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Rapid Communications

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Chiral hierarchies and chiral perturbations in technicolor theories

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It is shown that in an asymptotically free theory a slowly running coupling enhances the value of the appropriately defined fermion condensate while keeping the Goldstone-boson decay constant essentially unaltered. In technicolor theories, including the necessary effective four-fermion interactions which explicitly break some of the chiral symmetries, the condensate directly determines fermion and pseudo-Goldstone-boson masses through chiral perturbation theory. An enhanced condensate can increase these masses substantially without affecting M_W and M_Z. Some gauge-theory examples exhibiting this behavior are tabulated.

In a recent Letter[1] (paper I), it was observed that a slowly running coupling in an asymptotically free gauge theory can have an important effect on the dynamics of spontaneous chiral-symmetry breaking. The dynamically generated fermion mass $\Sigma(p)$ can fall much less rapidly over a substantial momentum range than its ultimate asymptotic form might suggest. Certain quantities of physical interest, such as the masses of quarks and leptons in technicolor theories, are especially sensitive to these higher-momentum components. These masses require the existence of additional interactions at some large scale M, which typically also lead to flavor-changing neutral currents[2] (FCNC's). For a certain range of gauge-theory parameters, the quark and lepton masses were estimated to be sufficiently large (and much larger than naively expected), even with M large enough to adequately suppress FCNC's.

The purpose of this Rapid Communication is to describe more generally the quantum dynamics underlying this kind of enhancement effect, to catalogue some of the theories that exhibit it, and to extend the quantitative estimates to include the masses of the pseudo-Goldstone bosons (PGB's) that are also often present in technicolor theories. An increase of pseudomasses is a consequence of a large-momentum-component enhancement in chiral perturbation theory. Other possible consequences of a slowly running coupling are also suggested.

Consider an asymptotically free gauge theory with a vectorlike coupling to N_f massless fermions. At a large enough momentum q, the running coupling will take the form

$$\alpha(q) = 1/b \ln q/\Lambda' , \tag{1}$$

where

$$q\frac{\partial}{\partial q}\alpha = \beta(\alpha) = -b\alpha^2 - c\alpha^3 + \cdots , \quad b > 0 .$$

At some scale μ, spontaneous breaking of the global chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry to $SU(N_f)_{L+R}$ is expected to take place. A common fermion mass $\Sigma(p)$ will develop with a computable dependence on the Euclidean momentum p. $\Sigma(p)$ will have some value Σ_0 of order μ for $p \lesssim \mu$ and then fall monotonically for larger p. For momentum p below μ, the condensing fermions will decouple and the running coupling $\alpha(p)$ can be expected to evolve more rapidly than above μ. This evolution will determine the relation between the chiral-symmetry-breaking scale μ and the physical confinement scale Λ. The point made in paper I was that if a small parameter is present in the theory, in the form of a slowly running coupling for $p > \mu$, a hierarchy of scales associated with spontaneous chiral-symmetry breaking can be generated. This hierarchy can in turn have important experimental consequences. In the class of theories being considered, however, fermion condensation will eliminate the small parameter for $p < \mu$. For this reason, one expects that $\mu/\Lambda = O(1)$. This feature of these theories is confirmed by our analytical and numerical studies.

The slow running of the coupling for $p > \mu$ can lead to a $\Sigma(p)$ that initially falls rather slowly. This has its most direct impact on the size of the condensate $\langle 0 | \bar{\psi}\psi | 0 \rangle_M$, which in turn determines the values of fermion and PGB masses in technicolor theories. The high-momentum components of the condensate can be computed perturbatively to give

$$\langle 0 | \bar{\psi}\psi | 0 \rangle_M \simeq \frac{N}{2\pi^2} \int^M p\, dp\, \Sigma(p) , \tag{2}$$

where M is the ultraviolet cutoff and where the fermions have been taken to be in the fundamental representation of an SU(N) gauge group. The cutoff is both physical and necessary to define the condensate. In the theories being considered here, however, M will be sufficiently large that the bulk of the contribution to the condensate will come from momenta $\mu < p \ll M$. Because $\langle 0|\bar{\psi}\psi|0\rangle_M$ is especially sensitive to momenta in this range, a slow fall of $\Sigma(p)$ over a substantial portion of the range can give it a much larger value than might be anticipated on naive dimensional grounds. In particular, $\langle 0|\bar{\psi}\psi|0\rangle_M$ can be enhanced in this way, while $\Sigma_0 \equiv \Sigma(0)$ and F_π, the Goldstone-boson decay constant, are not. The physical predictions to be described here will all follow from this fact.

