PHYSICAL REVIEW D VOLUME 35, NUMBER 3 1 FEBRUARY 1987

Kobayashi-Maskawa angles and SU(3) breaking in hyperon beta decay

John F. Donoghue, Barry R. Holstein, and Stefan W. Klimt*
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003
(Received 5 September 1986)

The determination of the Kobayashi-Maskawa matrix element V_{us} from hyperon β decays has long had a hidden uncertainty due to the almost universal assumption of SU(3) invariance in Cabibbo-type fits, especially since the data definitely indicate the presence of SU(3) breaking. We have reanalyzed the hyperon-decay data using the pattern of symmetry breaking predicted by the quark model including the center-of-mass correction. We find that the SU(3)-broken picture is far superior to the assumption of perfect SU(3), and provides a good fit to experiment. The sensitivity of V_{us} to the breaking is not large and we find $V_{us}=0.220\pm0.001\pm0.003$ (the errors are experimental and theoretical, respectively), in agreement with the results from kaon decay by Leutwyler and Roos.

I. INTRODUCTION

The seemingly arcane topic of SU(3) breaking in hyperon β decay is in fact important to the evaluation of the Kobayashi-Maskawa (KM) angles. The only high-precision determination of the KM elements comes from β decays.[1] For V_{ud}, the $\Delta S=0$, 0^+-0^+ nuclear β decays provide a measure of $\cos\theta_1$ at the level of 0.1% accuracy. In the case of V_{us} one turns to hyperon and kaon β decay. Individual hyperon decay rates are known to about 5% (2% in some cases) and, when used in combination, can provide a determination of V_{us} with an experimental error of about 1%. However, these data are generally analyzed under the assumption of perfect SU(3) invariance for the $\Delta S=1$ weak form factors. We expect SU(3) breaking to be significantly larger than 1% (indeed electromagnetic effects enter at this level) and such corrections could significantly modify the value of V_{us}. For example, a common shift in f_1 and g_1 for $\Delta S=1$ decays (caused, for example, by a lack of perfect overlap between s- and u-quark wave functions) would be exactly compensated by a corresponding increase in the apparent value of V_{us}. Until we understand the role of such SU(3) breaking, we cannot claim to have measured V_{us} precisely.

The purpose of this paper is to provide the needed description of SU(3) breaking and to test the sensitivity of V_{us} to such effects. We will use as our primary tool the quark model. Experience with quark-model calculations has shown that they provide a good guide to the size of first-order SU(3) breaking caused by the s and u,d quark mass difference. For example, baryon magnetic moments show a pronounced dependence on this mass splitting, which is well reproduced by the quark model. There remains some discrepancy in the description of the moments, but this is much smaller in magnitude than the dominant SU(3) breaking which is accurately portrayed by the quark model. We would expect correspondingly in predictions of weak form factors that the quark model would do better than the use of exact SU(3) and would provide a good estimate of the size and pattern of the leading SU(3)-breaking effects. The combined data on hyperon decay do indeed indicate the presence of such SU(3) breaking.[2] (As described below in Sec. III, we do not feel that this is an artifact of combining different experiments.) Unfortunately, previous use of the quark-model symmetry breaking[3] did *not* lead to an improved fit to the hyperon-β-decay data; indeed the fit was worse.[2,4] We have realized, however, that earlier calculations have omitted a crucial ingredient—center-of-mass or recoil corrections—-which modify both the magnitude and pattern of symmetry breaking. In Sec. II we calculate these heretofore neglected effects, and in Sec. III redo the Cabibbo fits.

We find that the quark-model pattern of symmetry breaking does in fact lead to a significantly improved description of hyperon β decay. Most importantly, we find that the fitted value of V_{us} depends far less sensitively on SU(3) breaking than might have been anticipated. We find only a 2% shift in V_{us} even though some form factors change by as much as 6% from their exact-SU(3) value and though the χ^2 of the fit changes by 11 units for 7 degrees of freedom. This reassures us both that the SU(3) breaking is under control and that it is not a major uncertainty in the determination of V_{12}. Our final value

$$V_{us}=0.220\pm0.001\pm0.003$$

(the first error is experimental and the second is our estimate of theoretical uncertainties) agrees well with that determined from kaon semileptonic decays by Leutwyler and Roos.[5] Our conclusion is that we do have a precise value of V_{us}.

