On a new low-mass pseudoscalar boson

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The production of a new low-mass pseudoscalar boson of mass $\simeq 1.6$ MeV in the strong Coulomb field of the supernucleus and its subsequent decay into an electron-positron pair in heavy-ion collisions are discussed. For the production mechanism we propose a bremsstrahlung-type process induced by an electromagnetic interaction. The energy spectrum and the production cross section of the positron are calculated in linear QED with an effective Lagrangian. The results are compared with those of the experiments of the GSI groups, and $g_p < 10^{-1}$, the boson-nucleon coupling constant, is evaluated. These results imply that the proposed mechanism requires a larger value for g_p than the currently accepted value $g_p < 10^{-4}$. The present g_p, however, is smaller by a few orders of magnitude than the value obtained in a semiclassical calculation of the same mechanism. This suggests that the proposed mechanism requires a larger value for the scalar and nucleon coupling constants to explain the GSI data. These and results from other proposed mechanisms induced by electromagnetic interaction lead us to conclude that the origin of the positron production, contrary to current popular belief, may not be of electromagnetic nature. As an alternative production mechanism, we propose a bremsstrahlung-type process induced by strong interaction (e.g., $pp \to pp\phi$, $np \to np\phi$). We estimate the production cross section for the positron. Its value agrees well with that of the observed cross section.

I. INTRODUCTION

The narrow positron peak at about 300 keV observed[1] in heavy-ion collisions by the GSI (Gesellschaft für Schwerionenforschung, Darmstadt) groups was thought to be the verification of the QED prediction of the spontaneous e^+e^- pair creation from the vacuum in a supernucleus with an ultrastrong electrostatic field.[2,3] From subsequent analysis[4,5] of the line shape, one comes to the conclusion that the positron peak originates neither from the nuclear transition in the individual final-state nuclei nor from the spontaneous e^+e^- pair emission from the decay of the QED vacuum. In search of the origin of the positron peak, Balantekin *et al.*[5] and Schäfer *et al.*[4] suggest that it may be due to the production of a light pseudoscalar boson followed by its decay into an e^-e^+ pair. Balantekin *et al.* estimate the upper limit of the coupling parameters from the available GSI data. Consequently, $m_\phi \simeq 1.6$ MeV has been estimated. They conclude that the new boson in question could not be the standard axion[6] introduced in gauge field theories to resolve the strong *CP*-violation problem, because the coupling parameter is found to be too high to be consistent with negative results of axion searches of the past. Schäfer *et al.*,[4] while discussing whether the positron peaks are caused by the decay of a light boson of mass $\simeq 1.6$ MeV produced in these heavy-ion collisions, examine the possibility of new particles with adjusted mass and coupling constants. In most cases they are found to be in conflict with the established precision data of atomic and nuclear physics. The only alternative seemed to be the production of a pseudoscalar neutral boson of mass ~ 1.6 MeV from nuclear current. The neutral character of the speculated boson is further confirmed by the detection of the coincident e^+e^- lines in recent experiment by the EPOS group.[7] The coupling constant to the electron is estimated to be $10^{-10} < \alpha_e < 10^{-9}$. The lower limit is consistent with the negative outcome of the axion searches and the lifetime is estimated to be in the region of $\sim 5\times 10^{-13}-10^{-10}$ sec.

Balantekin *et al.* did not discuss any specific mechanism for the production of the ϕ particle—rather they assume a certain *ad hoc* parametric form for the production cross section. The parameters are then determined from the location and width of the positron yield and the total integrated cross section of the spectrum. Subsequently Reinhardt *et al.*[8] investigate the scalar bremsstrahlung mechanism and Chodos and Wijewardhana[9] discuss a specific electromagnetic[10] interaction—all in semiclassical approaches to explain the positron spectra via the light boson. In this paper we assume that in the heavy-ion collision, a short-lived ($\tau \sim 10^{-20}$ sec) supernucleus of charge $Z = Z_1 + Z_2$ is formed. For the production mechanism[11] we propose a bremsstrahlung process induced by an electromagnetic interaction. The energy spectrum and production cross section of the positron are calculated in linear quantum electrodynamics (QED) with an effective Lagrangian. It is just like the bremsstrahlung process in QED—the only difference is that the photons are replaced by light pseudoscalar bosons. We examine whether the limit on coupling constants obtained from this analysis is consistent with the limits set in Refs. 4, 5, and 8.

We assume the nucleon-pseudoscalar interaction is of the form $L = g_p \bar{\psi}_p \gamma_5 \psi_p \phi$, and the lepton-pseudoscalar interaction of the form $L = g_e \bar{\psi}_e \gamma_5 \psi_e \phi$. In addition, there is the usual nucleon-photon interaction. ψ_p, ψ_e, and ϕ are proton, lepton, and pseudoscalar fields, respectively. The coupling constants g_p, g_e are unknown but their values can be estimated from the heavy-ion-collision data and from the value of the theoretical cross section. We calculate the differential cross section $d\sigma/dT_e$. The energy spectrum dN/dT_e is obtained by dividing the cross section by the total classical cross section $\sigma_{\rm cl}$, i.e., $dN/dT_e=(1/\sigma_{\rm cl})d\sigma/dT_e$ where the typical value of $\sigma_{\rm cl}=12.6$ b. The integrated cross section σ_I is obtained by numerically integrating $\int(d\sigma/dT_e)dT_e$ with the given data from the positron peak. From the value of the integrated cross section σ_I and the observed $\sigma\simeq200$ μb[1], we were able to determine the upper limit of $g_p<10^{-1}$. However, from a brief analysis we show that if the bremsstrahlung is induced by strong interaction, a much smaller value of g_p ($\simeq10^{-3}$) can be obtained to explain the observed cross section.

We present the concept behind the model of ϕ production in Sec. II and calculations in Sec. III. Results are discussed in Sec. IV and Sec. V is dedicated to our conclusion. Appendixes A, B, and C are added to support the calculations in details.

