Vacuum energy and dilaton tadpole for the unoriented closed bosonic string

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(Received 25 August 1986)

In oriented-closed-bosonic-string theory a dilaton tadpole (and vacuum energy) first develops at the one-loop level from performing the path integration over manifolds with the topology of the torus. We show that in the case of the unoriented closed bosonic string the leading contribution to the dilaton tadpole arises at the tree level from the path integral over manifolds with the topology of the projective plane. We explicitly compute the vacuum energy and the dilaton tadpole using Polyakov's formulation of string theory.

Polyakov's[1] path-integral formulation for string theory provides a convenient method for computing string-theory S-matrix elements. Recently considerable progress has been made in understanding the appropriate measure for the integration over metrics.[2–5] Polchinski[6] has used the path-integral formulation to compute the leading contribution to the vacuum energy for the closed oriented bosonic string. It arises from performing the functional integration over manifolds with the topology of the torus. Rohm[7] has also computed the one-loop vacuum energy using the operator formalism. In this paper we note that in the case of closed-unoriented-bosonic-string theory the leading contribution to the vacuum energy occurs at the tree level from performing the functional integration over manifolds with the topology of the projective plane (i.e., the sphere with opposite points identified). The vacuum energy and the resulting dilaton tadpole are explicitly computed.

The path integral for closed-unoriented-bosonic-string theory[8] contains a sum over both orientable and unorientable two-manifolds. For our calculation only the contributions from manifolds with the topology of S_2 and P_2 need to be considered. Then the partition function is

$$Z \simeq \sum_{S_2,P_2} \frac{(\det' P^\dagger P)^{1/2}}{\tilde{d} V_{\rm CKV}} \int [dx] \exp[-(S+S_{\rm ct})] \;. \qquad (1)$$

In Eq. (1) P is a differential operator that maps vectors v_d into second-rank tensors,

$$(Pv)_{ab} = (\delta_a^c \delta_b^d + \delta_a^d \delta_b^c - g_{ab} g^{cd}) \nabla_c v_d \;, \qquad (2)$$

and det′ denotes the determinant excluding the zero modes. The factor $V_{\rm CKV}$ in the denominator of Eq. (1) is the volume of the group generated by the conformal Killing vectors, $\tilde{d}$ is the order of the group[6] $\tilde{D}$ of diffeomorphism classes. The elements of $\tilde{D}$ represent the connected components of the group of conformal diffeomorphisms, including those which do not preserve orientation. For the sphere $\tilde{d}=2$ while for the projective plane $\tilde{d}=1$. The action is

$$S = \tfrac{1}{2} \int d^2\xi \sqrt{g}\, g^{ab} \frac{\partial}{\partial \xi^a} x^\mu(\xi) \frac{\partial}{\partial \xi^b} x_\mu(\xi) \qquad (3a)$$

and the local counterterms are

$$S_{\rm ct} = \mu^2 \int d^2\xi \sqrt{g} + \frac{\ln\lambda}{4\pi} \int d^2\xi \sqrt{g} R \;. \qquad (3b)$$

The constant μ^2 is chosen so that conformal invariance is preserved. The second term in (3b) gives rise in Z to a factor of λ^{-2} for S_2 and a factor of λ^{-1} for P_2 since

$$\frac{1}{4\pi} \int d^2\xi \sqrt{g} R$$

is the Euler number of the manifold.

Connected parts of S-matrix elements are computed by inserting the appropriate vertex operators into the path integral. Conditions that the vertex operators must satisfy in the oriented-string case have been given by Weinberg.[9] To these we must add the condition that the vertex operators be definable on P_2. This condition forbids the vertex operator for the antisymmetric tensor field, for example.[10] Therefore in the unoriented bosonic string the states with mass squares less than or equal to zero are the tachyon, dilaton, and graviton.

