Gauge structures beyond the standard model and 100-GeV mass region

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(Received 27 May 1986)

We study various forward-backward and polarization asymmetries evaluated near the Z^0 resonance for theories with $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge structures. Extension to other gauge structures is very simple in our formalism. We construct a linear combination of polarized forward-backward asymmetry and polarization asymmetry with initial-state-electron longitudinal polarization whose deviation from the value of the standard model can measure the effects of new currents directly. The analysis is exact at the tree level of the theory and enables one to study any model with any Higgs sector in terms of a *fixed* number of parameters. The results show that for a typical class of models the measurement of different asymmetries to 1% will impose a lower bound on $M_{Z'}$, the mass of an additional neutral gauge boson, to be of order 10 M_Z. Even much less accurate measurements will yield interesting information about new gauge structures. We also examine the implications of extended gauge structures for the precise value of the $W^{\pm}$ mass.

I. INTRODUCTION

The standard Glashow-Weinberg-Salam[1] (GWS) model of electroweak interactions based on $SU(2)_L \times U(1)_Y$ has achieved important success in describing neutral- and charged-current processes and determining the mass of W and Z gauge bosons. However, this theory contains many undetermined parameters. If these parameters are not to be put in *ad hoc* but rather to be determined by theory, then we must look for a still more fundamental theory of electroweak interactions which reduces to the GWS model at low energies. These more fundamental theories in general predict the existence of many new particles and the search for these novel excitations has been a major preoccupation of physicists working at the highest e^+e^- and $p\bar{p}$ colliders. In the late 1980s the CERN collider LEP, the Stanford Linear Collider (SLC), and the Fermilab Tevatron will explore the mass region up to about 100 GeV. Further direct exploration must await the very-high-energy hadron-hadron colliders planned for the late 1990s.

We may hope to evade the need to obtain increasingly higher center-of-mass energies by searching for indirect effects of the new particles. A previous paper[2] showed how to search for indirect effects of new heavy scalars and fermions which couple to the gauge bosons of $SU(2)_L \times U(1)_Y$; by studying the various polarization and forward-backward asymmetries on Z^0 resonance in $e^+e^- \to f\bar{f}$ processes at the 1% level, experimentalists at LEP/SLC could see the virtual quantum effects of the new particles and place limits on the scalar and fermion particle spectrum in the 100 GeV$-\frac{1}{2}$ TeV region. In this paper, we show how to look for indirect effects of new *gauge bosons* in the 100 GeV$-$1 TeV mass region.

One of the more interesting theoretical proposals is the possibility of an enlarged electroweak gauge group structure. Some of the new gauge bosons arising from such an enlarged gauge symmetry can have a mass of order 2$-$3 M_Z without contradicting present experimental bounds. One such class consists of left-right-symmetric gauge theories[3] based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. A left-right-symmetric theory is appealing since it allows for spontaneous breakdown of parity.[4] Another class has an extra U(1) gauge group; i.e., the gauge structure is $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$. This might appear as a low-energy electroweak symmetry[5] arising from string theories.[6] Both gauge groups can appear as an intermediate gauge structure within a grand-unified theory.

Because of the new gauge structure there are new currents; the particles have quantum numbers under the new group. Further, the Z and $W^{\pm}$ currents are modified because of the admixture of the new currents and gauge bosons, thus changing the physics even at the energy scales of the W and Z masses.

In this paper we show that a new gauge structure can be tested by measuring various asymmetries in e^+e^- collisions at energies around the Z resonance. Namely, the admixture of new currents changes the prediction of the standard model. Thus, SLC/LEP physics near the Z resonance offers a very important opportunity to test for new gauge structures beyond the standard model. SLC/LEP experiments will be done with high precision, large statistics, and good detectors. Also, e^+e^- physics is theoretically "clean," since it minimizes theoretical strong-interaction uncertainties. This could enable SLC and LEP to measure deviations of various asymmetries from the standard model to a precision of about 1% (Ref. 7).