II. THE FOUNDATION OF THE MODEL

Bremsstrahlung-type production of the ϕ particle has been discussed in a semiclassical approach in Ref. 8, where the coherence of each ion with respect to its constituent nucleons was assumed. In this paper we propose a slightly different model of the scattering mechanism and our entire calculation will be based on these assumptions. In doing so we stress the importance of the energy $\simeq6$ MeV/nucleon as a critical energy near the Coulomb barrier. When the incident ion just barely touches the target ion, we assume that for a very short period of time, there is a momentary fusion of two ions forming a supernucleus[11] of $A=A_1+A_2$ and $Z=Z_1+Z_2$. What happens as a result is that each of the incident protons (Z_1 protons in incident ion) emits a ϕ inside the supernucleus through the bremsstrahlung mechanism in the strong Coulomb field of the target ion. At the end we are left with the ϕ's whose lifetimes are much longer than the time of interaction between the two ions and they decay into e^+e^- pairs. If the ϕ's are produced nearly at rest, experimentally observed positron spectra can be produced. To calculate the cross section for the entire process, we can treat the scattering of each incident proton independently. If the production process is induced by an electromagnetic interaction, all Z protons will be able to participate in the internuclear electromagnetic interaction (due to its long-range characteristic). The bremsstrahlung amplitude will, therefore, be proportional to Z_1Z_2 and the cross section will vary as $(Z_1Z_2)^2$. But if the process is induced by strong interaction, only those nucleons within a distance of 2—3 fm from the surface of the interacting nucleus(ion) will be effective. In such a case the cross section will be proportional to $(A_1A_2)_{\rm eff}{}^2$ where a simple estimate of $(A_1A_2)_{\rm eff}{}^2$ will be given in the text.

If the incident energy is much smaller than the critical energy value of $\simeq6$ MeV, we cannot visualize a mechanism as explained above. Also if their energy is much larger, the collision will be violent and the energy distribution of the produced ϕ's will be chaotic and there will be no clean positron line. Therefore our assumptions are consistent with the experimental fact that the positron line is observed only at the critical value of the energy near the Coulomb barrier. To check this model one can measure the photon bremsstrahlung in *ion-ion* collisions near the Coulomb barrier. However, experimental data exist for nuclear photon bremsstrahlung for a single incident charged particle with energy of a few MeV. In that case our model obviously would predict the same value as any other calculation without any contradiction.

III. THE PRODUCTION CROSS SECTION AND ENERGY SPECTRUM OF THE POSITRON

The transition amplitude of the bremsstrahlung-type production of the ϕ particle followed by the e^+e^- pair production mechanism is represented in Fig. 1. The production amplitude is

$$\begin{aligned} S_{21}&=\bar{u}(p_2)\left[\gamma_0\frac{1}{\not p_1-\not k-m}\gamma_5+\gamma_5\frac{1}{\not p_2+\not k-m}\gamma_0\right]u(p_1)x\frac{1}{k^2-m_\phi{}^2+i\Gamma m_\phi}\\ &\quad\times\bar{u}(k_2)\gamma_5 v(k_1)\frac{-iZ_1Z_2e^2g_eg_p}{\mathbf{q}^2}F(\mathbf{q}^2)2\pi\delta(E_2-E_1+w)\\ &=M_{21}^{p}\bar{u}(k_2)\gamma_5 v(k_1)[(Z_1Z_2e^2)/\mathbf{q}^2](g_eg_p)\frac{F(\mathbf{q}^2)}{k^2-m_\phi{}^2+i\Gamma m_\phi}2\pi\delta(E_2-E_1+w)\ , \end{aligned} \tag{1}$$

where p_2 is the final-proton four-momentum, p_1 the initial-proton four-momentum. k is the four-momenta of ϕ particle, k_1,k_2 the four-momenta of the positron-electron pair, ϵ_1 and ϵ_2 being their energies. g_e is the ϕ and lepton coupling constant, g_p the ϕ and proton coupling constant. $\mathbf{q}=\mathbf{p}-\mathbf{k}-\mathbf{p}_2$. The effect of external nucleus recoil will be neglected, i.e., no energy transfer. $F(\mathbf{q}^2)$ is the electromagnetic form factor for the nucleus. E_1,E_2 is the total energies of the initial and final proton. w is the energy of the ϕ particle, m the mass of proton, m_e the mass of electron, and m_ϕ the mass of ϕ particle. In Eq. (1), we summed the amplitudes of all Z_1 protons of the projectile ion to reflect the coherence of the nucleus assuming all amplitudes are equal.

We assume that the ϕ particle is produced on shell and subsequently decays into e^+e^-. Therefore the propagator is replaced by the Breit term $1/(k^2-m_\phi{}^2+im_\phi\Gamma)$, where Γ is the total width of the ϕ particle. The kinematic approximations considered will be under the assumptions

$$E_1 \gg |\mathbf{p}_1| , \quad E_2 \gg |\mathbf{p}_2| , \quad w \gg |\mathbf{k}|$$

and

$$E_1, E_2 \gg w, \quad \text{i.e.,} \quad m \gg m_\phi .$$

Averaging over the initial and summing over the final proton spins we have (see Appendix A)

$$|\overline{M}{}^p_{21}|^2 = 2\langle s\rangle(2/E_1E_2)[(E_1E_2-m^2) + (\mathbf{p}_1-\mathbf{p}_2)\cdot\mathbf{k} - \mathbf{p}_1\cdot\mathbf{p}_2] , \tag{2}$$

where $\langle s\rangle$ is the average spin of the initial proton. Since

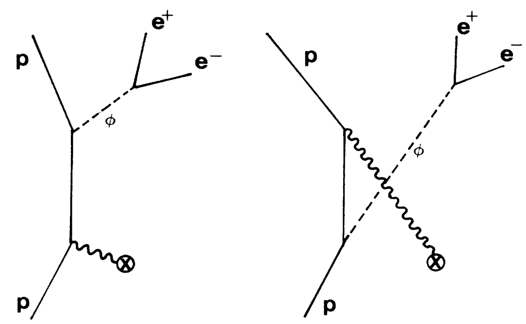

FIG. 1. The Feynman diagrams for the production of ϕ and subsequent decay into e^+e^- in a strong Coulomb field.

the nucleon spins in heavy nuclei are usually paired, the average spin $\langle s\rangle$ of the nuclei may be much smaller than $\frac{1}{2}$ effectively. The differential cross section for the e^+e^- pair production can be written as

$$d\sigma = (Z_1Z_2e^2)^2(g_e{}^2g_p{}^2)/(2\pi)^7 \frac{m_\phi{}^2}{4} \frac{|\mathbf{p}_2|}{|\mathbf{p}_1|} \frac{d^3k_2}{4\pi\epsilon_2 w} \frac{1}{(k^2-m_\phi{}^2)^2+m_\phi{}^2\Gamma^2} k^2dk \int \frac{|\overline{M}{}^p_{21}|^2}{\mathbf{q}^4} d\Omega\, d\Omega_2 . \tag{3}$$

To obtain Eq. (3) we made the following transformation for the invariant phase spaces $e^+e^- \to e^+\phi$ as

$$\frac{d^3k_1}{2\epsilon_1}\frac{d^3k_2}{2\epsilon_2} \to \frac{d^3k_2}{2\epsilon_2}\frac{d^3k}{2w}$$

which can be easily obtained [see Eq. (8)].

