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Analysis of $g_T(2050\text{—}2350)$ as glueball candidate

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We estimate coupling constants of g_T's to two pions as well as two gluons from the experimental ratio of $\sigma(\pi^- p \to g_T n)$ to $\sigma(\pi^- p \to f' n)$. We can evaluate decay rates of g_T into pairs of pseudoscalar and vector nonets as well as $\Gamma(\psi/J \to g_T \gamma)/\Gamma(\psi/J \to f'\gamma)$ in a rather model-independent way. Our estimates are consistent with the hypothesis that g_T's are all glueballs, or at least one g_T is a glueball. We also estimate $B(\theta \to \pi\pi) B(\theta \to \eta\eta) < 1.3 \times 10^{-3}$ for the product of decay branching ratios of θ(1700 MeV) meson.

I. INTRODUCTION

Quantum chromodynamics (QCD) is now the standard theory of strong interactions at least at the composite level of quarks. Owing to our lack of ability to perform reliable computations in most problems, the need for concrete verification of QCD remains open—especially for a convincing demonstration of the non-Abelian gauge nature of the theory. The existence of gluons, the gauge bosons of the theory, carrying color, almost certainly requires the existence[1,2] of color-singlet bound states composed of gluons. It is universally accepted that the discovery of glueballs would be a most significant verification of QCD.

There are now several resonances which are possible candidates for glueballs. These include[2–6] $0^{++}(750)$, $\iota(1440)$, $G(1590)$, $\theta(1710)$, $\xi(2220)$, $g_T(2050)$, $g_T(2300)$, and $g_T(2350)$. The lack of theoretical understanding of glueball properties has, however, led to much controversy and there is, as yet, no clear consensus on this question. It is generally believed that the best source for production of glueballs is the radiative decays of ψ/J into glueballs. Perturbative QCD calculation gives the branching ratio $B(\psi/J \to \gamma gg) \simeq 6\text{—}10\,\%$ and hence one is led to believe that glueballs may be copiously produced in this radiative decay. All glueball candidates except for $0^{++}(750)$, $G(1590)$, and the three $g_T(2050\text{—}2350)$'s were indeed discovered in this way. Arguments based upon $1/N$ expansions have led people to believe that glueballs would be mostly pure gluonic states with fairly narrow widths. Since glueballs must be flavor singlets, their hadronic decays would also be flavor symmetric. Unfortunately, practically all of the above-mentioned glueball candidates do not experimentally satisfy these stated conditions. This may be in part due to the unreliability of $1/N$ expansion for the realistic case of $N=3$. At any rate, we have to abandon, more or less, the strict criteria stated above by admitting relatively large quarkonium contents in physical glueballs. This clearly reflects our present lack of understanding of glueball properties and hence makes their identification more difficult.

In this paper we focus our attention on perhaps the most controversial candidates, the g_T's. The controversy is centered on the following issues. The g_T's are seen[6,7] in only one production reaction, namely, pion-nucleon collisions, and in only one decay mode $g_T \to \phi\phi$. There is to date no unambiguous signature for g_T's produced in the radiative decay of ψ/J, although indirect evidence for it exists as will be discussed shortly. This fact in itself is sometimes regarded as an indication[2,8] that the g_T's are not really glueballs but simply manifestations of some kinematical flukes. Even accepting g_T's to be genuine resonances, there are several possible explanations other than the most tempting possibility—that the g_T's are glueballs. Indeed, at least one of the g_T's may be a four-quark $(s\bar{s})(s\bar{s})$ state[9,10] or a hybrid state (Ref. 11) $s\bar{s}g$. It is possible also in principle that at least one of the three g_T's is a $s\bar{s}$ bound state, a radial excitation of $f'(1550\text{ MeV})$, which has been predicted[12] to exist in this same mass region by some theorists.

The purpose of this paper is an attempt to resolve some of these issues by determining the magnitude of the coupling parameters of g_T's to two mesons as well as to two gluons, assuming them to be genuine resonances. We note that the g_T's would be produced via two-gluon exchange in the reaction $\pi^- p \to g_T n$, irrespective of whether the g_T's are glueballs, four-quark states $(s\bar{s})(s\bar{s})$, and $s\bar{s}$ quarkonium states. Before proceeding further, we will first review the experimental evidence for the g_T's. There are three possible resonances[6,7] resolved at Brookhaven in reaction $\pi^- p \to g_T n$ at 22 GeV/c by phase-shift analysis. Two of these same structures have been subsequently seen[13] at CERN. Their spin (J), parity (P), and charge-conjugation (C) assignments are consistent with $J^{PC} = 2^{++}$ for all three g_T's. The masses, widths, and production cross sections are listed in Table I for ease of reference.

Our analysis is based upon the following observation. First, the g_T's are experimentally produced[6,7] in $\pi^- p \to g_T n$ reaction followed by $g_T \to \phi\phi$ at 22 GeV/c, dominantly (almost 90%) via the one-pion-exchange

TABLE I. Experimental data on g_T resonances.

	$g_T(2050)$	$g_T(2300)$	$g_T(2350)$
Mass (MeV)	2050^{+90}_{-50}	2300^{+20}_{-100}	2350^{+20}_{-30}
Width (MeV)	200^{+160}_{-50}	200^{+60}_{-50}	270^{+90}_{-20}
Relative production rates (%)	50^{+10}_{-10}	20^{+20}_{-20}	30^{+40}_{-20}
$\sigma(\pi^- p \to g_T n)B(g_T\to\phi\phi)$ (nb)	~ 10	~ 4	~ 6
Orbital angular wave of $\phi\phi$ system	S	D	D

mechanism. Similarly, a recent CERN experiment[14] on $\pi^- p \to Xn$ reaction followed by $X\to\eta\eta$ at 100 GeV/c for $X=f(1270)$, $\epsilon(1300)$, $f'(1525)$, $G_T(1590)$, and $h(2030)$ indicates that these also proceed dominantly via the same one-pion-exchange mechanism. Moreover, an older experiment[15] on $\pi^- p\to fn$ from 4 up to 50 GeV/c is also consistent with the one-pion-exchange hypothesis. Therefore, we assume that all these reactions $\pi^- p \to Xn$ are dominated by the one-pion-exchange mechanism in the energy range indicated. Note also that the one-pion-exchange mechanism appears to be still dominant at 175 GeV/c (Ref. 16). When we further note that f, f', and g_T have the same $J^{PC}=2^{++}$ quantum numbers, this implies that

$$\frac{M(\pi^- p\to g_T n)}{M(\pi^- p\to fn)} \simeq \frac{G(g_T\pi\pi)}{G(f\pi\pi)} , \tag{1.1}$$

$$\frac{M(\pi^- p\to f' n)}{M(\pi^- p\to fn)} \simeq \frac{G(f'\pi\pi)}{G(f\pi\pi)} , \tag{1.2}$$

where $G(X\pi\pi)$ is the effective coupling constant of the X meson to two pions. In particular, the ratios in Eqs. (1.1) and (1.2) are independent of the incoming pion momenta at high energy. First consider Eq. (1.1). We denote the three g_T's as $g_T^{(j)}$ $(j=1,2,3)$ hereafter whenever we want to indicate a particular g_T. Also, the present experimental values for $\sigma(\pi^- p\to g_T n)B(g_T\to\phi\phi)$ in Table I may have experimental errors[17] of a factor of 2 or so. Because of this, we introduce a fudge factor λ $(2>\lambda>\frac{1}{2})$ by multiplying λ into the values of $\sigma(\pi^- p\to g_T n)B(g_T\to\phi\phi)$ given in Table I. Using the experimental value[15] of $\sigma(\pi^- p\to fn)\simeq 20$ μb at 22 GeV/c, Eq. (1.1) suggests that

$$\left[\frac{G(g_T^{(j)}\pi\pi)}{G(f\pi\pi)}\right]^2 B(g_T^{(j)}\to\phi\phi) \simeq \begin{cases} 5\times10^{-4}\lambda, & g_T^{(1)}=g_T(2050) ,\\ 2\times10^{-4}\lambda, & g_T^{(2)}=g_T(2300) ,\\ 3\times10^{-4}\lambda, & g_T^{(3)}=g_T(2350) . \end{cases} \tag{1.3}$$

From Eq. (1.3), we can compute the decay rate of $g_T\to\pi\pi$ from

$$\frac{\Gamma(g_T\to\pi\pi)}{\Gamma(f\to\pi\pi)} = \left[\frac{G(g_T\pi\pi)}{G(f\pi\pi)}\right]^2 \left[\frac{M(f)}{M(g_T)}\right]^2 \left[\frac{k'}{k}\right]^5 , \tag{1.4}$$

where $M(f)$ and $M(g_T)$ are the masses of f and g_T, respectively, and k and k' are the magnitudes of pion momenta for decays $f\to\pi\pi$ and $g_T\to\pi\pi$ at rest, respectively. Using the experimental value of $\Gamma(f\to\pi\pi)\simeq 153$ MeV, this together with Eq. (1.3) gives

$$\Gamma(g_T^{(j)}\to\pi\pi)B(g_T^{(j)}\to\phi\phi) = \begin{cases} 0.34\lambda \text{ MeV}, & g_T^{(1)}=g_T(2050) ,\\ 0.19\lambda \text{ MeV}, & g_T^{(2)}=g_T(2300) ,\\ 0.30\lambda \text{ MeV}, & g_T^{(3)}=g_T(2350) . \end{cases} \tag{1.5}$$