II. SYMMETRY BREAKING IN THE QUARK MODEL

In this section we describe the calculation of the vector and axial-vector form factors in the quark model and identify the mechanisms of SU(3) breaking. Particular attention is given to the "center-of-mass" effect since this is the new feature of this calculation. We employ the wave-packet technique of Donoghue and Johnson,[6] although similar results can be obtained by the recoil-correction methods used by other authors.[7]

The method starts off by noting that a static extended object at rest, such as obtained in a quark-model picture of a hadron, cannot itself be a momentum eigenstate, but can at best be a superposition of momentum eigenstates with some weighting factor $\phi_B(p)$:

$$|B\rangle_{\rm QM}=\int d^3p\,\phi_B(p)\,|B(p)\rangle\ . \tag{1}$$

Such a superposition will have $\langle \mathbf{p}\rangle=0$ but $\langle \mathbf{p}^2\rangle\neq 0$. Given the normalization

$$\begin{aligned}&{}_{\rm QM}\langle B\,|\,B\rangle_{\rm QM}=1\ ,\\ &\langle B(p')\,|\,B(p)\rangle=(2\pi)^3\delta^3(p-p')\ ,\end{aligned} \tag{2}$$

one constrains $\phi_B(p)$ (the wave packet) to be normalized as

$$\int d^3p\,|\phi_B(p)|^2(2\pi)^3=1\ . \tag{3}$$

We are interested in calculating the weak form factors f_1 and g_1 defined by

$$\begin{aligned}\langle B_b\,|\,V_\mu(x)\,|\,B_a\rangle&=\bar u(p')(f_1^{ab}\gamma_\mu+\cdots)u(p)e^{-i(p-p')\cdot x}\ ,\\ \langle B_b\,|\,A_\mu(x)\,|\,B_a\rangle&=\bar u(p')(g_1^{ab}\gamma_\mu\gamma_5+\cdots)u(p)\\ &\quad\times e^{-i(p-p')\cdot x}\ .\end{aligned} \tag{4}$$

The ellipses in Eq. (4) refer to additional vector and axial-vector form factors which contribute to the decay rates only in order q/M or higher. These terms are adequately treated in Ref. 3, and their contribution to hyperon-decay rates are so small that, although possible, it is pointless to add very small modifications to that analysis.

In order to project out the leading form factors f_1 and g_1, we consider the $\mu=0$ component of V_μ and the $\mu=3$ component of A_μ (all baryons are treated with spin up). Then we have

$$\begin{aligned}\int d^3x\,\langle B_b\,|\,V_0(x)\,|\,B_a\rangle&=f_1^{ab}\int d^3p\,\phi_b^*(p)\phi_a(p)(2\pi)^3\bar u_b(p)\gamma_0u_a(p)\\ &\equiv f_1^{ab}\rho_V^{ab}\ ,\\ \int d^3x\,\langle B_b\,|\,A_3(x)\,|\,B_a\rangle&=g_1^{ab}\int d^3p\phi_b^*(p)\phi_a(p)(2\pi)^3\bar u_b(p)\gamma_3\gamma_5u_a(p)\\ &\equiv g_1^{ab}\rho_A^{ab}\ .\end{aligned} \tag{5}$$

In the limit that $p^2/m^2\ll 1$, both wave-packet integrals reduce to the normalization integral and $\rho_A=\rho_V=1$. This is nothing more than the usual nonrelativistic-quark-model method of calculating form factors.[8] However, for our analysis we properly perform the full calculation, including p^2/m^2 corrections.

There are two places wherein SU(3) breaking enters the calculation. On the left-hand side of Eq. (5) we need to employ specific wave functions for the quarks within the weak current. If there exists a slightly different spatial dependence for u, d, and s quarks, then the $\Delta S=1$ wave-function overlap of s and u will be somewhat different than the $\Delta S=0$ overlap with d and u. In the case of the vector current, the Ademollo-Gatto theorem requires that SU(3) breaking be second order in the symmetry-breaking parameter,[9] and hence should be small. In Ref. 3 this effect was calculated in the MIT bag model,[10] and it was found that all $\Delta S=1$ vector couplings were reduced by a factor 0.987 compared to their exact SU(3) predictions, while $\Delta S=0$ couplings were unchanged. A similar effect occurs for the case of the axial-vector current, where now, however, the breaking is first order and hence larger. It was determined that the $\Delta S=1$ axial-vector couplings are increased by a common factor of 1.08 compared to their usual predictions. Aside from this overall shift in the scale of $\Delta S=1$ form factors, the SU(3) structure of the matrix elements remains unchanged. Explicit derivation of these results is presented in Ref. 3.

