$\xi(2.2)$: A $\Lambda\bar{\Lambda}$ bound state rather than an orbitally excited quarkonium state

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We point out that a certain state (e.g., $^3P_0$) of $\Lambda\bar{\Lambda}$ bound system is rather narrow and can be a good candidate of $\xi(2.23)$. By using the quark-pair-creation model, we also study the decay widths of higher excited quarkonium states. We conclude that none of the quarkonium states is a likely candidate of $\xi(2.23)$.

I. INTRODUCTION

A narrow state called $\xi(2.2)$ was found[1] in $J/\psi \to \gamma X \to \gamma K^+K^-, \gamma K_S^0\bar{K}_S^0$ by the Mark III Collaboration. The spin of ξ is not known but it must be of even spin, parity, and charge conjugation ($J^{PC}=0^{++}, 2^{++}, 4^{++}, \ldots$) since it decays into $K\bar{K}$.

The experimental data for the ξ are[1]

$$\begin{aligned}
m(\xi) &= 2.232 \pm 0.007 \pm 0.007 \text{ GeV}/c^2 \quad \text{from } K_S^0 K_S^0 , \\
\Gamma(\xi) &= 0.018^{+0.023}_{-0.015} \pm 0.010 \text{ GeV}/c^2 \\
B(J/\psi \to \gamma\xi) B(\xi \to K_S^0 \bar{K}_S^0) &= (3.1^{+1.6}_{-1.3} \pm 0.7) \times 10^{-5} , \\
B(J/\psi \to \gamma\xi) B(\xi \to \mu^+\mu^-) &< 5 \times 10^{-6} , \\
B(J/\psi \to \gamma\xi) B(\xi \to K^*\bar{K}) &< 2.5 \times 10^{-4} , \\
B(J/\psi \to \gamma\xi) B(\xi \to K^*\bar{K}^*) &< 3 \times 10^{-4} , \\
J^{PC} &= (\text{even})^{++} .
\end{aligned} \tag{1.1}$$

Various authors studied the properties of ξ by identifying ξ as a Higgs boson,[2] a hybrid state,[3] a high-spin $q\bar{q}$ meson,[4] etc.[5]

There are difficulties with the identification of $\xi(2.2)$ as a physical neutral Higgs boson of the minimal standard model (see Refs. 2, 6, and 7). The nonobservation[8] of $\Upsilon \to \gamma\xi$, coupled with the upper limit on $B(\xi \to \mu^+\mu^-)$ seems to rule out most of the nonminimal models.

We studied[3] the possibility of identifying ξ as $s\bar{s}g$. We concluded that the possible quantum numbers for ξ as $s\bar{s}g$ are 0^{++} or 2^{++}. The hybrid state with $l_{s\bar{s}}=0$ ($l_{s\bar{s}}$ is the angular momentum between s and $\bar{s}$) seems excluded since it should be much broader than observed. An $l_{s\bar{s}}=2$ (2^{++}) hybrid may fit the width, but coupling to $\psi\gamma$ seems to be too weak.

Godfrey, Kokoski, and Isgur[4] identified ξ as an $L=3$, $s\bar{s}$ state. However, we will perform here an independent analysis by using the quark-pair-creation model which can reproduce widths of most known resonances. We show that the widths of all quarkonium states including the $L=3$, $s\bar{s}$ state are too large.

In the present paper we point out that the $\Lambda\bar{\Lambda}$ bound state is a good candidate for $\xi(2.23)$. One might think that all states become broad due to a large annihilation rate. However, in some models even deeply bound $p\bar{p}$ states become narrow. One should note that the situation is better in our case. The annihilation takes place very near the origin; thus such annihilation becomes less important for a loosely bound orbitally excited state. If $\xi(2.23)$ is a $\Lambda\bar{\Lambda}$ bound state the binding energy is almost zero. In this case Λ and $\bar{\Lambda}$ are just bound and the wave function must be spread out. Λ and $\bar{\Lambda}$ probably stay much of the time outside the potential range; thus, the annihilation rate becomes small.

Furthermore, we find a selection rule which forbids $K\bar{K}^*$ + c.c. decay mode for $\Lambda\bar{\Lambda}$, $^3P_{0,2}$ states. A similar selection rule is found for the decay of $p\bar{p}$ states.[9] One also expects a very small decay width $\xi \to \mu^+\mu^-$ in our model. If our interpretation is correct we expect $^3S_1$ and $^1S_0$, $\Lambda\bar{\Lambda}$ states at around 2.1 GeV with larger widths.

This paper is organized as follows. In Sec. II we show the $\Lambda\bar{\Lambda}$ potential used in our analysis and compute energy levels of various $\Lambda\bar{\Lambda}$ states. In Sec. III the total decay widths of various $\Lambda\bar{\Lambda}$ states are computed by using the optical model. A selection rule on a two-body decay mode is also discussed. In Sec. IV we criticize the interpretation of ξ as an $L=3$, $s\bar{s}$ state. Using the quark-pair-creation model we show that none of the states become narrow enough. A summary and conclusions are presented in Sec. V.

