Inelasticity and leading-particle effect: Momentum and mass distribution of the central fireball in high-energy hadronic interactions

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We calculate the distribution of the invariant mass and momentum of the system left by the leading particles in high-energy hadronic reactions on the assumption that it is determined by gluon-gluon interactions. Comparisons with existing data are presented and predictions for $\sqrt{s}=540$, 2000, and 40 000 GeV are made.

The concept of inelasticity plays an important role in the understanding of multiparticle production.[1] The idea is that only a part $M=K\sqrt{s}$ of the whole invariant energy $\sqrt{s}$ is used for multiparticle production in the central region while the rest of it is taken by the leading particles in the forward and backward regions. References 2–4 emphasized the importance of the leading-particle effect in high-energy collisions and used it in the context of statistical models for multiparticle production, which have become popular in connection with the quark-gluon-plasma investigations and the observation of violation of Koba-Nielsen-Olesen (KNO) scaling.[5]

The physical picture which we have in mind[6] is that of two colliding hadrons h_1 and h_2 interacting only through their gluon contents while the valence quarks go through and eventually form leading particles.[7] If x_1 and x_2 are the fractional momenta of gluons deposited in the central region and if the interacting glue leads finally to one central fireball then this fireball will have an energy W and momentum P (in the overall c.m. frame)

$$W=\tfrac{1}{2}(x_1+x_2)\sqrt{s}\ ,$$
$$P=\tfrac{1}{2}(x_1-x_2)\sqrt{s}\equiv x_c\frac{\sqrt{s}}{2}\ , \tag{1}$$

or invariant mass M and rapidity Y (in the same frame)

$$M=\sqrt{x_1x_2s}=K\sqrt{s}\ ,$$
$$Y=\frac{1}{2}\ln\frac{x_1}{x_2} \tag{2}$$

(all transverse-momentum dependence is ignored in what follows).

Here x_c denotes the fractional momentum of our central fireball. In Ref. 6 we have calculated the inelasticity distribution function $\chi(K)$ which provides a measure of the probability for the creation of a fireball with fractional energy K averaged over its momentum. [In fact, in Eq. (3) of Ref. 6 we made the approximation $\times\sum W_i{}^2$ where W_i is the energy of a minifireball.] Here we are interested in the more general distribution function $\chi(x_1,x_2)$. It provides the measure of the probability for the creation of a central fireball when fractions x_1 and x_2 of the energy of incoming hadrons h_1 and h_2 were deposited (and possibly equilibrated) in the central region. Note that $\chi(x_1,x_2)$ has to be normalized to unity over accessible phase space, i.e.,

$$\int_0^1 dx_1\int_0^1 dx_2\chi(x_1x_2)\theta(x_1x_2-K_{\min}{}^2)=1\ , \tag{3}$$

where $K_{\min}=0.6\ \mathrm{GeV}/\sqrt{s}$ (Ref. 8) is the minimal inelasticity. Once $\chi(x_1,x_2)$ is known we can easily write the distribution in other variables of potential interest by using the transformations (1) and (2). The corresponding distributions in the variables (K,x_c) and (K,Y) are denoted $\tilde{\chi}$ and $\tilde{\tilde{\chi}}$, respectively. The P (or Y or x_c) dependence of the fireball is relevant among other things in calculation of the forward-backward correlation, in studies of hadron-nucleus and nucleus-nucleus collisions, and in the interpretation of the Bose-Einstein correlations, in which the knowledge of the velocity of the emitting source $v\simeq\tanh Y$ is important for a proper interpretation of data.[9] Furthermore, it enables us to consider more accurately the distributions in one variable only, by integrating over the other.