Next, let us consider the implications of Eq. (1.2) for later purposes. Using experimental values[14] of

$$\begin{aligned} &\sigma(\pi^- p\to fn)B(f\to\eta\eta)=2.2\pm0.5 \text{ nb} ,\\ &\sigma(\pi^- p\to f'n)B(f'\to\eta\eta)=0.14\pm0.05 \text{ nb} , \end{aligned} \tag{1.6}$$

together with

$$B(f\to\eta\eta)=(2.8\pm0.7)\times10^{-3} , \tag{1.7}$$

we estimate

$$\left[\frac{G(f'\pi\pi)}{G(f\pi\pi)}\right]^2 B(f'\to\eta\eta)\simeq 1.8\times10^{-4} , \tag{1.8}$$

where we have neglected all experimental errors which may amount to a 40% correction in our estimate. To date, the experimental value for the branching ratio $B(f'\to\eta\eta)$ is not known. However, a QCD-based model calculation[12] predicts

$$B(f'\to\eta\eta)\simeq 0.12 \tag{1.9}$$

while the standard nonet mixing model with exact flavor-SU(3) symmetry gives essentially the same value as this. Hence, accepting Eq. (1.9), we obtain

$$\left[\frac{G(f'\pi\pi)}{G(f\pi\pi)}\right]^2 \simeq 1.5\times10^{-3} \tag{1.10}$$

which is compatible with old value obtained by Pawlicki *et al.*[18] Combining this with Eq. (1.3), we find

$$\left[\frac{G(g_T^{(j)}\pi\pi)}{G(f'\pi\pi)}\right]^2 B(g_T^{(j)}\to\phi\phi) = \begin{cases} 0.33\lambda, & g_T^{(1)}=g_T(2050)\ , \\ 0.13\lambda, & g_T^{(2)}=g_T(2300)\ , \\ 0.20\lambda, & g_T^{(3)}=g_T(2350)\ . \end{cases} \quad (1.11)$$

In particular, we estimate

$$|G(f\pi\pi)|:|G(f'\pi\pi)|:|G(g_T^{(1)}\pi\pi)| \simeq 26:1:0.57\left[\frac{\lambda}{B(g_T^{(1)}\to\phi\phi)}\right]^{1/2}. \quad (1.12)$$

The large value of $G(f\pi\pi)$ in comparison to $G(f'\pi\pi)$ is of course expected because of the quark-line rule.[19]

The CERN experiment at 100 GeV/c does not appear to have detected any signal for $\pi^- p\to g_T n$ followed by $g_T\to\eta\eta$. However, for the high-mass region of the spin-2 waves, they find two solutions: either a broad structure (solution I) or a narrow peak (solution II). The characteristics of solution I are

$$\langle M\rangle = 1950\pm 50 \text{ MeV}, \quad \text{FWHM} = 600\pm 50 \text{ MeV}\ ,$$

$$\sigma(\pi^- p\to Xn)B(X\to\eta\eta) = 15\pm 4 \text{ nb}\ ,$$

where FWHM means full width at half maximum. Solution II, on the other hand, shows a structure with a Breit-Wigner shape with

$$M = 1870\pm 40 \text{ MeV}, \quad \Gamma = 250\pm 30 \text{ MeV}\ ,$$

$$\sigma(\pi^- p\to Xn)B(X\to\eta\eta) = 10\pm 3 \text{ nb}\ .$$

If we accept solution I and interpret it to be partially due to production of three g_T's, then this could imply

$$\sigma(\pi^- p\to g_T^{(j)} n)B(g_T^{(j)}\to\eta\eta) \lesssim 5 \text{ nb}\ . \quad (1.13)$$

Repeating the same argument as before, this would give

$$B(g_T^{(j)}\to\pi\pi)B(g_T^{(j)}\to\eta\eta) \lesssim 2.5\times 10^{-2} \quad (1.14)$$

for each of the g_T's.

A similar method can be used to give an upper bound of

$$B(\theta\to\pi\pi)B(\theta\to\eta\eta) < 1.29\times 10^{-3} \quad (1.15)$$

for the product of branching ratios of $\theta(1690)$. Its physical implications will be discussed in Sec. IV.

Returning to the original problem, we have mentioned the fact that there is no convincing experimental data showing radiative decay of ψ/J into g_T. However, there is an indirect evidence of the following kind. The Mark III group[20] has observed a peak in $\psi/J\to M\gamma$, followed by $M\to\pi\pi$ with mass $M(M)=2086\pm 15$ MeV and $\Gamma(M)=210$ MeV. Unfortunately, the spin-parity assignment of this possible resonance is unknown. However, it is quite unlikely to be identified with $h(2030)$ with $\Gamma=400\pm 100$ MeV, which is a 4^{++} state. First, their widths differ. Second, a QCD-based phase-shift analysis[21] indicates that a high-mass 4^{++} wave would not be copiously produced in ψ/J radiative decays. Third, there is an experimental indication[20] for $M\to K\bar{K}$ mode which appears to be slightly less prominant than that of the $M\to\pi\pi$ decay. If this is so, then M cannot be identical to h, since we know $B(h\to K\bar{K})\approx 0.05B(h\to\pi\pi)$. It is thus quite tempting to identify the M with $g_T^{(1)}=g_T(2050)$, because of similarities in their masses and widths. Here, we will adopt this viewpoint. In that case, the experiment indicates

$$B(\psi/J\to g_T^{(1)}\gamma)B(g_T^{(1)}\to\pi\pi) \simeq (3.0\pm 1.1)\times 10^{-4}\ . \quad (1.16)$$

Also, we have[22,23]

$$B(\psi/J\to\rho\rho\gamma) \lesssim 6\times 10^{-4} \quad (2.1 < M < 2.4 \text{ GeV})\ ,$$

$$B(\psi/J\to\omega\omega\gamma) \lesssim 6\times 10^{-4} \quad (2.1 < M < 2.4 \text{ GeV})\ ,$$

$$B(\psi/J\to\phi\phi\gamma) \lesssim (3.1\pm 1.0)\times 10^{-4} \quad (2.0 < M < 2.8 \text{ GeV})\ ,$$

in the mass range $M(V_9V_9)$ specified above. Since production reactions such as $\psi/J\to g_T^{(j)}\gamma\to\rho\rho\gamma$ are partially responsible for the observed ratio, we estimate

$$\begin{aligned} &B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\rho\rho) < 2\times 10^{-4}\ , \\ &B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\omega\omega) < 0.87\times 10^{-4}\ , \\ &B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\phi\phi) < 1\times 10^{-4}\ , \end{aligned} \quad (1.17)$$

for each $j=1,2,3$.

Therefore, accepting the bound Eq. (1.17) for $j=1$, Eq. (1.16) implies

$$\begin{aligned} &B(g_T^{(1)}\to\pi\pi) \gtrsim 3.0B(g_T^{(1)}\to\phi\phi)\ , \\ &B(g_T^{(1)}\to\pi\pi) \gtrsim 3.4B(g_T^{(1)}\to\omega\omega)\ , \\ &B(g_T^{(1)}\to\pi\pi) \gtrsim 1.5B(g_T^{(1)}\to\rho\rho)\ . \end{aligned} \quad (1.18)$$

Combining this with Eq. (1.5), we find that

$$\begin{aligned} &B(g_T^{(1)}\to\phi\phi) \lesssim 0.024\lambda^{1/2}\ , \\ &B(g_T^{(1)}\to\pi\pi) \gtrsim 0.071\lambda^{1/2}\ , \end{aligned} \quad (1.19)$$

so that we also estimate

$$B(\psi/J\to g_T^{(1)}\gamma) \lesssim 4.2\times 10^{-3}\lambda^{-1/2} \quad (1.20)$$

for $g_T^{(1)}=g_T(2050)$.

Using information gathered above, we will make more detailed analysis of g_T's in the following sections, assuming that g_T's are either glueballs, $s\bar{s}$, or $s\bar{s}s\bar{s}$ states. We will show that all g_T's are consistent with the glueball hypothesis but that the $s\bar{s}$ interpretation is highly unlikely for any of g_T's. It is difficult to rule out the possibility that one or both of the second $g_T^{(2)}$ and the third $g_T^{(3)}$ may be $s\bar{s}s\bar{s}$ bound state (or states). However, the first $G_T^{(1)}=g_T(2050)$ is consistent only with the glueball hypothesis.

Although our main conclusion depends critically on the validity of Eq. (1.16), many results can be obtained largely independent of this assumption. In Sec. II we analyze the g_T decays into two-meson decay modes and show that the present experimental data is consistent with g_T being glueballs. In Sec. III we calculate the radiative decay rate of $\psi/J\to g_T\gamma$ and show also that it is consistent with the present data. Sec. IV is devoted to a related analysis of θ and ξ mesons.

II. MAIN ANALYSIS

In the previous section we extracted some information on decay rates of g_T's from the presently available experimental data. They are Eq. (1.5) and possibly Eq. (1.16). Here, on the basis of this information, we will analyze its consequences for the following three possible models: (1) g_T's are glueballs, (2) one of the g_T's is a $s\bar{s}$ state which is a radial excitation of f', and (3) one of g_T's may be a $(s\bar{s})(s\bar{s})$ bound state.

We first consider the possibility (1) that the g_T's are all glueballs. Let P_9 and V_9 designate the standard nonets of pseudoscalar and vector mesons, respectively. If g_T's are pure glueballs, then the decay rates of $g_T\to P_9P_9$ and V_9V_9 would be uniquely determined by the validity of flavor-SU(3) symmetry in terms of $B(g_T\to\phi\phi)$. Also, the decay $g_T\to P_9V_9$ would be forbidden by the general G-parity[24] consideration. In reality, however, g_T may mix with some $s\bar{s}$ state and/or the flavor-SU(3) symmetry may be badly broken. Because of these possibilities, it may be necessary to take into account possible violation of the SU(3) symmetry. In such a case we will have

$$\begin{aligned}
&M(g_T\to\rho_0\rho_0)=M(g_T\to\omega\omega)\ ,\\
&M(g_T\to\phi\phi)=(1+\alpha)M(g_T\to\omega\omega)\ ,\\
&M(g_T\to K^{*0}\bar{K}^{*0})=(1+\tfrac{1}{2}\alpha)M(g_T\to\omega\omega)\ ,\\
&M(g_T\to\omega\phi)=0\ ,
\end{aligned}\tag{2.1}$$

for the decay $g_T\to V_9V_9$, and

$$\begin{aligned}
&M(g_T\to K^0\bar{K}^0)=(1+\tfrac{1}{2}\beta)M(g_T\to\pi^0\pi^0)\ ,\\
&M(g_T\to\eta\eta)=[1+\beta\cos^2(\theta_0-\theta_P)]M(g_T\to\pi^0\pi^0)\ ,\\
&M(g_T\to\eta'\eta')=[1+\beta\sin^2(\theta_0-\theta_P)]M(g_T\to\pi^0\pi^0)\ ,\\
&M(g_T\to\eta\eta')=[M(g_T\to\pi^0\pi^0)-M(g_T\to K^0\bar{K}^0)]\\
&\qquad\qquad\times\sin2(\theta_0-\theta_P)\ ,
\end{aligned}\tag{2.2}$$