II. THE $\Lambda\bar{\Lambda}$ POTENTIAL

The $\Lambda\bar{\Lambda}$ one-boson-exchange potential (OBEP) is based upon t-channel exchanges of the mesons η, η', ω, and ϕ. In addition one adds a scalar meson σ_0 with isospin $I=0$ to simulate part of the 2π exchange. The $\Lambda\bar{\Lambda}$ potential can be derived by modifying the $N\bar{N}$ potential appropriately:

$$V(r)=-g^2\mu\left[\left[1-\frac{\mu^2}{2M^2}\right]F(\mu r)+\frac{\mu^2}{2M^2}G(\mu r)\mathbf{L}\cdot\mathbf{S}\right] \text{ for } \sigma_0 ,$$

$$=g^2\frac{\mu^3}{12M^2}[F(\mu r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+H(\mu r)S_{12}] \text{ for } \eta,\eta' ,$$

$$=-g^2\mu\left\{\left[1+\frac{\mu^2}{2M^2}\left[1+\frac{f}{g}\right]\right]F(\mu r) + \frac{\mu^2}{12M^2}\left[1+\frac{f}{g}\right]^2[2F(\mu r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-H(\mu r)S_{12}]-\frac{\mu^2}{2M^2}\left[3+\frac{4f}{g}\right]G(\mu r)\mathbf{L}\cdot\mathbf{S}\right\} \text{ for } \phi,\omega , \quad (2.1)$$

where

$$F(x)=x^{-1}e^{-x},\quad G(x)=(x^{-2}+x^{-3})e^{-x} ,$$
$$H(x)=(x^{-1}+3x^{-2}+3x^{-3})e^{-x} , \quad (2.2)$$
$$S_{12}=3\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 .$$

When the Schrödinger equation is used these potentials are too singular at the origin to have physically meaningful states. In order to avoid such bad behavior as $r\to 0$ one can, e.g., multiply the momentum-space version of the potential by $\Lambda_1{}^2/(\Lambda_1{}^2+q^2)$. This corresponds to introducing a form factor (see Ref. 11) to Λ-meson interaction. The effect of the cutoff factor is to transform each OBEP $V^{(i)}(r,m_i)$ to

$$[V^{(i)}(r,m_i)-V^{(i)}(r,\Lambda_1)][\Lambda_1{}^2/(\Lambda_1{}^2-m_i{}^2)] .$$

However, near the center we expect a large annihilation whose mechanism is not well known. Anyway the OBEP is not meaningful in the short-range region. Here we follow Dover and Richard[12] and simply cut off the potential at r_c:

$$V_R(r)=\begin{cases}\sum_i V^i(r_c,m_i), & r<r_c,\\ \sum_i V^i(r,m_i), & r\ge r_c .\end{cases} \quad (2.3)$$

This is the real part of our potential. The imaginary part will be discussed in Sec. III.

The $\Lambda\bar{\Lambda}$-meson coupling constants are computed by assuming SU(3)-nonet invariance:

$$G_{\Lambda\Lambda\eta}=-(\tfrac{4}{3})^{1/2}\alpha G_{NN\pi}(\cos\theta+\sin\theta) ,$$
$$G_{\Lambda\Lambda\eta'}=(\tfrac{4}{3})^{1/2}\alpha G_{NN\pi}(-\sin\theta+\cos\theta) ,$$
$$G_{NN\pi}{}^2/4\pi=14.2 ,$$
$$\theta=-11^\circ$$

(quadratic Gell-Mann –Okubo mass formula is used) ,

$$\alpha\equiv D/(D+F)=0.6 \ [\mathrm{SU}(6)] ,$$
$$g_{\Lambda\Lambda\eta}{}^2=G_{\Lambda\bar{\Lambda}\eta}{}^2/4\pi=4.26 ,$$
$$g_{\Lambda\Lambda\eta'}{}^2=G_{\Lambda\bar{\Lambda}\eta'}{}^2/4\pi=9.37 , \quad (2.4)$$
$$g_{\Lambda\Lambda\phi}{}^2=\tfrac{1}{9}g_{NN\omega}{}^2=24.0/9 ,$$
$$g_{\Lambda\Lambda\omega}{}^2=\tfrac{4}{9}g_{NN\omega}{}^2=24.0\times\tfrac{4}{9} ,$$
$$g_{\Lambda\bar{\Lambda}\sigma_0}{}^2=\lambda^2 g_{NN\sigma_0}{}^2,\quad \lambda=\frac{6-10\alpha}{9-12\alpha}$$

($\alpha=0$ for σ_0 is assumed) .

In the following we list the parameters used in our model (some of them are taken from Refs. 13 and 14):

Meson	Mass (MeV)	g^2
η	548.8	4.26
η'	957.57	9.37
ω	782.6	$24.0\times\frac{4}{9}$
ϕ	1019.5	$24.0\times\frac{1}{9}$
σ_0	550	4.20

(2.5)

The parameters of σ_0 are not well known. For example, we do not have any restriction on α for σ_0. One finds $\lambda=0$ for $\alpha=0.6$ [SU(6)], $\lambda=\frac{2}{3}$ for $\alpha=0$ ($D=0$), $\lambda=\frac{4}{3}$ for $\alpha=1$ ($F=0$). $\lambda=\frac{2}{3}$ is assumed here. We believe that our results obtained by using these parameters are not very sensitive to them. The expectation values of $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, S_{12} (tensor), and $\mathbf{L}\cdot\mathbf{S}$ are given by (for $l\neq 0$)

	j	$\mathbf{L}\cdot\mathbf{S}$	S	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$
$S=1$	$l+1$	l	$-2l/(2l+3)$	1
	l	-1	2	1
	$l-1$	$-l-1$	$-2(l+1)/(2l-1)$	1
$S=0$	l	0	0	-3

(2.6)

We plot the shape of the $\Lambda\bar{\Lambda}$ potential for ${}^3S_1$, ${}^1S_0$, ${}^3P_J$, ${}^1P_1$, ${}^3F_J$, and ${}^1F_3$ states in Fig. 1. V_{total} corresponds to the summation of all contributions ($\omega,\phi,\eta,\eta',\sigma_0$). The contributions from these five mesons are also plotted separately in this figure.