As was already mentioned the central fireball arises from the (undefined) number of minifireballs,[10] each of which is the result of gluon-gluon interactions. The probability to create such a fireball, $\chi(x_1,x_2)$ is

$$\chi(x_1,x_2)=\sum_{\{n_i\}}\delta\left[x_1-\sum n_ix_{1i}\right]\times\delta\left[x_2-\sum n_ix_{2i}\right]\prod_{\{n_i\}}P(n_i)\ , \tag{4}$$

where the sum extends over all minifireball numbers which are assumed to be independently produced so that n_i follows a Poisson distribution:

$$P(n)=\frac{\bar{n}^n}{n!}e^{-n}\ . \tag{5}$$

Using the standard procedure of expressing δ functions as integrals one can easily perform all summations and finally obtain

$$\mathcal{X}(x_1,x_2)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}dt_1dt_2\exp[i(x_1t_1+x_2t_2)]$$
$$\times\exp\left[\int_0^{x_1}dx_1'\int_0^{x_2}dx_2'\theta(x_1'x_2'-K_{\min}{}^2)\omega(x_1',x_2')(e^{-i(x_1't_1+x_2't_2)}-1)\right]$$
$$\times\exp\left[-\int_{x_1}^{1}dx_1'\int_{x_2}^{1}dx_2'\omega(x_1',x_2')\right]. \tag{6}$$

The last term represents the probability that there is no energy deposit from incoming hadrons beyond fraction x_1 and x_2. The distribution $\mathcal{X}(x_1,x_2)$ is fully specified once the spectrum of the minifireballs

$$\omega(x_1,x_2)=\frac{d\bar{n}}{dx_1dx_2}$$

is given. As before (cf. Ref. 6) we assume it to be a product

$$\omega(x_1,x_2)=G_{h1}(x_1)G_{h2}(x_2)C(x_1,x_2) \tag{7}$$

of gluon distributions $G_{h1,2}$ in hadrons $h_{1,2}$, respectively, and a factor $C(x_1,x_2)$ representing the probability that all minifireballs will form one central fireball. C can be taken equal to the ratio σ_{gg}/σ_{hh} where σ_{gg} is the total cross section for gluon-gluon interactions (taken at $M=K\sqrt{s}$) and σ_{hh} is the total inelastic hadron-hadron cross section (taken at $\sqrt{s}$). We shall parametrize σ_{gg} as

$$\sigma_{gg}=\begin{cases}\dfrac{\alpha}{M^2}+\delta\ln\dfrac{M^2}{\mu^2}, & M>M_0\,,\\[2ex] \dfrac{\alpha}{M_0{}^2}+\delta\ln\dfrac{M_0{}^2}{\mu^2}, & M\le M_0\,,\end{cases} \tag{8}$$

$$\alpha=(9.4\ \text{GeV}^2)\sigma_{hh}(\sqrt{s}=16.5\ \text{GeV})\,,$$
$$\delta=0.08\sigma_{hh}(\sqrt{s}=16.5\ \text{GeV}),\quad \mu=1.0\ \text{GeV}\,,$$

so that σ_{gg} behaves according to the presently accepted lore, i.e., like $\sim M^{-2}$ at low energies and like $\sim\ln M^2$ at high energies. The "freezing point" $M_0=5$ GeV is necessary because the first of Eq. (9) diverges as $M\to 0$. Its value is determined by requiring that $\langle K\rangle=0.49$ at $\sqrt{s}=16.5$–62.5 GeV and $\langle K\rangle=0.3$ at $\sqrt{s}=540$ GeV (Ref. 4).

This parametrization is the simplest possible one which incorporates the most general trends of the cross section as a function of the effective energy $M=\sqrt{x_1x_2s}$. With it σ_{gg} turns out to be equal to 25 mb at low energies, which is not an unreasonable result.

