# Effect of transmission through the Earth on neutrino oscillations

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Large transformation and regeneration phenomena are calculated to result from transmission through the Earth of neutrinos whose $E(\text{MeV})/\Delta m^2(\text{eV}^2)$ lie in the vicinity of $10^6$–$10^7$. As a result large time-of-night and seasonal variations are predicted for solar neutrinos in this parameter range.

## INTRODUCTION

Mikheyev and Smirnov[1] revolutionized ideas about neutrino oscillations[2] by noting that the electron interaction that occurs during transit through matter[3] could cancel out mass differences, produce a net local degeneracy, and so give rise to large mixing effects. The effects of the solar medium on neutrinos emitted in the Sun's central region may well provide the basis for understanding the "solar-neutrino puzzle" posed by the $^{37}$Cl experiment.[4] Such an explanation implies a number of characteristic effects that are open to experimental test and would serve as clear confirmations—special distortions of the neutrino energy spectra, appearance of fluxes of transformed neutrinos—that will be reviewed elsewhere.

In this paper we examine the effect of the Earth's matter on the transmission of neutrinos. As will be seen the effects are dramatically large for some regions of the ratio of neutrino energy to the neutrino mass squared difference. Such transmission phenomena translate for solar neutrinos into time-of-night effects that would be observable in real-time experiments that record event by event, and into time-of-year effects that could appear in these as well as in radio-chemical experiments. The importance of such shutter effects is clear. Furthermore, calculations show large effects for neutrinos created at the surface of the Earth and passing through it.

## FORMALISM

The necessary formalism can be taken directly from Wolfenstein;[3] we shall here consider only two-neutrino mixing. Then, the general state, a mixture of the two-neutrino species, $|\nu_e\rangle$ and $|\nu_X\rangle$,

$$\Psi(t)=C_e(t)\,|\,\nu_e\rangle+C_X(t)\,|\,\nu_X\rangle\ ,$$

is described by the transmission equation

$$i\frac{d}{dt}\begin{bmatrix}C_e\\C_X\end{bmatrix}=\begin{bmatrix}\frac{m_1{}^2}{2E}\cos^2\theta+\frac{m_2{}^2}{2E}\sin^2\theta+\sqrt{2}Gn_e & \left[\frac{m_2{}^2}{2E}-\frac{m_1{}^2}{2E}\right]\sin\theta\cos\theta\\ \left[\frac{m_2{}^2}{2E}-\frac{m_1{}^2}{2E}\right]\sin\theta\cos\theta & \frac{m_2{}^2}{2E}\cos^2\theta+\frac{m_1{}^2}{2E}\sin^2\theta\end{bmatrix}\begin{bmatrix}C_e\\C_X\end{bmatrix}. \tag{1}$$

The physical combinations $|\nu_e\rangle$, $|\nu_X\rangle$ are understood to be the combinations of the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$:

$$\begin{aligned}|\,\nu_e\rangle&=\cos\theta\,|\,\nu_1\rangle+\sin\theta\,|\,\nu_2\rangle\ ,\\ |\,\nu_X\rangle&=-\sin\theta\,|\,\nu_1\rangle+\cos\theta\,|\,\nu_2\rangle\ .\end{aligned} \tag{2}$$

The transmission equation is governed by the energy differences (for the same momenta) between the two mass components and by the interaction between the electron neutrino, $|\nu_e\rangle$, and the electrons of the medium, $-\sqrt{2}Gn_e$—an interaction not available to the other neutrino species. $G$ is the Fermi coupling constant and $n_e$ is the number of electrons per cubic centimeter.

The large mixing effects, the basis of the Mikheyev-Smirnov phenomenon, occur at or near the degeneracy that occurs when the neutrino-electron interaction balances out the mass effects. This equality between the diagonal elements,

$$\frac{m_1{}^2-m_2{}^2}{2E}\cos2\theta=-\sqrt{2}Gn_e\ , \tag{3}$$

requires $m_2>m_1$, which we shall assume to be the case; in familiar units, this optimum mixing condition is

$$\frac{E(\text{MeV})}{\Delta m^2(\text{eV}^2)}\approx\frac{7\times10^6}{\rho(\text{g/cm}^3)y_e}\cos2\theta\ , \tag{4}$$

where $\rho$ is the density of the matter and $y_e$ is the number of electrons per amu. Since the density of the Earth varies from $\sim3$ at the surface to $\sim13$ at the center[5] and $y_e\sim\frac{1}{2}$, neutrinos for which $E/\Delta m^2$ lies in the region

 

$\sim 10^6$ to $\sim 10^7$ should show interesting effects.

Numerical solution of the fundamental equations (1), appears to be the most satisfactory way to get trustworthy results. The form of the first-order coupled differential equations for probabilities as written by Mikheyev and Smirnov[1] are solved using the Bashforth-Adams-Milne predictor-corrector method.[6] The physically interesting initial conditions at the surface of the Earth differ, however, from those usually considered; instead of limiting the discussion to the familiar one in which the initial neutrino state is pure $|\nu_e\rangle$, arbitrary initial mixtures will be included in order to accommodate studies of solar neutrinos and their oscillation in transit through the solar medium and in space. To handle the varying initial conditions in a uniform way, $\Psi(t)$ will be written in terms of two functions that solve the Earth-transmission problem, with the orthogonal boundary conditions:

$$\begin{aligned} &\Psi_1(t),\ \ \Psi_1(0)=|\nu_e\rangle\ , \\ &\Psi_2(t),\ \ \Psi_2(0)=|\nu_X\rangle\ , \end{aligned} \tag{5}$$

then

$$\Psi(t)=\beta_1\Psi_1+\beta_2\Psi_2 \tag{6}$$

is equipped to meet general boundary conditions via a choice of $\beta_1,\beta_2$, with the properties of $\Psi_1,\Psi_2$ fixed and known from the separate calculations to be outlined in the next section.