The form of $\Sigma(p)$ in the perturbative regime is governed by the gap equation

$$\Sigma(p) = \frac{3C_2(R)}{2\pi}\left[\int^{p}\frac{kdk}{p^2}\alpha(p)\frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)} + \int_p^{\infty}\frac{kdk}{k^2}\alpha(k)\frac{k^2\Sigma(k)}{k^2+\Sigma^2(k)}\right], \quad (3)$$

where $\alpha(q)$ is the running coupling constant and $C_2 = (N^2-1)/2N$ for fermions in the fundamental representation of an SU(N) gauge theory. For $p \gg \mu$, this equation can be linearized, and with $\alpha(q)$ given by Eq. (1), the ultimate asymptotic form of $\Sigma(p)$ is $\Sigma_{\rm asy}(p) = (\ln p)^{A/2-1}/p^2$, where $A = 3C_2(R)/\pi b$. It is clear from this behavior that an ultraviolet cutoff M is required to define $\langle 0|\bar{\psi}\psi|0\rangle_M$.[3] Furthermore, if $\Sigma(p)$ falls less rapidly than $\Sigma_{\rm asy}$ over a substantial range below M, $\langle 0|\bar{\psi}\psi|0\rangle_M$ can be considerably enhanced.

By contrast, it can be seen from Eq. (3) that $\Sigma(p)$ for $p \rightarrow 0$ (the dynamical or "constituent" fermion mass) is not terribly sensitive to the high-k behavior of $\Sigma(k)$. Even a logarithmic fall with k will converge the integral equation for $k > \Sigma_0 \simeq \Lambda$, and eliminate the need for a cutoff. F_π is similarly insensitive to the high-momentum behavior of $\Sigma(k)$. This contribution can be computed to give[4]

$$F_\pi^2\Big|_{\substack{\text{high-}k\\ \text{components}}} \simeq \frac{N}{2\pi^2}\int^{\infty}\frac{dp}{p}\left[\Sigma^2(p) - \tfrac{1}{4}p\Sigma(p)\frac{d\Sigma}{dp}\right]. \quad (4)$$

Thus F_π is also much less sensitive to the high-k behavior of $\Sigma(k)$ than $\langle 0|\bar{\psi}\psi|0\rangle_M$.[5] In particular, the behavior of $\Sigma(k)$ expected with a slowly running coupling can substantially increase $\langle 0|\bar{\psi}\psi|0\rangle_M$ without increasing F_π. In a technicolor theory, this means that this mechanism will not substantially increase the weak-gauge-boson mass $M_W \equiv gF_\pi$.

We next summarize the behavior of $\Sigma(p)$ throughout the range $\mu \le p \le M$. At $p \simeq \mu$, the coupling α_μ will equal or exceed a critical value[6] α_c given by $3\alpha_c C_2(R)/\pi = 1$. If the condition $b c_\mu + c\alpha_\mu^2 \ll 1$ is satisfied (assuming convergence, $|c|\alpha_\mu^2 \ll b\alpha_\mu$), the coupling will evolve very slowly for a range of momenta above μ. By integrating the evolution equation for α and expanding about $q \simeq \mu$, one finds

$$\alpha(p) \simeq \alpha_\mu[1 - (b\alpha_\mu + c\alpha_\mu^2)\ln p/\mu + \cdots].$$