We solve the Schrödinger equation numerically with these potentials. Energy levels are plotted in Fig. 2 as a function of cutoff radius r_c. Since ξ has a positive parity the angular momentum between Λ and $\bar{\Lambda}$ should be odd. The lowest one is the P state. As seen from Fig. 2 three

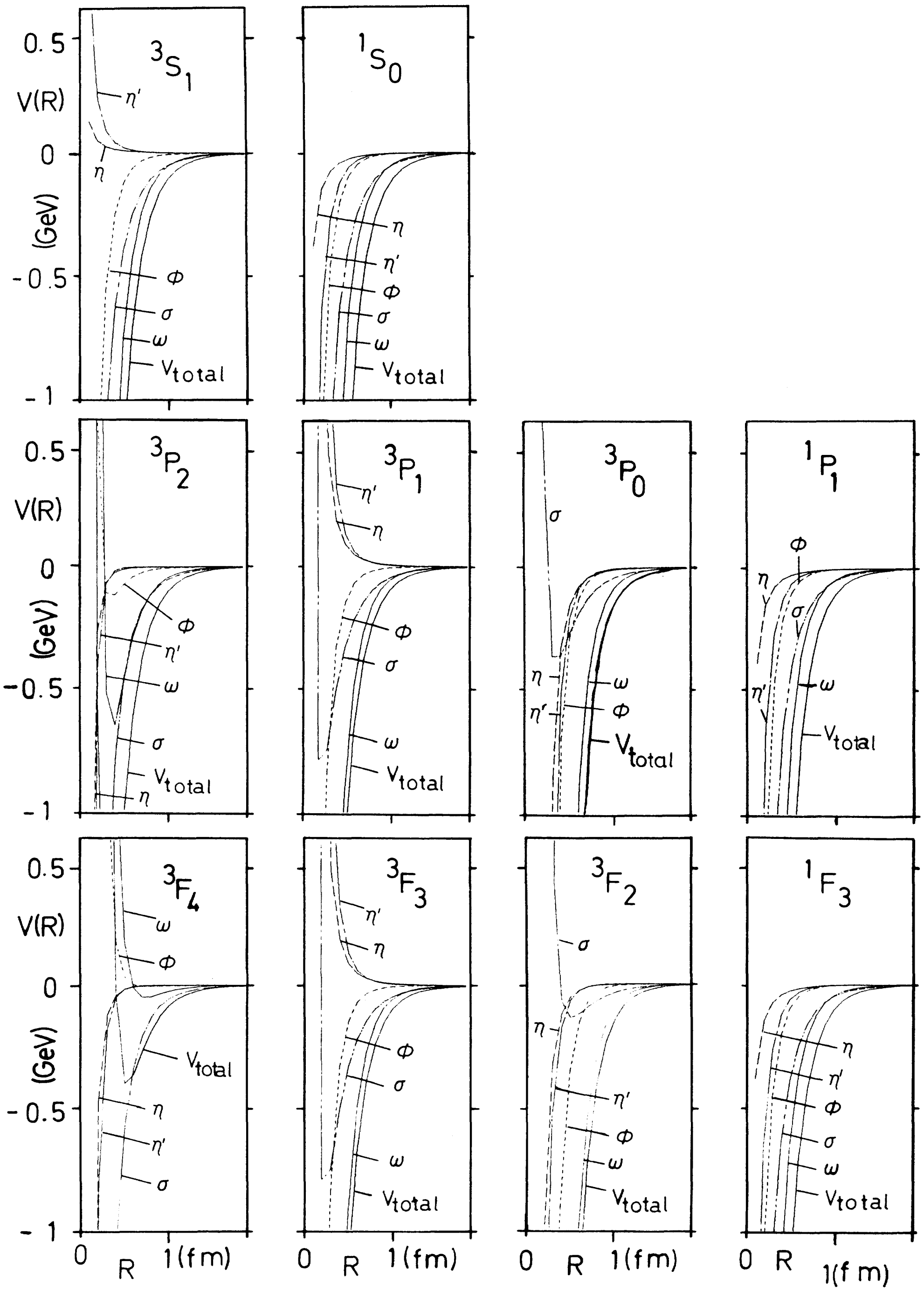

FIG. 1. Real part of potential between Λ and $\overline{\Lambda}$ as a function of the radius R. Contributions from ω, ϕ, η, η', and σ_0 are plotted separately. $V_{\text{total}} = V_\omega + V_\phi + V_\eta + V_{\eta'} + V_{\sigma_0}$.

levels $^3P_2$, $^3P_1$, and $^1P_1$ cluster and the $^3P_0$ state becomes much lower than others. From Fig. 1 one can also check that the potential for $^3P_0$ is by far the most attractive of all.