In the present work we evaluate various asymmetries in e^+e^- collisions for theories with a gauge group larger than the one of the standard model. In particular, we give results for the left-right-symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the gauge group with an extra U(1), i.e., $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$. However, this approach can be used for any gauge group beyond $SU(2)_L \times U(1)_Y$. The fermionic currents and the gauge-boson mass eigenstates are determined at the tree level *ex-*

actly. The results are valid for any Higgs-field content and any vacuum expectation value pattern which breaks the original symmetry via $SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$. We reparametrize the models in terms of a *fixed* number of parameters. Such an approach enables us to study *any* model within a proposed gauge group over the whole range of permitted values of $M_{Z'}$, the mass of an additional gauge boson.

As $M_{Z'} \to \infty$ these models reduce to $SU(2)_L \times U(1)_Y$ irrespective of the representation of the Higgs fields, i.e., decoupling takes place. Thus, by measuring a deviation of the polarization and forward-backward asymmetries from the standard model one can exclude a whole range of models with additional symmetries and impose a lower bound on $M_{Z'}$.

A particularly interesting quality is $\Delta^{c,b}$ (defined in Sec. II), which is a particular linear combination of the deviation from GWS of the polarized forward-backward asymmetry for $e^-(L)e^+ \to \bar{c}c$, $\bar{b}b$, and the deviation from GWS of the initial-state longitudinal-polarization asymmetry for $e^+e^-_{\rm pol} \to \mu^+\mu^-$. An important observation is that $\Delta^{c,b}$ measured on Z resonance, is identically zero in $SU(2)_L \times U(1)_Y$ even when the oblique[2,8] quantum corrections due to new scalars and fermions are included. Thus, $\Delta^{c,b} \neq 0$ is a clear indication that new undiscovered particles couple to e, μ, c, b, i.e., that there are new currents. At the tree level this can only be due to new gauge structures.

Hollik[9] has considered the shifts in the left-right and forward-backward asymmetries in $e^+e^- \to \bar{f}f$, $f=u$, d, μ, τ, for specific extended gauge groups with a very specific set of Higgs representations and symmetry-breaking parameters. We generalize on his work in the following ways.

(1) We show the effects of new gauge structures on all neutral- and charged-current processes at all energies; it is then clear how to compare SLC/LEP experiments to low-energy neutrino scattering or even production of new as-yet undiscovered fermions at LEP 2.

(2) We show that the number of new parameters entering these processes is fixed by the gauge structure alone and the quantum numbers of fermions under the new groups. We are then able to fix a subset of these (e.g., α, G_μ, and M_Z) in all models so as to display clearly the effects of new parameters and thus constrain them by experiment.

(3) We display exact formulas for $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ for any set of Higgs fields with any symmetry-breaking pattern. The generalization to other gauge groups is then obvious in our formalism.

(4) We show how to distinguish, at the Z resonance, effects of new gauge structures from quantum corrections in $SU(2)_L \times U(1)_Y$ by studying specific combinations of asymmetries. We show further that a certain combination is only sensitive to the quantum numbers of e, μ, c, b under G when the gauge group is $SU(2)_L \times U(1)_Y \times G$.

(5) There is another quantity which might be measured to high accuracy in the near future: the $W^\pm$ mass. We also show how it changes in an observable way from the GWS prediction in an extended gauge structure.

The paper is organized as follows. In Sec. II we define the measurable asymmetries. In Sec. III we summarize the results for $SU(2)_L \times U(1)_Y$; we comment on the choice of measurable parameters of the theory, and the effect of radiative corrections. In Sec. IV A we present the exact form of the currents and determine parameters for a theory with an $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ local gauge group and in Sec. IV B the results for the various asymmetries are presented. In Sec. V we repeat the analysis of Sec. IV, but this time for theories with $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group. In Sec. VI we summarize our results.