The relations between $\Psi_1$ and $\Psi_2$ make it necessary to compute only one of these, even though they are orthogonal to one another. It is easiest if we rewrite Eq. (1) in schematic form and use the standard Pauli matrices

$$i\frac{d}{dt}\begin{bmatrix} C_e \\ C_X \end{bmatrix}=(\alpha\sigma_x+\beta\sigma_x+\gamma 1)\begin{bmatrix} C_e \\ C_X \end{bmatrix}\ , \tag{7}$$

and with

$$\begin{bmatrix} C_e \\ C_X \end{bmatrix}=\exp\left[-i\int_0^t \gamma\, dt\right]\begin{bmatrix} C_e' \\ C_X' \end{bmatrix}\ , \tag{8}$$

$$i\frac{d}{dt}\begin{bmatrix} C_e' \\ C_X' \end{bmatrix}=(\alpha\sigma_x+\beta\sigma_z)\begin{bmatrix} C_e' \\ C_X' \end{bmatrix}\ . \tag{9}$$

Then, if $(C_e',C_X')$ is a solution, complex conjugation followed by multiplication by $\sigma_y$ demonstrates that $(C_X'^*,C_e'^*)$ is also a solution and one that obeys the $t=0$ conditions of $\Psi_2$. We have then

$$\Psi_1=\exp\left[-i\int_0^t \gamma\, dt\right]\begin{bmatrix} C_e' \\ C_X' \end{bmatrix}=\begin{bmatrix} C_e^{(1)} \\ C_X^{(1)} \end{bmatrix}\ , \tag{10}$$

$$\Psi_2=\exp\left[-i\int_0^t \gamma\, dt\right]\begin{bmatrix} -C_X'^* \\ C_e'^* \end{bmatrix}=\begin{bmatrix} C_e^{(2)} \\ C_X^{(2)} \end{bmatrix}\ , \tag{11}$$

and can see at once that

$$|C_e^{(2)}|=|C_X^{(1)}|=(1-|C_e^{(1)}|^2)^{1/2}\ . \tag{12}$$

With the notation $P_{E_1}$ for the fraction that remains $\nu_e$ at the end of the transmission after starting out as $|\nu_e\rangle$,

$$P_{E_1}=|C_e^{(1)}(t_f)|^2=1-|C_e^{(2)}(t_f)|^2\ . \tag{13}$$

Arbitrary initial conditions are, then, easily met. The most general form, appropriate for a source outside the Earth (not yet including the effect of the Earth) is

$$\begin{aligned} \Psi(\text{source})&=\exp(iE_1t)\cos\alpha\,|\nu_1\rangle-\exp[i(E_2t-\phi)]\sin\alpha\,|\nu_2\rangle \\ &=\exp(iE_1t)\left[|\nu_e\rangle\left\{\cos\alpha\cos\theta-\sin\alpha\sin\theta\exp\left[i\left(\frac{\Delta m^2}{2E}t-\phi\right)\right]\right\}\right. \\ &\qquad\left.+\,|\nu_X\rangle\left\{-\cos\alpha\sin\theta-\sin\alpha\cos\theta\exp\left[i\left(\frac{\Delta m^2}{2E}t-\phi\right)\right]\right\}\right]\ . \end{aligned} \tag{14}$$

Then the probability of an electron neutrino coming from the Sun is

$$\begin{aligned} P_S&=|\langle\nu_e|\Psi(\text{source})\rangle|^2 \\ &=(\cos^2\alpha\cos^2\theta+\sin^2\alpha\sin^2\theta)-\tfrac{1}{2}\sin2\alpha\sin2\theta\cos\left(\frac{\Delta m^2}{2E}t-\phi\right)\ . \end{aligned} \tag{15}$$

Ignoring any overall phase, we can take

$$\begin{aligned} \beta_1&=\cos\alpha\cos\theta-\sin\alpha\sin\theta\exp\left[i\left(\frac{\Delta m^2}{2E}t-\phi\right)\right]\ , \\ \beta_2&=-\cos\alpha\sin\theta-\sin\alpha\cos\theta\exp\left[i\left(\frac{\Delta m^2}{2E}t-\phi\right)\right]\ . \end{aligned} \tag{16}$$

Then the probability fraction $P_{SE}$ of remaining electron neutrino after transmission through the Earth is

$$\begin{aligned} P_{SE}(\beta_1,\beta_2)=&\,|\beta_1|^2|C_e^{(1)}(t_f)|^2+|\beta_2|^2|C_e^{(2)}(t_f)|^2+\tfrac{1}{2}(\beta_1^*\beta_2+\text{c.c.})[C_e^{(1)*}(t_f)C_e^{(2)}(t_f)+\text{c.c.}] \\ &+\tfrac{1}{2}(\beta_1^*\beta_2-\text{c.c.})[C_e^{(1)*}(t_f)C_e^{(2)}(t_f)-\text{c.c.}]\ , \end{aligned} \tag{17}$$

and recognizing that

$$P_S=|\beta_1|^2=1-|\beta_2|^2,\quad P_{E_1}=|C_e^{(1)}(t_f)|^2=1-|C_e^{(2)}(t_f)|^2\ ,$$
$$P_{SE}(\beta_1,\beta_2)=1+2P_SP_{E_1}-P_{E_1}-P_S+\tfrac{1}{2}(\beta_1^*\beta_2+\text{c.c.})(C_e^{(1)*}C_e^{(2)}+\text{c.c.})+\tfrac{1}{2}(\beta_1^*\beta_2-\text{c.c.})(C_e^{(1)*}C_e^{(2)}-\text{c.c.})\ . \tag{18}$$