By using the Breit-Fermi Hamiltonian one can also find such a pattern in the P states of quarkonium.[15,16] This pattern is indeed observed experimentally, e.g., (S^*,H,D,f) or (δ,B,A_1,A_2). Parametrizing the spin-dependent potentials by $H^{LS}=A\mathbf{L}\cdot\mathbf{S}$, $H^T=BS_{12}$, $H^{SS}=C\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ one always finds such a pattern for $2B \gtrsim A \gtrsim B \gg C > 0$. This comes from strong attractive L-S force and tensor force for $S=1, j=l-1$ state.

From Fig. 1 one can also see that the potential for $^3F_2$ is the most attractive of all F states. Because of the repul-

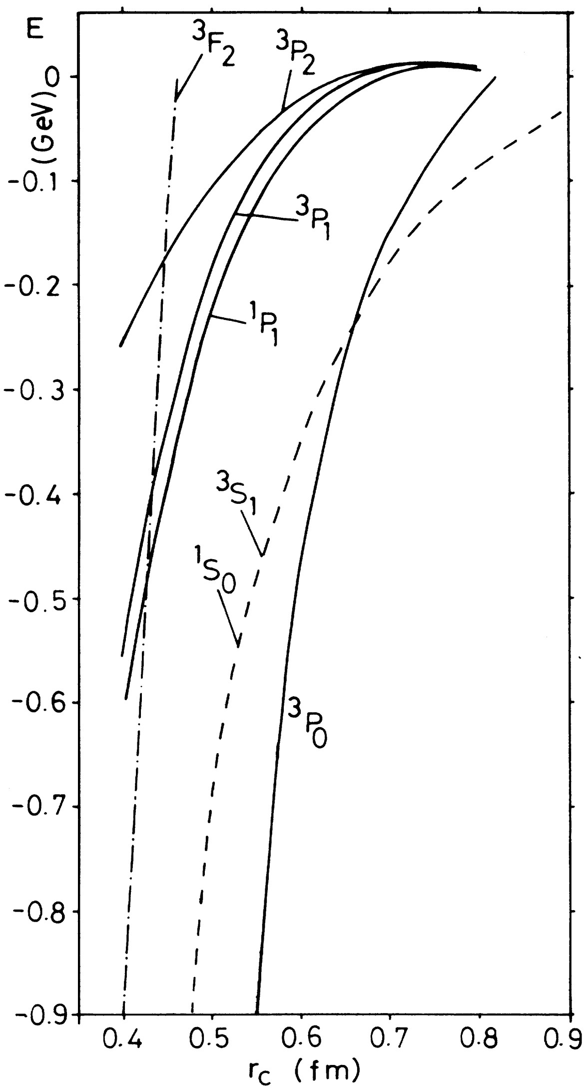

FIG. 2. The $\Lambda\bar{\Lambda}$ spectroscopy as a function of r_c defined in Eq. (2.3). Imaginary part of the potential is omitted. The splitting between ${}^3S_1$ and ${}^1S_0$ is too small to show in this figure.

sive centrifugal force P and F states become unbound for large r_c. In our model we find that ${}^3S_1$ and ${}^1S_0$ states are almost degenerate. The difference between these two states are too small to show in this figure. Since the spin-spin force is of short range, and since the central part of the potential is cut off in our model, the hyperfine splitting becomes very small in our model. In order to find the level splitting between ${}^3S_1$ and ${}^1S_0$ states one must study the short-range behavior of the potential seriously.

III. DECAY OF $\Lambda\bar{\Lambda}$ STATES

While very little is known about the decay of $\Lambda\bar{\Lambda}$ states, that of $p\bar{p}$ state was studied by various authors (e.g., Refs. 12, 14, and 17–21). In the optical potential approach it is necessary to add a strong imaginary potential to reproduce the experimental $p\bar{p}$ cross section.[17] This implies a large imaginary part in the energy. As a result we only find broad $p\bar{p}$ resonances. On the other hand, much narrower states are predicted in other approaches.[19,22]

Because of the lack of enough experimental data of $\Lambda\bar{\Lambda}$ scattering cross section in the present paper we compute various levels of $\Lambda\bar{\Lambda}$ states only in the optical model as a function of the strength of the imaginary potential. Dover and Richard[12] included the following imaginary part in their $p\bar{p}$ potential:

$$V_I(r) = -iW_0/(1+e^{(r-R)/a}) \tag{3.1}$$

with $a^{-1} = 5\ \text{fm}^{-1}$, $R=0$. Not very different values of these parameters a,R were also found by other authors.[23]

One can guess that the shape of the imaginary potential will be closely related to the shape of the quark wave function inside the proton or Λ. Schöberl, Falkensteiner, and Ono[24] computed the baryon spectrum by a variational method with a trial function

$$\psi = N\sum_{i=1}^{n} C_i \frac{(\alpha^i)^3 x^{3/4}}{\pi^{3/2}} \exp\left[-\frac{(\alpha^i)^2}{2}(\rho^2 + x\lambda^2)\right]. \tag{3.2}$$

In the case of two Gaussian approximation ($n=2$) they found

$$\begin{aligned} \alpha_1 &= 0.4018 \text{ for } p \text{ and } 0.4211 \text{ for } \Lambda\,, \\ \alpha_2 &= 0.6901 \text{ for } p \text{ and } 0.7291 \text{ for } \Lambda\,. \end{aligned} \tag{3.3}$$

From this result one can guess that the rms radius of Λ is around 5% smaller than that of p.