II. MEASURABLES: ASYMMETRIES

We shall study processes $e^+e^- \to \bar{f}f$ at the center-of-mass energies around Z resonance. When the mass of the final-state fermions f is much smaller than M_Z helicity is approximately conserved even at the one-loop level at each gauge-boson vertex. This holds well for all the known fermions except the top quark. Also when $f \neq e^-, \nu_e$, the t-channel scattering graph is absent. In the following we shall concentrate for simplicity on processes with $f \neq e, \nu_e, t$ with t the top quark. Also, we shall not include the effects of final-state hadronization processes for individual $f=u,d,s,c,b$ quarks. We will, however, consider the initial-state polarization asymmetry for the total cross section $e^+e^-_{\rm pol} \to$ hadrons (for $m_{\rm top} > M_Z/2$) since the hadronization for this process is understood.[10]

For the processes subject to the above approximations the reaction $e^+e^- \to \bar{f}f$ can be cast in the following form:[2]

$$\frac{d\sigma(e^+e^-(P) \to \bar{f}f(P'))}{d\Omega} = \frac{s}{4\pi} k_{PP'}{}^2 \left| [\mathcal{M}(-s)]^{ef}_{PP'} \right|^2 . \tag{2.1}$$

Here P and P' denote longitudinal polarizations L or R. A kinematic factor, $k_{PP'}{}^2$, is equal to $(u/s)^2$ for $P=P'=L,R$ and to $(t/s)^2$ for $P=L$, $P'=R$ and $P=R$, $P'=L$. Here, u,s,t are the Mandelstam variables. The matrix element $[\mathcal{M}(q^2)]^{ff'}_{PP'}$ is a properly normalized invariant amplitude which carries all the nontrivial information about the coupling. We shall write $[\mathcal{M}(q^2)]^{ff'}_{PP'}$ for the general case of three neutral gauge bosons—photon, Z, and Z':

$$[\mathcal{M}(q^2)]^{ff'}_{PP'} = \frac{(J_{\rm EM})^f_P (J_{\rm EM})^{f'}_{P'}}{q^2} + \frac{(J_Z)^f_P (J_Z)^{f'}_{P'}}{q^2 + M_Z{}^2 - i\,{\rm Im}\Pi^{1\,\rm loop}_{ZZ}(q^2)} + \frac{(J_{Z'})^f_P (J_{Z'})^{f'}_{P'}}{q^2 + M_{Z'}{}^2 - i\,{\rm Im}\Pi^{1\,\rm loop}_{Z'Z'}(q^2)} . \tag{2.2}$$

(The generalization to more than three neutral gauge bosons is obvious.) Here we have used the Euclidean metric, and $(J)^f_P$ refers to a particular fermionic current with fermion f having polarization P. For example, the electromagnetic current is written

$$J_{\rm EM}=eJ_Q\ , \tag{2.3}$$

$$J_Q=\bar{\psi}\gamma_\mu Q\psi\ , \tag{2.4}$$

$$(J_Q)^b_L=(J_Q)^b_R=Q_b=-\tfrac{1}{3}\ , \tag{2.5}$$

with ψ a fermion, $e^2=4\pi\alpha$, and Q the electric-charge operator so that $Q_e=-1$, $Q_c=\frac{2}{3}$. J_Z and $J_{Z'}$ are obviously the Z and Z' currents analogous to (2.3). The tree-level width of the Z (which, of course, is the imaginary part of the one-loop Z self-energy), $\mathrm{Im}\Pi^{1\,\rm loop}_{ZZ}$, reduces in the case where only light quarks and leptons are produced at $q^2=-s=-M_Z{}^2$ to form

$$M_Z\Gamma_Z\equiv\mathrm{Im}\Pi^{1\,\rm loop}_{ZZ}(-M_Z{}^2)=\frac{\alpha M_Z{}^2}{4}\sum_f\left[[(J_Z)^f_L+(J_Z)^f_R]^2\left[1+2\frac{m_f{}^2}{M_Z{}^2}\right]+[(J_Z)^f_L-(J_Z)^f_R]^2\left[1-4\frac{m_f{}^2}{M_Z{}^2}\right]\right]\times\left[1-4\frac{m_f{}^2}{M_Z{}^2}\right]^{1/2}c_{\rm QCD}\ , \tag{2.6}$$