Thus $\alpha(p)$ will only drop appreciably below α_μ when $(b\alpha_\mu + c\alpha_\mu^2)\ln p/\mu \rightarrow 1$. For lower momenta, a reasonable first approximation is $\alpha(q) = \alpha_\mu$. With $\alpha_\mu \simeq \alpha_c$, the solution to Eq. (3) is approximately $\Sigma(p) \sim 1/p$, a much slower fall than $\Sigma_{\rm asy}$. The asymptotic form will only emerge in the limit $b\,\alpha_\mu \ln p/\mu \gg 1$ which may or may not be reached for $p \le M$. It is worth remarking that the form of $\Sigma_{\rm asy}$ can also be derived using the operator-product expansion. The coefficient is then proportional to $\langle 0|\bar{\psi}\psi|0\rangle_R$, where the subscript denotes definition by subtraction at some scale less than p. The full coefficient is, of course, independent of the subtraction point.

Quantitative estimates of fermion and PGB masses will be given for a specific SU(N) technicolor gauge theory. Other possible theories exhibiting a similar behavior are listed in Table I. For an SU(N) theory with fermions in the fundamental representation, the coefficients b and c are given by

$$b = (11N/3 - 2N_f/3)/2\pi,$$

$$c = [34N^2/3 - 10NN_f/3 - (N^2-1)N_f/N]/(8\pi^2),$$

where N_f is the number of fermions. As an example, the gauge group will be taken to be SU(4) and it will be assumed that there are $N_f = 14$ fermions in the fundamental representation of this group. Then $\alpha_c \simeq 0.56$, $b = 0.85$, $c = -0.73$.

It will turn out numerically that $\alpha_\mu \simeq \alpha_c$. Thus the convergence of the β-function expansion does not seem bad even at $q \simeq \mu$. The slowness of the running coupling is en-

TABLE I. Theories with a walking coupling. Conditions: (1) asymptotic freedom ($b > 0$), (2) reasonable convergence ($|c|\alpha/b \lesssim 0.6$) for $\alpha \le \alpha_c$, and (3) fermions in the fundamental representation.

Gauge group	N_f	α_c	b	($b_{\text{no fermions}}$)	c	$b\alpha_c + c\alpha_c^2$
SU(3)	10	0.79	0.69	(1.75)	−0.31	0.35
SU(4)	14	0.56	0.85	(2.33)	−0.73	0.25
SU(7)	24	0.31	1.33	(4.08)	−2.14	0.27
SO(7)	8	0.70	0.61	(1.45)	−0.25	0.30
SO(11)	16	0.42	0.72	(2.63)	−1.65	0.19
SO(15)	24	0.30	1.25	(3.79)	−2.65	0.14
SO(16)	24	0.28	1.53	(4.08)	−2.34	0.25
E(6)	6	0.72	0.53	(1.17)	−0.15	0.30

sured by the fact that

$$b\alpha_\mu + c\alpha_\mu^2 \simeq 0.47 - 0.22 = 0.25 \ .$$

To be more explicit about the model, the 14 technicolored fermions can be composed of a colored weak doublet (U,D), a colorless weak doublet (E,N), and two colored weak singlets P and Q. An interesting feature of this model is that the asymptotic freedom of QCD is lost above the technicolor scale. Below the technicolor scale, the b parameter for QCD is given by $b_{\rm QCD} = (1/2\pi) \times (11 - \frac{2}{3} \times 6) = 7/2\pi$, while above, $b_{\rm QCD} = -(11/6\pi)$. Although the coupling then begins to grow, it will still be well below its low-energy ($\simeq 1$ GeV) value at momentum scales on the order of the cutoff M, providing $M \lesssim 10^3$ TeV.