We simply neglect this difference and use the imaginary potential (3.1) with $a^{-1} = 5\ \text{fm}^{-1}$ and $R=0$ for $\Lambda\bar{\Lambda}$. The method to solve the Schrödinger equation with optical potential numerically is known (see, e.g., Ref. 25). We use the computer program developed by Morimatsu. In Figs. 3 and 4 we show how the poles of the S matrix for the P state and for the F state move in the energy plane ($E = -E_{\text{BE}} - i\Gamma/2$). As seen from these figures the decay width becomes smaller with decreasing binding energy. This is understandable since if the particle is loosely bound it spends most of the time far away from the center and rarely touches the annihilation potential which is concentrated near the center.

Since the mass of ξ is almost equal to twice the mass of Λ, the binding energy is very small. In Eq. (2.2) we adjust r_c so that we find small binding energy. We find $r_c = 0.6$ fm if ξ is the ${}^3P_2$ state and $r_c = 0.8$ fm if ξ is the ${}^3P_0$ state. As seen from Figs. 3 and 4, the ${}^3P_0$ state is substantially narrower than the ${}^3P_2$ state if the same W_0 is assumed.

If $\xi(2.2)$ is a ${}^3P_2$ state, the ${}^3P_0$ state must also be found below this and the mass will be 2.1–2.2 GeV (see Fig. 2). The ${}^3P_0$ state should be narrower than ξ. If nothing is seen below ξ, a more attractive option is to identify $\xi(2.2)$ as a ${}^3P_0$ state. In this case all other P states, ${}^3P_2$, ${}^3P_1$, and ${}^1P_1$ states are all above ${}^3P_0$ and unbound and will not be observed. Both ${}^3S_1$ and ${}^1S_0$ states are below $\xi(2.2)$. Their masses will be around 2.1 GeV and widths are larger.

A selection rule is known for the P-wave $p\bar{p}$ annihilation.[9] The ${}^3P_0$ and ${}^3P_2$ $p\bar{p}$ states cannot decay into a pseu-