Instead of carrying the full messiness of this general form, the remainder of the discussion is based on using an average and so dropping the rapidly oscillating terms—those that are proportional to

$$\exp\left[\pm i\left[\frac{\Delta m^2}{2E}t-\phi\right]\right]\ .$$

The reason for proceeding in this way can be quickly understood on evaluating the phase

$$\chi=2\pi\left[\frac{\Delta m^2}{4\pi E}t\right]$$
$$\simeq 2\pi\left[\frac{\Delta m^2(\text{eV}^2)}{E(\text{MeV})}\frac{t}{2.5\times10^2}\right]\ ; \tag{19}$$

taking $t$ as the Earth-Sun distance, $\sim 1.5\times10^{13}$ cm,

$$\frac{\Delta m^2(\text{eV}^2)}{E(\text{MeV})}\sim 10^6\ ,$$

results in $\chi=2\pi(6\times10^4)$; binning the energy spectrum in fractions of 1% is clearly sufficient to achieve a good approximation to zero. Then,

$$\bar{P}_S=(\cos^2\alpha\cos^2\theta+\sin^2\alpha\sin^2\theta)\ ,$$
$$\overline{\beta_1^*\beta_2}=-\tfrac{1}{2}\cos 2\alpha\sin 2\theta=(\tfrac{1}{2}-\bar{P}_S)\tan 2\theta=\text{real}\ , \tag{20}$$

and

$$\bar{P}_{SE}=1+2\bar{P}_SP_{E_1}-P_{E_1}-\bar{P}_S$$
$$+\tan 2\theta(\tfrac{1}{2}-\bar{P}_S)(C_e^{(1)*}C_e^{(2)}+\text{c.c.})\ . \tag{21}$$

The combination $(C_e^{(1)*}C_e^{(2)}+\text{c.c.})$ must either be extracted from the calculation of $\Psi_1$, via Eq. (11), or by using the Mikheyev-Smirnov equations to calculate the particular $P_{SE}$ (to be called $P_{E_2}$) that starts with the conditions $P_{E_1}\equiv\frac{1}{2}$ and $C_e^*C_X=\frac{1}{2}$ at the Earth. This latter calculation corresponds to the choice $\beta_1=\beta_2=1/\sqrt{2}$ in Eq. (17), and leads at once to the relation

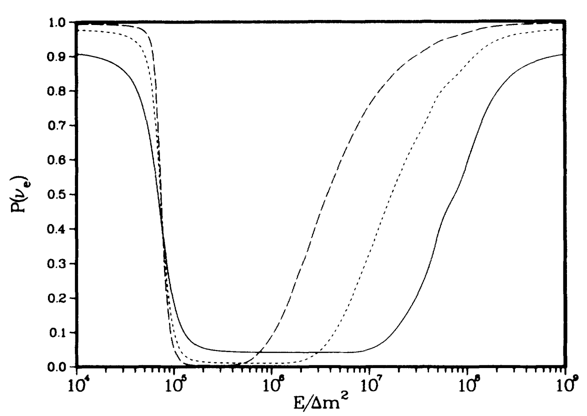


FIG. 1. A recalculation of the Mikheyev-Smirnov solution for the probability $P(\nu_e)$ that an electron neutrino created in the central solar region, will avoid an oscillation transformation and will survive as an electron neutrino in its transit through the solar medium and space to the Earth. The distribution of the neutrino points of origin is that of the $^8$B neutrinos as given by the "standard model" of Ref. 7. An average over neutrino energies that corresponds to binning the energy spectrum in small intervals has been included; this has the effect of averaging out very rapid oscillations. Three values of the mixing parameter are shown: medium dashes, $\sin 2\theta=0.1$; short dashes, $\sin 2\theta=0.2$; solid line, $\sin 2\theta=0.4$.

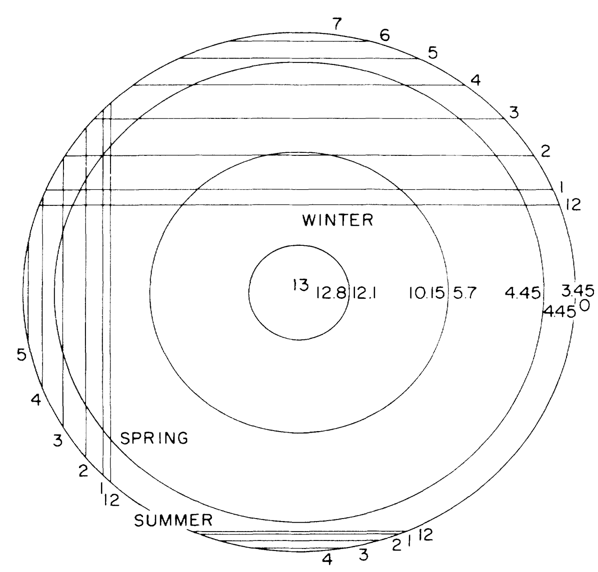


FIG. 2. Trajectories through the Earth, followed by a solar neutrino to reach a detector located at 43° north latitude at various times of the night at the winter and summer solstices and the spring-fall equinoxes. "Midnight" or "12" is taken as the time corresponding to the longest trajectory, and the other early morning hours indicated are measured from it. Of course, similar trajectories for symmetrical evening times would be followed. The principal density zones of the Earth are marked together with the densities (in g/cm$^3$) at the beginning and ends of these zones; the data are from Ref. 5.

