Radiative corrections to neutrino indices of refraction

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Quantum loop corrections to coherent forward neutrino scattering and indices of refraction n_{ν_l}, $l=e,\mu,\tau$ are examined in the standard $SU(2)_L \times U(1)$ model. For a neutral unpolarized medium with particle densities $N_e=N_p,N_n$ we find $p_\nu(n_{\nu_e}-n_{\nu_\mu})=-\sqrt{2}G_\mu N_e[1+O(\alpha m_\mu^2/m_W^2)]$ and

$$p_\nu(n_{\nu_\tau}-n_{\nu_\mu})=\frac{G_\mu}{\sqrt{2}}\frac{3\alpha}{2\pi\sin^2\theta_W}\frac{m_\tau^2}{m_W^2}[(N_p+N_n)\ln(m_\tau^2/m_W^2)+(N_p+\tfrac{2}{3}N_n)] .$$

Implications of our results for neutrino matter oscillations and elastic scattering are briefly discussed.

The effect of coherent forward scattering on neutrino oscillations in matter was investigated a number of years ago by Wolfenstein.[1] More recently, Mikheyev and Smirnov[2] employed that analysis to show how for a realistic range of neutrino masses and mixing parameters, neutrino matter oscillations between ν_e and ν_μ or ν_τ in the Sun's interior could be significantly enhanced and thus modify the spectrum of solar ν_e neutrinos. Such a scenario (henceforth referred to as the MSW effect) provides a natural solution to the solar neutrino puzzle,[3] i.e., why only about $\frac{1}{3}$ of the ν_e flux predicted by the standard solar model is experimentally observed.

The basic mechanism responsible for the MSW effect is contained in the coupled evolution equations[1,2,4] for left-handed neutrino states $\nu_l(t)$ propagating through matter with neutrino indices of refraction[5–8] n_{ν_l}, $l=e,\mu,\tau$,

$$i\frac{d}{dt}\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix}=\left[V\begin{bmatrix}(m_1^2-m_2^2)/2p_\nu & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & (m_3^2-m_2^2)/2p_\nu\end{bmatrix}V^{-1}-p_\nu\begin{bmatrix}n_{\nu_e}-n_{\nu_\mu} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & n_{\nu_\tau}-n_{\nu_\mu}\end{bmatrix}\right]\begin{bmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix} , \quad (1)$$

where V is a 3×3 unitary matrix which relates weak-interaction states ν_e, ν_μ, and ν_τ with vacuum mass eigenstates ν_i (with masses m_i), $i=1,2,3$, via

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}=V\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}=\begin{bmatrix}c_1 & s_1c_3 & s_1s_3\\ -s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta}\\ -s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} , \quad (2)$$

$c_i\equiv\cos\theta_i$, $s_i\equiv\sin\theta_i$, $i=1,2,3$,

and $p_\nu \simeq E_\nu \gg m_i$ is the neutrino momentum or energy. [The same equation governs right-handed-antineutrino evolution, but in that case $(n_{\bar\nu_l}-1)=-(n_{\nu_l}-1)$.] In Eq. (1) we have generalized the two-neutrino-mixing example usually considered to three neutrino species[4] and have neglected a $p_\nu(1-n_{\nu_\mu}+m_2^2/2p_\nu^2)$ 1 term which would only give rise to an overall phase that does not affect oscillations. Indeed, the only quantities that govern oscillatory behavior in matter are the mixing angles in Eq. (2), the mass-squared differences $m_i^2-m_j^2$, the neutrino momentum, and differences in the indices of refraction $n_{\nu_e}-n_{\nu_\mu}$ and $n_{\nu_\tau}-n_{\nu_\mu}$. The last parameters describe differences between interactions of the distinct neutrino flavors with the medium. Those refraction indices can be obtained from the neutrino scattering amplitudes

$$M(\nu_l f\to\nu_l f)=-i\frac{G_\mu}{\sqrt{2}}\bar\nu_l\gamma^\alpha(1-\gamma_5)\times\nu_l\bar f\gamma_\alpha(C^V_{\nu_l f}+C^A_{\nu_l f}\gamma_5)f , \quad (3)$$

where f is a generic fermion. For an unpolarized medium of normal matter one finds[7,8]

$$p_\nu(n_{\nu_l}-1)=-\sqrt{2}G_\mu\sum_{f=e,u,d}C^V_{\nu_l f}N_f , \quad (4)$$

where N_f is the particle number density of f's in the medium.

