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Gauge theories at finite temperature and chemical potential

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The relativistic partition function is studied respecting the internal symmetries. We consider generally the SU(N) gauge theories, and, in particular, the SU(3) symmetry relating to the quark-gluon plasma. The implications of the complex chemical potentials are analyzed and discussed in relation to the lattice gauge theories. It is explicitly shown that the physical partition function is real, which can be obtained from a complex Euclidean partition function by imposing Gauss's law.

In recent times there has been much discussion of the relation of a constant potential in gauge theories to the thermodynamical properties at finite temperature and chemical potential. The calculation of an effective potential for the order parameter of gauge theories at finite temperature in the one-loop approximation demonstrates an unusual oscillating structure. This has been explicitly calculated[1] for the SU(N) symmetry. On the other side, the relation between Bose-Einstein condensation and spontaneous symmetry breaking in gauge theories[2] has brought attention to the role of the chemical potential. However, the introduction of a chemical potential μ in the Abelian gauge theory with a constant external potential has been the point of some recent controversy.[3,4] In this work we shall consider the problem of introducing the chemical potential for the color SU(3) symmetry in relation to the quark-gluon plasma.

This problem demands consideration of the dynamical aspects of the relativistic quantum gases[5] in the presence of internal symmetries.[6] Very recently a general approach involving a group-theoretical projection technique[6-8] has been presented for this problem of quantum statistics with internal symmetry. Here we apply this approach to a model of a colorless quark-gluon plasma for the calculation of the relativistic partition function with a finite quark chemical potential μ_q. The presence of μ_q introduces an imaginary part into the generating function in the presence of the vacuum gauge potentials. However, we demonstrate here for our model that the final partition function is a real quantity. This fact may have important implications for the lattice gauge calculations at finite temperature and density.

First we shall generally discuss the evaluation of the partition function $Z_Q(T,V)$ in relation to a globally conserved total charge Q. The general form of $Z_Q(T,V)$ relates to a trace over the charge states of the type

$$Z_Q(T,V)=\operatorname*{Tr}_{Q} e^{-\beta\hat{H}} , \tag{1}$$

where β is the inverse temperature T and $\hat{H}$ is the Hamiltonian operator. In terms of the occupation-number representation for the particles of momentum k and charge q each state has a single-particle energy ϵ_k with n_k particles and $\bar{n}_k$ antiparticles. Whereby we may represent this trace as

$$Z_Q(T,V)=\sum_{\{n_k,\bar{n}_k\}}\delta\left[Q-q\sum_k(n_k-\bar{n}_k)\right]\exp\left[-\beta\sum_k\epsilon_k(n_k-\bar{n}_k)\right] \tag{2a}$$

$$=\int_0^{2\pi}\frac{d\phi}{2\pi}e^{-iQ\phi}\sum_{\{n_k\}}\exp\left[-\sum_k(\beta\epsilon_k-iq\phi)n_k\right]\sum_{\{\bar{n}_k\}}\exp\left[-\sum_k(\beta\epsilon_k+iq\phi)\bar{n}_k\right] . \tag{2b}$$

Thus we see already that the constraint on the total charge Q can give rise to a representation in terms of complex quantities. At this point we introduce the quantum statistics ($\sigma=+1$ for Fermi-Dirac and $\sigma=-1$ for Bose-Einstein). The partition function can now be generally written as

$$Z_Q^{(\sigma)}(T,V)=\int_0^{2\pi}\frac{d\phi}{2\pi}e^{-iQ\phi}\tilde{Z}^{(\sigma)}(T,V,\phi) , \tag{3}$$

where $\tilde{Z}^{(\sigma)}(T,V,\phi)$ is the generating function for specific statistics denoted by σ and a given parameter (angle) ϕ. For an ideal quantum gas with a single type of particle and its antiparticle the result is simply

$$\ln\widetilde{Z}^{(\sigma)}(T,V,\phi)=\sigma\sum_k [\ln(1+\sigma e^{-\beta\epsilon_k+iq\phi}) + \ln(1+\sigma\epsilon^{-\beta\epsilon_k-iq\phi})] \ . \tag{4}$$

We shall generalize this known result to situations containing more charges as well as the additional properties from the gauge potentials and the chemical potentials through the use of the group-theoretical projection technique[6] in order to describe the internal symmetry of the quark-gluon plasma.