The leading contribution to tachyon scattering amplitudes comes from the contribution of S_2 to Z. To set our conventions we briefly review the calculation of the sphere's contribution to tachyon scattering amplitudes. The vertex operator for the tachyon is

$$V_T(p) = \kappa_T \int d^2\xi \sqrt{g}\, e^{ip\cdot x(\xi)} \;. \qquad (4)$$

The contribution of S_2 to the n-point tachyon scattering amplitude is given by

$$A_n(p_1,\ldots,p_n) = \frac{(\det' P^\dagger P)^{1/2}}{2V_{\rm CKV}} \times \int [dx] e^{-(S+S_{\rm ct})} V_T(p_1) \cdots V_T(p_n) \;. \qquad (5)$$

We perform the functional integration over x^μ by expanding in normalized eigenfunctions of the Laplacian on the unit sphere:

$$x^\mu = \sum_l \sum_{m=-l}^{l} c^\mu_{lm} Y_{lm}(\xi) , \tag{6}$$

using

$$[dx] = \prod_{\mu,l,m} \frac{dc^\mu_{lm}}{\sqrt{2\pi}} . \tag{7}$$

There is arbitrariness in the choice of measure that cancels out in physical quantities.[3,6,11] Since the action is quadratic in the variable x^μ the functional integral over $[dx]$ can be evaluated yielding

$$A_n(p_1, \ldots, p_n) = (2\pi)^{26}\delta(p_1 + \cdots + p_n) a_n , \tag{8a}$$

where

$$a_n = \frac{2^{13}\kappa_T{}^n \lambda^{-2}(\det' P^\dagger P)^{1/2}[\det'(-\nabla^2)]^{-13}}{2V_{\text{CKV}}} \times \int \left[\prod_{i=1}^{n} d^2\xi_i \sqrt{g(\xi_i)} \right] \times \exp\left[-\tfrac{1}{2} \sum_{i,j} p_i^\mu p_j^\nu \langle x_\mu(\xi_i) x_\nu(\xi_j) \rangle \right] . \tag{8b}$$

In Eq. (8b) $\langle x_\mu(\xi_i) x_\nu(\xi_j)\rangle$ is the propagator excluding the zero modes

$$-\nabla_{\xi_i}{}^2 \langle x_\mu(\xi_i) x_\nu(\xi_j) \rangle = \delta_{\mu\nu} \left[\frac{\delta^2(\xi_i - \xi_j)}{\sqrt{g}} - \frac{1}{4\pi} \right] . \tag{9}$$

It is convenient at this point to map the sphere onto the complex plane by stereographic projection onto a plane cutting through the sphere's equator. The northern hemisphere is mapped onto the region $|z| > 1$ and the southern hemisphere is mapped onto the region $|z| < 1$. In complex coordinates the propagator is

$$\langle x_\mu(z_1) x_\nu(z_2) \rangle = -\frac{\delta_{\mu\nu}}{4\pi} \left[\ln \left[\frac{|z_1 - z_2|^2 + \frac{1}{2}\epsilon^2 \exp\left[-\phi \left[\frac{z_1 + z_2}{2} \right] \right]}{(1 + |z_1|^2)(1 + |z_2|^2)} \right] + 1 \right] . \tag{10}$$

Here ϵ^2 is an invariant short-distance cutoff and we have written the metric as $g_{z\bar{z}} = e^\phi$. Dependence on the choice of conformal gauge cancels if the tachyons are on the mass shell:

$$p_i{}^2 = 8\pi, \quad i = 1, \ldots, n .$$

Then we find that

$$a_n(p_1, \ldots, p_n) = \frac{2^{12}\lambda^{-2}(\kappa_T \epsilon^2)^n (\det' P^\dagger P)^{1/2}[\det'(-\nabla^2)]^{-13}}{V_{\text{CKV}}} \int d^2 z_1 \cdots \int d^2 z_n \exp\left[\frac{1}{4\pi} \sum_{i<j} p_i \cdot p_j \ln |z_i - z_j|^2 \right] . \tag{11}$$