One other possible decay channel for ϕ is $\phi\to\gamma\gamma$. However, $\Gamma(\phi\to\gamma\gamma)$ is small; hence, for Γ, the theoretical value of $\Gamma(\phi\to e^+e^-)$ alone can be used. It is given by

$$\Gamma(\phi\to e^+e^-) = g_e{}^2 m_\phi/4\pi(1-4m_e{}^2/m_\phi{}^2)^{1/2} . \tag{4}$$

Under the narrow-width approximation, the Breit factor can be replaced by the δ function:

$$\frac{1}{(k^2-m_\phi{}^2)^2+m_\phi{}^2\Gamma^2} \to \frac{\pi}{m_\phi\Gamma}\delta(k^2-m_\phi{}^2) , \tag{5}$$

where the δ function again can be written as

$$\delta(k^2-m_\phi{}^2) \to \delta((k_1+k_2)^2-m_\phi{}^2) = \delta(2k_{20}w - 2|\mathbf{k}_2|\,|\mathbf{k}|\cos\theta'' - m_\phi{}^2) , \tag{6}$$

where θ'' is the angle of the positron with respect to $\mathbf{k}$.

$$d^3k_2 = \mathbf{k}_2{}^2 dk_2\, d(\cos\theta'')2\pi .$$

Therefore the angular integration

$$\int_{-1}^{+1} d(\cos\theta'')\delta(2k_{20}w - 2|\mathbf{k}_2|\,|\mathbf{k}|\cos\theta'' - m_\phi{}^2) \tag{7}$$

$$= \int_{-1}^{+1} \frac{1}{2|\mathbf{k}|\,|\mathbf{k}_2|}\delta\left[\cos\theta'' - \frac{2k_{20}w - m_\phi{}^2}{2|k|\,|\mathbf{k}_2|}\right] d(\cos\theta'') \tag{8}$$

can be easily done using the δ function and the integration limits on Eq. (7) gives the factor

$$\frac{1}{2|\mathbf{k}|\,|\mathbf{k}_2|}$$

and the condition

$$-1 \le \frac{2k_{20}w - m_\phi{}^2}{2|\mathbf{k}|\,|\mathbf{k}_2|} \le 1 .$$

The boundary condition generates limits on w, i.e., $w_- < w < w_+$ where

$$w_\pm = \frac{m_\phi{}^2}{2m_e{}^2}\left[\epsilon_2 \pm |\mathbf{k}_2| \left[1 - \frac{4m_e{}^2}{m_\phi{}^2}\right]^{1/2}\right] . \tag{9}$$

Also, it can further be shown that (see Appendix B) the leading contribution for the angular integration in Eq. (3) can be obtained as

$$\int\int \frac{d\Omega\, d\Omega}{\mathbf{q}^4}|\overline{M}{}^p_{21}|^2 = 2\langle s\rangle \frac{2}{E_1E_2}\frac{4\pi^2}{(E_1E_2-m^2)}\ln\frac{4(E_1E_2-m^2)}{m_\phi{}^2} ,$$

where $E_2 = E_1 - w$. Substitute $\epsilon_2 = E$ and $\epsilon_1 = w - E$ and we have finally (taking into the $Z_{1,2}$ factor explicitly)

$$d\sigma/dT_e = 2\langle s\rangle (Z_1{}^2 Z_2{}^2 e^4 g_p{}^2)/(2\pi)^3 \frac{1}{4E_1(E_1{}^2-m^2)^{1/2}} \times \frac{1}{2(1-4m_e{}^2/m_\phi{}^2)^{1/2}} \int_{w_-}^{w_+} dw \frac{[(E_1-w)^2-m^2]^{1/2}}{(E_1E_2-m^2)(E_1-w)} \ln \frac{4[E_1(E_1-w)-m^2]}{M_\phi{}^2}, \quad (10)$$

where $E_1 = T_p + m$, $E = T_e + m$, and $T_p \simeq 6$ MeV, and T_p and T_e are the kinetic energies of the proton and electron, respectively. The expression (10) can be rewritten as

$$f^e(E) = \frac{d\sigma}{dT_e} = \frac{1}{2(1-4m_e{}^2/m_\phi{}^2)^{1/2}} \int_{w_-}^{w_+} \frac{dw}{(w^2-m_\phi{}^2)^{1/2}} \frac{d\sigma^B}{dw}, \quad (11)$$

where $d\sigma^B/dw$ is the cross section for ϕ production alone as given in Appendix C. w is the boson energy and it has the overall kinematic limit $m_\phi < w < E_1 - m$. In the integration of Eq. (10), which is done numerically, the allowed range of $w_\pm$ had to be properly chosen to reflect this overall kinematic limit on w.

IV. RESULTS

Before we start to discuss our results we would like to make a few comments on the kinematic approximations we made to derive our final results. To make sure that the approximations did not deviate grossly from the actual result, we calculated cross sections for two more processes: (1) photoproduction of ϕ particle only; (2) production of ϕ particle only in a bremsstrahlung-type process, due to the accelerated proton by an external Coulomb field. We consider only the Born term. These two processes are just like Compton scattering and bremsstrahlung in QED—with the replacement of the final photon by a light pseudoscalar boson. The calculated cross sections are listed in Appendix C.