For the gluon distributions $G(x)$ we used the parametrization of Glück, Hoffmann, and Reya[11] in which scaling violations have been included. This parametrization is good even for small values of x where perturbative QCD calculations are not applicable. For very high energies ($\sqrt{s}=40$ TeV) the parametrization of Duke and Owens[11] was used instead as more suitable. In both cases it is necessary to reduce the number of gluons at low x (this is consistent with similar expectations in Ref. 12); this was done by introducing a factor $\beta<1$ in the x^{-1} term of $G(x)$:

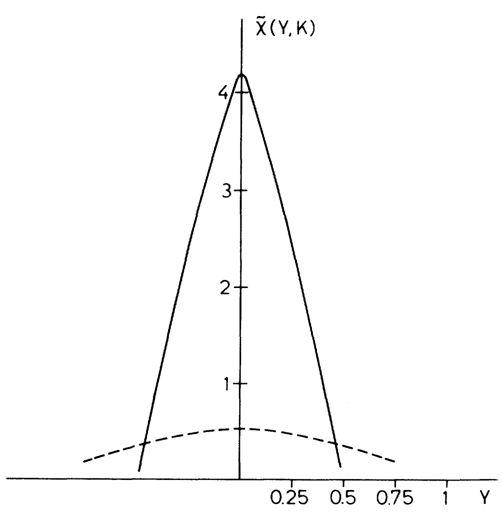

FIG. 1. The inelasticity distribution $\tilde{\mathcal{X}}(K=0.2,Y)$ (dotted curve) and $\tilde{\mathcal{X}}(K=0.5,Y)$ (solid curve).

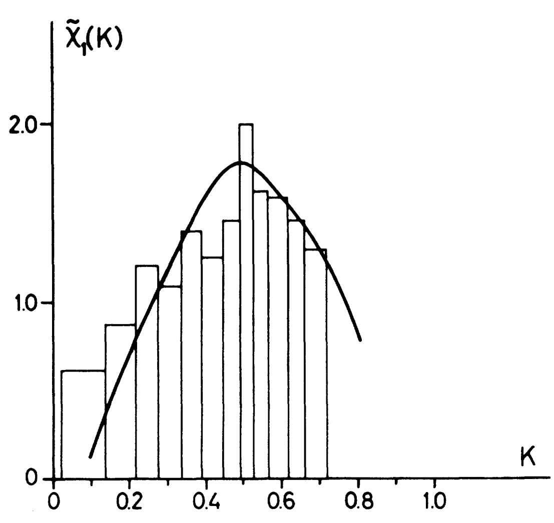

FIG. 2. $\tilde{\mathcal{X}}_1(K)$ at ISR energies. Comparison with data from Ref. 13.

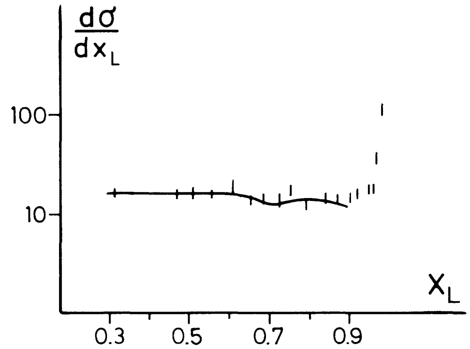

FIG. 3. $d\sigma/dx_L$ for the leading particle at ISR energies. Comparison with data from Ref. 2.

$$G(x) \sim \frac{\beta}{x} \quad \text{for } x \ll 1 \,. \tag{9}$$

This makes sense only in the context of our representation of $\omega(x_1,x_2)$ in terms of separate product of G's and $C(x_1,x_2)$. Numerically all results depend on the products $\beta\alpha$ and $\beta\delta$. The separation of α,β,δ and their interpretation should be always done with the above-mentioned picture in mind.