The integration over three of the complex z's is equivalent to an integration over the invariant measure for the group generated by the conformal killing vectors. This is the group SL(2,C) of transformations

$$z' = \frac{Az + B}{Cz + D}, \quad AD - BC = 1 . \tag{12}$$

The integrand in Eq. (11) is a scalar density with respect to these transformations. For infinitesimal transformations

$$A = 1 + \epsilon_1/2, \quad B = \epsilon_0 , \quad C = -\epsilon_2, \quad D = 1 - \epsilon_1/2 , \tag{13}$$

Eq. (12) becomes

$$\delta z = \epsilon_0 + \epsilon_1 z + \epsilon_2 z^2 . \tag{14}$$

The infinitesimal diffeomorphisms generated by the conformal Killing vectors v_j^a take the form $\delta\xi^a = \sum_j \alpha_j v_j^a$. The volume of the group generated by the conformal Killing vectors V_{CKV} is defined to be the integral over the invariant group volume normalized so that when the v_j^a are chosen orthonormal it reduces to $\prod_j d\alpha_j$ near the identity. Using the metric

$$|v|^2 = \int d^2\xi \sqrt{g} g_{ab} v^a v^b \tag{15}$$

we find that the six properly normalized conformal Killing vectors, using the notation

$$\begin{bmatrix} v^z \\ v^{\bar{z}} \end{bmatrix} ,$$

are

$$\left[\frac{3}{8\pi}\right]^{1/2} \begin{bmatrix} z \\ \bar{z} \end{bmatrix}, \quad \left[\frac{3}{8\pi}\right]^{1/2} \begin{bmatrix} z \\ -\bar{z} \end{bmatrix},$$
$$\left[\frac{3}{16\pi}\right]^{1/2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad i\left[\frac{3}{16\pi}\right]^{1/2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \tag{16}$$
$$\left[\frac{3}{16\pi}\right]^{1/2} \begin{bmatrix} z^2 \\ \bar{z}^2 \end{bmatrix}, \quad i\left[\frac{3}{16\pi}\right]^{1/2} \begin{bmatrix} z^2 \\ -\bar{z}^2 \end{bmatrix} .$$

It follows that the invariant group measure near the identity is

$$dg = 4\left[\frac{8\pi}{3}\right]^3 d^2\epsilon_0 d^2\epsilon_1 d^2\epsilon_2 \,. \tag{17}$$

Trading the integrations over z_1, z_2, and z_3 for an integration over dg (which cancels the factor $V_{\rm CKV}$ in the denominator) we find that

$$a_n(p_1,\ldots,p_n)=\lambda^{-2}(\kappa_T\epsilon^2)^n Q_S \times \int d^2z_4\cdots\int d^2z_n\,|z_1-z_2|^2|z_2-z_3|^2|z_3-z_1|^2\exp\left[\frac{1}{4\pi}\sum_{i<j}p_i\cdot p_j\ln|z_i-z_j|^2\right], \tag{18}$$

where

$$Q_S=2^{13}\left[\frac{3}{16\pi}\right]^3(\det{}'P^\dagger P)^{1/2}[\det{}'(-\nabla^2)]^{-13} \tag{19}$$

and the values of z_1, z_2, and z_3 are arbitrary. In particular the three- and four-point tachyon scattering amplitudes are

$$a_3(p_1,p_2,p_3)=\lambda^{-2}(\kappa_T\epsilon^2)^3Q_S \,, \tag{20a}$$

$$a_4(p_1,p_2,p_3,p_4)=\pi\lambda^{-2}(\kappa_T\epsilon^2)^4Q_S\frac{\Gamma(3-(t+s)/8\pi)\Gamma(t/8\pi-1)\Gamma(s/8\pi-1)}{\Gamma((s+t)/8\pi-2)\Gamma(2-t/8\pi)\Gamma(2-s/8\pi)}\,, \tag{20b}$$

where $s=(p_1+p_2)^2$ and $t=(p_1+p_3)^2$.