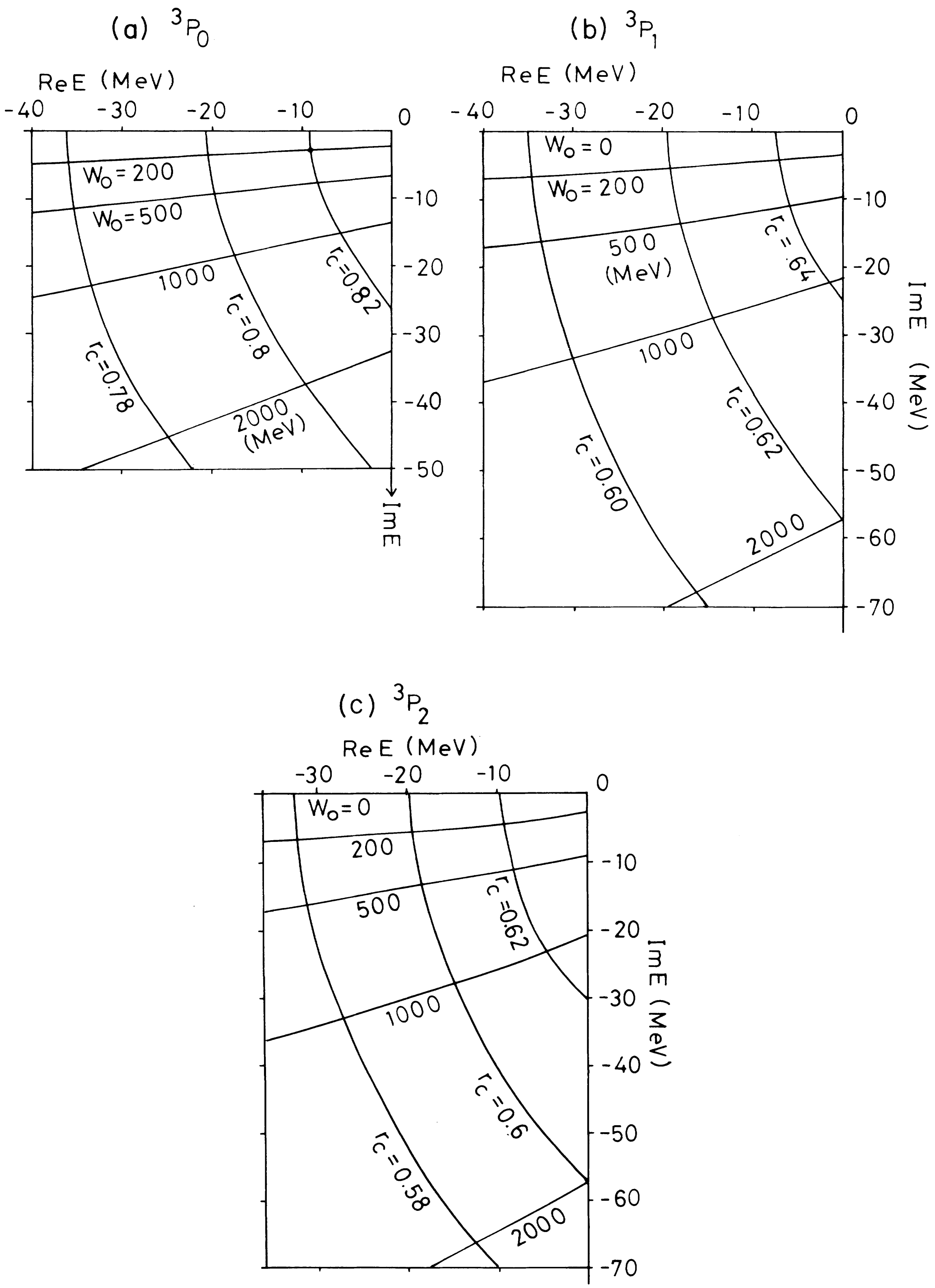

FIG. 3. Positions of poles of S matrix in E plane for P state are plotted as functions of W_0 [Eq. (3.1)] and r_c [Eq. (2.3)]. $E=-E_{BE}-i\Gamma/2$. (a) $^3P_0$; (b) $^3P_1$; (c) $^3P_2$. For example, in (a) the intersection between the line for $r_c=0.82$ fm and that for $W_0=1000$ MeV corresponds to the position of the pole in E plane ($E\simeq-5.9-15.2i$).

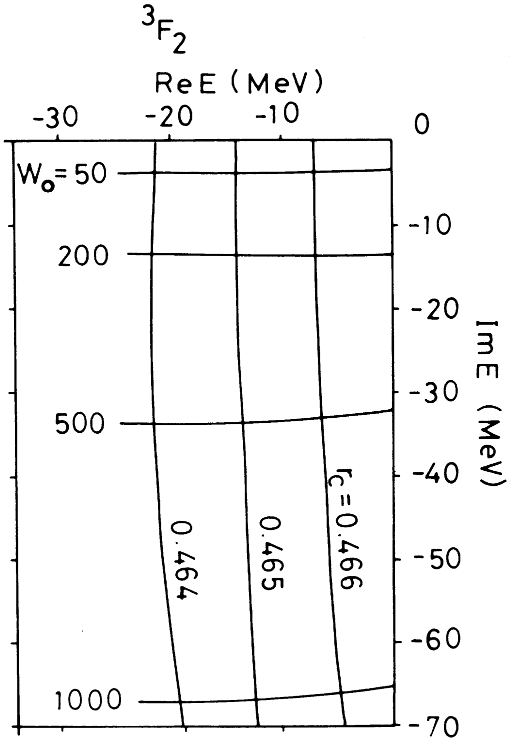

FIG. 4. Same as Fig. 3 but for F state.

doscalar meson and a vector meson, i.e.,

$$p\bar{p}, {}^3P_0, {}^3P_2 \to \rho^{\pm}\pi^{\mp}, \omega\pi, \eta\rho, \text{ etc. }, \quad (3.4)$$

is forbidden. If this selection rule can be generalized to $\Lambda\bar{\Lambda}$ one finds

$$\Lambda\bar{\Lambda}, {}^3P_0, {}^3P_2 \to K^*\bar{K} + \text{c.c.} \quad (3.5a)$$

is forbidden,

$$\Lambda\bar{\Lambda}, {}^3P_1 \to K^*\bar{K} + \text{c.c.} \quad (3.5b)$$

is allowed. The experimental upper limit for $B(\xi \to K^*\bar{K} + \text{c.c.})$ is[1] an order of magnitude larger than that for the observed $K\bar{K}$ final states.

In our model $\xi(2.2)$ is difficult to decay into $\mu^+\mu^-$ since three pairs of $q\bar{q}$ must annihilate into a single photon, thus the small branching ratio $B(\xi \to \mu^+\mu^-)$ is understandable. We are not trying to study all decay modes of $\xi(2.2)$ here, but we only make a comment on the $\pi\pi$ decay mode. The experimental upper limit is[1]

$$B(J/\psi \to \gamma\xi) B(\xi \to \pi\pi) < 2\times 10^{-5} . \quad (3.6)$$

This decay mode has more phase space than $K\bar{K}$ but the branching is smaller than or of the same order of $K\bar{K}$.

However, since we have a very large decay momentum in these processes, the simple phase-space relation $\Gamma \propto k^{2l+1}$ breaks down. The hadrons which we consider have a typical size of 0.8 fm. The relation $\Gamma \propto k^{2l+1}$ holds only for $k\times$ (hadron size) $\ll 1$, i.e., $k \ll 0.25$ GeV. The overlap integral which appears in the decay rate formula contains a factor which oscillates rapidly for large k. When this is integrated, it becomes small due to cancellation. Because of such a factor, the $\pi\pi$ decay mode is relatively more suppressed than $K\bar{K}$. Furthermore, there are two light quarks and one s quark inside Λ. In this respect light quarks annihilate more often than s. Therefore, we believe the present experimental limit of $\pi\pi$ is not a problem in our model.

IV. CAN ξ BE A QUARKONIUM STATE?

One of the most widely used models for the strong decay of the quarkonium states is the quark-pair-creation (QPC) model.[27,28] This model is so successful

TABLE I. Decay amplitudes for $^3P_0$ and for $^3F_4$ in terms of invariant amplitudes $\mathscr{L}_P(\pm)$ and $\mathscr{L}_F(\pm)$ which are defined in Eqs. (2.2) and (2.3) in Ref. 33.