Because $\omega(x_1,x_2)$ is strongly peaked at very small $x_{1,2}$ it is justified to make in Eq. (7) the approximation

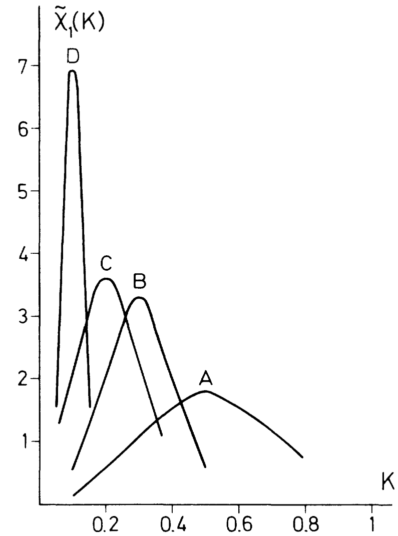

FIG. 4. $\tilde{\chi}_1(K)$ at various energies ($A=16.5-62.5$ GeV, $B=540$ GeV, $C=2000$ GeV, $D=40\,000$ GeV).

$$e^{-i(x_1t_1+x_2t_2)}-1=-i(x_1t_1+x_2t_2) -\tfrac{1}{2}(x_1t_1+x_2t_2)^2 \,. \tag{10}$$

This leads to a Gaussian integral for $\chi(x_1,x_2)$ which can easily be evaluated and gives

$$\chi(x_1,x_2)=\frac{\exp(-I)}{2\pi h}\exp\left\{-\frac{1}{2h^2}[I_{20}(x_1-I_{10})^2+I_{02}(x_2-I_{01})^2-2I_{11}(x_1-I_{10})(x_2-I_{01})]\right\}, \tag{11}$$

where

$$I_{nm}=\int_0^{x_1}dx_1'x_1'^n\int_0^{x_2}dx_2'x_2'^m\theta(x_1x_2-K_{\min}^2)\omega(x_1',x_2'),$$
$$I=\int_x^1 dx_1'\int_{x_2}^1 dx_2'\omega(x_1',x_2')\,, \tag{12}$$
$$h=|I_{20}I_{02}-I_{11}^2|^{1/2}\,.$$

From $\chi(x_1,x_2)$ one can immediately get distributions in other variables and calculate single-variable distributions such as

$$\tilde{\chi}_1(K)=\int_0^1 dx_1\int_0^1 dx_2\chi(x_1,x_2)\delta(\sqrt{x_1x_2}-K)\,,$$
$$\tilde{\tilde{\chi}}(Y)=\int_0^1 dx_1\int_0^1 dx_2\chi(x_1,x_2)\delta\left[\frac{1}{2}\ln\frac{x_1}{x_2}-Y\right], \tag{13}$$
$$\tilde{\chi}_2(x_c)=\int_0^1 dx_1\int_0^1 dx_2\chi(x_1,x_2)\delta(x_1-x_2-x_c) =\int_{K_{\min}}^1 dK\,\tilde{\chi}(K,x_c)\,.$$

TABLE I. Results for different moments of χ for various energies.

$\sqrt{s}$ (GeV)	$\langle K\rangle$	$\langle x_c\rangle$	ΔK	Δx_c	$\langle Kx_c\rangle$	ΔY
16.5–62.5	0.49[a]	0.00	0.19	0.20	0.00	0.20
540	0.30[a]	0.00	0.12	0.11	0.00	0.18
2000	0.20	0.00	0.10	0.07	0.00	0.17
40000	0.10	0.00	0.055	0.04	0.00	0.15

[a]Input.

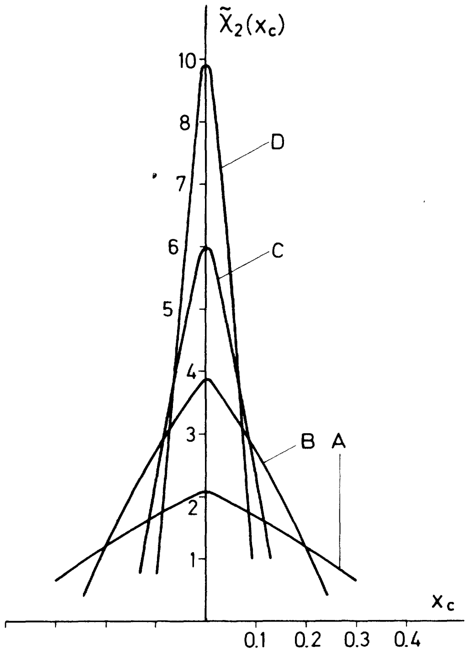

FIG. 5. $\widetilde{\chi}_2(x_c)$ at various energies.