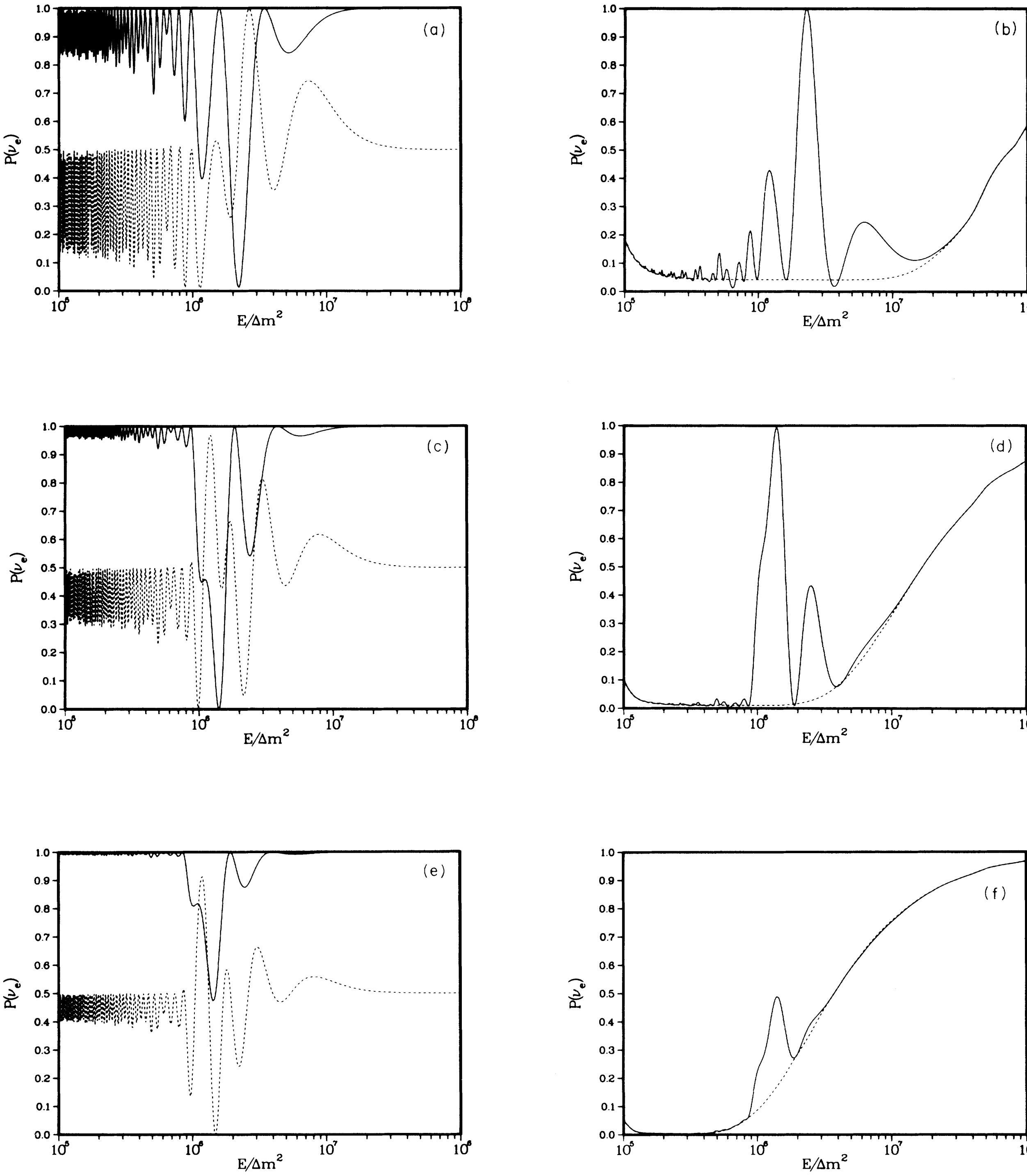


FIG. 3. (a) Solid line, the probability for an electron neutrino created at the Earth's surface to survive untransformed at the antipode, $P_{E_1}$, for a diametrical trajectory. Dashed line, $P_{E_2}$ for this same diametrical trajectory. The mixing parameter $\sin 2\theta = 0.4$. (b) Solid line, the probability that a solar neutrino manifests itself as an electron neutrino after passage through the Earth on this same diametrical trajectory, $\overline{P}_{SE}$. Dashed line, the probability that a solar neutrino manifests itself as an electron neutrino, from Fig. 1, as seen directly without the Earth effect, shown for comparison. The mixing parameter $\sin 2\theta = 0.4$. (c) and (d) Similar plots for $\sin 2\theta = 0.2$. (e) and (f) Similar plots for $\sin 2\theta = 0.1$. (g) $P_{E_1}$ calculated on a trajectory 45° from the nadir for $\sin 2\theta = 0.4$. (h) Similar to (g) for $\sin 2\theta = 0.2$. (i) Similar to (g) for $\sin 2\theta = 0.1$.