The masses of ordinary fermions arise from the assumed existence of effective four-fermion couplings. With the strength taken to be g_M^2/M^2, and the closed technicolor loop assumed to cut off at the same scale,

$$m_f \simeq \frac{g_M^2}{M^2} \langle 0 | \bar{\psi}\psi | 0 \rangle_M \ , \qquad (5)$$

where the high-momentum part of $\langle 0 | \bar{\psi}\psi | 0 \rangle_M$ is given by Eq. (2). To estimate the overall size of this quantity, we note that since there are four $SU(2)_W$ doublets, $F_\pi = 250$ GeV/2 = 125 GeV. Σ_0 can be estimated either by scaling up from QCD[7] or equivalently by applying Eq. (4), remembering that the integral is dominated by momenta $k \simeq \Sigma_0$. The result is $\Sigma_0 \simeq 350$ GeV. Then,

$$\langle 0 | \bar{\psi}\psi | 0 \rangle_M \simeq \frac{2(350 \text{ GeV})^3}{\pi^2} \int^{M/\Sigma_0} \chi d\chi \sigma(\chi) \ , \qquad (6)$$

where $\sigma = \Sigma(p)/\Sigma_0$ and $\chi = p/\Sigma_0$. To adequately suppress flavor-changing neutral currents, M must be at least 300 TeV.[8]

A rough estimate of the integral can be obtained by recalling that $\sigma(\chi)$ will behave approximately like $1/\chi$ until $\chi \rightarrow \exp[1/(b\alpha_\mu + c\alpha_\mu^2)]$. Thus the relevant chiral hierarchy in the theory can be expected to be

$$\langle 0 | \bar{\psi}\psi | 0 \rangle_M / (2\Sigma_0^3/\pi^2) = \exp[1/(b\alpha_\mu + c\alpha_\mu^2)] \ .$$

For the present model, this indicates a value of about 50 for the integral and, therefore, a fermion mass on the order of 200 MeV. For a more precise estimate, we have numerically determined $\Sigma(k)$ as a function of k and then numerically evaluated the integral in Eq. (6). The integral turns out to be approximately 200, and thus, with $g_M^2/4\pi^2 \simeq 1$, $m_f \simeq 800$ MeV.

The numerical analysis leading to this result begins with the gap equation [Eq. (3)], cut off in the ultraviolet at M. A chiral-symmetry-breaking scale Σ_0 is assumed to exist, and below $\mu \equiv 2\Sigma_0$, the running coupling will evolve without the retarding effect of the condensed fermions. A physical confinement scale Λ is assumed to exist at which $\alpha(q)$ reaches some value α_Λ above α_c. We have here taken α_Λ to be between 0.6 and 0.9. Below Λ, we have simply taken $\alpha(q)$ to be a constant in the gap equation. Out of this comes, first of all, a relation between Σ_0 and Λ. As expected, $\mu/\Lambda = O(1)$ for any α_Λ in the above range. The gap equation [Eq. (3)] is, of course, basically perturbative and cannot be expected to completely govern chiral-symmetry breaking. It is being used here as a qualitative guide to the relative order of magnitude of Σ_0 and Λ. The result $\Sigma_0/\Lambda = O(1)$ is expected since there is no small parameter at the low momentum scales. Our fundamental result, the enhancement of $\langle 0 | \bar{\psi}\psi | 0 \rangle_M$, comes, fortunately, only from the momentum regime $p > \mu$, where the gap equation is more reliable.

The masses of PGB's can be similarly estimated.[9] With $N_f = 14$, there will be 195 Goldstone bosons. Of these, there will remain 16 color singlets unabsorbed by the $W^\pm$ and Z^0. Although some of these will get electroweak masses expected to be on the order of a few GeV, only the effective four-fermion interactions can lift their masses beyond this range. If it is assumed that the separate chiral symmetries responsible for these Goldstone bosons are explicitly broken by effective four-technifermion interactions, masses will be generated. A standard application of chiral perturbation theory gives[10]

$$M_p^2 \simeq \frac{a^2}{M^2} \frac{g_M^2}{F_\pi^2} (\langle 0 | \bar{\psi}\psi | 0 \rangle_M)^2 \ , \qquad (7)$$

where a^2 is a coefficient of order unity. Using Eq. (5), the pseudomass can be written in terms of the fermion mass: $M_p = aMm_f/g_M F_\pi$. For the present model, this gives $M_p = a \times 300$ GeV.