with $c_{\rm QCD}=1$ for leptons and

$$c_{\rm QCD}\simeq 3\left[1+\frac{\alpha_{\rm strong}(-M_Z{}^2)}{\pi}\right]$$

for quarks. We put in this width and a similar Z' width [obtained by replacing J_Z by $J_{Z'}$ in (2.6)] so that the Z and Z' propagators remain finite on resonance.

Having the explicit form for the partial cross section (2.1) one defines the left-right initial-state polarization asymmetry, the forward-backward asymmetry and the polarized forward-backward asymmetry in the following way:

$$A_{LR}^{e^+e^-\to f\bar{f}}(-s)=\frac{\sigma(e^-(L)e^+\to\bar{f}f)-\sigma(e^-(R)e^+\to\bar{f}f)}{\sigma(e^-(L)e^+\to\bar{f}f)+\sigma(e^-(R)e^+\to\bar{f}f)}\ , \tag{2.7}$$

$$A_{FB}^{e^+e^-\to f\bar{f}}(-s)=\frac{\int d\phi\left[\int_0^1-\int_{-1}^0\right]d\cos\theta\,\frac{d\sigma(e^+e^-\to\bar{f}f)}{d\Omega}}{\sigma(e^+e^-\to\bar{f}f)}\ , \tag{2.8}$$

$$A_{FB}^{e^+e^-_L\to f\bar{f}}(-s)=\frac{\int d\phi\left[\int_0^1-\int_{-1}^0\right]d\cos\theta\,\frac{d\sigma(e^-(L)e^+\to\bar{f}f)}{d\Omega}}{\sigma(e^-(L)e^+\to\bar{f}f)}\ , \tag{2.9}$$

with θ the angle between e and f. We also define

$$A_{LR}^{e^+e^-\to\Sigma f\bar{f}}(-s)$$

in the following way:

$$A_{LR}^{e^+e^-\to\Sigma f\bar{f}}(-s)=\frac{\sum_{f\neq e,\nu_e,t}[\sigma(e^-(L)e^+\to\bar{f}f)-\sigma(e^-(R)e^+\to\bar{f}f)]}{\sum_{f\neq e,\nu_e,t}[\sigma(e^-(L)e^+\to\bar{f}f)+\sigma(e^-(R)e^+\to\bar{f}f)]}\ . \tag{2.10}$$

In Eq. (2.7), $f=t$ is not included because of the mixing of helicity amplitudes in the cross sections for final-state top quarks.

Also of interest at SLC/LEP is the τ polarization asymmetry

$$A_{\tau\rm pol}=\frac{\sigma(e^+e^-\to\tau^+\tau^-(L))-\sigma(e^+e^-\to\tau^+\tau^-(R))}{\sigma(e^+e^-\to\tau^+\tau^-(L))+\sigma(e^+e^-\to\tau^+\tau^-(R))}\ . \tag{2.11}$$

On Z resonance this is equal to the left-right polarization asymmetry if e-τ universality holds.

The above quantities can readily be measured in the SLC/LEP experiments. On the Z resonance these asymmetries take on particularly simple forms because the first and third terms in (2.2) are negligible and the Z propagator in the second term drops out of the final expressions for asymmetries (which are *ratios* of cross sections). For example, if we define the following ratio of left- and right-handed couplings of fermion f to the Z at $q^2=-s=-M_Z{}^2$:

$$\mathscr{A}^f=\frac{[(J_Z)^f_L]^2-[(J_Z)^f_R]^2}{[(J_Z)^f_L]^2+[(J_Z)^f_R]^2}\ , \tag{2.12}$$