Process	M^2	Process	M^2
$^3P_0, u\bar{u}+d\bar{d}\to\eta\eta$	$\frac{1}{36}\sin^4\delta_P \mathscr{L}_P(-)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\eta\eta$	$\frac{1}{36}\sin^4\delta_P \mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to\eta'\eta'$	$\frac{1}{36}\cos^4\delta_P \mathscr{L}_P(-)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\eta'\eta'$	$\frac{1}{36}\cos^4\delta_P \mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to\eta\eta'$	$\frac{1}{9}\sin^2\delta_P\cos^2\delta_P \mathscr{L}_P(-)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\eta\eta'$	$\frac{1}{9}\sin^2\delta_P\cos^2\delta_P \mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to\pi\pi$	$\frac{1}{12}\mathscr{L}_P(-)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\pi\pi$	$\frac{1}{12}\mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to\rho\rho$	$\frac{1}{27}\mathscr{L}_P(-)^2+\frac{20}{27}\mathscr{L}_P(+)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\rho\rho$	$\frac{5}{21}\mathscr{L}_F(-)^2+\frac{23}{168}\mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to K\bar{K}$	$\frac{1}{36}\mathscr{L}_P(-)^2$	$^3F_4, u\bar{u}+d\bar{d}\to K\bar{K}$	$\frac{1}{36}\mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to K\bar{K}^*+\text{c.c.}$	0	$^3F_4, u\bar{u}+d\bar{d}\to K\bar{K}^*+\text{c.c.}$	$\frac{5}{72}\mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to K^*\bar{K}^*$	$\frac{1}{108}\mathscr{L}_P(-)^2+\frac{5}{27}\mathscr{L}_P(+)^2$	$^3F_4, u\bar{u}+d\bar{d}\to K^*\bar{K}^*$	$\frac{5}{63}\mathscr{L}_F(-)^2+\frac{23}{504}\mathscr{L}_F(+)^2$
$^3P_0, u\bar{u}+d\bar{d}\to\omega\omega$	$\frac{1}{27}\mathscr{L}_P(-)^2+\frac{20}{27}\mathscr{L}_P(+)^2$	$^3F_4, u\bar{u}+d\bar{d}\to\omega\omega$	$\frac{5}{63}\mathscr{L}_F(-)^2+\frac{23}{504}\mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to\eta\eta$	$\frac{1}{18}\cos^4\delta_P \mathscr{L}_P(-)^2$	$^3F_4, s\bar{s}\to\eta\eta$	$\frac{1}{18}\cos^4\delta_P \mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to\eta'\eta'$	$\frac{1}{18}\sin^4\delta_P \mathscr{L}_P(-)^2$	$^3F_4, s\bar{s}\to\eta'\eta'$	$\frac{1}{18}\sin^4\delta_P \mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to\eta\eta'$	$\frac{1}{9}\sin^2\delta_P\cos^2\delta_P \mathscr{L}_P(-)^2$	$^3F_4, s\bar{s}\to\eta\eta'$	$\frac{1}{9}\sin^2\delta_P\cos^2\delta_P \mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to K\bar{K}$	$\frac{1}{18}\mathscr{L}_P(-)^2$	$^3F_4, s\bar{s}\to K\bar{K}$	$\frac{1}{18}\mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to K\bar{K}^*+\text{c.c.}$	0	$^3F_4, s\bar{s}\to K\bar{K}^*+\text{c.c.}$	$\frac{5}{36}\mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to K^*\bar{K}^*$	$\frac{1}{54}\mathscr{L}_P(-)^2+\frac{10}{27}\mathscr{L}_P(+)^2$	$^3F_4, s\bar{s}\to K^*\bar{K}^*$	$\frac{10}{63}\mathscr{L}_F(-)^2+\frac{23}{252}\mathscr{L}_F(+)^2$
$^3P_0, s\bar{s}\to\phi\phi$	$\frac{1}{54}\mathscr{L}_P(-)^2+\frac{10}{27}\mathscr{L}_P(+)^2$	$^3F_4, s\bar{s}\to\phi\phi$	$\frac{10}{63}\mathscr{L}_F(-)^2+\frac{23}{252}\mathscr{L}_F(+)^2$

TABLE II. Predicted widths (in MeV) of various quarkonium states. We assume $m_{q\bar{q}}=2.23$ GeV.

Decay product \ Initial state	$3\,^3P_0$ $u\bar{u}+d\bar{d}$	$3\,^3P_2$ $u\bar{u}+d\bar{d}$	$1\,^3F_2$ $u\bar{u}+d\bar{d}$	$1\,^3F_4$ $u\bar{u}+d\bar{d}$
$\eta\eta$	3.3	1.5	0.7	5.6
$\eta'\eta'$	5.5	0	1.9	0.2
$\eta\eta'$	26.1	0.3	4.0	8.6
$\pi\pi$	3.3	73	0.1	131
$\rho\rho$	396	196	1919	634
$K\bar{K}$	18	1.8	5.5	15.8
$K\bar{K}^*$+c.c.	0	3.6	13.5	43
$K^*\bar{K}^*$	307	3.4	48.4	62
$\omega\omega$	389	63	613	205
Decay product \ Initial state	**$3\,^3P_0$ $s\bar{s}$**	**$3\,^3P_2$ $s\bar{s}$**	**$1\,^3F_2$ $s\bar{s}$**	**$1\,^3F_4$ $s\bar{s}$**
$\eta\eta$	11.0	0.1	5.9	11.1
$\eta'\eta'$	1.6	0.05	6.3	0.21
$\eta\eta'$	13.0	0.5	14.2	6.1
$K\bar{K}$	41.6	0.1	6.9	97.4
$K\bar{K}^*$+c.c.	0	38	22.9	286
$K^*\bar{K}^*$	56.9	23	374	290
$\phi\phi$	2.7	7.6	15.2	61.9