We have factored out the muon decay constant

$$G_\mu=1.166\,36\pm0.000\,02\times10^{-5}\ \text{GeV}^{-2} \quad (5)$$

in Eqs. (3) and (4). It is defined by the muon lifetime formula[9,10]

$$\tau^{-1}(\mu\to\text{all})=\frac{G_\mu{}^2 m_\mu{}^5}{192\pi^3}f\left[\frac{m_e{}^2}{m_\mu{}^2}\right]\left[1+\frac{3}{5}\frac{m_\mu{}^2}{m_W{}^2}\right]\left\{1+\frac{\alpha}{2\pi}\left[\frac{25}{4}-\pi^2\right]\left[1+\frac{2\alpha}{3\pi}\ln\left[\frac{m_\mu}{m_e}\right]\right]\right\}, \tag{6}$$

$$f(x)=1-8x+8x^3-x^4-12x^2\ln x\ ,$$

such that $O(\alpha)$ photonic corrections originally computed[19] in the local $V-A$ theory have been separated out. They are not absorbed into the definition of G_μ partly for historical reasons, but also because they are not universal to other low-energy weak processes. That normalization of G_μ is very convenient for coherent neutrino scattering because there are then no $O(\alpha)$ corrections[11] to the charged-current contributions to $C^V_{\nu_e e}$ in Eq. (4). (We subsequently discuss this point further.) There are corrections of $O(\alpha m_\mu{}^2/m_W{}^2)$; but they are negligible. [Throughout this paper we distinguish $O(\alpha)$ from $O(\alpha m_f{}^2/m_W{}^2)$ corrections.]

In the standard $SU(2)_L\times U(1)$ electroweak model, one finds, at tree level,[5,10]

$$C^V_{\nu_l f}=T_{3f}-2Q_f\sin^2\theta_W,\quad l\neq f\ , \tag{7a}$$

$$C^V_{\nu_l l}=+1+T_{3l}-2Q_l\sin^2\theta_W\ , \tag{7b}$$

where (Ref. 12) $\sin^2\theta_W\equiv 1-m_W{}^2/m_Z{}^2\simeq 0.23$ while Q_f and T_{3f} are the electric charge and weak isospin of the generic fermion f, i.e., $Q_f=(0,-1,\frac{2}{3},-\frac{1}{3})$ and $T_{3f}=(\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2})$ for $f=(\nu_e,e,u,d,)$. The $+1$ in Eq. (7b) comes from the charged-current contribution to $\nu_l l\to\nu_l l$ scattering.[5,11] That important term distinguishes ν_e interactions with electrons in matter from ν_μ and ν_τ interactions and gives rise to the MSW effect.[1,2] (That term corresponds to an additional repulsive interaction.[7,13]) Because of it, neutrino energy levels in matter can cross as the electron density varies [see Eq. (1)] and reasonance phenomena can greatly enhance oscillations.[2] From Eqs. (4) and (7), one finds the lowest-order result[1,7]

$$p_\nu(n_{\nu_e}-n_{\nu_\mu})=-\sqrt{2}G_\mu N_e\ , \tag{8a}$$

$$p_\nu(n_{\nu_\tau}-n_{\nu_\mu})=0\ , \tag{8b}$$

which has been extensively employed in (1) to study neutrino matter oscillations.[1,2,4,13,14]

The potential importance of the MSW effect and the possibility of applying a similar analysis to other astrophysical problems such as stellar collapse, neutron-star cooling, etc., has motivated us to scrutinize more closely coherent forward neutrino scattering in matter. In particular, we have examined one-loop electroweak radiative corrections to the expressions in Eq. (8). Results of that study are reported here.