The process (1) was calculated exactly without making any kinematic approximation, and the cross section of the process (2) was calculated in the present scheme of kinematic approximations. Then we compared their ratio with the ratio of Compton to bremsstrahlung cross section in QED. We find that the ratios are of the same order of magnitude under the same values of parameters. This enhances our confidence in our scheme of kinematic approximation. As expected we find the cross section for the photoproduction to be a few orders of magnitude larger—after subtracting the Z_1 factor.

Equation (9) for $w_\pm$ is plotted in Fig. 2 as a function of T. In fact, it is easy to show that w_- has a minimum at $T_e^{\min}$ given by

$$T_e^{\min} + m_e = m_\phi/2 \,. \quad (12)$$

Detailed analytic and numerical investigations of Eq. (10) or Eq. (11) shows that the peak of the spectrum $f^e(E)$ occurs at $T_e^{\min}$ provided $d\sigma^B/dw$ is a smooth function of w. Experimentally it is found to be at 300 keV. Therefore from Eq. (12), we must have

$$m_\phi = 2(T_e^{\min} + m_e) = 2\,(0.300+0.51)\ \text{MeV} = 1.62\ \text{MeV}\,.$$

If ϕ is produced exactly at rest in any reference frame and decays into e^+e^- pair, the energy of the positrons in that frame is fixed and Eq. (12) is straightforward. Following this observation, some authors[9,10] have argued that if the ϕ's are produced at rest in the heavy-ion c.m. frame, then if the c.m. moves with a speed say, $c/20$, then the observed width of the positron spectrum in the laboratory frame can be explained entirely in terms of the Doppler broadening. However, in our case, since every kinematics is defined in the laboratory frame, the width of the positron spectrum is determined by the momentum distribution of ϕ's alone, as in Appendix C.

It can be pointed out at this stage that this overall kinematic limit on w results from the assumption that all nucleons equally share the energy of the supernucleus. However, it is more likely that there is a nontrivial distribution of energy among the protons. In this case one

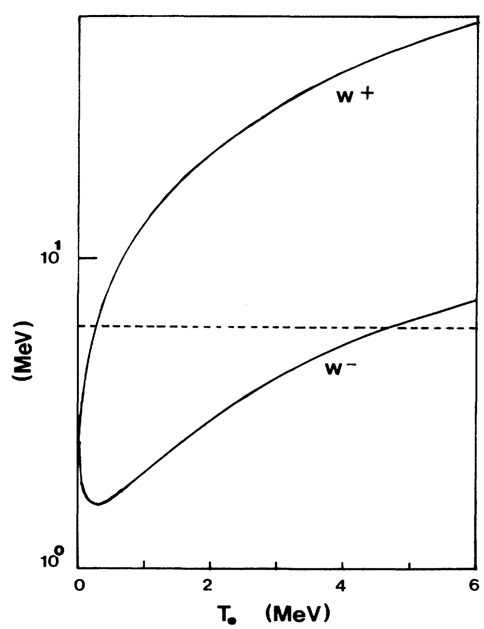

FIG. 2. The plot of w_+ and $w_\mp$ as a function of the kinetic energy T_e of the positron. The Y axis is in logarithmic scale. The broken line corresponds to $w = E_1 - m$, the overall kinematic limit on w as discussed in the text.

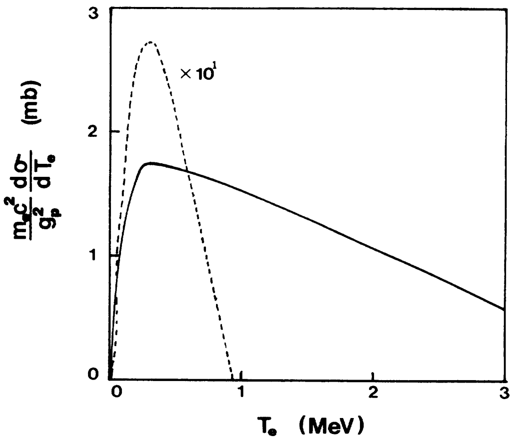

FIG. 3. The positron spectrum versus T_e. The solid curve is for $T_p \simeq 6.0$ MeV and the dashed curve is for $T_p \simeq 2.0$ MeV.

needs to convolute the above cross section with a proton energy distribution function in this and all other known works it is taken to be just a δ function.[12] That will also vary the overall kinematic limit of w depending on the interacting proton energy.

We have taken the form factor $F(\mathbf{q}^2)$ to be unity. The energy spectrum of the positron is shown in Fig. 3. We see that our calculation does not produce a narrow peak for the spectrum in agreement with other authors although the method of treatment of the process in this paper is quite different from earlier works. The broad nature of the spectrum is of course expected since the ϕ production alone before decaying into e^+e^- pair has a peak at higher value of the ϕ energy and is suppressed at low momentum. This ϕ production cross section is shown in Fig. 4. If the kinetic energy of the initial proton is taken to be smaller than 6 MeV, the spectrum in Fig. 3 becomes narrower. The magnitude of the cross section becomes smaller too. As, for example, at $T_p \simeq 2$ MeV, the spectrum has a width of only 500 keV.

It is not difficult to find the maximum value of $T_e^{\max}$ at which the spectrum ends by using Eq. (9) and Fig. 2 for a given T_p. It is given by

$$T_e^{\max}=\tfrac{1}{2}\{T_p+[(T_p{}^2-m_\phi{}^2)(1-4m_e{}^2/m_\phi{}^2)]^{1/2}\}-m_e \ .$$

With $T_p \simeq 2.0$ MeV, $m_\phi \simeq 1.62$ MeV, $T_e^{\max} \simeq 0.94$ MeV, and for $T_e^{\max} \simeq 6.0$ MeV, $T_e^{\max} \simeq 4.73$ MeV. This shows how the width of the e^+ spectrum depends on the value of T_p.