for $g_T\to P_9P_9$. Here, α and β are the first-order SU(3)-breaking parameters for the decays $g_T\to V_9V_9$ and P_9P_9 respectively. We have assumed ideal mixing for the ω-ϕ system, while θ_P is the octet-singlet mixing angle for the η-η' system, and $\theta_0=\arctan 1/\sqrt{2}\simeq 35°16'$. From these we can compute $B(g_T\to V_9V_9)$ and $\Gamma(g_T\to P_9P_9)$ in terms of $B(g_T\to\phi\phi)$ since $\Gamma(g_T\to\pi\pi)$ is experimentally given by Eq. (1.5). These are tabulated in Tables II and III for $\alpha=\beta=0$, i.e., no SU(3) breaking. The actual decay rate $\Gamma(g_T\to K\bar{K})$, for example, is obtained from the value in Table III by multiplying by a factor $(1+\frac{1}{2}\beta)^2$ as is evident from Eq. (2.2). For these calculations, it may be noted that we have used in Table II the S-wave phase volume for $g_T^{(1)}$ and D-wave phase volumes for $g_T^{(2)}$ and $g_T^{(3)}$ in accordance with the experiment for V_9V_9 decays. Also, we have set, for simplicity,

$$B_j=B(g_T^{(j)}\to\phi\phi)\tag{2.3}$$

in Table II. Also, we have taken into account the finite width correction of g_T's. If we neglect the width of g_T's, then the value of $B(g_T^{(1)}\to\rho\rho)$, for example, would become $20B_1$ which is almost twice larger than the value of $11.23B_1$ in the Table II. Also, if we allow the SU(3) violation, then $g_T\to\bar{K}K^*$ decay is now allowed, although all other P_9V_9 modes such as $g_T\to\pi\rho$ and $\eta\omega$ are still forbidden by the G parity. From Tables II and III, we see that contributions for $\Gamma(g_T\to V_9V_9)$ will decrease for smaller values of B_j, while this is opposite for $\Gamma(g_T\to P_9P_9)$. In order to obtain more definite answers, we will assume here $\alpha=\beta=0$, i.e., no SU(3) breaking. Although this may not be a good approximation, this will not affect our final conclusions greatly as long as $|\alpha|$ and $|\beta|$ are less than 0.2 or so. Let ϵ_j be all decay branching ratio other than modes $g_T^{(j)}\to V_9V_9$ and P_9P_9, i.e.,

$$\epsilon_j=B(g_T^{(j)}\to\text{all decay modes other than }V_9V_9\text{ and }P_9P_9)\ .\tag{2.4}$$

Moreover set

$$\begin{aligned}
&B(g_T^{(j)}\to P_9P_9)=\frac{N_j}{B_j}\ ,\\
&B(g_T^{(j)}\to V_9V_9)=l_jB_j\ .
\end{aligned}\tag{2.5}$$

Then N_j and l_j $(j=1,2,3)$ are numerical numbers independent of $B_j=B(g_T^{(j)}\to\phi\phi)$. For example, we find $l_j=26.7$ and $N_1\simeq 0.31\times10^{-2}\lambda$ from values of Tables II and III. Now, conservation of the probability requires

TABLE II. Calculated branching ratio for $g_T\to V_9V_9$.

	$g_T^{(1)}=g_T(2050)$	$g_T^{(2)}=g_T(2300)$	$g_T^{(3)}=g_T(2350)$
$B(g_T\to\phi\phi)$	B_1	B_2	B_3
$B(g_T\to\omega\omega)$	$3.64B_1$	$12.25B_2$	$9.25B_3$
$B(g_T\to\rho\rho)$	$11.23B_1$	$39.74B_2$	$29.95B_3$
$B(g_T\to K^*\bar{K}^*)$	$10.84B_1$	$22.04B_2$	$17.52B_3$
$B(g_T\to V_9v_9)$	$26.71B_1$	$75.03B_2$	$57.72B_3$

TABLE III. Calculated decay rates for $g_T \to P_9 P_9$ in MeV.

	$g_T(2050)$	$g_T(2300)$	$g_T(2350)$
$\Gamma(g_T \to \pi\pi)$	$0.34\lambda/B_1$	$0.19\lambda/B_2$	$0.30\lambda/B_3$
$\Gamma(g_T \to K\bar{K})$	$0.24\lambda/B_1$	$0.16\lambda/B_2$	$0.25\lambda/B_3$
$\Gamma(g_T \to \eta\eta)$	$0.05\lambda/B_1$	$0.035\lambda/B_2$	$0.05\lambda/B_3$
$\Gamma(g_T \to \eta\eta')$	$5\times10^{-4}\lambda/B_1$	$3\times10^{-3}\lambda/B_2$	$4.7\times10^{-3}\lambda/B_3$
$\Gamma(g_T \to P_9 P_9)$	$0.63\lambda/B_1$	$0.39\lambda/B_2$	$0.61\lambda/B_3$

$$\frac{N}{B}+lB+\epsilon=1 ,$$

where we have dropped the suffix j for simplicity. Since this is a quadratic equation for B, we can solve this as

$$B=\frac{1}{2l}\{(1-\epsilon)\pm[(1-\epsilon)^2-4Nl]^{1/2}\} , \tag{2.6a}$$

with

$$(1-\epsilon)^2 \geq 4Nl . \tag{2.6b}$$

In particular, Eqs. (2.6a) and (2.6b) also require

$$\frac{1}{2l}(1-\sqrt{1-4Nl}\,) \leq B \leq \frac{1}{2l}(1+\sqrt{1-4Nl}\,) . \tag{2.7}$$

Hereafter we ignore all experimental errors by taking central values with $\lambda=1$. Also, assuming the exact validity of the SU(3) symmetry (so that $\alpha=\beta=0$), Eqs. (2.6) and (2.7) require

$$\begin{aligned} &3.4\times10^{-3} \lesssim B_1 = B(g_T^{(1)} \to \phi\phi) \lesssim 0.035, \quad \epsilon_1 \lesssim 0.42 , \\ &2.3\times10^{-3} \lesssim B_2 = B(g_T^{(2)} \to \phi\phi) \lesssim 0.011, \quad \epsilon_2 \lesssim 0.24 , \\ &2.6\times10^{-3} \lesssim B_3 = B(g_T^{(3)} \to \phi\phi) \lesssim 0.015, \quad \epsilon_3 \lesssim 0.28 , \end{aligned} \tag{2.8}$$

where we have used the values of l and N in the Tables II and III. The lower bounds in Eq. (2.8) are better than those calculated from Eq. (1.5) together with $\Gamma(g_T \to \pi\pi) \leq \Gamma(g_T \to \text{all})$. The values of B as a function of the parameter ϵ are given in Fig. 1, assuming again that there is no SU(3) breaking with $\lambda=1$. Note that these estimates are independent of the validity of Eq. (1.16), i.e., we did *not* make the identification of the particle $M(2086)$ with $g_T^{(1)}(2050)$, which has been discovered by the Mark III group in $\psi/J \to M\gamma$, followed by $M \to 2\pi$. If we assume this identification with Eq. (1.16) and if we use the experimental upper bounds of Eqs. (1.14) and (1.17), we can improve the bound for B_1 further as

$$0.0041\lambda \lesssim B_1 \lesssim 0.012\lambda^{1/2} . \tag{2.9}$$

Here, we have now explicitly taken account of the experimental fudge factor λ for definiteness, although we still assume the exact validity of the SU(3). Then the most likely value for $B(g_T^{(1)} \to \phi\phi)$ lies somewhere between 2% and 0.2%. We also note that Eqs. (2.9) and (1.12) require

$$8.9 \geq \left| \frac{G(g_T^{(1)}\pi\pi)}{G(f'\pi\pi)} \right| \geq 5.7\lambda^{1/4} . \tag{2.10}$$

Since both $G_T \to \pi\pi$ and $f' \to \pi\pi$ will dominantly proceed via the two-gluon-exchange mechanism, and since the coupling constants of a glueball with two gluons is expected to be much larger than that of a quarkonium coupling with two gluons, the ratio in Eq. (2.10) may also be regarded to be consistent with $g_T^{(1)}$ being a glueball.

As an exercise, let us choose $B(g_T^{(1)} \to \phi\phi)=1\%$ with $\lambda=1$. This will give $B(g_T^{(1)} \to \rho\rho)=11\%$. $B(g_T^{(1)} \to \omega\omega) \simeq 3.6\%$, $B(g_T^{(1)} \to K^*\bar{K}^*) \simeq 10\%$, $B(g_T^{(1)} \to \pi\pi) \simeq 17\%$, $B(g_T^{(1)} \to K\bar{K}) \simeq 12\%$, and $B(g_T^{(1)} \to \eta\eta) \simeq 2.5\%$, which account for 58% of the total decay rate of $g_T^{(1)}$, provided that we can ignore SU(3)-violating effects.