One can also easily calculate all relevant moments of $\widetilde{\chi}_1(K)$, $\widetilde{\chi}_2(Y)$, $\widetilde{\chi}_2(x_c)$, etc.

It is worthwhile to mention here that for the simplified case of gluon distributions $G(x)=\beta/x$ [i.e., neglecting terms important in the limit $x\to 1$ because the inelasticity distribution $\chi(x_1x_2)$ is most sensitive to the behavior of $G(x)$ in the $x\to 0$ limit] all calculations can be done analytically and one finds that in terms of the K,Y variables the distribution in rapidity Y of the central fireball is independent of the value of K. The departure from uniformity of $\widetilde{\widetilde{\chi}}(K=\text{const},Y)$ as a function of Y is then a measure of the importance of other factors in the $G(x)$ function (see Fig. 1).

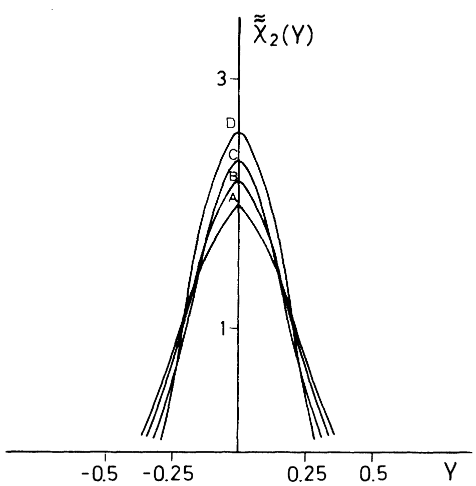

FIG. 6. $\widetilde{\widetilde{\chi}}_2(Y)$ at various energies.

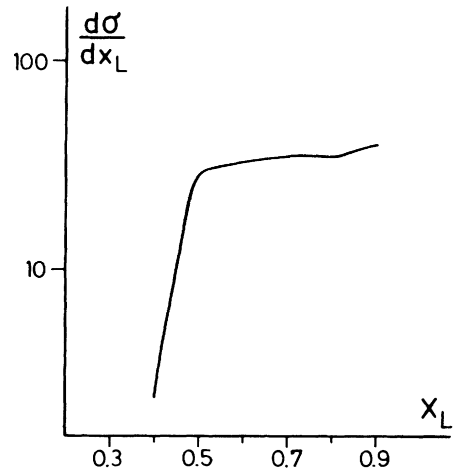

FIG. 7. $d\sigma/dx_L$ for the leading particle at 540 GeV.

Once $\chi(x_1,x_2)$ is given it is straightforward to calculate the spectrum of leading particles, say in the forward direction:

$$\frac{1}{\sigma}\frac{d\sigma}{dx_L}=\int_0^1 dx_1\int_0^1 dx_2\chi(x_1,x_2)\delta(x_L-1+x_1)\ . \quad (14)$$

The comparison with existing data is presented in Figs. 2 and 3. The leading-particle spectrum of Basile *et al.*[2] and inelasticity distribution of Brick *et al.*[13] plus the requirement that the mean inelasticity