We now discuss the analysis for the color group SU(3) with chemical potentials μ_k for each quark flavor. The addition of the chemical potentials is readily accomplished in the usual way by replacing the Hamiltonian operator $\hat{H}$ in (1) by a modified Hamiltonian operator for the f quark flavors

$$\hat{H}(\mu_1,\ldots,\mu_f)=\hat{H}-\sum_{k=1}^{f}\mu_k\hat{q}_k \ , \tag{5}$$

where $\hat{q}_k$ are the external charge generators, which do not break the exact internal symmetry[7] of the SU(3) color.

The projection technique has been previously investigated[7,8] for arbitrary multiplets (p,q) of color SU(3). The fundamental orthogonality relationship for multiplets $(p,q),(p',q')$ with the associated group characters $\chi_{pq}(\varphi,\psi)$, $\chi_{p'q'}(\varphi,\psi)$ is given by

$$\int d\varphi\, d\psi\, M(\varphi,\psi)\chi_{pq}(\varphi,\psi)\chi_{p'q'}(\varphi,\psi)=\delta_{pp'}\delta_{qq'} \ , \tag{6}$$

where $M(\varphi,\psi)$ is the Haar measure for SU(3) and φ and ψ are the two integration (group) parameters.

At this point we look more closely at a model for a colorless quark-gluon plasma. The two commuting generators of the maximal Abelian subgroup of color SU(3) are the isospin $\hat{I}_3$ and the hypercharge $\hat{Y}_8$ operators. In order to project out the color-singlet partition function, we need the general SU(3) generating function relating to the baryon-number operator $\hat{A}$ (Refs. 9—11);

$$\widetilde{Z}(T,V,\mu,\varphi,\psi)=\mathrm{Tr}\exp[-\beta(\hat{H}_0-\mu\hat{A})+i\varphi\hat{I}_3 + i\psi\hat{Y}_8] \ , \tag{7a}$$

where μ is associated with the baryon-number conservation. The color-singlet partition function can then be obtained by

$$Z(T,V,\mu)=\int d\varphi\, d\psi\, M(\varphi,\psi)\widetilde{Z}(T,V,\mu,\varphi,\psi) \ . \tag{7b}$$

If we now explicitly consider the particular SU(3) representations relating to the noninteracting quarks (1,0), antiquarks (0,1), and gluons (1,1), then $\widetilde{Z}$ can be written as a simple product:

$$\widetilde{Z}=\prod_{i=r,g,b}\widetilde{Z}_Q^{(i)}\prod_{j=\lambda,\mu,\nu,\delta}\widetilde{Z}_G^{(j)} \ . \tag{8}$$

We write out formally the different contributions:

(i) gluon,

$$\widetilde{Z}_G^{(j)}(T,V,\alpha_j)=\sum_{\{n_k\}}\exp\left[\sum_k(-\beta\epsilon'_k+i\alpha_j)n_k\right] \times\sum_{\{\bar{n}_k\}}\exp\left[\sum_k(-\beta\epsilon'_k-i\alpha_j)\bar{n}_k\right] \ ; \tag{9a}$$

(ii) quark,

$$\widetilde{Z}_Q^{(i)}(T,V,\alpha_i)=\sum_{\{n_k\}}\exp\left[\sum_k[-\beta(\epsilon_k-\mu_q)+i\alpha_i]n_k\right] \times\sum_{\{\bar{n}_k\}}\exp\left[\sum_k[-\beta(\epsilon_k+\mu_q) - i\alpha_i]\bar{n}_k\right] , \tag{9b}$$

where the quark chemical potential μ_q is one-third of that for the baryons. The parameters α_i are for the different quark colors, while the α_j are for the different gluons, which may be related to the angels φ and ψ in (7a) in the following way:

$$\alpha_r=\tfrac{1}{2}\varphi+\tfrac{1}{3}\psi,\quad \alpha_g=-\tfrac{1}{2}\varphi+\tfrac{1}{3}\psi,\quad \alpha_b=-\tfrac{2}{3}\psi \ , \tag{10a}$$

$$\alpha_\lambda=\varphi,\quad \alpha_\mu=\tfrac{1}{2}\varphi+\psi,\quad \alpha_\nu=-\tfrac{1}{2}\varphi+\psi,\quad \alpha_\delta=0 \ . \tag{10b}$$