$$\begin{aligned}A_{LR}^{e^+e^-\to\mu^+\mu^-}(-M_Z{}^2)&=A_{RR}^{e^+e^-\to f\bar{f}}(-M_Z{}^2),\quad f\neq e,\nu_e,t\\&=A_{LR}^{e^+e^-\to\rm hadrons}(-M_Z{}^2)\\&=\mathscr{A}^e\ ,\end{aligned} \tag{2.13}$$

so that initial-state left-right polarization asymmetries to any final-state fermions (except t,e,ν_e) gives information on resonance only about the initial-state electrons.[10] This means that we can use all hadronic data, with the increase in statistics, to measure A_{LR}, the quantity of most interest in this paper.

Similarly, the forward-backward asymmetries factorize

$$A_{FB}^{e^+e^-\to f\bar{f}}(-M_Z{}^2) \simeq \tfrac{3}{4}\mathcal{A}^e\mathcal{A}^f \ , \tag{2.14}$$

$$A_{FB}^{e^+e_L^-\to f\bar{f}}(-M_Z{}^2) \simeq \tfrac{3}{4}\mathcal{A}^f \ . \tag{2.15}$$

In this paper we will assume that all of these asymmetries have been calculated in the GWS model with a Higgs doublet and three generations of quarks and leptons including all radiative $O(\alpha_{\rm EM})$ corrections—initial- and final-state bremsstrahlung and weak and QED one-loop effects—and that the GWS predictions are known to much better than 1% accuracy. Furthermore, we will assume that the asymmetries could eventually be measured to 1% accuracy. These two statements are of course the object of much controversy in the literature. (There is a small hadronic uncertainty even in purely leptonic processes[11] from the photon vacuum polarization of Fig. 3.) Also, we will be interested in the forward-backward asymmetries for $e^+e^-\to c\bar{c}$ and $e^+e^-\to b\bar{b}$ with and without electron polarization. Although a measurement of the asymmetry to b quarks to high accuracy seems feasible, an accurate measurement of the asymmetry to c quarks could be very difficult because of the contamination of c due to b decay. We use the 1% accuracy figure here as a *goal* in measurement; the reader should be forewarned that the true experimental accuracy will only be known when the experiments are actually done. Also, theoretical uncertainties in the hadronization of final-state quarks might result in large uncertainties. Nevertheless we will assume that the various asymmetries are known to ± 0.01 in what follows.

This paper will concentrate on the *shifts* of the various asymmetries from their values in the GWS model. Thus we define

$$\delta A_{LR}^{e^+e^-\to f\bar{f}} = A_{LR}^{e^+e^-\to f\bar{f}}\Big|_{\substack{\text{experimentally}\\ \text{measured}}} - A_{LR}^{e^+e^-\to f\bar{f}}\Big|_{\rm GWS} \ , \tag{2.16}$$

$$\delta A_{FB}^{e^+e^-\to f\bar{f}} = A_{AB}^{e^+e^-\to f\bar{f}}\Big|_{\substack{\text{experimentally}\\ \text{measured}}} - A_{LR}^{e^+e^-\to f\bar{f}}\Big|_{\rm GWS} \ , \tag{2.17}$$

and similarly forward-backward asymmetries with left-handed electrons $\delta A_{FB}^{e^+e_L^-\to f\bar{f}}$ and left-right asymmetry to hadrons $\delta A_{LR}^{e^+e^-\to \rm hadrons}$. We imagine that δA_{LR} is due to new physics from beyond the GWS model. We mention three possible sources of such physics.

(i) One-loop radiative corrections due to new scalars and fermions in $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ in which the new particles do not couple directly to light leptons and quarks but only enter in $W^\pm$, Z, and A(photon) self-energies, the so-called "oblique" loop corrections[2] (Fig. 1).

(ii) One-loop radiative corrections due to new scalars and fermions in $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ in which the new particles couple directly to light leptons and quarks; the so-called "direct" corrections[2] (Fig. 2).