Next, let us consider another possibility—that one of the g_T's, say, $g_T^{(j)}$, is a $s\bar{s}$ state. In that case, we expect to have

$$\Gamma(g_T^{(j)} \to \omega\omega) \approx \Gamma(g_T^{(j)} \to \rho\rho) << \Gamma(g_T^{(j)} \to K^*\bar{K}^*) . \tag{2.11}$$

However, the ratio $\Gamma(g_T \to K^*\bar{K}^*)/\Gamma(g_T \to \phi\phi)$ is still given by Table II. Also, we would expect

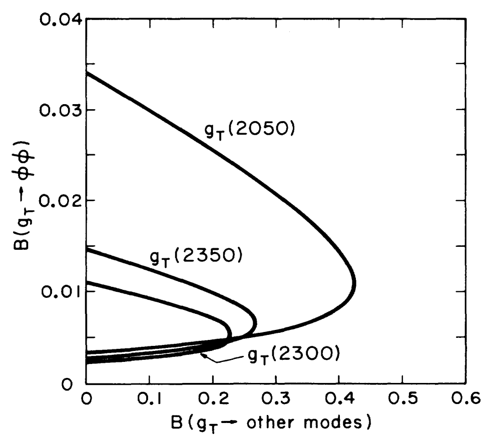

FIG. 1. The values of B_j as functions of ϵ_j, where $B_j=B(g_T^{(j)} \to \phi\phi)$ is the branching ratio of $g_T^{(1)}$ into $\phi\phi$ mode and $B(g_T \to \text{other modes})$ represents all branching ratios other than V_9V_9 and P_9P_9 decay modes.

$$\Gamma(g_T^{(j)}\to\pi\pi) \ll \Gamma(g_T^{(j)}\to K\bar{K})$$

and

$$\Gamma(g_T^{(j)}\to\pi\pi) \ll \Gamma(g_T^{(j)}\to\eta\eta) ,$$

since decays $g_T^{(j)}\to K\bar{K}$ and $\eta\eta$ can proceed now via continuous quark-line diagrams in contrast with two-gluon exchange for the two-pion decay mode. Since such a $s\bar{s}$ state must be a radially excited state of the f' meson, we would expect to have

$$\frac{\Gamma(f'\to\pi\pi)}{\Gamma(f'\to\text{all})} \approx \frac{\Gamma(g_T^{(1)}\to\pi\pi)}{\Gamma(g_T^{(1)}\to\text{all})} ,$$

which is badly violated. We also expect the validity of

$$\frac{M(g_T^{(j)}\to K\bar{K})}{M(g_T^{(j)}\to\pi\pi)} \approx \frac{M(f'\to K\bar{K})}{M(f'\to\pi\pi)} , \tag{2.12}$$

as well as

$$M(g_T^{(j)}\to\eta\eta) \simeq 2\sin^2(\theta_0-\theta_P) M(g_T^{(j)}\to K_0\bar{K}_0) . \tag{2.13}$$

Setting

$$\gamma = \left| \frac{M(f'\to K\bar{K})}{M(f'\to\pi\pi)} \right|^2 , \tag{2.14}$$

then we estimate

$$\frac{B(g_T^{(j)}\to K\bar{K})}{B(g_T^{(j)}\to\pi\pi)} = \begin{cases} 0.71\gamma & (j=1) , \\ 0.82\gamma & (j=2) , \\ 0.84\gamma & (j=3) , \end{cases} \tag{2.15a}$$

as well as

$$\frac{B(g_T^{(j)}\to\eta\eta)}{B(g_T^{(j)}\to\pi\pi)} = \begin{cases} 0.14\gamma & (j=1) , \\ 0.18\gamma & (j=2) , \\ 0.18\gamma & (j=3) , \end{cases} \tag{2.15b}$$

where we assumed $\theta_0-\theta_P \simeq 45°$. Since the value of $B(g_T^{(j)}\to\pi\pi)$ can be evaluated from Eq. (1.5) in terms of $B(g_T^{(j)}\to\phi\phi)$, we can repeat essentially the same analysis as before. But it turns out that the inequality Eq. (2.7) is badly violated for the present $s\bar{s}$ case. In order to pinpoint the difficulty, let us now assume $\Gamma(g_T^{(j)}\to\text{all})$ to be a free parameter. Moreover, we estimate γ conservatively to be

$$368 \gtrsim \gamma \gtrsim 141 , \tag{2.16}$$

corresponding to a very conservative estimate of

$$0.5 < B(f'\to K\bar{K}) < 1 . \tag{2.17}$$

Then the analogue of Eq. (2.6) becomes

$$\begin{aligned} &\frac{1}{\lambda}\Gamma(g_T^{(1)}\to\text{all})(1-\epsilon_1)^2 > 1066-2758 \text{ MeV} , \\ &\frac{1}{\lambda}\Gamma(g_T^{(2)}\to\text{all})(1-\epsilon_2)^2 > 1321-3412 \text{ MeV} , \\ &\frac{1}{\lambda}\Gamma(g_T^{(3)}\to\text{all})(1-\epsilon_3)^2 > 1214-3136 \text{ MeV} , \end{aligned} \tag{2.18}$$

which require too large values of $\Gamma(g_T^{(j)}\to\text{all})$ even for the minimum case of $\lambda=\frac{1}{2}$ and $\epsilon_j=0$. Therefore, we believe that for any $g_T^{(j)}$ $(j=1,2,3)$ to be a pure $s\bar{s}$ state is very unlikely.

Finally, let us consider the case of $g_T^{(j)}$ to be a $(s\bar{s})(s\bar{s})$ state. In this case, the dominant decay modes would be only $\Gamma(g_T\to\phi\phi)$ and $\Gamma(g_T\to\eta\eta)$. We can rewrite Eqs. (1.13) or (1.14) as

$$\lambda\frac{B(g_T^{(j)}\to\eta\eta)}{B(g_T^{(j)}\to\phi\phi)} \lesssim \begin{cases} 14.7, & j=1 , \\ 26.3, & j=2 , \\ 32.2, & j=3 , \end{cases} \tag{2.19}$$

if we use Eq. (1.5). For $j=1$, Eq. (1.19) implies, however,

$$B(g_T^{(1)}\to\phi\phi) \lesssim 0.024\lambda^{1/2} ,$$

so that Eq. (2.19) also gives

$$B(g_T^{(1)}\to\eta\eta) \lesssim 0.35\lambda^{-1/2} .$$

Therefore, we believe that $g_T^{(1)}$ is likely not a $(s\bar{s}s\bar{s})$ state, since we will not be able to account for the total decay rates in terms of supposed dominant modes $g_T^{(1)}\to\eta\eta$ and $\phi\phi$, not to mention the smallness of $B(g_T^{(1)}\to\phi\phi)$. Also, we note the following: If $g_T^{(j)}$ is a $(s\bar{s}s\bar{s})$ state, it is quite plausible that we have

$$\left[\frac{G(g_T^{(j)}\pi\pi)}{G(f'\pi\pi)}\right]^2 \ll 1 \tag{2.20}$$

in contrast with Eq. (1.12) since two-pion decay of a $(s\bar{s}s\bar{s})$ system is doubly forbidden in the sense of the quark-line rule.[19] Of course, it may be argued that both $g_T^{(j)}$ and f' couple with two pions via two-gluon exchange and hence there is no firm basis for the validity of Eq. (2.20). However, the $s\bar{s}s\bar{s}$ system is a kind of a molecular system, and its representative wave function will have a small value at the origin. Thus Eq. (2.20) may still be plausible. If we believe this, then it is a little difficult to reconcile it with Eq. (1.11),

$$\left[\frac{G(g_T^{(j)}\pi\pi)}{G(f'\pi\pi)}\right]^2 B(g_T^{(j)}\to\phi\phi) = \begin{cases} 0.33\lambda, & j=1 , \\ 0.13\lambda, & j=2 , \\ 0.20\lambda, & j=1 , \end{cases} \tag{1.11}$$

unless $B(g_T^{(j)}\to\phi\phi)\approx 1$ and $\lambda\approx\frac{1}{2}$. Since we cannot evaluate the ratio of the coupling constants accurately, our statement is really a caveat. The best possible way to discriminate glueballs from quarkonia (either $s\bar{s}$ or $s\bar{s}s\bar{s}$) is to directly measure the ratio $B(g_T\to\rho\rho)/B(g_T\to\phi\phi)$ and/or $B(g_T\to\omega\omega)/B(g_T\to\phi\phi)$, since the glueball model predicts large values for these ratios, but the quarkonium cases will give conversely very small values for the same.

III. $\psi/J\to g_T\gamma$ DECAY

The decays $\psi/J\to g_T\gamma$ and $f'\gamma$ must proceed in angular momentum states with $l=0$, 2, or 4. So far, there appears to be no experimental data available concerning this. To be definite, we assume here that they occur dominantly only in the S wave ($l=0$). The calculation of the decay rates $\Gamma(\psi/J\to g_T\gamma)$ and $\Gamma(\psi/J\to f'\gamma)$ is very difficult. However, their ratio may be estimated within an accuracy of a factor of 4 or so in the following way. We compare the Feynman diagrams for decays $\psi/J\to g_T\gamma$ and $f'\gamma$ as in Fig. 2. Then, we see that the ratio of the decay matrix elements may be estimated by

$$\frac{M(\psi/J\to g_T^{(j)}\gamma)}{M(\psi/J\to f'\gamma)}\simeq\frac{G(g_T^{(j)}gg)}{G(f'gg)}\ , \tag{3.1}$$

where $G(Xgg)$ is a coupling constant of the X meson ($X=g_T^{(j)}$ or f') with two gluons. However, the validity of Eq. (3.1) is at best approximate by the following reason. A general vertex function for a system consisting of a tensor meson and two virtual gluons contains in general five independent form factors. Therefore $G(Xgg)$ in Eq. (3.1) must be understood as a kind of average of these five-independent coupling parameters with coefficients which depend upon dynamics. Next, by a similar reasoning, we would also have

$$\frac{M(g_T^{(j)}\to\pi\pi)}{M(f_0'\to\pi\pi)}=\frac{G(g_T^{(j)}\pi\pi)}{G(f_0'\pi\pi)}\simeq\frac{\widetilde{G}(g_T^{(j)}gg)}{\widetilde{G}(f_0'gg)}\ , \tag{3.2}$$

where $\widetilde{G}(Xgg)$ is another average of these five coupling constants in general with different coefficients. In Eq. (3.2) we used f_0' instead of f', where f_0' is the bare $s\bar{s}$ component contained in the physical f' meson. The reason for it is due to the fact that $u\bar{u}+d\bar{d}$ component contained in f' may directly decay into two pions without exchange of two gluons. Strictly speaking, we should also use the bare $(g_T)_0$ instead of the physical g_T in Eq. (3.2) by the same reason. However, we assume in this note that the physical g_T does not contain an appreciable amount of $u\bar{u}+d\bar{d}$ component, although it may still contain a sizeable $s\bar{s}$ component in addition to the possibly dominant gluonic constituent. Although we would not have $\widetilde{G}(Xgg)=G(Xgg)$, we still expect to have its approximate validity, barring some accidental situations. Hence, setting

$$\mu_j=\frac{G(g_T^{(j)}gg)}{\widetilde{G}(g_T^{(j)}gg)}\frac{\widetilde{G}(f_0'gg)}{G(f'gg)}\ , \tag{3.3}$$

we expect that μ_j is at least of order of the unity, since we would show later $\widetilde{G}(f_0'gg)\simeq\widetilde{G}(f'gg)$. In this paper we tentatively assume $\frac{1}{2}\le\mu_j\le 2$, allowing a possible variation of $\widetilde{G}/G$ by a factor of 2 or so. From Eqs. (3.1)–(3.3), we find

$$\frac{M(\psi/J\to g_T^{(j)}\gamma)}{M(\psi/J\to f'\gamma)}=\mu_j\frac{M(g_T^{(j)}\to\pi\pi)}{M(f_0'\to\pi\pi)}\ . \tag{3.4}$$