It is convenient to parametrize the one-loop $O(\alpha)$ radiative corrections to the $C^V_{\nu_l f}$ in Eq. (7) by[15,16]

$$C^V_{\nu_l f}=\rho^{(\nu_l;f)}T_{3f}-2Q_f\lambda^{(\nu_l;f)}\sin^2\theta_W,\quad f\neq l\ , \tag{9a}$$

$$C^V_{\nu_l l}=+1+\rho^{(\nu_l;l)}T_{3l}-2Q_l\lambda^{(\nu_l;l)}\sin^2\theta_W\ , \tag{9b}$$

where $\rho^{(\nu_l;f)}$ and $\lambda^{(\nu_l;f)}=1+0(\alpha)$ corrections. Note, there are no $O(\alpha)$ corrections to the important charged-current $+1$ term in Eq. (9b). That is due to our use of G_μ in Eq. (6) which incorporates all short-distance electroweak radiative corrections into its definition and the fact that photonic corrections to the vector part of the $\nu_l l$ effective local charged-current amplitude cancel at $q^2=0$ due to Ward's identity. There are corrections of $O(\alpha m_\mu{}^2/m_W{}^2)$ to the muon-decay charged-current amplitude which are effectively absorbed into G_μ and therefore would contribute to a renormalization of the $+1$ term in Eq. (7b); however, since they are very small, we neglect them.

The complete $O(\alpha)$ electroweak corrections to $C^V_{\nu_l f}$ can be obtained from Ref. 12. Rather than reproducing those results, we comment on features important for this discussion. First note, that the decomposition of the $O(\alpha)$ corrections in Eq. (9) between $\rho^{(\nu_l;f)}$ and $\lambda^{(\nu_l;f)}$ is quite arbitrary.[15] However, from the explicit results in Ref. 12, one finds that it is possible and convenient to divide them such that to $O(\alpha)$ [but not yet including $O(\alpha m_l{}^2/m_W{}^2)$ terms], the $\rho^{(\nu_l;f)}$ depend on f but are independent of ν_l

$$\rho^{(\nu_e;f)}=\rho^{(\nu_\mu;f)}=\rho^{(\nu_\tau;f)} \tag{10a}$$

while the $\lambda^{(\nu_l;f)}$ depend on ν_l but are independent of f such that, at $q^2=0$,

$$\lambda^{(\nu_e;f)}=\lambda^{(\nu_\mu;f)}-\frac{\alpha}{3\pi}\frac{1}{\sin^2\theta_W}\ln\left[\frac{m_\mu}{m_e}\right]=\lambda^{(\nu_\tau;f)}-\frac{\alpha}{3\pi}\frac{1}{\sin^2\theta_W}\ln\left[\frac{m_\tau}{m_e}\right]. \tag{10b}$$

The differences in Eq. (10b) are due to neutrino charge radii effects. In the case of elastic coherent scattering on spin-0 isoscalar nuclear targets, Sehgal found,[17] for small $|q^2|$, Eq. (10b) implies

$$\sigma(\nu_e N):\sigma(\nu_\mu N):\sigma(\nu_\tau N)::1:1.04:1.06\ . \tag{11}$$

Those differences may have observable consequences in low-energy scattering processes.

In the case of the neutrino index of refraction, for an electrically neutral medium with $N_e=N_p$ and N_n arbitrary, one finds using $N_u=2N_p+N_n$, $N_d=N_p+2N_n$ that terms proportional to $\lambda^{(\nu_l;f)}$ in Eq. (9) cancel in the summation of Eq. (4). On the other hand, the $\rho^{(\nu_l;f)}T_{3f}$ terms in Eq. (9) do modify the individual n_{ν_l} by $O(\alpha)$ corrections which turn out to be of order 0.5% for $m_t\simeq 40$ GeV, $\sin^2\theta_W=0.23$, and $m_{\text{Higgs}}\simeq m_Z$. However, because of the equality in Eq. (10a), one finds that the differences $n_{\nu_e}-n_{\nu_\mu}$ and $n_{\nu_\tau}-n_{\nu_\mu}$ are not modified by $O(\alpha)$ corrections. Indeed, the leading corrections to those differences are terms of $O(\alpha m_l{}^2/m_W{}^2)$ which we have so far neglected. For $\nu_{\nu_e}-n_{\nu_\mu}$, such $O(\alpha m_\mu{}^2/m_W{}^2)$ corrections to the tree level result in Eq. (8a) are negligible. In the case of $n_{\nu_\tau}-n_{\nu_\mu}$, the $O(\alpha m_\tau{}^2/m_W{}^2)$ corrections to $\rho^{(\nu_\tau;f)}$ not in