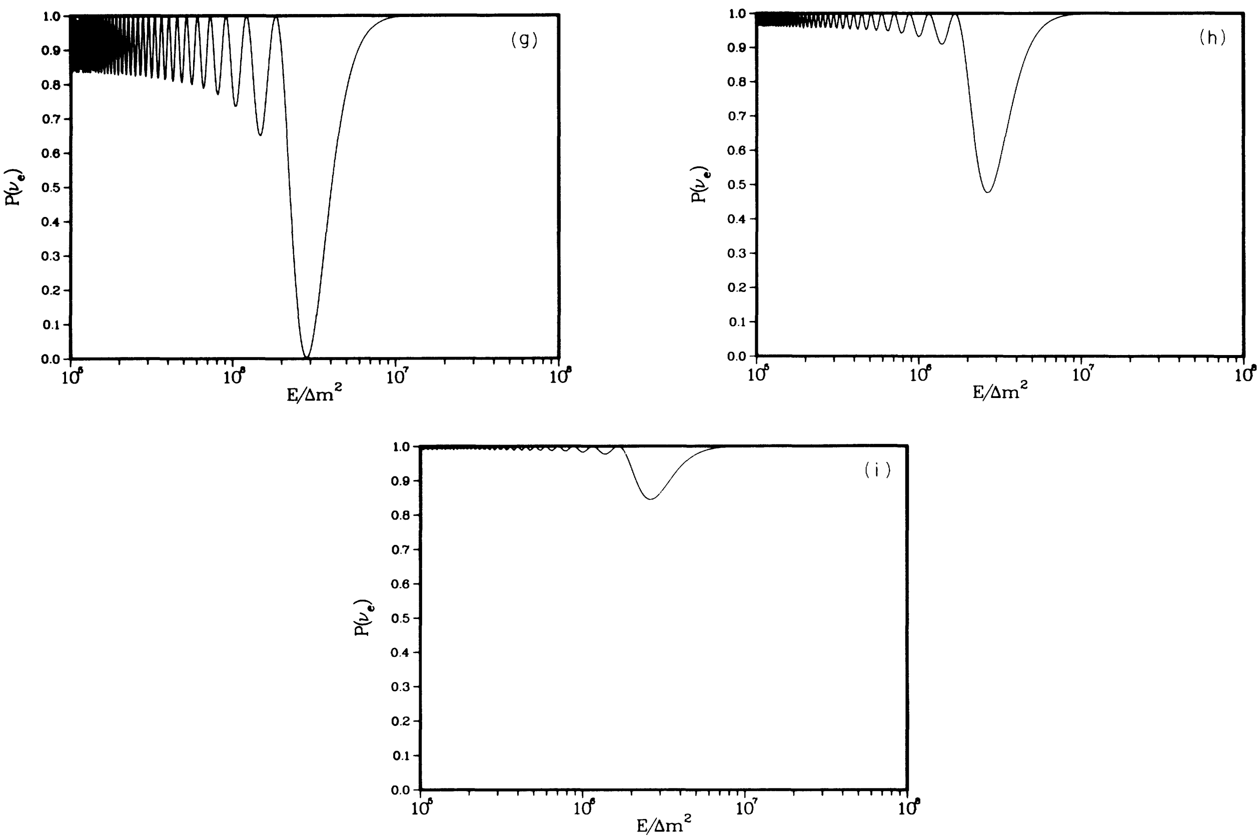


FIG. 3. (*Continued*).

$$2P_{E_2}-1=2P_{SE}\left[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]-1$$
$$=[C_e^{(1)*}(t_f)C_e^{(2)}(t_f)+\text{c.c.}] \ . \tag{22}$$

$P_{E_2}$ is a solution beginning at the surface of the Earth like $P_{E_1}$ but with the boundary condition of equal parts of both species of neutrinos and real relative phases. The results described in the next sections were obtained using this $P_{E_2}$ and the relation

$$\overline{P}_{SE}=1+2\overline{P}_S P_{E_1}-\overline{P}_S-P_{E_1}$$
$$-\tfrac{1}{2}(2\overline{P}_S-1)(2P_{E_2}-1)\tan 2\theta \ . \tag{23}$$

## RESULTS OF COMPUTATIONS

In this section will be reported the results of calculations on the effect of the Earth on the oscillation transformation of neutrinos emitted from each of two sources: one located at the Earth's surface; the other located in the central region of the Sun and surrounded by the solar medium which acts to transform the neutrinos in their transit. The solar source will be taken as distributed over the central solar region with the profile of $^8$B neutrinos as calculated in the "standard model."[7] $\overline{P}_S$, a function of $E/\Delta m^2$ and $\theta$, contains the effect of the solar medium. Figure 1 contains a recalculation of that originally given by Mikheyev and Smirnov.[1] As discussed in the preceding sections, an averaging over small energy intervals has been performed to match the physical situation; this averaging or binning in small energy intervals has the effect of eliminating fast oscillations.

The function $P_{E_1}$ has a double role. Apart from its use in the calculation of $\overline{P}_{SE}$, it directly expresses the probability that an electron neutrino created at the Earth's surface in a reactor, accelerator, or a cosmic-ray event would survive as an electron neutrino in its travel through the Earth to a detector at some other point on the Earth's surface. Of course, $P_{E_1}$ is a function of the trajectory, as well as the neutrino parameters, $E_\nu$, $\Delta m^2$, and $\theta$. Similarly $P_{E_2}$ will depend on this same set of trajectory and neutrino properties.