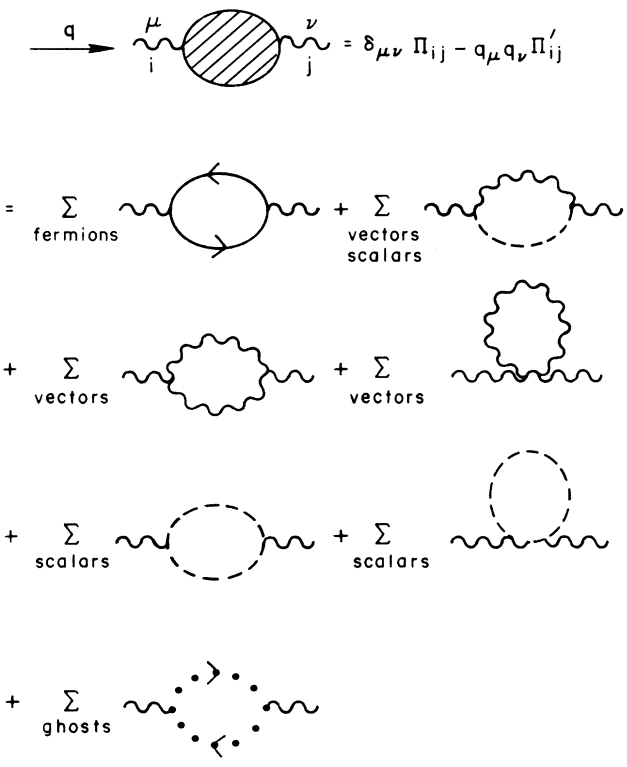

FIG. 1. One-loop radiative correction due to the new scalars and fermions in $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ in which the new particles do not couple to light leptons and quarks; the so-called oblique corrections.

(iii) Physics due to the existence of new gauge bosons in theories which are based on extended gauge structures such as $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$, $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_{Y'}$ or even something more complicated.

We will show in Secs. IV and V that the particular combination of shifts in asymmetries evaluated on Z^0 resonance,

$$\Delta^f = \delta A_{LR}^{e^+e^-\to\mu^+\mu^-}(-M_Z{}^2) - \frac{4}{3}\frac{a_e}{a_f}\delta A_{FB}^{e^+e_L^-\to f\bar{f}} \ , \quad f=b,c,\mu \ , \tag{2.18}$$

and the *definition* in the GWS model

$$\sin^2\theta_W\cos^2\theta_W \equiv \frac{\pi\alpha}{\sqrt{2}G_\mu M_Z{}^2(1-0.06)} \ , \tag{2.19}$$

and a_f and a_e calculable in the GWS model

$$a_f = \frac{-4\sin^2\theta_W I_{3L}^f(Q^f)^2(I_{3L}^f - \sin^2\theta_W Q^f)}{[(I_{3L}^f - \sin^2\theta_W Q^f)^2 + (-Q^f\sin^2\theta_W)^2]^2}$$
$$\simeq \begin{cases} -3.8, & u \text{ quark} \ , \\ -0.71, & d \text{ quark} \ , \\ -7.5, & e \ , \end{cases} \tag{2.20}$$