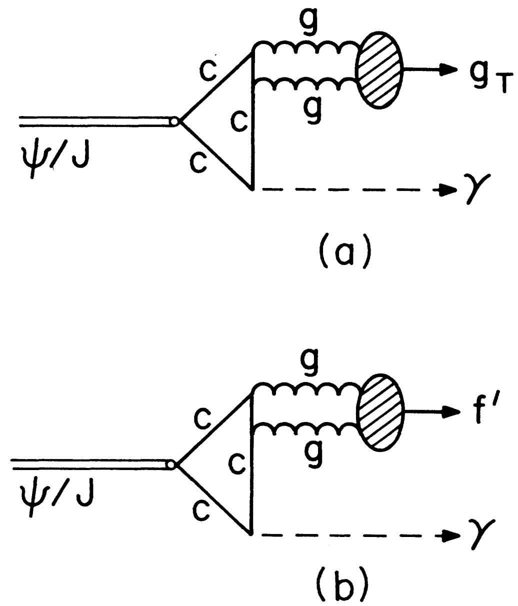

FIG. 2. Feynman diagrams responsible for $\psi/J\to g_T\gamma$ and $\psi/J\to f'\gamma$. The wavy and broken lines represent the gluon and photon, respectively.

For numerical evaluations of $B(\psi/J\to g_T^{(j)}\gamma)$ and $B(g_T^{(j)}\to\pi\pi)$, it is convenient to introduce R_j by

$$\frac{1}{R_j}=\left|\frac{M(\psi/J\to g_T^{(j)}\gamma)}{M(\psi/J\to f'\gamma)}\right|[B(g_T^{(j)}\to\phi\phi)]^{1/2} \tag{3.5a}$$

or equivalently

$$\frac{1}{R_j}=\left|\frac{\mu_j M(g_T^{(j)}\to\pi\pi)}{M(f_0'\to\pi\pi)}\right|[B(g_T^{(j)}\to\phi\phi)]^{1/2}\ . \tag{3.5b}$$

Then, we can express

$$B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\phi\phi)=\frac{k_j}{k}\left[\frac{1}{R_j}\right]^2 B(\psi/J\to f'\gamma)\ , \tag{3.6}$$

where k_j and k are magnitudes of the photon momenta for the (S-wave) radiative decays $\psi/J\to g_T^{(j)}\gamma$ and $f'\gamma$ at rest, respectively. For $j=1$, we assume the validity of Eq. (1.16) and rewrite

$$[B(g_T^{(1)}\to\pi\pi)]^2=\frac{[\Gamma(g_T^{(1)}\to\pi\pi)B(g_T^{(1)}\to\phi\phi)][B(\psi/J\to g_T^{(1)}\gamma)B(g_T^{(1)}\to\pi\pi)]}{\Gamma(g_T^{(1)}\to\text{all})B(\psi/J\to g_T^{(1)}\gamma)B(g_T^{(1)}\to\phi\phi)}\ .$$

We numerically evaluate the numerator of this equation from Eqs. (1.5) and (1.16). Then together with Eq. (3.6), it gives

$$B(g_T^{(1)}\to\pi\pi)\simeq 8.29\times10^{-4}\lambda^{1/2}R_1[B(\psi/J\to f'\gamma)]^{-1/2}\ , \tag{3.7}$$

where we neglected (and will neglect hereafter) for simplicity all experimental errors as in Eq. (1.16) and in

$\Gamma(g_T^{(1)}\to\text{all})=200^{+160}_{-50}$ MeV, using their central values. We calculate also $B(g_T^{(1)}\to\phi\phi)$ and $B(\psi/J\to g_T^{(1)}\gamma)$ from Eqs. (1.5), (1.16), and (3.7) to be

$$B(g_T^{(1)}\to\phi\phi)\simeq 2.05[B(\psi/J\to f'\gamma)]^{1/2}\lambda^{1/2}R_1{}^{-1}\ , \tag{3.8a}$$

$$B(\psi/J\to g_T^{(1)}\gamma)\simeq 0.362[B(\psi/J\to f'\gamma)]^{1/2}\lambda^{-1/2}R_1{}^{-1}\ . \tag{3.8b}$$

At the present time, the values of $B(\psi/J\to f'\gamma)$ quoted in the literature are

$$B(\psi/J\to f'\gamma)B(f'\to K\bar{K})=\begin{cases}(1.6\pm0.64)\times10^{-4}, & \text{Mark II}\ ,\\ (2.1\pm1.1)\times10^{-4}, & \text{DM 2}\ ,\\ (3.0\pm1.5)\times10^{-4}, & \text{Mark III}\ .\end{cases}$$

Here, we adopt the average of the three measurements

$$B(\psi/J\to f'\gamma)B(f'\to K\bar{K})\simeq 2.1\times10^{-4}$$

for definiteness. Assuming $B(f'\to K\bar{K})\simeq 0.87$ corresponding to $B(f'\to\eta\eta)\simeq 0.12$ of Eq. (1.9), this gives

$$B(\psi/J\to f'\gamma)\simeq 2.40\times10^{-4}$$

so that Eqs. (3.7) and (3.8) give

$$B(g_T^{(1)}\to\pi\pi)\simeq 0.0535\lambda^{1/2}R_1\ , \tag{3.9a}$$

$$B(g_T^{(1)}\to\phi\phi)\simeq 0.0318\lambda^{1/2}R_1{}^{-1}\ , \tag{3.9b}$$

$$B(\psi/J\to g_T^{(1)}\gamma)\simeq 5.61\times10^{-3}\lambda^{-1/2}R_1{}^{-1}\ . \tag{3.9c}$$

On the other side, Eq. (3.6) leads to

$$B(\psi/J\to g_T^{(1)}\gamma)B(g_T^{(1)}\to\phi\phi)\simeq 1.78\times10^{-4}(R_1)^{-2}\ , \tag{3.10a}$$

$$B(\psi/J\to g_T^{(2)}\gamma)B(g_T^{(2)}\to\phi\phi)\simeq 1.42\times10^{-4}(R_2)^{-2}\ , \tag{3.10b}$$

$$B(\psi/J\to g_T^{(3)}\gamma)B(g_T^{(3)}\to\phi\phi)\simeq 1.34\times10^{-4}(R_3)^{-2}\ . \tag{3.10c}$$

Again, Eqs. (3.9) but not Eqs. (3.10) presuppose the validity of Eq. (1.16).

It remains now that values of R_1, R_2, and R_3 have to be evaluated. To this end, we start with Eq. (3.5b) which can be rewritten as

$$\left[\frac{1}{R_j}\right]^2=(\mu_j)^2\left[\frac{M(g_T^{(j)})}{M(f')}\right]^2\left[\frac{k'}{k_j'}\right]^5\left|\frac{M(f'\to\pi\pi)}{M(f_0'\to\pi\pi)}\right|^2\times\frac{\Gamma(g_T^{(j)}\to\pi\pi)B(g_T^{(j)}\to\phi\phi)}{\Gamma(f'\to\pi\pi)}\ . \tag{3.11}$$

Here, k' and k_j' are magnitudes of the pion momenta for decays $f'\to\pi\pi$ and $g_T^{(j)}\to\pi\pi$ at rest, respectively, and $M(X)$ is the mass of the tensor boson X ($X=f'$ and $g_T^{(j)}$). Since the value of $\Gamma(g_T^{(j)}\to\pi\pi)B(g_T^{(j)}\to\phi\phi)$ is given in Eq. (1.5), the only unknown quantity in Eq. (3.11) is $M(f'\to\pi\pi)/M(f_0'\to\pi\pi)$ whose evaluation requires a specific f-f' mixing model. Although we can easily consider a general f, f', and θ mixing model,[25] it introduces many unknown parameters. Because of it, we consider here the simplest standard f-f' mixing theory with

$$\begin{aligned}|f_0\rangle&=\cos(\theta_0-\theta_T)|f\rangle+\sin(\theta_0-\theta_T)|f'\rangle\ ,\\ |f_0'\rangle&=\cos(\theta_0-\theta_T)|f\rangle-\cos(\theta_0-\theta_T)|f'\rangle\ ,\end{aligned} \tag{3.12}$$

where θ_T is the f-f' mixing angle and

$$\theta_0=\arctan 1/\sqrt{2}=35°16'$$

is the ideal mixing angle. We then find

$$\begin{aligned}\frac{M(f_0'\to\pi\pi)}{M(f'\to\pi\pi)}&=\frac{M(f\to\pi\pi)}{M(f'\to\pi\pi)}\left[\sin(\theta_0-\theta_T)-\frac{M(f'\to\pi\pi)}{M(f\to\pi\pi)}\cos(\theta_0-\theta_T)\right]\\ &=\frac{G(f\pi\pi)}{G(f'\pi\pi)}\left[\sin(\theta_0-\theta_T)-\frac{G(f'\pi\pi)}{G(f\pi\pi)}\cos(\theta_0-\theta_T)\right]\ ,\end{aligned} \tag{3.13}$$

where we note

$$M(f'\to\pi\pi)/M(f\to\pi\pi)=G(f'\pi\pi)/G(f\pi\pi)\ .$$

Now, Eq. (1.10) gives

$$G(f'\pi\pi)/G(f\pi\pi)=\pm0.038\ .$$

However, only the positive solution is permissible in view of results of the quark-line rule[19] as well as of the experimentally observed f-f' interference effect.[18] From Eqs.