The second origin of SU(3) breaking is on the right-hand side of Eq. (5), i.e., in the wave packets. Again the Ademollo-Gatto theorem guarantees that the deviation of ρ_V from unity must be second order in symmetry breaking. In fact, we find, using the wave packets described below, that ρ_V is never more than 0.3% away from unity and hence we will henceforth use $\rho_V=1$. In the case of the axial-vector current we will consider the leading correction in $\langle p^2\rangle/m^2$, using the relation

$$\bar u(p)\gamma_3\gamma_5u(p)\simeq 1-\frac{p^2}{3m_am_b}\left[\frac{1}{4}+\frac{3m_b}{8m_a}+\frac{3m_a}{8m_b}\right]\ . \tag{6}$$

With this we obtain

$$\rho_A^{ab}=1-\frac{\langle p^2\rangle_{ab}}{3m_am_b}\left[\frac{1}{4}+\frac{3m_b}{8m_a}+\frac{3m_a}{8m_b}\right] \tag{7}$$

and, finally then,

$$g_A^{ab}=\frac{1}{\rho_A^{ab}}\left\langle B_b\,\middle|\,\int d^3x\,A_3(x)\,\middle|\,B_a\right\rangle\ . \tag{8}$$

However, we are interested primarily in SU(3) breaking, *not* in the absolute value of g_A predicted within a given quark model. In our fits then we normalize the axial-vector couplings by that of the neutron proton transition. Hence, the entire SU(3) breaking is contained in the factor

$$\eta_{ab}=\frac{\rho_A^{np}}{\rho_A^{ab}}\ . \tag{9}$$

For a given value of $\langle p^2\rangle$, the SU(3) breaking is a simple function of the baryon masses. The baryon wave packets have been previously evaluated in the bag model,[11] and the resulting values of η^{ab}, corresponding to $\langle p^2\rangle=0.43$ GeV^2, are shown in Table I. Although we have utilized the bag model in obtaining specific numbers, we expect that other models will yield very similar results. Indeed

TABLE I. Semileptonic-decay form factors, calculated for $\langle p^2 \rangle = 0.43$ GeV^2.

Decay	η	$g_1/g_1^{SU(3)}$	$f_1/f_1^{SU(3)}$
$\Delta S = 0$			
$n \to pe\bar{\nu}_e$	1.000	1.00	1.00
$\Sigma^- \to \Lambda e\bar{\nu}_e$	0.9383	0.9383	1.00
$\Sigma^+ \to \Lambda e^+\nu_e$	0.9390	0.9390	1.00
$\Delta S = 1$			
$\Lambda \to pe\bar{\nu}_e$	0.9720	1.050	0.987
$\Lambda \to p\nu\bar{\nu}_\mu$	0.9720	1.050	0.987
$\Sigma^- \to ne\bar{\nu}_e$	0.9628	1.040	0.987
$\Sigma^- \to n\mu\bar{\nu}_\mu$	0.9628	1.040	0.987
$\Xi^- \to \Lambda e\bar{\nu}_e$	0.9287	1.003	0.987
$\Xi^- \to \Lambda\mu\bar{\nu}_\mu$	0.9287	1.003	0.987
$\Xi^- \to \Sigma^0 e\bar{\nu}_e$	0.9216	0.9954	0.987

quark-model calculations have been performed in a wide variety of versions and experience has shown that current matrix element ratios such as these depend only on basic aspects of the quark wave functions, so that very little difference between models emerges. Nevertheless, we will in our final analysis allow sufficient theoretical uncertainty to cover these potential differences.

To summarize the corrections then we find only small modifications for the vector currents,

$$f_1^{ab} = (f_1^{ab})_{SU(3)}, \quad \Delta S = 0 \; , \qquad = 0.987 (f_1^{ab})_{SU(3)}, \quad \Delta S = 1 \; , \tag{10}$$

while the axial-vector currents have somewhat larger corrections:

$$g_1^{ab} = \eta^{ab} (g_1^{ab})_{SU(3)}, \quad \Delta S = 0 \; , \qquad = 1.08 \eta^{ab} (g_1^{ab})_{SU(3)}, \quad \Delta S = 1 \; , \tag{11}$$

where in both cases $(\;)_{SU(3)}$ indicates the usual value obtained in exact SU(3). These results are summarized in Table I. Notice that in the case of g_1 the two SU(3)-breaking effects go in *opposite* directions and the overall amount of SU(3) breaking is less than previously thought. In addition the *pattern* of breaking is modified. In particular there is a decrease in the magnitude of the axial-vector coupling for the $\Sigma \to \Lambda$ transition. This has been favored by the data for a long time[2] and will play an important role in the improvement found in the experimental fit.