$\rho^{(\nu_\mu;f)}$ provide the leading effect and are therefore potentially interesting.[16] Such terms arise from the one-loop diagrams in Fig. 1. The contribution of those diagrams, neglecting heavy-quark mixing[18] and strong interactions, can be extracted from Ref. 19. We find that the WW box and Z exchange diagrams give rise to nonuniversal $O(\alpha m_\tau^2/m_W^2)$ corrections that modify $\rho^{(\nu_\tau;f)}$ as follows [for $E_\nu \lesssim 5$ GeV (Ref. 20)]:

$$\Delta\rho^{(\nu_\tau;f)} \equiv \rho^{(\nu_\tau;f)} - \rho^{(\nu_\mu;f)} , \tag{12a}$$

$$\Delta\rho^{(\nu_\tau;e)} \equiv \Delta\rho^{(\nu_\tau;d)} = \frac{\alpha}{8\pi\sin^2\theta_W} J(m_\tau^2/m_W^2) , \tag{12b}$$

$$\Delta\rho^{(\nu_\tau;u)} = \frac{-\alpha}{8\pi\sin^2\theta_W} K(m_\tau^2/m_W^2) , \tag{12c}$$

where

$$J(x) = \frac{(2+x)x}{1-x} + \frac{3(2-x)x}{(1-x)^2}\ln x \sim \begin{cases} 2x + 6x\ln x & \text{as } x \to 0 , \\ -x & \text{as } x \to \infty ; \end{cases} \tag{13a}$$

$$K(x) = \frac{(4-x)x}{1-x} + \frac{3x^2\ln x}{(1-x)^2} \sim \begin{cases} 4x & \text{as } x \to 0 , \\ +x & \text{as } x \to \infty . \end{cases} \tag{13b}$$

For a neutral unpolarized medium with $N_e = N_p$ and N_n arbitrary, one finds[21] from Eqs. (12) and (13)

$$p_\nu(n_{\nu_\tau} - n_{\nu_\mu}) = \frac{G_\mu}{\sqrt{2}} \frac{3\alpha}{2\pi\sin^2\theta_W} \frac{m_\tau^2}{m_W^2} \times \left[(N_p+N_n)\ln\left[\frac{m_\tau^2}{m_W^2}\right] + N_p + \tfrac{2}{3}N_n + O\left[\frac{m_\tau^2}{m_W^2}\ln\frac{m_\tau^2}{m_W^2}\right]\right] . \tag{14}$$

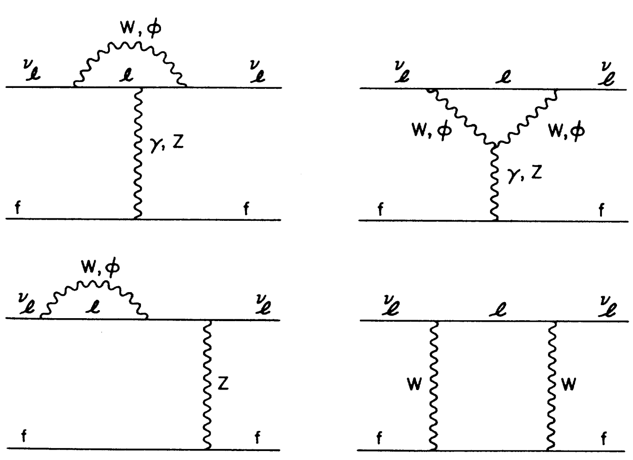

FIG. 1. One-loop diagrams, which give rise to nonuniversal (ν_l-dependent) radiative corrections to the scattering $\nu_l f \to \nu_l f$, $f = u$, d, or e. The ν_l-self-energy diagram is meant to denote corrections to both ν_l external legs.