(1.5), (3.11), and (3.13), we estimate

$$(R_j)^2=\frac{1}{\lambda(\mu_j)^2}[\sin(\theta_0-\theta_T)-0.038\cos(\theta_0-\theta_T)]^2 a_j\ , \tag{3.14a}$$

$$a_j=\begin{cases}2\times 10^3, & j=1\ ,\\ 5\times 10^3, & j=2\ ,\\ 3.3\times 10^3, & j=3\ .\end{cases} \tag{3.14b}$$

The right-hand side of Eq. (3.14a) is very sensitive for values assumed for θ_T because of a large cancellation. We note that the standard linear and quadratic SU(3) mass formula give $\theta_T \simeq 29°$ and $\theta_T \simeq 31°$, respectively, so that $|\theta_0-\theta_T| \ll 1$. This is the reason why we cannot set $M(f_0'\to\pi\pi)\simeq M(f'\to\pi\pi)$. However, for the evaluation of $\widetilde{G}(f_0'gg)$, we encounter no such cancellation so that we may set $\widetilde{G}(f_0'gg)\simeq\widetilde{G}(f'gg)$ in Eq. (3.3). Returning to the original problem, we adopt here the value of $\theta_T\simeq 28°$ by the following reason although its exact value is not essential. The validity of both the flavor-SU(3) and the quark-line rule[19] leads to

$$B(f'\to\eta\eta)\simeq 0.08[\sin(\theta_0-\theta_T)-\sqrt{2}\cos(\theta_0-\theta_T)]^2\ , \tag{3.15}$$

where we used the known decay rate of $\Gamma(A_2\to K\bar{K})\simeq 5.4$ MeV and $\theta_0-\theta_P\simeq 45°$, θ_P being the η-η' mixing angle. The value of $B(f'\to\eta\eta)\simeq 0.12$ used in Eq. (1.9) can be reproduced with $\theta_T\simeq 28°$.

In order to show that our calculations are compatible with the present experimental data, we tentatively assume here a large value of

$$|\sin(\theta_0-\theta_T)-0.038\cos(\theta_0-\theta_T)|\simeq 0.11(\lambda\mu_j{}^2)^{1/2} \tag{3.16}$$

which is consistent with $\theta_T\simeq 28°$ and $\lambda\mu_j^2\simeq 0.65$. However, the case of $\theta_T\simeq 31°$ is probably incompatible with Eq. (3.16), since it requires $\lambda\mu_j^2\simeq 0.11$. Note that we are implicitly assuming $\mu_1=\mu_2=\mu_3$. At any rate, Eqs. (3.14) give then

$$R_j=\begin{cases}4.94, & j=1\ ,\\ 7.80, & j=2\ ,\\ 6.37, & j=3\ ,\end{cases} \tag{3.17}$$

so that Eqs. (3.9) and (3.10) now predict

$$\begin{aligned}&B(g_T^{(1)}\to\pi\pi)\simeq 0.26\lambda^{1/2}\ ,\\ &B(g_T^{(1)}\to\phi\phi)\simeq 6.4\times 10^{-3}\lambda^{1/2}\ ,\\ &B(\psi/J\to g_T^{(1)}\gamma)\simeq 1.14\times 10^{-3}\lambda^{-1/2}\ ,\end{aligned} \tag{3.18}$$

as well as

$$B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\phi\phi)=\begin{cases}7.3\times 10^{-6}, & j=1\ ,\\ 2.33\times 10^{-6}, & j=2\ ,\\ 3.31\times 10^{-6}, & j=3\ .\end{cases} \tag{3.19}$$

Neglecting possible SU(3)-violating effects, our present solution with assumption of $g_T^{(1)}$ being a pure glueball implies

$$\begin{aligned}&B(g_T^{(1)}\to\pi\pi)\simeq 26\%,\quad B(g_T^{(1)}\to K\bar{K})\simeq 19\%\ ,\\ &B(g_T^{(1)}\to\eta\eta)\simeq 3.8\%,\quad B(g_T^{(1)}\to\rho\rho)\simeq 7.2\%\ ,\\ &B(g_T^{(1)}\to K^*\bar{K}^*)\simeq 6.4\%,\quad B(g_T^{(1)}\to\omega\omega)\simeq 2.3\%\ ,\end{aligned}$$

etc., with $\lambda=1$, accounting for 65% of the total decay rate of $g_T^{(1)}$. This gives

$$B(g_T^{(1)}\to\pi\pi)B(g_T^{(1)}\to\eta\eta)\simeq 0.01$$

which is consistent with the bound of Eq. (1.14). Also, our value for $B(g_T^{(1)}\to\phi\phi)$ is, of course, consistent with the bounds of Eq. (2.8).

Now, if we assume g_T's to be glueballs, then we can calculate $B(\psi/J\to g_T\gamma)B(g_T\to V_9V_9)$ from Eq. (3.19), since the value of $B(g_T\to V_9V_9)$ is given in Table II. The resulting computation is listed in Table IV with $\lambda=1$ and without the SU(3)-violating factors.

As we see from Table IV, our calculation is consistent with the present experimental upper bound without even considering the effects of possible SU(3) violation. We emphasize that the values of Table IV are independent of the validity of Eq. (1.16). If one of the g_T's is a $s\bar{s}s\bar{s}$ state, then we have to set values for $V_9V_9=\omega\omega$ and $\rho\rho$ to be essentially zero in Table IV. This case is also consistent with the present experiment. If the glueball interpretation of g_T's is correct, then the present upper limit for $B(\psi/J\to g_T\gamma)B(g_T\to\rho\rho)$ is only 1.85 times above our result in Table IV. Therefore, if more data for this quantity becomes available, then we will be in a good position to test our theory.

In Sec. I we noted that we have probably observed the decay $\psi/J\to g_T^{(1)}\gamma$ followed by $g_T^{(1)}\to\pi\pi$ decay, but that the same mode has not been observed for $g_T^{(2)}$ and $g_T^{(3)}$. From Eqs. (3.19) and (1.5) with values of R_2 and R_3 listed in Eq. (3.17), we calculate

$$\begin{aligned}&B(\psi/J\to g_T^{(2)}\gamma)B(g_T^{(2)}\to\pi\pi)\simeq 2.22\times 10^{-9}\frac{\lambda}{(B_2)^2}\ ,\\ &B(\psi/J\to g_T^{(3)}\gamma)B(g_T^{(3)}\to\pi\pi)\simeq 2.58\times 10^{-9}\frac{\lambda}{(B_3)^2}\ .\end{aligned} \tag{3.20}$$

The experimental lack of the decay mode may probably be interpreted as

$$B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\pi\pi)\lesssim 10^{-4}\quad\text{for } j=2 \text{ and } 3\ .$$

Then this requires

$$\begin{aligned}&B_2=B(g_T^{(2)}\to\phi\phi)\gtrsim 4.71\times 10^{-3}\lambda^{1/2}\ ,\\ &B_3=B(g_T^{(3)}\to\phi\phi)\gtrsim 5.08\times 10^{-3}\lambda^{1/2}\ ,\end{aligned}$$

which in turn lead to

$$\begin{aligned}&B(\psi/J\to g_T^{(2)}\gamma)\lesssim 0.49\times 10^{-3}\lambda^{-1/2}\ ,\\ &B(\psi/J\to g_T^{(3)}\gamma)\lesssim 0.65\times 10^{-3}\lambda^{-1/2}\end{aligned} \tag{3.21}$$

from Eq. (3.19). Unfortunately, the upper bounds of Eq.

TABLE IV. $B(\psi/J\to g_T\gamma)B(g_T\to V_9V_9)$.

V_9V_9	$g_T(2050)$	$g_T(2300)$	$g_T(2350)$	Total	Total experimental bound
$\omega\omega$	2.66×10^{-5}	2.85×10^{-5}	4.59×10^{-5}	1.01×10^{-4}	$<2.6\times10^{-4}$
$\rho\rho$	8.20×10^{-5}	9.26×10^{-5}	14.87×10^{-5}	3.23×10^{-4}	$<6\times10^{-4}$
$\phi\phi$	0.73×10^{-5}	0.23×10^{-5}	0.49×10^{-5}	0.15×10^{-4}	$<(3.1\pm1.0)\times10^{-4}$
$K^*\bar{K}^*$	7.92×10^{-5}	5.12×10^{-5}	8.70×10^{-5}	2.17×10^{-4}	

(3.21) are probably a little too small for interpretations of $g_T^{(2)}$ and $g_T^{(3)}$ being glueballs as we will see from the discussions in Sec. V. However, there is another possibility to explain the unobservability of the $\pi\pi$ mode. Because of the proximity of masses of $g_T^{(2)}$ and $g_T^{(3)}$, decay amplitudes for $g_T^{(2)}\to\pi\pi$ and $g_T^{(3)}\to\pi\pi$ in $\psi/J\to g_T^{(j)}\gamma$ decays may interfere destructively with each other to make them small.[26] In this case, there is no reason for accepting Eq. (3.21). We may mention that the experimental smallness of

$$B(\psi/J\to g_T^{(j)}\gamma)B(g_T^{(j)}\to\pi\pi)$$

for $j=2$ and 3 readily follow also if $g_T^{(2)}$ and $g_T^{(3)}$ are $(s\bar{s})(s\bar{s})$ states.

We may remark also that the present solution $B(g_T^{(1)}\to\phi\phi)\simeq0.64\%$ is not the only consistent one. For example, the case of $B(g_T^{(1)}\to\phi\phi)=1\%$ discussed in the previous section gives also a barely consistent solution with $R_1\simeq3.18$ and

$$|\sin(\theta_0-\theta_T)-0.038\cos(\theta_0-\theta_T)|\simeq0.071(\lambda\mu^2)^{1/2}\ .$$

In that case, we have to multiply a factor 2.4 to values of Table IV.