The expressions ϵ_k and ϵ'_k are the quark and gluon single-particle energies. Thus for both the gluons and the quarks we are able to evaluate the generating function of (9a) and (9b) in terms of a sum over the known single-particle partition functions Z' for the relativistic quantum gases,[5] which yields

$$\widetilde{Z}_G^{(j)}(T,V,\alpha_j)=\exp\left[2\sum_{n=1}^{\infty}\frac{1}{n}Z'_G(n\beta,V)\cos n\alpha_j\right] , \tag{11a}$$

$$\widetilde{Z}_Q^{(i)}(T,V,\alpha_i)=\exp\left[2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}Z'_Q(n\beta,V) \times\cos n(\alpha_i-i\beta\mu_q)\right] . \tag{11b}$$

We note here that these results may be readily compared to the Abelian case[4] with a single constant "vacuum parameter" α. For the massless gluons the usual Stefan-Boltzmann temperature dependence is found for Z'_G, whereupon (11a) simply becomes

$$\widetilde{Z}_G^{(j)}(T,V,\alpha_j)=\exp\left[\frac{4VT^3}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^4}\cos n\alpha_j\right] . \tag{12}$$

The above series may be summed to give a known result.[9] Furthermore, for the massive quarks the analysis of (11b) leads to somewhat more lengthy expressions. The explicit evaluation of the indicated integrals over color SU(3) will be considered in a following work[12] for the more general cases. However, if we now consider only the special case of the light quarks, the analytical results may be obtained[9] by use of the single-particle partition function as for the gluons, however, by suitably shifting the angular variable. We summarize this special case of (11a) and (11b) for the sake of later discussions as follows:

$$\ln\widetilde{Z}_G(T,V,\alpha_j)=\frac{\pi^2}{6}VT^3\sum_j\left[-\frac{7}{30}+\left[\frac{\alpha_j}{\pi}-1\right]^2-\frac{1}{2}\left[\frac{\alpha_j}{\pi}-1\right]^4\right], \tag{13a}$$

$$\ln\widetilde{Z}_Q(T,V,\mu_q,\alpha_i)=\frac{\pi^2}{3}VT^3\sum_i\left[\tfrac{7}{30}-(\alpha_i-i\beta\mu_q)^2\pi^{-2}+\tfrac{1}{2}(\alpha_i-i\beta\mu_q)^4\pi^{-4}\right]. \tag{13b}$$

Now we want to discuss the implications of these results in relation to gauge theories at finite-temperature and chemical potentials. It has been previously established for Abelian gauge as well as nongauge theories[4] that a constant potential may appear as an imaginary chemical potential in the partition function. The SU(3) case exhibits a similar behavior. We start from the usual QCD Lagrangian in the *Minkowski* metric where the gluon fields are considered to contain a classical external part[9] given by the constant real potentials A_3^{ext} and A_8^{ext} which are related to the diagonal generators of SU(3). These terms contribute to the general potential A_a^μ by replacing

$$A_a^\mu \to A_a^\mu+\delta_{a3}A_3^{\mu\,\text{ext}}+\delta_{a8}A_8^{\mu\,\text{ext}}. \tag{14}$$

After artificially turning off all interactions of the gluons except with the external potential, it has been shown by canonical quantization in the Hamiltonian formalism that the resulting Hamiltonian for the gluons is simply given by

$$\hat{H}_G=\hat{H}_{0G}+gA_3^{0\,\text{ext}}\hat{I}_3+gA_8^{0\,\text{ext}}\hat{Y}_8, \tag{15}$$

which can be expected for charged particles interacting with a constant external potential. Thus, if one *chooses*

$$gA_3^{0\,\text{ext}}=-i\varphi/\beta, \tag{16a}$$

$$gA_8^{0\,\text{ext}}=-i\psi/\beta, \tag{16b}$$

which amounts to rotating $A^{0\,\text{ext}}$ to purely imaginary values, then the total Hamiltonian operator for the gluons in the external potential becomes

$$\hat{H}_G=\hat{H}_{0G}-i\frac{\varphi}{\beta}\hat{I}_3-i\frac{\psi}{\beta}\hat{Y}_8, \tag{17}$$

which corresponds to the effective Hamiltonian to be used in the generating function (7a). A corresponding result holds for the quarks.[9]