It is interesting to note that the leading-logarithmic term depends only on the total nucleon density $N_p + N_n$. For an isoscalar medium with $N_n = N_p = N_e$, this becomes

$$p_\nu(n_{\nu_\tau} - n_{\nu_\mu}) = \frac{G_\mu}{\sqrt{2}} \frac{3\alpha}{\pi\sin^2\theta_W} \frac{m_\tau^2}{m_W^2} \left[\ln\left[\frac{m_\tau^2}{m_W^2}\right] + \frac{5}{6}\right] N_e . \tag{15}$$

Comparing Eqs. (15) and (8a), one finds using $m_\tau = 1.78$ GeV, $m_W = 81$ GeV,

$$\frac{n_{\nu_\tau} - n_{\nu_\mu}}{n_{\nu_e} - n_{\nu_\mu}} = \frac{-3\alpha}{2\pi\sin^2\theta_W} \frac{m_\tau^2}{m_W^2} \left[\ln\left[\frac{m_\tau^2}{m_W^2}\right] + \frac{5}{6}\right] \simeq 5\times 10^{-5} . \tag{16}$$

As one might expect, the induced-loop effect for $n_{\nu_\tau} - n_{\nu_\mu}$ is much smaller than the important n_{ν_e} tree-level result, but not completely negligible.

The parameters $\lambda^{(\nu_\tau;f)}$ are also modified by $O(\alpha m_\tau^2/m_W^2)$ corrections. From the Z and γ exchange diagrams in Fig. 1, one finds[19]

$$\Delta\lambda^{(\nu_\tau;f)} \equiv \lambda^{(\nu_\tau;f)} - \lambda^{(\nu_\mu;f)} , \tag{17a}$$

$$\Delta\lambda^{(\nu_\tau;f)} = \frac{-\alpha}{8\pi\sin^2\theta_W} L(m_\tau^2/m_W^2) + \frac{\alpha}{3\pi\sin^2\theta_W}\ln\left[\frac{m_W}{m_\mu}\right] , \tag{17b}$$

$$L(x) = \frac{x(7-x)}{1-x} + \frac{2}{3}\frac{(2+x)(4x-1)}{(1-x)^2}\ln x . \tag{17c}$$

Those corrections are, however, independent of f; so they cancel out of n_{ν_τ} for a neutral medium.

If a fourth generation of fermions with lepton doublet (ν_L, L) exists, we know from recent UA1 bounds[22] that $m_L > 42$ GeV, so $m_L^2/m_W^2 > \frac{1}{4}$. In that case $n_{\nu_L} - n_{\nu_\mu}$ and $n_{\nu_L} - n_{\nu_\tau}$ are not so small. Indeed, making the replacement $m_\tau \to m_L$ in Eqs. (12)–(17), we find for $N_p = N_e$,

$$p_\nu(n_{\nu_L} - n_{\nu_\mu}) = \frac{G_\mu}{\sqrt{2}} \frac{\alpha}{4\pi\sin^2\theta_W} \times [(N_p+N_n)J(x_L) + (N_p + \tfrac{1}{2}N_n)K(x_L)] , \quad x_L = m_L^2/m_W^2 , \tag{18}$$

$$\frac{n_{\nu_L} - n_{\nu_\mu}}{n_{\nu_e} - n_{\nu_\mu}} = \frac{-\alpha}{16\pi\sin^2\theta_W} \times \left[\frac{20x_L + x_L^2}{1-x_L} + \frac{24x_L - 3x_L^2}{(1-x_L)^2}\ln x_L\right] . \tag{19}$$

Allowing for a range $42 \leq m_L \leq 360$ GeV (the upper bound comes from phenomenological neutral-current constraints on fermion-loop corrections), one finds

$$\frac{n_{\nu_L}-n_{\nu_\mu}}{n_{\nu_e}-n_{\nu_\mu}}=(0.5\text{--}3)\times 10^{-2}, \quad \tfrac{1}{4}\leq m_L{}^2/m_W{}^2 \leq 20 . \tag{20}$$