We must point out that our discussion rests on the assumption that the distribution of the proton energy is like a δ function as mentioned in the preceding paragraph. However, if the proton energy distribution is broad which is very probable as the proton is a member of a coherent nucleus, the maximum kinetic energy of a proton can be $T_p \times A$ with a finite probability, however small. This will be reflected in $T_e^{\max}$ through the above equation. Then, of course, the positron spectrum, even for $T_p \simeq 2$ MeV, may become broader depending on the shape of the proton kinetic energy distribution function. In other words, as the coherence of the nucleus will be maintained through the exchange of virtual mesons, e^+e^- emitted from a certain proton may share the energy of many other nucleons. However their maximum energy $T_e^{\max}$ may be large to make the spectrum broad even when T_p of the emitting proton is small. In our present discussion we report the result in Fig. 3, however, without considering a detailed sharing of energy among the nucleons. Reinhardt *et al.*[8] reported a broad spectrum obtained by considering the coherent nucleus from an alternative approach.

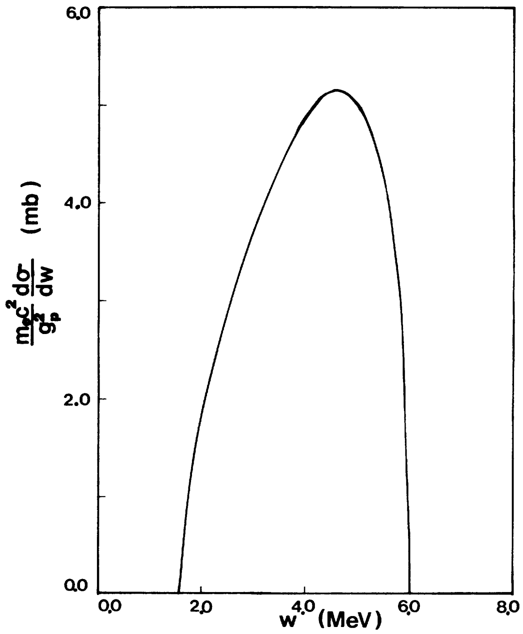

FIG. 4. The cross section of ϕ alone as a function of energy of the boson for $T_p \simeq 6$ MeV. Note the breadth and the position of the peak which affects the positron peak.

Next we integrate the differential cross section $d\sigma/dT_e$ and obtain, after substituting the various experimental values,

$$\sigma=\int(d\sigma/dT_e)dT_e$$
$$=2\langle s\rangle(Z_1{}^2Z_2{}^2e^4g_p{}^2)5.13\times10^{-12}\ \text{MeV}^{-2} \ . \qquad (13)$$

Experimentally σ_I has been reported to be equal to ~ 200 μb. If we take $\langle s\rangle=\frac{1}{2}$, this gave us the limit $g_p \lesssim 10^{-1}$ with $Z=Z_1+Z_2=180$–188. And if $\langle s\rangle$ equals, say, $\frac{1}{10}$, then $g_p \simeq 1$. Since we have no way to estimate exactly the effective spin of a proton in heavy nuclei, we will take $\langle s\rangle=\frac{1}{2}$ for the rest of the discussion of this paper. In other words, we find that if the bremsstrahlung-type process is the mechanism for producing the ϕ particle, the coupling constant would have to satisfy the above bound. The positron yield definitely received a big boost due to the presence of $(Z_1Z_2)^2$ factors reflecting the strength of the Coulomb field in the supernucleus and thereby affecting the limit on g_p.

The same bremsstrahlung mechanism has been considered, albeit semiclassically, by Reinhardt *et al.*[18] They obtain a limit on $g_p{}^2/4\pi \simeq 10^4$. To understand this a few orders of magnitudes difference between the two independent calculations, we need to see how $d\sigma^B/dw$ in Eq. (11) from the two calculations compare. The result of Ref. 8 provides

$$d\sigma^B/dw \simeq k[(k/w)^2(k/m)^2](Z_1Z_2)^2e^4 \cdots \quad (14)$$

which should be compared with the result of this paper given in Appendix C. Both of them contain the leading factor k coming from the phase space. However, Eq. (14) contains an additional strong dependence on k coming from matrix elements which makes the total cross section small. In fact, as was shown in Appendix A, the matrix elements in QED calculations contained similar k/w or k/m factors which have been neglected compared to other larger terms. These larger terms could not be obtained in a purely semiclassical calculation of Ref. 8. Besides, it should be mentioned that the calculation of Reinhardt *et al.* includes the pseudovector derivative coupling of the boson to nucleon. Consequently, the estimated value of g_p was much larger than the value of our paper. In other words, the semiclassical approach provides a much smaller cross section than that of the value calculated from QED, for the same value of the coupling constant.

Although we estimate $g_p \lesssim 10^{-1}$, this value is much larger than the value of g_p estimated from other constraints. Reinhardt *et al.* estimate $g_p \simeq 10^{-2}-10^{-4}$. Therefore, if we take $g_p \simeq 10^{-3}$ as "acceptable," we see that the calculated bremsstrahlung cross section is smaller than the experimental one by a factor of 10^{-4}. Therefore, unless g_p is as large as 10^{-1}, the bremsstrahlung induced by electromagnetic interaction cannot explain the production mechanism of the observed positron line spectrum. This leads us to consider an alternative bremsstrahlung mechanism induced by strong interaction. The electromagnetic interaction being long ranged could involve all the protons of the interacting ions. However, in the case of the strong interaction only those nucleons (both proton and neutron) lying within a distance of $\simeq 2-3$ fm of the interacting surface can play an effective role. To estimate the order of magnitude of the cross section due to the strong interaction, we need, in Eq. (13), the replacement

$$(Z_1Z_2)^2e^4 \rightarrow (A_1A_2)_{\rm eff}{}^2 g_{\rm st}{}^4 ,$$

where $g_{\rm st}$ is a strong coupling parameter: $g_{\rm st}{}^2/4\pi \simeq 14$. Considering only the nucleons in two facing disks of radius R_1,R_2 where R_1,R_2 are the radius of the ions (e.g., for $U^{238}+U^{238}$, $R_{1,2}$ is $\simeq 8.5$ (fm) and thickness 2 fm we estimate

$$(A_1A_2)_{\rm eff} \simeq Z_1Z_2/5 \text{ with } A_1,A_2 \simeq 2.5Z_1, 2.5Z_2 ,$$

respectively. Now using the experimental value of $g_{\rm st}{}^2$, we get an enhancement of 10^5 over the electromagnetically induced bremsstrahlung mechanism. This is, of course, an overestimate of the $(A_1A_2)_{\rm eff}$ in the above calculation. However, the strong enhancement factor is of the right magnitude required to make the cross section comparable with the experimental value with g_p being as small as 10^{-3}. In other words if bremsstrahlung is the mechanism for ϕ production and if g_p is 10^{-3}, then the process must be induced by the strong interaction so as to make the theory agree with the experiment as far as the total integrated cross section is concerned.