We now can clearly see that the imaginary vacuum gauge potentials A_3^{ext} and A_8^{ext} lead automatically to a complex structure of the Euclidean partition function Z_E of (7a) in the presence of quarks with a chemical potential μ. Thus from Eqs. (14)—(17) we may write

$$Z_E(T,V,\mu,A_3^{\text{ext}},A_8^{\text{ext}})=\widetilde{Z}(T,V,\mu,i\beta gA_3^{0\,\text{ext}},i\beta gA_8^{0\,\text{ext}})=\widetilde{Z}(T,V,\mu,\varphi,\psi), \tag{18}$$

which is (7a). This fact has been considered as a major problem for the calculations on a lattice,[13] because the fermion determinant with a nonzero chemical potential is generally not real.[14]

Next we want to establish that in spite of the complex generating function the final color-singlet partition function is real. This fact has mainly to do with the quark-antiquark symmetry inherent to the colorless partition function.[9] The symmetry between quarks and antiquarks immediately imposes the condition on the partition function:

$$Z(T,V,-\mu_q)=Z(T,V,\mu_q), \tag{19}$$

which in view of the above equations (7a)—(13b) is necessary and sufficient for Z to be real. However, we may show this in more detail as follows. From (10b) and (12) we see that $\widetilde{Z}_G^{(j)}(T,V,\alpha_j)$ is invariant under the reflection $\varphi,\psi\to-\varphi,-\psi$. Thus we use (8) together with (7b), which is the relevant generalization of (3) for SU(3), to find $Z(T,V,\mu_q)$ in terms of the product of $\widetilde{Z}_G$ and $\widetilde{Z}_Q$. Then by performing the integration over the group SU(3) we find

$$Z^*(T,V,\mu_q)=\int dM(\varphi,\psi)\prod_{i,j}\widetilde{Z}_Q^{(i)*}(\varphi,\psi)\widetilde{Z}_G^{(j)}(\varphi,\psi) \tag{20a}$$

$$=\int dM(\varphi,\psi)\prod_{i,j}\widetilde{Z}_Q^{(i)}(-\varphi,-\psi)\widetilde{Z}_G^{(j)}(\varphi,\psi) \tag{20b}$$

$$=\int dM(-\varphi,-\psi)\prod_{i,j}\widetilde{Z}_Q^{(i)}(-\varphi,-\psi)\times\widetilde{Z}_G^{(j)}(-\varphi,-\psi). \tag{20c}$$

However, (20c) is just a representation for $Z(T,V,\mu_q)$, where in (20b) we have used (13b) and (10a). Therefore, we can conclude that $Z^*(T,V,\mu_q)=Z(T,V,\mu_q)$, which proves that $Z(T,V,\mu_q)$ is *real*.

The discussion which we presented in the above paragraphs can be naturally extended to arbitrary SU(N)-color internal-symmetry group. In this case the general expression for the generating function of the gluons and massive quarks can be obtained[6] as follows:

$$\ln\widetilde{Z}_Q(V,T,\mu_q,\gamma)=g_Q\frac{m^2VT}{2\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}K_2(n\beta m)\times\frac{1}{d_Q}\left[e^{n\beta\mu_q}\chi^Q(\gamma n)+e^{-n\beta\mu_q}\bar{\chi}^Q(\gamma n)\right], \tag{21a}$$

$$\ln\widetilde{Z}_G(V,T,\gamma)=g_G\frac{VT^3}{d_G\pi^2}\sum_n\frac{1}{n^4}\chi^G(n\gamma), \tag{21b}$$

where g_Q, g_G are, respectively, the quark and gluon degeneracy factors and m the quark mass. $\gamma\equiv(\gamma_1,\ldots,\gamma_r)$, where γ_i are the parameters (angles) and r the rank of the SU(N) group. The characters $\chi^Q(\gamma)$ and $\chi^G(\gamma)$ and dimensions d_Q and d_G are those of the fundamental (quark) and the adjoint (gluon) representations of the SU(N) group, where $\bar{\chi}^Q(\gamma)=[\chi^Q(\gamma)]^*$. From the above generating functions one can find the colorless partition function for SU(N) internal symmetry by means of integration over the group

$$Z(T,V,\mu_q)=\int d\gamma\, M(\gamma)\widetilde{Z}_Q(T,V,\mu_q,\gamma)\times\widetilde{Z}_G(T,V,\gamma)\ , \tag{22}$$

where $M(\gamma)$ now indicates the invariant measure on the group. Furthermore, it is easy to check for the particular structure of SU(3) that (21a), (21b) and (22) lead for massless quarks to the results of (11), (12), and (13a) and (13b).