So, if a fourth generation exists, our result for $n_{\nu_L}-n_{\nu_\mu}$ is large enough to be potentially important. We also note that the $O(\alpha m_L{}^2/m_W{}^2)$ correction to $\lambda^{(\nu_L;f)}$ is given by

$$\Delta\lambda^{(\nu_L;f)} \equiv \lambda^{(\nu_L;f)}-\lambda^{(\nu_\mu;f)}$$
$$=\frac{-\alpha}{8\pi\sin^2\theta_W}L(m_L{}^2/m_W{}^2)+\frac{\alpha}{3\pi}\frac{1}{\sin^2\theta_W}\ln\frac{m_W}{m_\mu} . \tag{21}$$

That shift together with $\Delta\rho^{(\nu_L;f)}$ modifies the ν_L-nucleon elastic-scattering cross section such that, for an isoscalar target,

$$\frac{\sigma(\nu_L N)}{\sigma(\nu_\mu N)}\simeq 1+\frac{\alpha}{3\pi\sin^2\theta_W}\ln\left[\frac{m_W}{m_\mu}\right]^2 - \frac{\alpha}{4\pi\sin^2\theta_W}\left[L(x_L)-\frac{3}{4\sin^2\theta_W}\times[K(x_L)+J(x_L)]\right] \tag{22a}$$

or

$$\frac{\sigma(\nu_L N)}{\sigma(\nu_\mu N)}\simeq 1.025\sim 0.942 \quad \text{for } \tfrac{1}{4}\leq x_L \leq 20 . \tag{22b}$$

In the above analysis, we ignored strong-interaction effects; however, their inclusion should not significantly alter our results. The ν_l-e one-loop amplitudes are free of strong interactions while the Z and γ exchange diagrams involving quarks in Fig. 1 are protected by CVC (conservation of vector current) from strong-interaction renormalization. That leaves the WW box diagrams which are, fortunately, dominated by high-frequency loop momenta $\gtrsim m_\tau$. We, therefore, expect QCD corrections to the leading

$$O(\alpha m_\tau{}^2/m_W{}^2\ln(m_\tau{}^2/m_W{}^2))$$

terms in Eqs. (12)–(17) to be of relative order $\alpha_s(m_\tau)/\pi \lesssim 0.1$. The low-frequency parts of those diagrams can be analyzed using nucleon form factors. Such an analysis indicates that the expression in square brackets in Eq. (14) is modified by $O(m_N{}^2/m_\tau{}^2)\simeq 0.3$ terms. So, in total, we expect our result for $n_{\nu_\tau}-n_{\nu_\mu}$ to be modified by $\approx 10\%$ due to strong interactions. In the case of a fourth generation with $m_L > 42$ GeV, strong-interaction corrections to the $n_{\nu_L}-n_{\nu_\mu}$ results in Eqs. (18)–(20) should be even smaller.

What are the implications of our results? First of all, the nonrenormalization of $p_\nu(n_{\nu_e}-n_{\nu_\mu})=-\sqrt{2}G_\mu N_e$ to $O(\alpha)$ means that previous analyses of $\nu_e-\nu_\mu$ oscillations in matter are insensitive to higher-order corrections as long as G_μ defined via Eq. (6) is employed. In addition, for oscillation phenomena involving ν_τ and ν_μ or all three neutrino species, our result for $n_{\nu_\tau}-n_{\nu_\mu}$ may have physical implications depending on the neutrino mixing angle parameters. For example, consider the (albeit unrealistic) case where $c_1=1$ in Eq. (2), so only $\overset{(-)}{\nu}_\mu-\overset{(-)}{\nu}_\tau$ oscillations in vacuum or matter are possible. We can then treat the ν_μ-ν_τ system as a two-neutrino oscillation problem with 1 mixing angle

$$\nu_\mu=\nu_2\cos\theta+\nu_3\sin\theta, \quad \nu_\tau=-\nu_2\sin\theta+\nu_3\cos\theta .$$