III. CABIBBO FIT TO HYPERON β DECAY

In this section we first comment on the hyperon data and then perform fits to beta decay with and without SU(3) breaking. The world-average data for hyperon decay[4,12] is displayed in Table II. Surprisingly, the channel which most deserves comment is neutron decay, $n \to pe\nu$. Because of the neutron's 10-min lifetime, the experiments to directly measure the lifetime are difficult to perform and at present yield contradictory answers.[13] Several experiments disagree by more than their quoted errors. By contrast the measurement of g_1 via the polarization asymmetry has been performed with higher precision and with good agreement among the various recent experiments.[14] Since g_1 and the neutron-decay rate provide essentially the same information (the theory connecting the two is solid and the KM element V_{ud} is well determined from nuclear transitions[1]) we have chosen to only use g_1 for neutron decay in the fits. We have included the recent measurement from Grenoble in the quoted value, leading to a slightly larger than usual value.

The experimental numbers for many of the decays modes are dominated by the data from the CERN SPS measurements.[4] This group has claimed that it is best not to average the world data but prefer to use only their own results in performing a Cabibbo fit. We do not, however, see any reason not to average the other available data. The only other high-statistics result is that on $\Lambda \to pe\nu$ by the Massachusetts-BNL-Columbia group.[15] For this mode the results are very similar and there is no conflict. Out of the ten decay channels only $\Xi^- \to \Lambda e\nu$ has any disagreement between the SPS data and the world average.

TABLE II. Fitted rates for hyperon-beta-decay channels. The g_1/f_1 values below the line were not included in the fit.

Quantity	Experiment	SU(3)-invariant fit: Fitted value	SU(3)-invariant fit: χ^2 ($\chi_{tot}^2 = 19.9$)	Broken-SU(3) fit: Fitted value	Broken-SU(3) fit: χ^2 ($\chi_{tot}^2 = 8.5$)
$\Gamma(\Lambda \to pe\nu)$	3.18 ± 0.053	3.24	1.63	3.22	0.59
$\Gamma(\Sigma^- \to Ne\nu)$	6.90 ± 0.23	6.50	3.22	6.75	0.57
$\Gamma(\Xi^- \to \Lambda e\nu)$	3.32 ± 0.36	2.90	1.31	2.65	3.40
$\Gamma(\Xi^- \to \Sigma^0 e\nu)$	0.53 ± 0.10	0.51	0.05	0.48	0.29
$\Gamma(\Sigma^- \to \Lambda e\nu)$	0.38 ± 0.048	0.45	11.6	0.42	2.84
$\Gamma(\Sigma^+ \to \Lambda e\nu)$	0.253 ± 0.06	0.272	0.10	0.254	0.00
$\Gamma(\Lambda \to p\mu\nu)$	0.597 ± 0.133	0.555	0.10	0.55	0.12
$\Gamma(\Sigma^- \to N\mu\nu)$	0.30 ± 0.27	0.307	0.005	0.318	0.33
$\Gamma(\Xi^- \to \Lambda\mu\nu)$	1.1 ± 1.1	0.83	0.06	0.76	0.01
$g_1(N \to pe\nu)$	1.2625 ± 0.007	1.253	1.84	1.258	0.41
$g_1/f_1(\Lambda \to pe\nu)$	0.72 ± 0.03	0.73		0.75	
$g_1/f_1(\Sigma^- \to Ne\nu)$	-0.37 ± 0.05	-0.32		-0.38	
$g_1/f_1(\Xi^- \to \Lambda e\nu)$	0.25 ± 0.05	0.20		0.18	
$f_1/g_1(\Sigma^- \to \Lambda e\nu)$	0.034 ± 0.090	0.0		0.0	

In this case we follow the Particle Data Group in using a scale factor of 2 in the error estimate.

Although in Table II we display the measured g_1/f_1 values, we do not include them in the fit. This is because their values were determined under the *assumption* of SU(3) invariance, and if, as we argue, SU(3) breaking is present, their values would change somewhat. In particular, the Dalitz-plot analysis (but not the rates) is sensitive to a nonzero value of the second-class axial-vector form factor g_2 (Ref. 4). In the presence of SU(3) breaking $g_2 \neq 0$, and this could shift the measured values of g_1/f_1. Actually, had we included these ratios, the quality of the fit would be even better (see below), but we feel on principle that they should not be used.