Then, of course, one is left with explaining the narrowness of the positron spectrum. In Fig. 4 we have shown that if the average kinetic energy of the interacting nucleons can be made smaller inside the supernucleus through some not-yet known mechanism (as, for example, the change in binding energy/u due to the supernucleus status, etc.), the ϕ's would be produced with very small kinetic energy, which would automatically result in a narrow peak. However, it would require a detailed calculation to show if that is indeed possible and will be deemed beyond the scope of this work. The position of the peak, as was shown, is controlled by m_ϕ alone.

V. DISCUSSION

The mechanism for the production of the ϕ particle in heavy-ion collisions we discussed in the preceding sections admittedly describes a very simplified picture of the complex situation existing in heavy-ion collisions. It has been pointed out by Balantekin *et al.* that, since the electromagnetic processes during the heavy-ion collisions are strongly time dependent, reliable calculations of any production mechanism requires nonperturbative methods and beyond. However, in this paper we only attempted to get an estimate of the yield of positron via the production of the ϕ particles in a simple model. We found that the experimental data can be matched to provide the coupling $g_p \simeq 10^{-1}$, as far as the integrated cross section is concerned. We believe that taking into account the exact kinematics without approximation and the effects of proton and nuclear form factors can alter the limit on the coupling constant g_p. The limit on g_p obtained from our rigorous calculation is more stringent than the limit available in the literature.[8,9] We obtain $\alpha_p \lesssim 10^{-3}$ independent of the value g_e compared to $\alpha_p < 10^4$ obtained by Reinhardt *et al.*[8] From the contribution of the ϕ particle to the anomalous magnetic moments of the electron, one expects $g_e \lesssim 10^{-4}$. This limit on g_e set by the anomalous magnetic moment may change due to the cancellations by introducing, if one is willing, more than one new particle. As a result a newer limit $\alpha_e < 10^{-7}-10^{-6}$ is reported from the analysis of the data on the hyperfine splitting in the positronium ground state.[13] In that respect our limit on g_p independent of g_e is indeed a plus. Various atomic phenomena also put a limit on $g_eg_p < 10^{-6}-10^{-8}$. How reliable these limits are depends on a detailed calculation which does not yet exist. However, Schäfer *et al.*[4] suggest that the production mechanism of the particle ϕ might be similar to nuclear bremsstrahlung which gives $\sigma(E_\gamma > 1.6 \text{ MeV}) \simeq 5\times 10^{-4}$ b. Without making detailed calculations, they estimate that $g_p \simeq 10^{-4}$ by simply replacing the coupling constant in the bremsstrahlung calculation to explain the positron line in heavy-ion collisions. In our calculation we find that due to γ_5 coupling of the particle ϕ (instead of γ_λ for bremsstrahlung photon) there is a large cancellation in the matrix element itself. Thus, a much larger value of g_p is needed, if the production of the ϕ particle in heavy-ion collisions is attributed to a bremsstrahlung-type production mechanism induced by electromagnetic interaction. We also showed that if the process is indeed induced by strong interaction, $g_p \simeq 10^{-3}$ can explain the observed cross section.

In summary, the production cross section of the positron in heavy-ion collisions calculated from the bremsstrahlunglike mechanism induced by electromagnetic interaction in the framework of a linear QED turns out to be a few orders of magnitude larger than the values obtained from the semiclassical approach of the same mechanism and results from other semiclassical methods. However, the value of the present calculation is still a few orders of magnitude smaller than the observed cross section. Thus we conclude from these and other analyses that the origin of the positron spectrum cannot be attributed to an electromagnetic interaction. We found, however, that an alternative production mechanism induced by strong interactions (e.g., $pp \to pp\phi$, $pn \to pn\phi$, $nn \to nn\phi$) provides a production cross section for the positron, that agrees well with the observed one, with $g_p \simeq 10^{-3}$.

As a comment, it can be mentioned that both theoretical[14] and experimental works[15] on a similar light boson have been carried out independently prior to the GSI discovery. Whether they all can be related by expanding the family of the boson requires further theoretical and experimental investigations.

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APPENDIX A

The matrix element from Eq. (1) is

$$M_{21}^{p} = \bar{u}_p(p_2)\left[\gamma_0 \frac{1}{\not{p}_1 + \not{k} - m}\gamma_5 + \gamma_5 \frac{1}{\not{p}_2 + \not{k} - m}\gamma_0\right] u_p(p_1) .$$

Averaging over initial and sum over final spins, we get

$$|\overline{M}_{21}^{p}|^2 = 2\{2[A^2(\tilde{p}_2 \cdot k)(p_1 \cdot k) + B(p_2 \cdot k)(\tilde{p}_1 \cdot k)] - (A^2 + B^2)[(p_1 \cdot \tilde{p}) m_\phi{}^2 + m^2 m_\phi{}^2]$$
$$+ 2AB[(p_2 \tilde{k})(\tilde{p}_1 \cdot k) - (p_2 \cdot \tilde{p}_1)(k \cdot \tilde{k}) + (p_2 \cdot k)(\tilde{p}_1 \cdot k) - m^2 k \cdot \tilde{k}]\} ,$$

$$(a \cdot b) = a_\mu b^\mu = a_0 b_0 - \mathbf{a} \cdot \mathbf{b} ,$$

where $\tilde{p} = (p_0, -\mathbf{p})$

$$A = \frac{1}{2(\mathbf{p}_1 \cdot \mathbf{k}) - m_\phi{}^2}, \quad B = \frac{1}{2(\mathbf{p}_2 \cdot \mathbf{k}) + m_\phi{}^2} .$$

We simplify as follows in the laboratory frame: $p_1 = (E_1, \mathbf{p}_1)$, $p_2 = (E_2, \mathbf{p}_2)$, $k = (w, \mathbf{k})$

$$2(p_1 \cdot k) - m_\phi{}^2 = 2E_1 w - 2\mathbf{k} \cdot \mathbf{p}_1 - m_\phi{}^2 = 2E_1 w \left[1 - \frac{\mathbf{k}}{w} \cdot \frac{\mathbf{p}_1}{E_1} - \frac{m_\phi{}^2}{2wE_1}\right] \simeq 2E_1 w .$$