It is well established that in the finite-temperature gauge theory the zero component of the gauge field A_0 takes on the role of the Lagrange multiplier which ensures that all the states satisfy Gauss's law. Because of the periodic boundary condition in Euclidean space one cannot eliminate A_0 by setting it equal to zero. Nevertheless, one can choose a gauge in such a way that $A_0^a(\mathbf{x},\tau)\lambda_a$ is a constant in Euclidean space-time, so that

$$A_0^{ab}=g^{-1}\alpha^{ab}\delta_{ab}\ . \tag{23}$$

In this case the obtained[1] effective potential V_{eff} for the order parameter (Wilson loop) of the SU(N) gauge group is essentially the same as the generating functions given in (21a) and (21b). Actually, of course, we have

$$V_{\text{eff}}=\frac{T}{V}(\ln\widetilde{Z}_Q+\ln\widetilde{Z}_G)\ . \tag{24}$$

It is also interesting to note in the above chosen gauge (23) that the Wilson loop defined as

$$L(x)=\frac{1}{N}\mathrm{Tr}P\exp\left[ig\int_0^\beta A_0(\mathbf{x},\tau)d\tau\right] \tag{25}$$

represents the character of the fundamental representation of the SU(N) group. For SU(2) the characters for an arbitrary J have the particularly simple form

$$\chi^{(J)}(\varphi)=\sin[(2J+1)/\varphi/2]/(\sin\varphi/2)\ . \tag{26}$$

This gives for the fundamental representation $J=\frac{1}{2}$ the form $2\cos(\varphi/2)$. Thus if one chooses $\varphi=\beta\alpha$, it can be immediately seen from Eqs. (23) and (25) that the Wilson loop $L(x)$ is simply $\frac{1}{2}\chi^{(1/2)}(\varphi/2)$.

With the identification of the Wilson loop as the character of the fundamental representation of the SU(N) group, one can write the generating function for the massive quarks as follows:

$$\ln\widetilde{Z}_Q=g_Q\frac{Vm^2T}{2\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\times[(\cosh n\beta\mu_q)\mathrm{Re}L+i(\sinh n\beta\mu_q)\mathrm{Im}L]K_2(n\beta m)\ . \tag{27}$$

The above equation with the assumed Boltzmann statistics ($n=1$) has the same structure as the fermion contribution to the partition function in the hopping-parameter expansion on the lattice.[15] In this case one has

$$\ln\det(1-KM)\simeq 4N_f(2\kappa)^{N_\beta}\times\sum_{\mathbf{x}}[(\cosh N_\beta\mu a)\mathrm{Re}L(\mathbf{x})+i(\sinh N_\beta\mu a)\mathrm{Im}L(\mathbf{x})]\ , \tag{28}$$

where N_f is the number of flavors, κ the hopping parameter, and a the lattice spacing.

We have indicated the above rather formal analogy between lattice gauge theory and the model under consideration in order to show that the problem with the complex determinant on the lattice is not due to the lattice structure but has a rather general nature. It appears when we deal with a nonzero chemical potential for a non-Abelian gauge theory. Nevertheless, as we have indicated in the previous paragraphs, the physical partition function is real even when the fermion contribution is expressed in complex quantities. This situation also holds for the lattice gauge theory.[15]

The exceptional situation appears when the gauge group is SU(2). Then the character of the fundamental representation of SU(2) as well as the Wilson loop are real. This, in turn, implies that the imaginary parts of the generating function in (21a) and in the fermion determinant (28) have vanished.

In conclusion we state that the presence of a chemical potential together with a constant imaginary gauge potential leads generally for SU(N) to a complex form of the Euclidean partition function. We have shown above in (18) that the Euclidean partition function is identical with the generating function for the projection onto a color-singlet partition function for the quark-gluon plasma. Furthermore, the imposition of the conservation laws expressed in terms of the internal symmetries yields a real partition function. In view of our results expressed in (16)–(18) we may state that a real partition function can be obtained unambiguously from the complex Euclidean partition function (including constant external gauge and chemical potentials) only by integrating over the external gauge potentials. This group-invariant integration effectively imposes the constraint by Gauss's law for color-singlet states only on the many-particle Hilbert space under consideration.

Two publications have appeared since this work was first written, which offer some further ideas for issues raised here. A recent study of the strong-coupling limit of SU(2) symmetry at finite baryon density using Monte Carlo simulation and mean-field methods[16] discusses the problem of a finite chemical potential in relation to the chiral phase transition. A further work on Abelian nongauge theories[17] provides another approach to this problem using the ζ-function regularization of high-temperature expansions.

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