The four-point amplitude has a tachyon pole

$$a_4\to\frac{8\pi^2\lambda^{-2}(\kappa_T\epsilon^2)^4Q_S}{s-8\pi}\quad\text{as } s\to 8\pi \tag{21}$$

which implies by factorization that

$$\lambda=|a_3|\,Q_S{}^{1/2}(8\pi^2)^{-3/2}\,. \tag{22}$$

The dilaton vertex operator is

$$V_D(p)=\kappa_D\int d^2\xi\sqrt{g(\xi)}\left[e^{\mu\nu}(p)g^{ab}\frac{\partial}{\partial\xi^a}x_\mu\frac{\partial}{\partial\xi^b}x_\nu+\frac{\sqrt{24}}{16\pi}R-\frac{\sqrt{24}}{\pi\epsilon^2}\right]e^{ip\cdot x}\,, \tag{23}$$

where[12]

$$e^{\mu\nu}(p)=\frac{1}{\sqrt{24}}(\delta^{\mu\nu}-p^\mu\bar p^{\,\nu}-p^\nu\bar p^{\,\mu}) \tag{24}$$

and $p\cdot\bar p=1$, $p^2=\bar p^{\,2}=0$. The counterterms in V_D are needed to preserve conformal invariance in scattering amplitudes involving the dilaton. The normalization of κ_D is determined by computing the dilaton-tachyon-tachyon scattering amplitude and comparing it with the dilaton pole in Eq. (20b). We find that

$$(\kappa_D\lambda^{-1})^2=(8\pi^2)^2Q_S{}^{-1}\,. \tag{25}$$

To compute the contribution of the manifold P_2 to scattering amplitudes it is necessary to expand x^μ in normalized eigenfunctions of the Laplacian on P_2, the unit sphere with antipodal points identified. Since x^μ is a scalar field, it must take the same value at antipodal points, and for P_2 we expand

$$x^\mu=\sqrt{2}\sum_{l\ \rm even}\sum_{m=-l}^{l}c^\mu_{lm}Y_{lm}(\xi) \tag{26}$$

and use the measure in Eq. (7) for the functional integral over x^μ. The factor of $\sqrt{2}$ in Eq. (26) occurs because the $Y_{lm}(\xi)$ have normalization one-half on P_2.

A vector field v^a on S_2 is defined on P_2 if each of its integral curves is mapped into itself by the identification of antipodal points. Suppose in some coordinate system points p on the unit sphere have coordinates ξ^a and their antipodal points $-p$ have coordinates $g^a(\xi)$. Then the condition for a vector field $v^a(\xi)$ to be well defined on P_2 is

$$v^b(g(\xi))=D^b_a(\xi)v^a(\xi)\,, \tag{27}$$

where the matrix $D^b_a(\xi)$ is given by

$$D^b_a(\xi)=\frac{\partial}{\partial\xi^a}g^b(\xi)\,. \tag{28}$$

For the complex coordinates z the map g takes z into $-1/\bar z$ and so

$$D=\begin{bmatrix}0 & 1/\bar z^2\\ 1/z^2 & 0\end{bmatrix}.$$

In these coordinates a vector field is defined on P_2 if

$$\frac{1}{z^2}v^z(p)=v^{\bar z}(-p)\,. \tag{29}$$

The group generated by the conformal Killing vectors is not the same on P_2 as on S_2. Only those transformations in (12) which satisfy $(-1/\bar z')=(-1/\bar z)'$ are defined on P_2. This restricts the transformations (12) to those that satisfy

$$A=\bar D,\quad C=-\bar B,\quad |A|^2+|B|^2=1\,. \tag{30}$$

Hence on P_2 the group generated by the conformal Killing vectors is SO(3) (Ref. 13). This corresponds to the group of rotations on the sphere which clearly leave antipodal points identified. Writing $A=(1-|B|^2)^{1/2}e^{i\alpha}$ the invariant group measure is proportional to $d^2B\,d\alpha$.