An important feature of any Cabibbo-type fit is a careful treatment of the radiative corrections, which are important given the accuracy of the present data. There are three components to this correction. (1) A model-independent and lepton-energy-dependent term has been calculated by Sirlin.[16] (2) The Coulomb interaction generates a well-known modification in those decays with charged baryons in the final state. (3) A small lepton-energy-independent and model-dependent correction has been evaluated by Abers, Dicus, Norton, and Quinn[17] and improved by Sirlin.[18] In addition to the radiative corrections we have employed the full energy dependence in the decay rate[19] and have included the small effects of the f_2 and g_3 form factors. For $\Delta S=0$ decays we use $V_{ud}=0.9747$ as determined by Sirlin and Zucchini.[20]

First in order to normalize our results we perform a standard SU(3) invariant fit to the data. The free parameters are V_{us}, $F+D$, and $\alpha \equiv D/F+D$. The latter two parameters refer to the SU(3) structure of the axial-vector current matrix element. The fit is rather poor with $\chi^2=19.9$ for 7 degrees of freedom. The best fit is obtained with

$$\begin{aligned} V_{us} &= 0.224 \pm 0.001 , \\ F+D &= 1.253 \pm 0.008 , \\ \alpha &= 0.629 \pm 0.008 . \end{aligned} \tag{12}$$

As reported in Ref. 2 the basic problem seems to be an incompatibility of the $\Delta S=0$ data (especially $\Sigma^- \rightarrow \Lambda e \nu$) which favors $\alpha \approx 0.58$ with the $\Delta S=1$ mode which are best fit by $\alpha=0.642$. The expert reader may wonder why our fit is rather poor while the authors of Ref. 4 obtain a better fit ($\chi^2=13$ for 7 DF). The reason is primarily the smaller errors on the $\Lambda \rightarrow p e \nu$ mode which come from the BNL data and which constrain the fit more than occurs if one uses only the SPS information. (We comment that, as a check, we have reproduced the fit of Ref. 4, when using their data set alone.) We conclude that the presence of SU(3) breaking in hyperon decay is evidenced by the poor fit in the SU(3) limit.

To see if we can understand the pattern of SU(3) breaking, we use the quark-model results given in Table I. Aside from these modifications, the parameters and procedures are identical to those in the case considered above. We find a much improved fit, with $\chi^2=8.5$ for 7 DF. The parameters of the fit are

$$\begin{aligned} V_{us} &= 0.220 \pm 0.001 , \\ F+D &= 1.258 \pm 0.007 , \\ \alpha &= 0.646 \pm 0.008 . \end{aligned} \tag{13}$$

One of the beneficial features of the breaking is to lower the $\Sigma^- \rightarrow \Lambda$ axial-vector form factor, which removes the clash between the $\Delta S=0$ and $\Delta S=1$ data sets. The drop in χ^2 by ten units is obviously significant statistically and it indicates that the quark model has captured the essence of the SU(3)-breaking pattern required by the data. Note that the center-of-mass corrections are crucial in obtaining this agreement. Without this modification but including the overlap effect the fit would have been much worse ($\chi^2=28$). It is gratifying to note that the parameters of the fit do not change much due to the inclusion of SU(3) breaking. Thus the shift in V_{us} is only 2%. This shift is larger than the purely experimental uncertainty, but is not as large as it could otherwise have been. We conclude that the SU(3) breaking *can* be understood, and that its effect on V_{us} is not large. As indicated by the quality of the fits, the value to be quoted is that which has been determined with SU(3) breaking included.

One might ask if an improved fit could be obtained using a wave packet with a different value of $\langle p^2 \rangle$ in the center-of-mass correction. In fact the data *do* prefer a larger value. The quality of fit crosses the $\chi^2=1$ per DF mark at $\langle p^2 \rangle=0.50$ ($V_{us}=0.221$) and reaches a best fit when $\langle p^2 \rangle=0.65$ ($\chi^2=5.0$, $V_{us}=0.223$). However, it is dangerous to draw conclusions based upon slight differences among fits all with $\chi^2<1$ per DF and so we can only state that good fits can be obtained for a wide range of $\langle p^2 \rangle$ with very little variation of V_{us}. The bag model does not favor the larger values of $\langle p^2 \rangle$ and so we will use our standard result for the central value of V_{us}, but will include theoretical error bars which include the full range of values, i.e.,