We have taken as a model of the Earth a somewhat simplified version of the current standard geophysical picture derived from seismological evidence.[5] Figure 2 shows the density zones within the Earth and the values of the density taken at the boundaries; inside each zone the density was assumed to vary linearly with radial distance, in approximate conformity with the geophysical model.

The Earth's effect on neutrino transformation has been pointed out by Barger *et al.*,[8] and LoSecco,[9] in a recent paper, discusses it in terms of cosmic-ray induced neutrino events. In this note we discuss a solution of the complete neutrino transport equation, with attention to the variation of neutrino trajectories that comes about either because of the Earth's rotations or because a neutrino direction is defined as part of the observation.

As a first orientation, we have the results for a trajectory through the center of the Earth and for a choice of neutrino parameter $\sin 2\theta = 0.4$ (an angle a little smaller than the Cabibbo angle). Figure 3(a) shows $P_{E_1}$ and $P_{E_2}$. In the $E/\Delta m^2$ region of $10^6$–$10^7$, $P_{E_1}$ (the probability that a neutrino starting at the Earth's surface as $\nu_e$ will remain $\nu_e$ in its transit to the antipodal point on the surface), is seen to vary from unity to nearly zero; this large variation down to the vicinity of zero is a matter-resonance effect. The behavior of $P_{E_1}$ in the $10^5$–$5\times 10^5$ region closely resembles matter oscillations in vacuum, with an oscillatory amplitude of approximately $\frac{1}{2}\sin^2 2\theta = 0.08$. $P_{E_2}$ also shown in Fig. 3(a) has a corresponding oscillatory behavior, although it begins at a value of 0.5.

Figure 3(b) shows the result, as obtained with Eq. (23), for a solar neutrino that passes through the Earth on this same diametrical trajectory; the appropriate Mikheyev-Smirnov probability, which serves as the source term, is also shown in the figure for a direct comparison. There is a clear, very large regeneration effect in the $E/\Delta m^2$ $10^6$–$10^7$ region. It is also worth noting that the rapid oscillations seen in $P_{E_1}$ and $P_{E_2}$ tend to cancel in the solar-neutrino combination $\overline{P}_{SE}$. This can be traced back to the averaging of the source term, $\overline{P}_S$, which acts to drop the fast oscillations associated with the quickly varying phase $X$, Eq. (19); as can be explicitly shown by using the vacuum-oscillation expressions for $P_{E_1}$ and $P_{E_2}$, the source-averaged expression, Eq. (23), also correctly eliminates the oscillations which would differ only in the replacement of the Earth-Sun distance by a distance increased by the Earth's diameter. Figures 3(c), 3(d), and 3(e), 3(f) show the results for this same diametrical trajec-