The solution of Eq. (1) is then essentially the same as the MSW (Refs. 1 and 2) treatment of ν_e-ν_μ oscillations in matter with $n_{\nu_e}-n_{\nu_\mu}$ now replaced by our much smaller $n_{\nu_\tau}-n_{\nu_\mu}$, i.e.,

$$i\frac{d}{dt}\begin{bmatrix}\nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix}=\begin{bmatrix}\dfrac{\Delta m^2{}_{32}s^2}{2p_\nu} & \dfrac{\Delta m^2{}_{32}sc}{2p_\nu}\\[2ex] \dfrac{\Delta m^2{}_{32}sc}{2p_\nu} & \dfrac{\Delta m^2{}_{32}c^2}{2p_\nu}-p_\nu(n_{\nu_\tau}-n_{\nu_\mu})\end{bmatrix}\begin{bmatrix}\nu_\mu(t)\\ \nu_\tau(t)\end{bmatrix}, \tag{23}$$

where $\Delta m^2{}_{32}=m^2{}_3-m^2{}_2$ and $s\equiv\sin\theta$, $c\equiv\cos\theta$. [The same basic equation governs $\bar\nu_\tau$-$\bar\nu_\mu$ oscillations in matter except that $(n_{\bar\nu_\tau}-n_{\bar\nu_\mu})=-(n_{\nu_\tau}-n_{\nu_\mu})$.] Defining the vacuum oscillation length

$$L_v=2\pi\left[\frac{2p_\nu}{\Delta m^2{}_{32}}\right] \tag{24a}$$

and refraction index length

$$L_0=-\frac{2\pi}{p_\nu(n_{\nu_\tau}-n_{\nu_\mu})}\simeq 6\times 10^{13}\ \text{cm}/\rho , \tag{24b}$$

where $\rho=(N_n+N_p)/6\times 10^{23}$ is the matter density in g/cm^3, one finds that the effective matter oscillation mixing angle θ_m and oscillation length L_m are given by[1]

$$\sin^2 2\theta_m=\frac{\sin^2 2\theta}{1\pm 2\dfrac{L_v}{L_0}\cos 2\theta+\dfrac{L_v{}^2}{L_0{}^2}} , \tag{25a}$$

$$L_m=\frac{L_v}{\left[1\pm 2\dfrac{L_v}{L_0}\cos 2\theta+\dfrac{L_v{}^2}{L_0{}^2}\right]^{1/2}} . \tag{25b}$$

The + and − signs in Eq. (25) refer to ν_τ-ν_μ and $\bar{\nu}_\tau$-$\bar{\nu}_\mu$ systems, respectively. An initial $\overset{(-)}{\nu}_\mu$ state traversing a medium of constant density ρ will have a transition probability of becoming a $\overset{(-)}{\nu}_\tau$ at distance x given by

$$P_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\tau} = \sin^2 2\theta_m \sin^2 \left[\frac{\pi x}{L_m} \right] \tag{26a}$$

$$= \left[\frac{L_m}{L_v} \right]^2 \sin^2 2\theta \sin^2 \left[\frac{\pi x}{L_m} \right] . \tag{26b}$$

For $L_v \ll L_0$, one finds $\theta_m \simeq \theta$ and $L_m \simeq L_v$; so in that case matter oscillations will be essentially the same as vacuum oscillations, i.e., the effect of $n_{\nu_\tau} - n_{\nu_\mu}$ is negligible. On the other hand, if $L_v \gg L_0$, one finds $L_m \simeq L_0$ and the oscillations in matter are highly suppressed for both neutrinos and antineutrinos. Intermediate scenarios are more interesting. In particular a resonance condition can exist for either $\bar{\nu}_\mu$-$\bar{\nu}_\tau$ or ν_μ-ν_τ oscillations depending on whether $\Delta m^2{}_{32}$ is positive or negative. Such a condition is satisfied (for $\Delta m^2{}_{32} > 0$) when

$$\frac{L_v}{L_0} = \cos 2\theta \tag{27a}$$

or

$$\frac{2p_\nu}{\Delta m_{32}{}^2} \simeq \frac{9.5 \times 10^{12} \text{cm}}{\rho} \cos 2\theta . \tag{27b}$$