IV. COMMENTS ON θ, ξ, AND G MESONS

In Sec. I we noted that many reactions $\pi^-p\to Xn$ with $X=f,f',\epsilon,G,h$ mesons at 100 GeV/c have been found[14] to proceed dominantly via a one-pion-exchange mechanism. However, they did not see any signal for $\pi^-p\to\theta n$, followed by $\theta\to\eta\eta$ at the same energy on the level of

$$\sigma(\pi^-p\to\theta n)B(\theta\to\eta\eta)<0.5\text{ nb}\ \ (95\%\text{ confidence level})\ .$$

Repeating the same reasoning such as we have used in the derivation of Eq. (1.5), this implies

$$B(\theta\to\pi\pi)B(\theta\to\eta\eta)<1.29\times10^{-3}\ , \tag{4.1}$$

where we have used $\Gamma(\theta\to\text{all})=153\pm10$ MeV and $M(\theta)=(1710\pm50)$ MeV. Since we have experimentally[5]

$$B(\psi/J\to\theta\gamma)B(\theta\to\eta\eta)\approx B(\psi/J\to\theta\gamma)B(\theta\to\pi\pi)\simeq2.4\times10^{-4}\ , \tag{4.2}$$

Eqs. (4.1) and (4.2) require

$$B(\theta\to\pi\pi)<0.036\ , \tag{4.3}$$

$$B(\psi/J\to\theta\gamma)>6.70\times10^{-3}\ . \tag{4.4}$$

Then from the experimentally known rate for $B(\theta\to K\bar{K})/B(\theta\to\pi\pi)$, this gives

$$B(\theta\to K\bar{K})<0.14\ . \tag{4.5}$$

However, the experimental samples for the $\theta\to\eta\eta$ decay may be contaminated from $G\to\eta\eta$ decay so that the actual rate of $B(\theta\to\eta\eta)$ may be easily smaller by a factor of 0.7 in comparison to $B(\theta\to\pi\pi)$. Then the bounds in Eqs. (4.3) and (4.4) may change by 20%. This large branching ratio for the radiative θ decay of ψ/J is consistent with the usual belief that the θ meson is dominantly a glueball. We may compare this with[27]

$$B(\psi/J\to\gamma\iota(1460))>(6.9\pm0.4\pm1.0)\times10^{-3}\ . \tag{4.6}$$

However, it is difficult to explain $B(\theta\to\pi\pi)\simeq B(\theta\to\eta\eta)$, unless the flavor-SU(3) symmetry is badly broken or the θ contains a sizable quarkonium content.[2,25] Also, we estimate

$$\left|\frac{G(\theta\pi\pi)}{G(f\pi\pi)}\right|\lesssim0.176,\quad\left|\frac{G(\theta\pi\pi)}{G(f'\pi\pi)}\right|\lesssim4.59 \tag{4.7}$$

from the equations analogous to Eq. (1.4).

Next, let us consider ϵ and G mesons. They are 0^{++} mesons with $M(G)=1590$ MeV, $\Gamma(G)\simeq280$ MeV, $M(\epsilon)=1220$ MeV, and $\Gamma(\epsilon)=320$ MeV. At 100 GeV/c, it has been found[14] that

$$\begin{aligned}&\sigma(\pi^-p\to Gn)B(G\to\eta\eta)\simeq3.8\pm0.7\text{ nb}\ ,\\&\sigma(\pi^-p\to\epsilon n)B(\epsilon\to\eta\eta)\simeq3.7\pm1.5\text{ nb}\ ,\end{aligned} \tag{4.8}$$

with

$$B(\epsilon\to\eta\eta)\simeq2\times10^{-2}\ . \tag{4.9}$$

Again, using the same reasoning as before, this indicates

$$\frac{B(G\to\pi\pi)B(G\to\eta\eta)}{B(\epsilon\to\pi\pi)}\simeq2\times10^{-2}\ . \tag{4.10}$$

If G is dominantly a glueball, then we would expect to have roughly

$$\sigma(\pi^-p\to Gn)\lesssim\tfrac{1}{10}\sigma(\pi^-p\to\epsilon n)\ .$$

This requires

$$B(G\to\eta\eta)/B(\epsilon\to\eta\eta)\gtrsim10$$

from Eq. (4.8) and hence $B(G\to\eta\eta)\gtrsim0.2$ and $B(G\to\pi\pi)\lesssim0.09$, where we used the experimental value of $B(\epsilon\to\pi\pi)\simeq 0.9$.

Finally, let us consider $\xi(2230 \text{ MeV})$ with $\Gamma<50$ MeV. It has not been observed at 100 GeV/c with the limit of

$$\sigma(\pi^- p\to\xi n)B(\xi\to\eta\eta)<0.5 \text{ nb } (95\% \text{ C.L.}) . \quad (4.11)$$

So far, the spin-parity assignment of ξ is not well known. If it is 0^{++} or 2^{++}, then we can compare Eq. (4.11) with the corresponding cross section for ϵ or f meson. If ξ has 4^{++} assignment, we have to compare it with that of $h(2030)$, where we have

$$\begin{aligned}&\sigma(\pi^- p\to hn)B(h\to\eta\eta)=1.32\pm 0.35 \text{ nb} ,\\ &B(h\to\eta\eta)\simeq(2.2\pm 1.0)\times 10^{-3} .\end{aligned} \quad (4.12)$$

From these, we estimate

$$\Gamma(\xi\to\pi\pi)B(\xi\to\eta\eta)<0.43 \text{ MeV } (0^{++}) , \quad (4.13a)$$

$$\Gamma(\xi\to\pi\pi)B(\xi\to\eta\eta)<0.60 \text{ MeV } (2^{++}) , \quad (4.13b)$$

$$\Gamma(\xi\to\pi\pi)B(\xi\to\eta\eta)<0.06 \text{ MeV } (4^{++}) . \quad (4.13c)$$

V. CONCLUDING REMARKS

In the previous sections we demonstrated that all $g_T(2050)$, $g_T(2300)$, and $g_T(2350)$ are consistent with the hypothesis of g_T's being glueballs, although $g_T(2300)$ or $g_T(2350)$ could be a $(s\bar{s}s\bar{s})$ state. The best way of distinguishing the two possibilities is to measure the ratio $B(g_T\to\rho\rho)/B(g_T\to\phi\phi)$ which will be a large (or small) number for the former (or latter) model. We have also shown that none of g_T's is likely to be a radially excited $s\bar{s}$ state. It is, however, difficult to rule out the possibility that some of the g_T's may be a hybrid state $s\bar{s}g$.

We may note also that the two-gluon system allows four states with $J^{PC}=2^{++}$, corresponding to $^5S_2$, $^1D_2$, $^5D_2$, and $^5G_2$. It may be tempting to identify $\theta(1710)$, $g_T(2050)$, $g_T(2300)$, and $g_T(2350)$ with linear combination of these four states. In this context, we note that Lee[28] has analytically calculated $J^{PC}=2^{++}$ glueballs in the strong-coupling limit and obtains a cluster of three glueball states which could correspond to the three g_T's. For more elaboration of this point, see Ref. 6. Also, the mass reversal inequality $m(A_2)>m(f)$ necessitates[29] most likely the existence of at least one glueball state with mass larger than 1.88 GeV, provided that the so-called β-parameter of the particle mixing theory is positive. An argument which favors the positive sign for β will be discussed elsewhere.

These facts are in favor of g_T's being glueballs. However, we must be cautious. First, accepting the value of R_1 given in Eq. (3.17), we must have

$$B(\psi/J\to g_T^{(1)}\gamma)\simeq 1.14\times 10^{-3}\lambda^{-1/2} \quad (5.1)$$

as in Eq. (3.18). Although this value even with $\lambda=\frac{1}{2}$ is somewhat smaller than

$$B(\psi/J\to\theta\gamma)>6.70\times 10^{-3} \quad (5.2)$$

estimated in Eq. (4.4), it is still above five times larger than $B(\psi J\to f'\gamma)\simeq 2.40\times 10^{-4}$. Also, it must be kept in mind that our estimate Eq. (5.1) is sensitive on many approximations and assumptions as well as many experimental uncertainties, and we can easily enhance its value by a factor of 2 possibly even up to 3 with luck without any serious problem. Nevertheless, it still remains to be a problem for us to explain why the radiative decay width of $\psi/J\to g_T^{(1)}\gamma$ is smaller possibly by a factor of 4 when compared with $\psi/J\to\theta\gamma$. After all, if both θ and $g_T^{(1)}$, with the same spin-parity assignments 2^{++}, are dominantly glueballs, then the coupling constants of these glueballs to two gluons ought to be approximately the same since both states possess essentially the similar total decay rates. But in that case, we will expect that the decay rates for $\psi/J\to\theta\gamma$ and $\psi/J\to g_T^{(1)}\gamma$ should be also approximately the same. We can however give the following two tentative explanations for this "anomaly." First, as we noted in Sec. III, the interaction of the spin-2 particles with two gluons contain 5 independent form factors. Therefore, a factor of 2 discrepancy between *effective* coupling constants of θ and $g_T^{(1)}$ bosons to two gluons may not be implausible, just as we introduced the theoretical fudge factor μ in Eq. (3.9). Second, the θ meson appears to contain a sizable quarkonium content[2,25] in addition to its dominant glueball component. If both glueball and quarkonium contents of the θ interfere constructively for $\psi/J\to\theta\gamma$ decay, then this may enhance $B(\psi/J\to\theta\gamma)$ further. At any rate, we believe that the smallness of $B(\psi/J\to g_T^{(1)}\gamma)$ in comparison to $B(\psi/J\to\theta\gamma)$ is not a serious obstacle for the glueball interpretation of $g_T^{(1)}$. Although we have assumed here θ to be a glueball, there is a possibility that it might be rather a hybrid. Note that the decay branching rates of $B(\theta\to P_9P_9)$ and of $B(\theta\to V_9V_9)$ are very small in comparison to those of $B(g_T^{(1)}\to P_9P_9)$, as we will see from Eqs. (4.3) and (4.5). It appears that the θ-meson decays dominantly into some unknown multibody channel.