phenomenologically that it has been used repeatedly (see, e.g., Refs. 29—35). We omit the detailed description of this model which was already given previously (see Refs. 33—35). We only list the decay amplitudes of $^3P_0$ and $^3F_4$ in Table I. Amplitudes for $^3P_2$ states and for $^3F_2$ states were already shown in Refs. 33 and 35, respectively. The formula to obtain the decay rate can be found in Ref. 33. The model used here is identical with that in Refs. 33 and 35.

In order to see which level is expected to be around 2.2 GeV we take the model of Ono and Schöberl[36,37] which can reproduce the quarkonium spectrum nicely. We show predicted masses of relevant states:

	Mass (MeV)
$u\bar{u}+d\bar{d},2^3P_2$	1913
$s\bar{s},2^3P_2$	2047
$u\bar{u}+d\bar{d},3^3P_2$	2466
$s\bar{s},3^3P_2$	2512
$u\bar{u}+d\bar{d},1^3F_2$	~2100
$s\bar{s},1^3F_2$	~2200
$u\bar{u}+d\bar{d},2^3F_2$	~2700

(4.1)

One can say that $2P$ states are too low and $2F$ states are too high. Thus $3P$ and $1F$ states both for $u\bar{u}+d\bar{d}$ and for $s\bar{s}$ can be candidates for $\xi(2.2)$. In Table II we show decay widths of these two states computed by using the QPC model. The mass of the initial particle is assumed to be 2.23 GeV. As seen from Table II the narrowest state is $3\,^3P_2,s\bar{s}$ (width ~70 MeV). However, this width is already much larger than the observed width of ξ. To make the matter worse the coupling to $K\bar{K}$ is too small. Other states including all $l=3$ states are far broader than observed. Thus we conclude that none of quarkonium states are likely candidates of $\xi(2.23)$.

We do not know the precise reason why the results in the Godfrey-Kokoski-Isgur (GKI) model[4] are so different from ours. We have a feeling that if both $^3F_2$ and $^3F_4$ are as narrow as $\xi(2.23)$ as predicted by the GKI model, many other quarkonium states with large l will also become narrow and are easy to be detected. Some of them should be seen even in $J/\psi\rightarrow\gamma X$. One should compute decay widths of all $q\bar{q}$ states with large l (e.g., $l=2,3,4$) in the GKI model and compare with what is observed.

On the other hand, in the QPC model $q\bar{q}$ states with large l are broad. This will naturally explain why they are so difficult to observe.

V. SUMMARY AND CONCLUSIONS

To summarize, we have discussed the nature of the $\xi(2.23)$. First we have studied the properties of $\Lambda\bar{\Lambda}$ states by using the one-boson-exchange potential with suitable modifications. We have shown $^3P_0$ state is the most likely candidate for $\xi(2.23)$. By using the optical potential we have shown that the $^3P_0$ state is relatively narrow.

It is natural to expect that the orbitally excited $\Lambda\bar{\Lambda}$ states will become relatively narrow since the wave function at the origin of these states is zero and the annihilation takes place near the origin. By using the optical model we have checked that the annihilation rate becomes still smaller for our case where the binding energy is very small. In our model $^3S_1$ and $^1S_0$, $\Lambda\bar{\Lambda}$ states must be there below $\xi(2.2)$. Since the wave function of the S state is not zero at the origin the width of S state is larger than that of P state.

One might think that orbitally excited $p\bar{p}$ states should be also narrow if one of the $\Lambda\bar{\Lambda}$ states is narrow. We can consider the following possibilities.

(i) Some of the $p\bar{p}$ states are indeed narrow. They are already found experimentally and some models predicted such narrow $p\bar{p}$ states theoretically.

(ii) In the case of $\xi(2.23)$ the binding energy is almost zero. If the $p\bar{p}$ potential is slightly less attractive, the corresponding orbitally excited state becomes unbound. The S state is substantially broader if it is bound. In this case there will be no narrow $p\bar{p}$ states.

We have given reasons why the branching ratio $B(\xi\rightarrow\mu^+\mu^-)$ is small in our model and we have shown a selection rule $\xi\not\rightarrow K\bar{K}^*$ + c.c. for $\xi={}^3P_{0,2},\Lambda\bar{\Lambda}$.

Second, we have studied the properties of quarkonium states. States which have masses around 2.2 GeV are $3P$ and $1F$ (both $u\bar{u}+d\bar{d}$ and $s\bar{s}$) states. By using the quark-pair-creation model we have computed decay widths of all these states. It is found that these are all too broad to fit ξ. Since this model fails very rarely by a factor 2 we conclude none of the quarkonium states are likely candidates for the ξ.

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