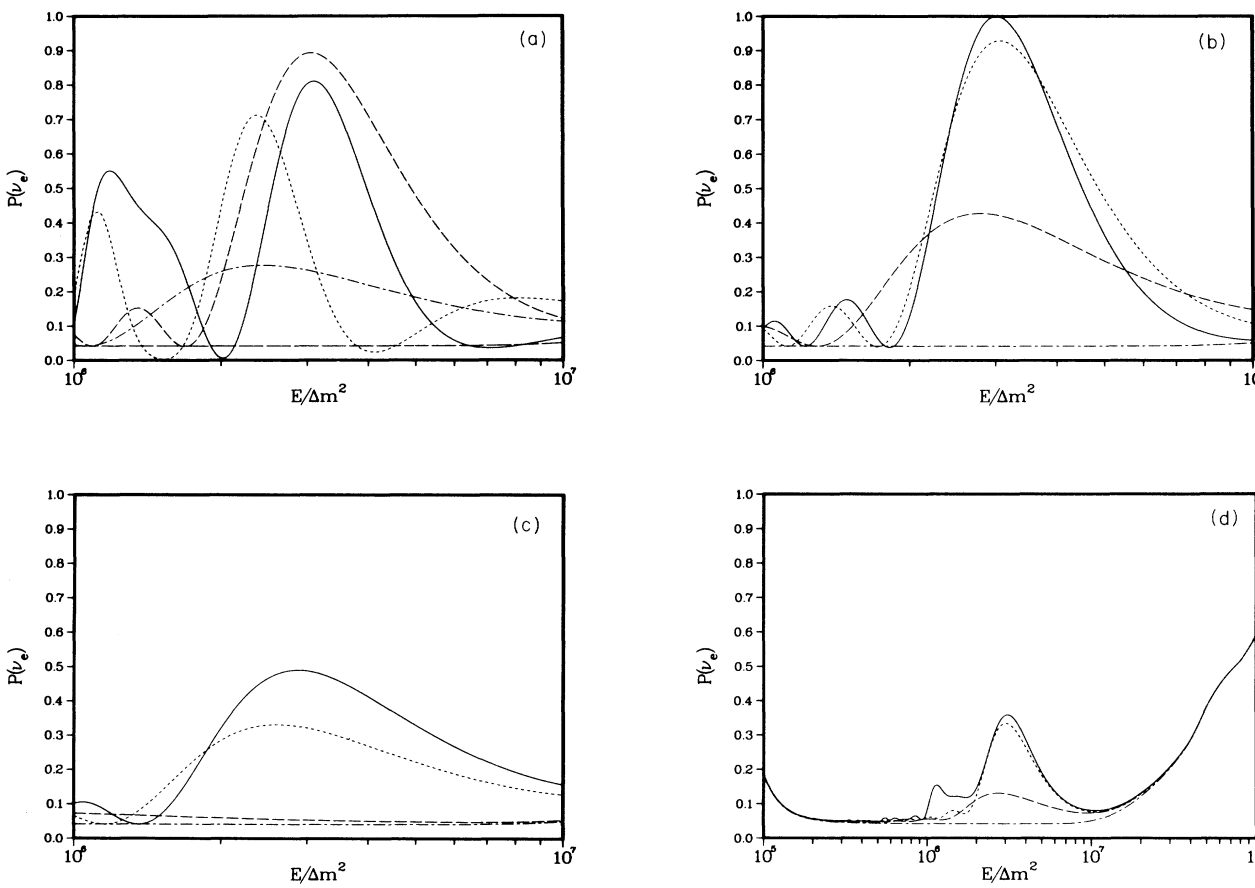


FIG. 4. (a) $\sin 2\theta = 0.4$, detector at 43° north latitude, winter solstice: solid line, the probability that a solar neutrino manifests itself as an electron neutrino at midnight; short-dashed line, midnight ±2 h; medium-dashed line, midnight ±4 h; medium- and short-dashed line, midnight ±6 h; long-dashed line, daylight hours. (b) Similar plot at the spring-fall equinox: solid line, midnight; short-dashed line, midnight ±2 h; medium-dashed line, midnight ±4 h; medium- and short-dashed line, daylight hours. (c) Similar to (b) at the summer solstice. (d) Solid line, 24 h average at the winter solstice; short-dashed line, at the spring-fall equinox; medium-dashed line, at the summer solstice; short- and medium-dashed line, daylight hours only.

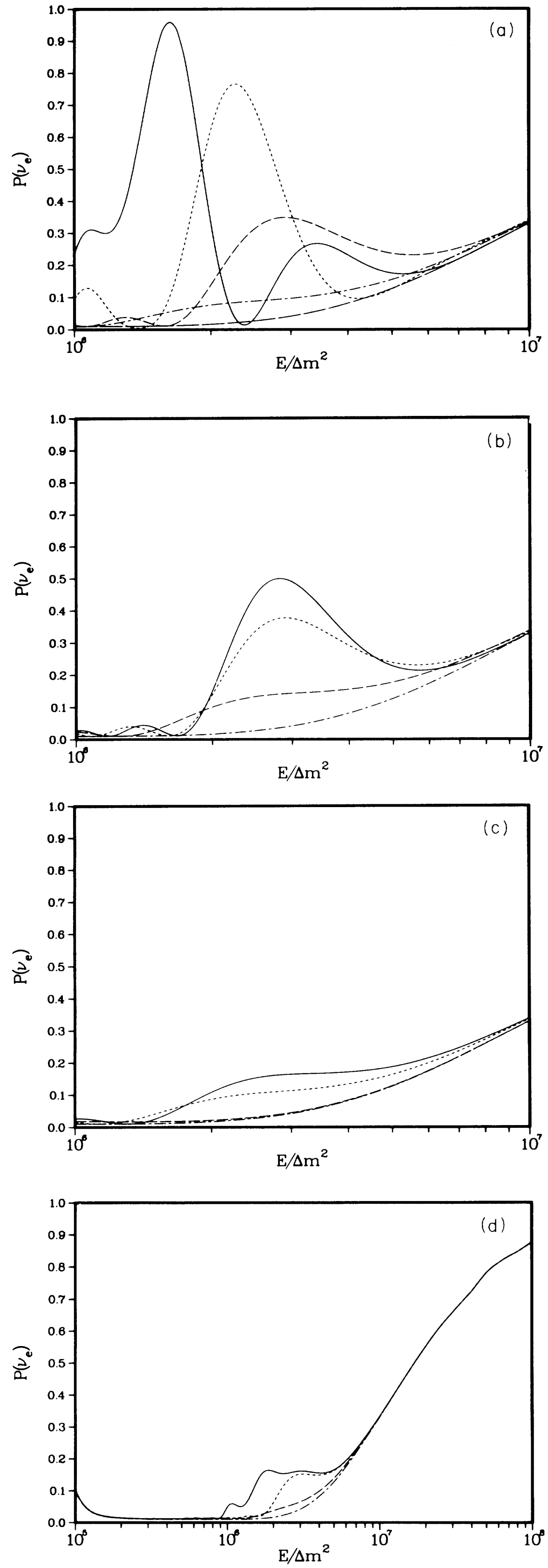


FIG. 5. Similar to Fig. 4 for $\sin 2\theta = 0.2$.