Next, our value of $B(g_T^{(1)}\to\phi\phi)\simeq 6.4\times 10^{-3}\lambda^{1/2}$ given in Eq. (3.18) implies

$$|G(f\pi\pi)|:G\,|(f'\pi\pi)|:|G(g_T^{(1)}\pi\pi)|\simeq 26:1:7.1\lambda^{1/4} \quad (5.3)$$

from Eq. (1.12). This is not unreasonable for the glueball hypothesis of $g_T^{(1)}$ as we have discussed already in connection with Eq. (2.10). We may compare this value with Eq. (4.7) which is rewritten as

$$|G(f\pi\pi)|:|G(f'\pi\pi)|:|G(\theta\pi\pi)|\simeq 26:1:\lesssim 4.6 . \quad (5.4)$$

Together with Eq. (5.3), this gives

$$\frac{|G(\theta\pi\pi)|}{|G(g_T^{(1)}\pi\pi)|}\lesssim 0.65\lambda^{-1/4} . \quad (5.5)$$

In this case, the situation is now opposite to the previous case for the radiative decay of ψ/J. However, for the same reasons as given before the ratio $G(\theta\pi\pi)/G(g_T^{(1)}\pi\pi)$ should not become too small. This fact could suggest that the upper bound in Eq. (5.5) and hence also in Eqs. (4.3) and (4.4) are probably almost equalities within 50% correction margins if θ is a glueball.

Of course, our predictions given in this paper could certainly change substantially, when more accurate experimental data, especially those of $B(g_T\to\phi\phi)$ and $B(g_T\to\rho\rho)$ become available. If our solution given in

Sec. III is correct with $B(g_T^{(1)}\to\pi\pi)B(g_T^{(1)}\to\eta\eta)\simeq 0.01$, then we would expect to have

$$\frac{\sigma(\pi^- p\to g_T^{(1)}n)B(g_T^{(1)}\to\eta\eta)}{\sigma(\pi^- p\to fn)B(f\to\eta\eta)}\simeq 1\ . \tag{5.6}$$

Therefore, if the 100-GeV/c CERN experiment[14] could improve its statistics by a factor of 2, then they could be able to observe $\pi^- p\to g_T^{(1)}n$ followed by $g_T^{(1)}\to\eta\eta$ in spite of large background due to the $h(2030)$ meson production. Similarly, high-energy experiments on $K^- p\to X\Lambda$ with $X=f,f',\theta$ and g_T may help to determine values of $B(g_T\to K\bar{K})$ and $B(\theta\to K\bar{K})$, not to mention $B(f\to K\bar{K})$ and $B(f'\to K\bar{K})$, assuming that the reaction at the high energies will take place dominantly via one K-meson exchange mechanism just as the one-pion-exchange process dominates $\pi^- p\to Xn$ reactions at high energies. For example, then we would find

$$\frac{\sigma(K^- p\to g_T^{(1)}\Lambda)B(g_T^{(1)}\to K\bar{K})}{\sigma(\pi^- p\to f'\Lambda)B(f'\to K\bar{K})}\simeq\left[\frac{k'}{k_G}\right]^5\left[\frac{M(g_T^{(1)})}{M(f')}\right]^2 \times\frac{\Gamma(g_T^{(1)}\to\text{all})}{\Gamma(f'\to\text{all})} \times\left[\frac{B(g_T^{(1)}\to K\bar{K})}{B(f'\to K\bar{K})}\right]^2, \tag{5.7}$$

where k' and k_G are magnitudes of the kaon momenta for $f'\to K\bar{K}$ and $g_T^{(1)}\to K\bar{K}$ decays at rest, respectively. Using $B(g_T^{(1)}\to K\bar{K})\simeq 0.19$ given in Sec. III, this will yield

$$\frac{\sigma(K^- p\to g_T^{(1)}\Lambda)B(g_T^{(1)}\to K\bar{K})}{\sigma(K^- p\to f'\Lambda)B(f'\to K\bar{K})}\simeq 0.028\ . \tag{5.8}$$

Also, we estimate

$$\frac{\sigma(K^- p\to g_T^{(1)}\Lambda)B(g_T^{(1)}\to\eta\eta)}{\sigma(K^- p\to f'\Lambda)B(f'\to\eta\eta)} \simeq 0.58\frac{B(g_T^{(1)}\to K\bar{K})B(g_T^{(1)}\to\eta\eta)}{B(f'\to K\bar{K})B(f'\to\eta\eta)} \simeq 0.039\ . \tag{5.9}$$

With respect to θ meson, we similarly calculate

$$\frac{\sigma(K^- p\to\theta\Lambda)B(\theta\to K\bar{K})}{\sigma(K^- p\to f'\Lambda)B(f'\to K\bar{K})}\lesssim 0.005\ ,$$

where we used $B(\theta\to K\bar{K})<0.14$ in view of Eq. (4.5). However, since we expect to have possible background due to production of $h(2030)$, it may be necessary to perform phase-shift analysis to eliminate the background for $g_T^{(1)}$.

Note added. After this paper was written, it came to our attention that the DM2 group has reported at the 1986 International Conference on High Energy Physics at Berkeley a broad structure in $\psi\to X\gamma$ followed by $X\to K_S K_S$ in the mass range of g_T's with $J^{PC}(X)=2^{++}$. Assuming that this is due to a single resonance, they find $M(X)=2198\pm 20$ MeV, $\Gamma(X\to\text{all})=217\pm 55$ MeV. It is therefore quite possible that this signature be due to the g_T's in accordance with Eq. (1.16). Also, C. A. Dominguez and N. Paver in a recent Trieste Report No. IC/86/84, 1984 (unpublished) conclude that QCD sum rules require existence of at least one tensor glueball with a mass $M\geq 2$ GeV and a width of about 200 MeV in addition to $\theta(1710)$.

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[1]See, for example, M. Chanowitz, in *Multiparticle Dynamics 1983*, proceedings of the XIV International Symposium, Lake Tahoe, 1983, edited by P. M. Yager and J. F. Gunion (World Scientific, Singapore, 1984); P. M. Fishbane and S. Meshkov, Commun. Nucl. Part. Phys. **13**, 325 (1984), S. Meshkov, in Proceedings of the Aspen Winter Physics Conference, 1986 (unpublished).

[2]J. F. Donoghue, in *The Quark Structure of Matter*, proceedings of the Yukon Advanced Study Institute, Whitehorse, Canada, 1984, edited by N. Isgur, G. Karl, and P. J. O'Donnell (World Scientific, Singapore, 1985).

[3]M. Svec, in *Hadron Spectroscopy—1985*, proceedings of the International Conference, College Park, Maryland, edited by S. Oneda (AIP Conf. Proc. No. 132) (AIP, New York, 1985).

[4]S. Cooper, in *Proceedings of the International Europhysics Conference on High Energy Physics*, Bari, Italy, 1985, edited by L. Nitti and G. Preparata (Laterza, Bari, 1985).

[5]K. Königsman, Phys. Rep. **139**, 243 (1986).

[6]S. J. Lindenbaum, Report No. BNL-37412, 1986 (unpublished); S. J. Lindenbaum and R. S. Longacre, in *Hadron Spectroscopy—1985* (Ref. 3).

[7]A. Eitken *et al.*, Phys. Rev. Lett. **49**, 1620 (1982); Phys. Rev. D **25**, 2446 (1982).

[8]H. Gomm, Phys. Rev. D **30**, 1120 (1984); G. Karl, W. Roberts, and N. Zagury, Phys. Lett. **149B**, 403 (1984).

[9]A. M. Badalyan and B. L. Ioffe, Institute of Theoretical and Experimental Physics Report No. ITEP-3, Moscow, 1986 (unpublished); Professor B. L. Ioffe (private communication).

[10]T. K. Kuo and N. Nakagawa, Phys. Rev. **32**, 306 (1985); B-A. Li and K-F. Liu, University of Kentucky report, 1985 (un-

published).
[11]F. E. Close, in *The Quark Structure of Matter* (Ref. 2) and references quoted therein.
[12]S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); J. H. Field, DESY Report No. 85-110, 1985 (unpublished).
[13]P. S. L. Booth *et al.*, Nucl. Phys. **B242**, 51 (1984); CERN Report No. EP/85-138, 1985 (unpublished).
[14]D. Alde *et al.*, Nucl. Phys. **B269**, 485 (1986).
[15]V. N. Bolotov *et al.*, Phys. Lett. **52B**, 489 (1974).
[16]C. Bromberg *et al.*, Phys. Rev. D **29**, 588 (1984).
[17]S. J. Lindenbaum (private communication).
[18]A. J. Pawlicki *et al.*, Phys. Rev. D **15**, 3196 (1977).
[19]S. Okubo, Prog. Theor. Phys. Suppl. **63**, 1 (1978).
[20]H. Willutzki, in *Hadron Spectroscopy—1985* (Ref. 3).
[21]A. Billoire *et al.*, Phys. Lett. **80B**, 387 (1979).
[22]R. M. Baltrasaitis *et al.*, Phys. Rev. Lett. **53**, 1723 (1985); Phys. Rev. D **32**, 566 (1985); **32**, 2883 (1985).
[23]B. Jean-Marie, in *Proceedings of the International Europhysics Conference in High Energy Physics* (Ref. 4).
[24]H. J. Lipkin, Phys. Lett. **124B**, 509 (1983).
[25]H. J. Schnitzer, Nucl. Phys. **B207**, 131 (1982); J. L. Rosner, Phys. Rev. D **27**, 1101 (1983); J. L. Rosner and S. F. Tuan, *ibid.* **27**, 1544 (1983); S. Ono and O. Pène, Z. Phys. C **21**, 109 (1983); A. Barmon, R. Casas, J. Casulleras, and F. Cornet, *ibid.* **28**, 573 (1985).
[26]S. Minami, Lett. Nuovo Cimento **41**, 289 (1984), has invoked a similar destructive interference mechanism to suppress the decay rate of $\psi/J \to g_T\gamma \to \phi\phi\gamma$.
[27]J. L. Rosner, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies,* Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1986).
[28]T. D. Lee, in Proceedings of the 21st Course of the International School of Subnuclear Physics, Erice, Trapani, Italy, 1983 (unpublished).
[29]R. Sinha, Nuovo Cimento **92A**, 83 (1986).