In that case $L_m = L_v / \sin 2\theta = L_0 / \tan 2\theta$ and $\theta_m = 45°$, i.e., effective maximal mixing independent of θ. Translated into mass differences, resonance occurs when

$$\Delta m^2{}_{32} = (4 \times 10^{-9} \text{ eV}^2) \rho \left[\frac{p_\nu}{1 \text{ GeV}} \right] / \cos 2\theta . \tag{28}$$

So, for ordinary matter where $\rho \simeq 2$ or even in the solar interior where $\rho \simeq 100$, one must have [assuming $\cos 2\theta \simeq O(1)$] either very small $\Delta m^2{}_{32}$ or very large p_v. In the latter case, our calculation of $n_{\nu_\tau} - n_{\nu_\mu}$ will be somewhat modified[20] and one must seriously question the use of coherent forward-scattering amplitudes and the evolution equation over distance scales of $\sim 10^{13}$ cm. Indeed, the interaction mean free path for high-energy neutrinos,

$$l_{int} \simeq (2.5 \times 10^{14} \text{ cm}) \{ \rho [p_\nu / (1 \text{ GeV})] \}^{-1} , \tag{29}$$

should be greater than $\approx L_m$ [see Eq. (25b)] if our analysis is to make sense. That constrains us to $p_\nu \lesssim (5 \text{ GeV}) \tan 2\theta$; so we should restrict our remarks to low-energy neutrinos.

Our $n_{\nu_\tau} - n_{\nu_\mu}$ result is perhaps most interesting for very dense media such as the late stages of a stellar collapse or the interior of a neutron star[1,17,23] where $\rho \gtrsim 10^{10}$ and the resonance condition in Eq. (28) would be satisfied (for $E_\nu \simeq 25$ MeV) when $\Delta m^2{}_{32} \simeq 1$ eV^2, a reasonable value. Since in that case $L_m = L_0 / \tan 2\theta \simeq 60 m / \tan 2\theta$, it is likely that $\bar{\nu}_\mu$-$\bar{\nu}_\tau$ neutrinos would resonate while ν_μ-ν_τ would not (their roles are reversed for $\Delta m^2{}_{32} < 0$). That could have consequences for the high-energy spectrum (Ref. 24) $E_\nu \gtrsim 100$ MeV, where we might expect somewhat different $\overset{(-)}{\nu}_\mu$ and $\overset{(-)}{\nu}_\tau$ flux. The above mechanism could cause an asymmetry in the $\nu_\mu / \bar{\nu}_\mu$ ratio. In addition, since high-energy $\overset{(-)}{\nu}_\tau$ can more easily escape the dense medium, such oscillation could have an effect on stellar collapse and neutron-star cooling scenarios. Of course, if a fourth generation exists, our larger value for $\nu_{\nu_L} - n_{\nu_\mu}$ could also have interesting consequences for dense media.

The above example was limited to $\overset{(-)}{\nu}_\mu$-$\overset{(-)}{\nu}_\tau$ mixing. If neutrinos have mass, we should expect all species to mix. In principle, the results illustrated above for $\overset{(-)}{\nu}_\mu$-$\overset{(-)}{\nu}_\tau$ oscillations in dense matter can be completely changed when $\overset{(-)}{\nu}_e$ is coupled in. It turns out, however, that $\overset{(-)}{\nu}_e$ effectively decouples from the other two neutrinos, in the $\overset{(-)}{\nu}_\mu$-$\overset{(-)}{\nu}_\tau$ resonance region, because the matrix in Eq. (1) is dominated by the relatively large value of $n_{\nu_e} - n_{\nu_\mu}$. The discussion given above then holds with some reinterpretation of the mixing angle θ and $\Delta m_{32}{}^2$. A complete three-neutrino mixing analysis will be presented in a future publication.

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