Similarly

$$2(p_2 \cdot k) + m_\phi{}^2 \simeq 2E_2 w ,$$

$E_1 = E_2 + w$ due to the δ function:

$$A^2 + B^2 = \frac{E_1{}^2 + E_2{}^2}{4w^2 E_1{}^2 E_2{}^2} \simeq \frac{1}{2w^2 E_1 E_2}, \quad 2AB = \frac{1}{2w^2 E_1 E_2}, \quad A^2 - B^2 = \frac{1}{4w^2} \frac{E_2{}^2 - E_1{}^2}{E_1{}^2 E_2{}^2} = -\frac{E_1 + E_2}{4wE_1{}^2 E_2{}^2} .$$

Simplification leads to

$$|\overline{M}_{21}^{p}|^2 \simeq \frac{2}{E_1 E_2}\left[(E_1 E_2 - m^2) + (\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k} - \mathbf{p}_1 \cdot \mathbf{p}_2 - \frac{w}{E_1} \mathbf{p}_1 \cdot \mathbf{k} - \frac{1}{w^2} (\mathbf{p}_1, \mathbf{k})(\mathbf{p}_2 \cdot \mathbf{k})\right] .$$

Assume $w \ll E_1$, $|\mathbf{k}| \ll w$,

$$|\overline{M}_{21}^{p}|^2 \simeq \frac{2}{E_1 E_2}[(E_1 E_2 - m^2) + (\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{k} - \mathbf{p}_1 \cdot \mathbf{p}_2] .$$

APPENDIX B

We need to evaluate the three integrals

$$I_1=\int\int\frac{d\Omega_2 d\Omega}{\mathbf{q}^4},\quad I_2=\int\int\frac{d\Omega_2 d\Omega}{\mathbf{q}^4}(\mathbf{p}_1\cdot\mathbf{p}_2),\quad I_3=\int\int\frac{d\Omega_2 d\Omega}{\mathbf{q}^4}(\mathbf{Q}\cdot\mathbf{k})\ .$$

We have

$$\begin{aligned}
I_1&=\int d\Omega_2\int\frac{d\Omega}{\mathbf{q}^4}\\
&=\int d\Omega_2\int\frac{d\Omega}{(\mathbf{Q}-\mathbf{k})^4},\quad \mathbf{Q}=\mathbf{p}_1-\mathbf{p}_2\\
&=2\pi\int d\Omega_2\int\frac{d(\cos\theta)}{(\mathbf{Q}^2+\mathbf{k}^2-2\,|\,\mathbf{Q}\,|\,|\,\mathbf{k}\,|\cos\theta)^2}\\
&=2(2\pi)\int d\Omega_2\frac{1}{(\mathbf{Q}^2+\mathbf{k}^2)^2-4\mathbf{Q}^2\mathbf{k}^2}\\
&=4\pi(2\pi)\int\frac{d(\cos\theta')}{(\mathbf{p}_1{}^2-\mathbf{p}_2{}^2-\mathbf{k}^2-2\,|\,\mathbf{p}_1\,|\,|\,\mathbf{p}_2\,|\cos\theta')^2}\\
&=\frac{(4\pi)^2}{(\mathbf{p}_1{}^2+\mathbf{p}_2{}^2-\mathbf{k}^2)^2-4\,|\,\mathbf{p}_1\,|^2\,|\,\mathbf{p}_2\,|^2}\\
&=\frac{(4\pi)^2}{[(\,|\,\mathbf{p}_1\,|+|\,\mathbf{p}_2\,|\,)^2-\mathbf{k}^2](\,|\,\mathbf{p}_1\,|-|\,\mathbf{p}_2\,|^2-\mathbf{k}^2)}\ ,
\end{aligned}$$

$$\begin{aligned}
D_1&=|\,\mathbf{p}_1\,|^2+|\,\mathbf{p}_2\,|^2-|\,\mathbf{k}\,|^2-2\,|\,\mathbf{p}_1\,|\,|\,\mathbf{p}_2\,|\\
&=E_1{}^2+E_2{}^2-2m^2-|\,\mathbf{k}\,|^2-2[(E_1{}^2-m^2)(E_2{}^2-m^2)]^{1/2}\\
&=E_1{}^2+E_2{}^2-2m^2-k^2-2[E_1{}^2E_2{}^2-m^2(E_1{}^2+E_2{}^2)+m^4]^{1/2}\ .
\end{aligned}$$

Now using $E_1{}^2+E_2{}^2=2E_1E_2+w^2=2E_1E_2$ in the square root, we have

$$D_1=E_1{}^2+E_2{}^2-2m^2-2(E_1E_2-m^2)-k^2\simeq(E_1-E_2)^2-\mathbf{k}^2\simeq m_\phi{}^2\ . \tag{B1}$$

Similarly

$$\begin{aligned}
D_2=|\,\mathbf{p}_1\,|^2+|\,\mathbf{p}_2\,|^2-|\,\mathbf{k}\,|^2+2\,|\,\mathbf{p}_1\,|\,|\,\mathbf{p}_2\,|&\simeq E_1{}^2+E_2{}^2-2m_2-\mathbf{k}^2+2(E_1E_2-m^2)\\
&\simeq 2\times 2(E_1E_2-m^2)-\mathbf{k}^2=4(E_1E_2-m^2)\ ,\\
I_1=\frac{4\pi^2}{m_\phi{}^2(E_1E_2-m^2)}\ .
\end{aligned}\tag{B2}$$

Integral I_2

$$I_2=\int d\Omega\,\mathbf{p}_1\cdot\mathbf{p}_2\int\frac{d\Omega}{\mathbf{q}^4}\ .$$

The integral over d was done in I; hence,

$$I_2=4\pi\int\frac{d\Omega_2\,|\,\mathbf{p}_1\,|\,|\,\mathbf{p}_2\,|\cos\theta'}{[(\mathbf{p}_1{}^2+\mathbf{p}_2{}^2-\mathbf{k}^2)-2\,|\,\mathbf{p}_1\,|\cdot|\,\mathbf{p}_2\,|\cos\theta']^2}\ .$$