Near the identity the transformations (30) are

$$\delta z = B^1(1+z^2)+iB^2(1-z^2)+2i\alpha z \ , \tag{31}$$

where $B=B^1+iB^2$. The normalized conformal Killing vectors are

$$\left[\frac{3}{16\pi}\right]^{1/2}\begin{bmatrix}1+z^2\\1+\bar{z}^2\end{bmatrix},\quad i\left[\frac{3}{16\pi}\right]^{1/2}\begin{bmatrix}1-z^2\\-1+\bar{z}^2\end{bmatrix},\quad i\left[\frac{3}{4\pi}\right]^{1/2}\begin{bmatrix}z\\-\bar{z}\end{bmatrix} \tag{32}$$

and so the group volume is

$$V_{\rm CKV}=\left[\frac{16\pi}{3}\right]^{3/2}\int_{\text{unit disk}}d^2B\int_0^{\pi}d\alpha = \left[\frac{16\pi}{3}\right]^{3/2}\pi^2 \ . \tag{33}$$

The finite volume for SO(3) gives rise to a qualitative difference in the contribution to amplitudes from S_2 and P_2. The manifold P_2 will give a nonzero contribution to the vacuum energy (the zero-point amplitude) and to the dilaton one-point function.

According to Eq. (1) the contribution of P_2 to the vacuum energy density is

$$\mathscr{E}_{\rm vac}=-\frac{1}{\pi^2}\left[\frac{3}{16\pi}\right]^{3/2}\times\lambda^{-1}(\det{}'P^{\dagger}P)^{1/2}[\det{}'(-\nabla^2)]^{-13} \ . \tag{34}$$

Since the determinants and λ are positive the vacuum energy is negative. Using Eqs. (19) and (22) the dependence on λ can be replaced by dependence on the physical three tachyon scattering amplitude a_3. This gives

$$\mathscr{E}_{\rm vac}=-\frac{1}{|a_3|}\left[\frac{\pi}{4}\right]\left[\frac{(\det{}'P^{\dagger}P)_{P_2}}{[(\det{}'P^{\dagger}P)_{S_2}]^{1/2}}\right]^{1/2}\times\left[\frac{[\det{}'(-\nabla^2)]_{P_2}}{\{[\det{}'(-\nabla^2)]_{S_2}\}^{1/2}}\right]^{-13} \tag{35}$$

where the subscripts have been placed on the determinants to signify what manifold the operator is defined on.

To compute the ratio of determinants of $P^{\dagger}P$ we note

$$(P^{\dagger}Pv)_b=-2\nabla^a\nabla_a v_b-2v_b \ . \tag{36}$$

It follows that on S_2 the eigenvectors of $P^{\dagger}P$ are

$$v^a=g^{ab}\frac{\partial}{\partial\xi^b}Y_{lm},\quad v^a=\frac{1}{\sqrt{g}}\epsilon^{ab}\frac{\partial}{\partial\xi^b}Y_{lm} \tag{37}$$

with eigenvalues $2(l-1)(l+2)$. In Eq. (37) ϵ^{ab} is the antisymmetric symbol. For P_2 only vector fields which satisfy $v^{\theta}(p)=-v^{\theta}(-p)$ and $v^{\phi}(p)=v^{\phi}(-p)$ are admissible. Hence for odd l only

$$v^a=\frac{1}{\sqrt{g}}\epsilon^{ab}\frac{\partial}{\partial\xi^b}Y_{lm}$$

is allowed and for even l only

$$v^a=g^{ab}\frac{\partial}{\partial\xi^b}Y_{lm}$$

is allowed. On P_2 every eigenvalue of $P^{\dagger}P$ that occurs on S_2 is present but its multiplicity $(2l+1)$ is only half what it is on S_2. Therefore

$$(\det{}'P^{\dagger}P)_{P_2}=[(\det{}'P^{\dagger}P)_{S_2}]^{1/2} \ . \tag{38}$$

Next we consider the ratio

$$r=\frac{[\det{}'(-\nabla^2)]_{P_2}}{\{[\det{}'(-\nabla^2)]_{S_2}\}^{1/2}}=\left[\frac{\prod_{l\ \text{even}}[l(l+1)]^{2l+1}}{\prod_{l\ \text{odd}}[l(l+1)]^{2l+1}}\right]^{1/2}=\frac{2\times4\times6\times8\times\cdots}{1\times3\times5\times7\times\cdots} \ . \tag{39}$$