The central conclusion of this Rapid Communication is that a slowly evolving coupling can considerably enhance the vacuum value of $\bar{\psi}\psi$ while keeping F_π essentially unaltered. For the technicolor theory considered in detail here, and for many of the theories in Table I, this can naturally produce fermion masses in the 100-MeV range and PGB masses in the 100-GeV range, while keeping the scale M large enough to adequately suppress FCNC's. While the raising of the fermion mass scale is probably welcome, it is still far from clear whether a realistic theory of fermion masses is possible in this context. A 100-GeV mass range for PGB masses puts them beyond current experimental lower limits but possibly within the reach of the next generation of accelerators.

A more systematic investigation of the effects of a slowly evolving coupling should be undertaken.[11] Another important question is the size of the next-order contribution to the gap equation. Even though the β-function expansion may converge and the running of the coupling might be neglected for a range of momenta, the coupling strength itself ($\sim C_2\alpha_\mu$) is not small. There could, therefore, be important higher-order contributions to the gap equation. The next-order term is formed by replacing the single-gluon contribution to the kernel by two crossed gluons. It is easy to see[12] that in SU(N) theories this contribution is down by two powers of $1/N$ in a $1/N$ expansion. While large N may indeed be relevant for technicolor theories, a slowly running coupling will not emerge in a simple $1/N$ expansion. The number of fermion flavors N_f must be simultaneously increased with N to keep b and c small. Whether the phenomena described in this paper can be studied in the context of some systematic expansion scheme is an interesting question.

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[1]T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[2]S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).

[3]The Landau gauge is employed and the theory is defined by the renormalization subtractions appropriate for the massless theory. It is perhaps more conventional to define $\langle 0 | \bar{\psi}\psi | 0 \rangle$ by subtraction at some scale close to Λ rather than M. This convention would seem less appropriate in the present context where $\langle 0 | \bar{\psi}\psi | 0 \rangle_M$ appears directly in physical mass formulas.

[4]R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973); H. Pagels and S. Stokar, *ibid.* **20**, 2947 (1979); R. Casalbuoni, in Proceedings of the 1985 International Symposium on Composite Models for Quarks and Leptons (unpublished).

[5]This observation has also been made in a recent report by J. M. Frere, University of Michigan Report No. 86-15, 1986 (unpublished).

[6]K. Higashijima, Phys. Rev. D **29**, 1228 (1984); P. Castorina and S. Y. Pi, *ibid.* **31**, 411 (1985); M. Peskin, in *Recent Advances in Field Theory and Statistical Mechanics,* Proceedings of the Les Houches Summer School Session XXXIX, edited by J. B. Zuber and R. Stora (North-Holland, Amsterdam, 1984).

[7]The scaling-up procedure is only approximately correct because of the slow running of the coupling in the technicolor theory. Since Σ_0 is not very sensitive to the high-momentum components, it should be fairly accurate. The condensate cannot, of course, be determined by scaling up from QCD.

[8]S. Dimopoulos, S. Raby, and K. Lane, Nucl. Phys. **B172**, 509 (1980); S. Dimopoulos and J. Ellis, *ibid.* **B182**, 505 (1981). It is assumed here that no Glashow-Iliopoulos-Maiani (GIM) mechanism or pseduo-GIM mechanism [M. A. B. Beg, Phys. Lett. **124B**, 403 (1983); **129B**, 113 (1983)] is operative.

[9]For a general discussion of pseudo-Goldstone-boson masses, see E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981). B. Holdom, Phys. Rev. D **24**, 1441 (1981), also discusses the raising of pseudo-Goldstone-boson masses in the context of a fixed-point scenario.

[10]The factorization of the matrix element $\langle 0 | \bar{\psi}\psi \; \bar{\psi}\psi | 0 \rangle$ is being assumed here. This result can be established for the high-momentum components of interest using the asymptotic freedom of the theory.

[11]See, for example, M. Peskin and B. Holdom, Nucl. Phys. **B208**, 397 (1982); M. Dine and N. Seiberg, City College of New York Report No. CCNY-HEP-86/2, 1986 (unpublished).

[12]We thank K. Lane for discussions on this point.