{}^2}{16}f_\pi{}^2\int d^3x\ \mathrm{Tr}[\lambda_3(U+U^\dagger)]\ , \tag{3.1}$$

where f_π is the pion decay constant and λ_3 is the third Gell-Mann matrix of SU(3). Sandwiching this term between the baryon wave functions, one obtains the em mass contribution to the baryons as given in column 4 of Table I, where μ is given[3] by

$$\mu=\tfrac{2}{3}\pi(m_{K^0}{}^2-m_{K^+}{}^2)f_\pi{}^2\int_0^\infty[1-\cos F(r)]r^2dr$$
$$\sim 3.3 \text{ MeV}\ . \tag{3.2}$$

This term yields the em mass splitting as displayed in column 2 of Table III. All the signs are right, but magnitudes differ considerably from experiment.[3]

TABLE II. Electromagnetic mass differences in the quark model.

Mass difference	Without photon-cloud energy ($\delta m=1.78$ MeV)	Photon-cloud contribution (±0.30 MeV)	With photon-cloud energy (MeV) ($\delta m=3.5$ MeV)	Experiment (Ref. 20) (MeV)
$n-p$	1.3[a]	-0.76	1.3[a]	1.293
$\Sigma^--\Sigma^+$	3.6	0.17	7.7	7.97 ± 0.07
$\Sigma^--\Sigma^0$	1.8	0.97	2.9	4.88 ± 0.06
$\Xi^--\Xi^0$	2.4	0.86	6.3	$6.4\ \pm0.6$
$\Delta\Sigma$	0	1.78	1.8	1.79 ± 0.19

[a]Input.

TABLE III. Electromagnetic mass differences in the Skyrme model.

Mass difference	Skyrme model only (MeV)	With quark mass term $\delta m=0.6$ MeV	With photon-cloud energy $\mu\simeq 8$ MeV	Experiment (Ref. 20) (MeV)
$n-p$	0.7	1.3[a]	0.9±0.3	1.293
$\Sigma^- - \Sigma^+$	3.3	4.6	8.0±0.3[a]	7.97±0.07
$\Sigma^- - \Sigma^0$	1.6	2.3	4.9±0.3	4.88±0.06
$\Xi^- - \Xi^0$	2.6	3.3	7.0±0.3	6.4 ±0.6
$\Delta\Sigma$	0	0	1.8±0.1	1.79±0.19

[a]Input.

It has been suggested that some new features may be required in the Skyrme model, since the reproduced phenomenology agrees with experiment within 30%. One idea has been to include quarks in this model by associating complementary roles to the Skyrmion and the quarks;[18] i.e., the quarks keep the Skyrmion from collapsing, while the Skyrmion keeps the quarks confined. Following this view, quark mass terms have been included to improve the hadron mass spectrum.[19] Recently, for em mass differences of N and Δ isomultiplets also, the down-up quark mass difference δm has been introduced in the Skyrme model.[5]

If this simple prescription is extended to the hyperon sector, the em mass difference would look like

$$\begin{aligned} n-p &= \frac{\mu}{5}+\delta m \ , \\ \Sigma^- - \Sigma^+ &= \mu+2\delta m \ , \\ \Sigma^- - \Sigma^0 &= \frac{\mu}{2}+\delta m \ , \\ \Xi^- - \Xi^0 &= \frac{4\mu}{5}+\delta m \ . \end{aligned} \tag{3.3}$$

The sum rules (2.3) and (2.4) are regained. The calculated numbers with $\delta m=0.63$ MeV are given in column 3 of Table III. Though a slight improvement is achieved, the major discrepancy remains.

B. With photon-cloud energy

In this section we find that when the photon-cloud energy contributions are subtracted from the observed mass differences, the subtracted mass difference appears in the same ratio (within errors) as predicted by the Skyrme model, i.e.,

$$\begin{aligned} &(n-p) : (\Sigma^- - \Sigma^+) : (\Sigma^- - \Sigma^0) : (\Xi^- - \Xi^0) \\ &= \mu/5 : \mu : \mu/2 : 4\mu/5 \\ &= 2.0\pm0.3 : 7.8\pm0.4 : 3.9\pm0.4 : 5.6\pm0.9 \ . \end{aligned}$$

Treating μ as a parameter, the calculated mass differences are given in column 4 of Table III. The values, with a choice of $\mu\sim 8$ MeV, are found to be in nice agreement.

IV. SUMMARY AND CONCLUSION

We have studied the electromagnetic mass differences of the $\frac{1}{2}^+$ baryon in the constituent-quark model and the Skyrme model with and without the photon-cloud energy. To conclude, we find that the mass differences seem to require a photon-cloud energy contribution. We have not included the $\frac{3}{2}^+$ decuplet baryons, since the errors in the data are rather large to decide upon the situation.

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