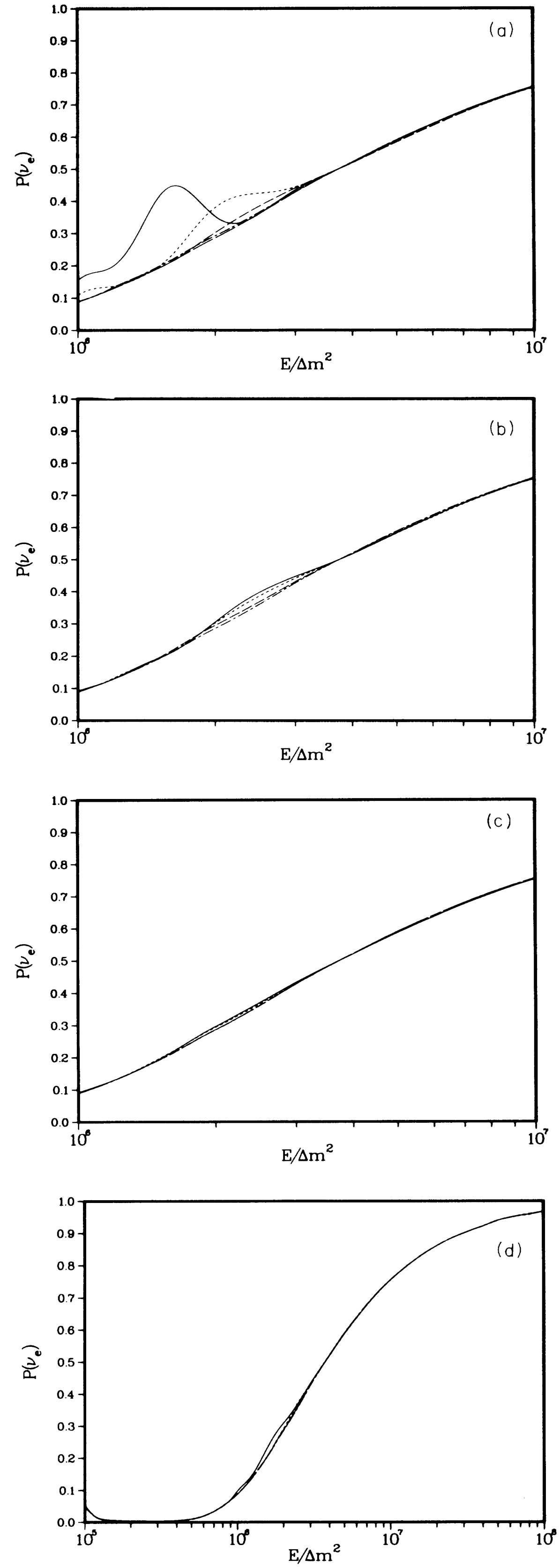


FIG. 6. Similar to Fig. 4 for $\sin 2\theta = 0.1$.

tory for $\sin 2\theta = 0.2$ and 0.1, respectively. There is some narrowing and diminution, but there is clearly a sizable effect even for these smaller values of the mixing angle.

As noted before, $P_{E_1}$ has a direct physical interpretation, since it tells us the survival probability of one species of neutrino created at the surface. Because the effects of transit on a diametrical trajectory are so striking, we add, in Figs. 3(g)—3(i), the analogous results for a trajectory 45° away from the nadir. The effects are diminished, but still interesting and markedly affected by the lesser densities encountered on this less deep trajectory. These results are directly applicable to accelerator or cosmic-ray-produced neutrinos, whether appearance or disappearance experiments and whichever the initial species. (An initial $X$ neutrino will have a survival probability given by $P_{E_1}$; the transformation probability is, of course, given by $1-P_{E_1}$ in this two-neutrino calculation.)

We turn now to calculate the time-of-night and seasonal responses of a detector located at 43° north latitude (between Rome and Homestake, South Dakota). Figure 2, which illustrates the density profile of the Earth, also shows the Earth path of a trajectory connecting the Sun and detector at various times of day at the winter and summer solstices and the spring-fall equinox. The qualitative nature of the effects can now be guessed. The deepest-going winter trajectory, midnight, while not as deep as the diametrical trajectory, is still deeper lying than the midnight trajectory of summer. To this must be added the increased hours of night in the winter, hours in which the solar neutrinos must come through the Earth to reach the detector.

Both the seasonal and diurnal effects are clearly visible for $E/\Delta m^2$ in the $10^6$—$10^7$ range. Figure 4 illustrates these for the mixing angle choice $\sin 2\theta = 0.4$. Figures 4(a)—4(c) show the behavior of $\overline{P}_{SE}$ as a function of $E/\Delta m^2$ for various times (midnight, midnight $\pm 2$ h, midnight $\pm 4$ h, etc., and daylight hours) on four characteristic days (winter solstice, spring-fall equinox, summer solstice). Figure 4(d) represents the 24 h average on each of these four seasonally significant days. The time-of-night variation is very striking, and would be a delightfully clear signal in a real-time experiment—should nature choose $\Delta m^2$ appropriately. The seasonal variations are also large. In fact, the overall regeneration effect, the average over the whole year is sufficiently large ($\overline{P}_{SE}$ averages to $\sim \frac{1}{4}$) as to add to the two Mikheyev-Smirnov solutions, a third, quasisolution to the "solar-neutrino puzzle"; that is, there is sufficient regeneration to provide a nonzero, suitably diminished signal. It does not seem to be an actual solution because the seasonal variations are not reported as having been observed.

Figures 5 and 6 show corresponding results for the choices of the mixing angle, $\sin 2\theta = 0.2$ and 0.1, respectively. The time-of-night effects, especially at the winter solstice, are still quite strong; seasonal differences (24 h averaging) become harder to detect as $\sin 2\theta$ is reduced.

In summary, the Earth has been shown to produce a large oscillation transformation effect for $E/\Delta m^2$ in the range $10^6$—$10^7$ for a wide range of the mixing parameter, $\sin 2\theta$. This large effect is present for both surface and solar-neutrino sources.

*Note added in proof.* After completion of this manuscript we noted the closely related work of E. D. Carlson [Phys. Rev. D **34**, 1454 (1986)] and J. Bouchez, M. Cribier, J. Rich, M. Spiro, D. Vignaud, and W. Hampel [Z. Phys. C (to be published)].


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