$$V_{us}=0.220 \pm 0.001 \pm 0.003 . \tag{14}$$

Finally, we comment on the inclusion of the g_1/f_1 values. The parameters of our fit would imply the values of g_1/f_1 listed in Table II. We emphasize that these are not themselves fit but are derived after the fit from the $F+D$ and α values. The four numbers would themselves have a $\Delta\chi^2=3.1$, and so it is clear that if included in our fit they would slightly increase its quality. One can use this information to conclude that there is no evidence for *large* values of the g_2 form factor.

IV. CONCLUSIONS

We have demonstrated that the SU(3) breaking present in hyperon β decay can be well understood in the quark model, when the full calculation, including the center-of-mass correction, is performed. The quark-model predictions are summarized in Table I. We have performed Cabibbo fits to the hyperon data which include this calculated SU(3) breaking. We find a much better fit than that obtained under the assumption of perfect SU(3) invariance. It is our opinion that in the future these symmetry-breaking corrections must be included in any treatment of hyperon decay.

The fits provide a determination of V_{us} as given in Eq. (1). This value is in good agreement with the corresponding number extracted from kaon decays ($V_{us}=0.2196\pm0.0023$) by a method which also included SU(3) breaking.[5] The modest variation of V_{us} with changes in assumptions used in the fit indicates that the value is reliable within the quoted errors. The combinations of the kaon and hyperon values

$$V_{us}=0.2198\pm0.001\pm0.002 \tag{15}$$

represents perhaps the best that can be done with the present knowledge. The most reliable value of V_{ud} comes from a recent analysis of the radiative corrections in nuclear β decay, which yielded[20]

$$V_{ud}=0.9747\pm0.0011 \ . \tag{16}$$

When combined with the bound[1] on V_{ub} ($V_{ub}<0.009$) one finds

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9984\pm0.002 \tag{17}$$

which is consistent with the unitarity of the KM matrix for three quark generations.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation. S.W.K. acknowledges the support of the Fulbright commission.

*Present address: Institut für Theoretical Physik, Universität Regensburg, 8400 Regensburg, Federal Republic of Germany.

[1]For a recent review, see W. Marciano, in Proceedings of the Twenty-Third International Conference on High Energy Physics, Berkeley, 1986, edited by S. Loken (World Scientific, Singapore, to be published).

[2]Early discussions of SU(3) breaking in hyperon β decay are given by J. F. Donoghue and B. R. Holstein, Phys. Rev. D **25**, 2015 (1982); A. Garcia and P. Kielanowski, Phys. Lett. **110B**, 498 (1982).

[3]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **25**, 206 (1982).

[4]J. M. Gaillard and G. Sauvage, Annu. Rev. Nucl. Part. Sci. **34**, 351 (1984).

[5]H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).

[6]J. F. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975 (1980).

[7]See, e.g., J. Bartelski *et al.*, Phys. Rev. D **29**, 1035 (1984).

[8]J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rep. **131**, 319 (1986).

[9]M. Ademollo and R. Gatto, Phys. Lett. **13**, 264 (1964).

[10]T. DeGrand, R. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).

[11]J. F. Donoghue and E. Golowich, Phys. Rev. D **26**, 3092 (1982).

[12]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[13]For a summary, see P. Bopp *et al.*, J Phys. (Paris) Colloq. **45**, C3-21 (1984).

[14]V. E. Krohn and G. R. Ringo, Phys. Lett. **55B**, 175 (1975); B. G. Erozolimskii *et al.*, Yad. Fiz. **30**, 692 (1979) [Sov. J. Nucl. Phys. **30**, 356 (1979)]; see Bopp *et al.*, Ref. 13.

[15]J. Wise *et al.*, Phys. Lett. **98B**, 123 (1981); **91B**, 165 (1980).

[16]A. Sirlin, Phys. Rev. **164**, 1767 (1967).

[17]E. Abers, D. Dicus, R. Norton, and H. Quinn, Phys. Rev. **167**, 1461 (1968).

[18]A. Sirlin, Rev. Mod, Phys. **50**, 576 (1978).

[19]M. M. Nieto, Rev. Mod. Phys. **40**, 140 (1968).

[20]A. Sirlin and R. Zucchini, Phys. Rev. Lett. **57**, 1994 (1986).