It is straightforward to relate[14] the logarithm of r to the Riemann ζ function $\zeta_R(s)$ and its first derivative evaluated at $s=0$:

$$\ln r=\sum_{k=1}^{\infty}(-1)^k\ln k=-\frac{d}{ds}\sum_{k=1}^{\infty}(-1)^k k^{-s}\Big|_{s=0}=\frac{d}{ds}[(1-2^{1-s})\zeta_R(s)]\Big|_{s=0} \ . \tag{40}$$

Thus $r=\sqrt{\pi/2}$ and our expression for the vacuum energy density simplifies to

$$\mathscr{E}_{\rm vac}=-\left[\frac{2^9}{\pi^{11}}\right]^{1/2}\frac{1}{|a_3|} \ . \tag{41}$$

To compute the dilaton tadpole the propagator $\langle x^{\mu}(\xi_i)x^{\nu}(\xi_j)\rangle$ is needed on P_2. It satisfies

$$-\nabla_{\xi_i}^{2}\langle x_{\mu}(\xi_i)x_{\nu}(\xi_j)\rangle=\delta_{\mu\nu}\left[\frac{\delta^2(\xi_i-\xi_j)}{\sqrt{g}}-\frac{1}{2\pi}\right] . \tag{42}$$

The factor of $-1/2\pi$ occurs because the zero-mode contribution is omitted. In complex coordinates

$$\langle x_\mu(z_1)x_\nu(z_2)\rangle = -\frac{1}{4\pi}\delta_{\mu\nu}\left[\ln\left[\frac{|z_1-z_2|^2+\frac{1}{2}\epsilon^2\exp\left[-\phi\left[\frac{z_1+z_2}{2}\right]\right]}{(1+|z_1|^2)(1+|z_2|^2)}\right]\right.$$
$$\left.+\ln\left[\frac{|1+\bar{z}_1z_2|^2}{(1+|z_1|^2)(1+|z_2|^2)}\right]+2\right]. \tag{43}$$

Performing the functional integral with a single insertion of the dilaton vertex operator gives

$$A_D(p) = -(2\pi)^{26}\delta(p)\frac{\sqrt{24}}{2\pi^3}\left[\frac{3}{16\pi}\right]^{3/2}\lambda^{-1}\kappa_D(\det{}'P^\dagger P)^{1/2}[\det{}'(-\nabla^2)]^{-13}$$
$$\times\int_{\text{unit disk}} d^2z \lim_{z'\to z}\left[\frac{\partial}{\partial z}\frac{\partial}{\partial\bar{z}'}+\frac{\partial}{\partial\bar{z}}\frac{\partial}{\partial z'}\right]\ln|1+\bar{z}z'|^2 \tag{44}$$

$$= -(2\pi)^{26}\delta(p)\frac{\sqrt{24}}{2\pi^2}\left[\frac{3}{16\pi}\right]^{3/2}\lambda^{-1}\kappa_D(\det{}'P^\dagger P)^{1/2}[\det{}'(-\nabla^2)]^{-13}. \tag{45}$$

Using Eqs. (25), (38), and (40) this becomes

$$A_D(p) = -\left[\frac{384}{\pi^{13}}\right]^{1/2}(2\pi)^{26}\delta(p). \tag{46}$$

Therefore, the effective action for the dilaton field D contains a term[15]

$$\mathscr{S}_{\text{eff}} = \left[\frac{384}{\pi^{13}}\right]^{1/2}\int d^{26}x\sqrt{g(x)}D(x). \tag{47}$$

There is a sign ambiguity for the coefficient which can be absorbed into the sign of the dilaton field.

We thank S. Das, J. Polchinski, S. Wadia, and our advisor R. Rohm for useful discussions.

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$$\left[\frac{\partial x^\mu}{\partial z}\right]\left[\frac{\partial x^\nu}{\partial\bar{z}}\right]-\left[\frac{\partial x^\nu}{\partial z}\right]\left[\frac{\partial x^\mu}{\partial\bar{z}}\right]=0.$$

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