$$\langle K\rangle=\int_{K_{\min}}^1 dK\,K\widetilde{\chi}_1(K)$$

shifts with energy from $\langle K\rangle=0.49$ at CERN ISR and Fermilab energies to $\langle K\rangle=0.3$ at collider energies[4] fixes our parameters: $(\alpha\beta^2)$, $(\beta^2\delta)$, and μ (in $K_{\min}$) completely. The energy dependence of $\langle K\rangle$ is crucial in establishing the rate of energy dependence of $c=\sigma_{gg}/\sigma_{hh}$ (i.e., α and β), while the leading-particle spectrum determines practically the factor β. The data on $\widetilde{\chi}_1(K)$ and the normalization requirement are more than enough to fix $(\alpha\beta^2)$, (β^2,δ), and μ completely. The resolution between α, β, and δ is then somehow arbitrary. If we take $\beta=1$ (i.e., "normal" gluon distribution) σ_{gg}/σ_{hh} turns out to be unphysically small. Assuming, on the other hand, that σ_{gg}/σ_{hh} is of the order of unity leads to $\beta=0.1$, i.e., the effective number of soft gluons is much lower than expected from a naive extrapolation of deep-inelastic scattering data, but in agreement with Ref. 12.

Table I summarizes our results for different moments of χ, specifically, $\langle K\rangle$, the dispersions of K, x_c, and Y (notice that $\langle x_c\rangle=\langle y\rangle=0$), and $\langle Kx_c\rangle$, for different energies. Because our $\chi(K)$ is a Gaussian, i.e., $\chi(K)=\exp[-(K-\langle K\rangle)^2/2(\Delta K)^2]$, $\langle K\rangle$ and the dispersion ΔK define completely the distribution.

In Figs. 4 and 5 the prediction for $\widetilde{\chi}_1(K)$ and $\widetilde{\chi}_2(x_c)$, for energies $\sqrt{s}=2$ and 40 TeV, are presented. We note the following.

(1) $\langle K\rangle$ continues to decrease[14] with $\sqrt{s}$ reaching at Superconducting Super Collider energies (40 TeV) the value of ~ 0.1. Also the width of $\widetilde{\chi}_1(K)$ decreases with

energy from ~ 0.19 at ISR energies to ~ 0.055 at 40 TeV. Notice, however, that this prediction is sensitive to the energy dependence of the ratio $c=\sigma_{gg}/\sigma_{hh}$, i.e., the relative importance of soft gluons in the hadronic inelastic cross section.

(2) The width of $\widetilde{\chi}_2(x_c)$, i.e., Δx_c decreases in the same energy range from 0.2 to 0.04.

(3) For the only energy ($\sqrt{s}=16.5$ GeV) where data for $\widetilde{\chi}_1(K)$ exist, the agreement with experiment is good (Fig. 6). Both $\widetilde{\chi}_1(K)$ and $\widetilde{\chi}_2(x_c)$ can be approximated at high energies by Gaussians defined by the parameters in Table I.

(4) The leading-particle spectrum $d\sigma/dx_L$ (Fig. 7) at ISR energies where data exist is in agreement with experiment. It is interesting to observe that the gluon-gluon interaction picture explains the flatness of $d\sigma/dx_L$ at ISR energies and predicts a change (at $x<0.6$) of $d\sigma/dx_L$ at $\sqrt{s}=540$ GeV.

It is seen that asymptotically $\widetilde{\chi}_1(K)$ and $\widetilde{\chi}_2(x_c)$ approach δ functions which means that at very high energies the averaging over K and x_c simplifies. The momentum spectrum of fireballs $\chi(x_c)$ is already very narrow at ISR energies and this justifies the approximation of Ref. 6 (cf. the text above) where the x_c spectrum was neglected altogether.

The knowledge of χ is an essential prerequisite in the search for a quark-gluon plasma since there the energy density is the decisive physical quantity. Similar considerations apply for the interpretation of multiplicity distributions.[5] Last but not least as shown above the determination of $\chi(x_1,x_2)$ could provide interesting and important information about gluon-gluon cross sections and gluon structure functions. While in the present study we were concerned only with hadron-hadron reactions, it is obvious that for hadron-nucleus and nucleus-nucleus collisions the inelasticity concept plays an equally important part. The extension of the calculations presented here to these cases is under consideration. However, it should be clear that there is also an urgent need for experimental tests of the results and predictions presented here and calorimeters appear to be a natural candidate for this purpose.

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