Substituting $a=(\mathbf{p}_1{}^2+\mathbf{p}_2{}^2-\mathbf{k}^2)$, $b=2\,|\,p_1\,|\,|\,p_2\,|$, and $\cos\theta'=x$,

$$I_2=4\pi^2b\int_{-1}^{+1}\frac{x\,dx}{(a-bx)^2}=4\pi^2b\left[\frac{1}{b^2}\ln\frac{a-b}{a+b}+\frac{a}{b}\frac{2}{a^2-b^2}\right]\ .$$

Restoring a and b and using the results from (A1) and (A2) we have

$$I_2=-\frac{2\pi^2}{E_1E_2-m^2}\ln\frac{4(E_1E_2-m^2)}{m_\phi{}^2}+(E_1E_2-m^2)I_1\ .$$

Integral I_3

$$I_3 = \int d\Omega_2 \int d\Omega \frac{\mathbf{Q}\cdot\mathbf{k}}{\mathbf{q}^4} \ ,$$

where $\mathbf{Q}=\mathbf{p}_1-\mathbf{p}_2$. The integration over $d\Omega$ can be done as in I_2 and obtain

$$I_3 = 2\pi \int d\Omega_2 \frac{\mathbf{Q}^2+\mathbf{k}^2}{(\mathbf{Q}^2+\mathbf{k}^2)^2-4\mathbf{k}^2\mathbf{Q}^2} + C_3 \ ,$$

where C_3 is another term containing a logarithm. It can be chosen that C_3, though it contains $\mathbf{p}_1\cdot\mathbf{p}_2$, can be neglected, being smaller. Hence

$$I_3 \simeq 2\pi d\Omega \frac{\mathbf{Q}^2+\mathbf{k}^2}{(\mathbf{Q}^2+\mathbf{k}^2)^2-4\mathbf{Q}^2\cdot\mathbf{k}^2} = 2\pi(\mathbf{p}_1{}^2+\mathbf{p}_2{}^2+\mathbf{k}^2)\int \frac{d\Omega_2}{(\mathbf{Q}^2-\mathbf{k}^2)^2} - 4\pi \int \frac{d\Omega_2 \mathbf{p}_1\cdot\mathbf{p}_2}{(\mathbf{Q}^2-\mathbf{k}^2)^2} \ .$$

From results of the I_1 and I_2 evaluations we can see that

$$I_3 \simeq (E_1E_2-m^2)I_1 - I_2 \ .$$

Then finally

$$\int |M_{21}^g|^2 \frac{1}{\mathbf{q}^4} d\Omega\, d\Omega_2 \simeq \frac{2}{E_1E_2}[(E_1E_2-m^2)I_1+I_3-I_2]$$

$$\simeq \frac{2}{E_1E_2}[(E_1E_2-m^2)+(E_1E_2-m^2)I_1-2I_2]$$

$$\simeq \frac{2}{E_1E_2}\frac{4\pi^2}{(E_1E_2-m^2)}\ln\frac{4(E_1E_2-m^2)}{m_\phi{}^2} \ .$$

APPENDIX C

The photoproduction cross section $\sigma(\gamma p \to p\phi)$ of the ϕ particle up to the Born diagram only is

$$\sigma = \int d\Omega \frac{d\sigma^\gamma}{d\Omega} \ ,$$

where (in the rest frame of the proton)

$$\frac{d\sigma^\gamma}{d\Omega} = \frac{(eg_p)^2}{(2\pi)^2}\frac{1}{16mq}\sum_{w=\pm w}\frac{(w^2-m_\phi{}^2)}{|(w^2-m_\phi{}^2)^{1/2}E-qw\cos\theta|}|M|^2 \ ,$$

where

$$|M|^2 = \{2w(E_2w-qk\cos\theta+k^2)[4m(q-w)(m_\phi{}^2+mq-mw)+m_\phi{}^4] + mm_\phi{}^2[2(4m^2q^2+(2mw-m_\phi{}^2))-4mq(2mw-m_\phi{}^2)] - E_2m_\phi{}^2[4m^2q^2+(2mw-m_\phi{}^2)]\}\frac{1}{2mq^2(2mw-m_\phi{}^2)^2} \ .$$

With $E_2=m+q-w$, q is the photon energy, m, m_ϕ is the proton and ϕ-particle mass, respectively, $w\pm=[(2ab\pm\sqrt{4ab-4AC})/2A]$, $a=m_\phi{}^2=2mq$ $b=2(q+m)\simeq 2E$, $A=2(m+q\alpha^+)(m+q\alpha^-)$, $C=a^2+4m_\phi{}^2q^2(1-\alpha^+\alpha^-)$, $\alpha^\pm=1\pm\cos\theta$, and $k=(w^2-m_\phi{}^2)^{1/2}$. The calculation is carried out without any approximation. The proton form factor was taken to be unity.

The corresponding cross section σ^B for the bremsstrahlung-type production of ϕ particle in the Coulomb field of charge Z_e is given by

$$\sigma^B = \int_{m_\phi}^{E_1-m} dw \frac{d\sigma^B}{dw} \ ,$$

where

$$\frac{d\sigma^B}{dw} = 2\langle s\rangle \frac{(Z_1Z_2e^2g_p)^2}{32\pi^3}\frac{\{(w^2-m_\phi{}^2)[(E_1-w)^2-m^2]\}^{1/2}}{(E_1{}^2-m^2)^{1/2}E_1(E_1-w)[E_1(E_1-w)-m^2]}\ln\frac{4[E_1(E_1-w)-m^2]}{m_\phi{}^2} \ ,$$

where $E_1=$ total energy of the proton. Other values are defined in the text. This expression was obtained under the same kinematical approximations as described in the text. $\langle s\rangle$ is the average effective spin of each nucleon in a heavy nucleus. This, in reality, can differ by a large amount from $\frac{1}{2}$ due to spin pairing of the nucleons.

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[12]We did not take the proton distribution into account, because we know very little about the proton state in small space-time dimensions when the collision takes place to form a supernucleus. The present calculated value, e.g., the production cross section, therefore, represents the upper bond of expected value.

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