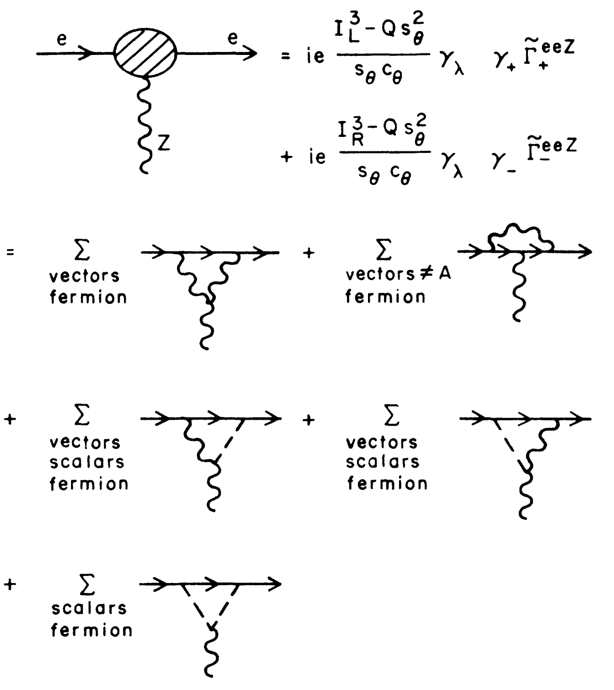

FIG. 2. One-loop radiative corrections due to the new scalars and fermions in $SU(2)_L \times U(1)_Y$ in which the new particles couple directly to light leptons and quarks; the so-called direct corrections.

for $M_Z = 94$ GeV is insensitive to the physics (i) and that a nonzero value for Δ^f is a clear signal that some new undiscovered particle couples directly to e, μ, c, or b; e.g., that physics (ii) or (iii) is operative. We will further show that the quantity Δ^b/Δ^c depends only on the quantum numbers of b, c, and e under the new gauge and further that its value can be used to distinguish between gauge groups.

So far we have concentrated entirely on the s-channel neutral-current processes. It must be emphasized that (2.2) may be used to calculate *any* neutral-current process. For example, the polarized Bhabha scattering cross section is easily written:

$$\frac{d\sigma}{d\Omega}(e^+e^-(L)\to e^+e^-) = \frac{s}{4\pi}[k_{LL}{}^2 \,|\, \mathcal{M}(-s)_{LL}^{ee} + \mathcal{M}(-t)_{LL}^{ee} \,|^2 + k_{LR}{}^2 \,|\, \mathcal{M}(-s)_{LR}^{ee} \,|^2 + |\, \mathcal{M}(-t)_{LR}^{ee} \,|^2] \; . \tag{2.21}$$

The dominant weak effects on Z^0 resonance in Bhabha scattering occur for large angle e's and, if e-μ universality holds, these should be the same as for final $\mu^+\mu^-$ pairs, which will be discussed extensively in this paper. We therefore will not discuss Bhabha scattering further but it should be remembered that this process could give bounds which can also be used to constrain enlarged gauge groups.

Similarly, low-energy neutral-current neutrino scattering is easily written in terms of (2.2); this is important in understanding the limits on $M_{Z'}$ from present neutral-current data (see Durkin and Langacker in Ref. 12). In the future, CHARM II will measure low-energy $\nu_\mu e$ scattering, thus avoiding hadronic uncertainties. Of course the processes $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ are easily written in terms of $\mathcal{M}(-t)_{LR}^{\nu e}$ and $\mathcal{M}(-t)_{LL}^{\nu e}$ and so our analysis is easily extended to this case.

We now address four-fermion charged-current processes. It is clearly simple to write an effective charged-current matrix element in analogy with (2.2) in terms of the charged-current J_W and $W^\pm$ mass and $W^\pm$ width.[2] In the case of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ we would obviously add a second charged-current $J_{W'}$ and $W'^\pm$ mass and width. Thus our analysis will suffice for all four-fermion charged-current processes as well.

The purpose of this paper is the following. We will first identify the full set of parameters describing the interaction of fermions and vector bosons in an extended group gauge theory after spontaneous symmetry breaking. We will keep α, G_μ, M_Z fixed by experiment. Note that M_Z is not allowed to vary with the other parameters; we will use the value $M_Z = 94$ GeV in the numerical work. We will also choose $M_{Z'}$ as an input parameter (the second mass scale). We will then *calculate* the neutral and charged currents J_Z, $J_{Z'}$, J_W, $J_{W'}$ as functions of the parameters α, G_μ, M_Z, $M_{Z'}, \ldots$ [where the ellipses represent other parameters of $O(1)$] thereby allowing precise experimental determinations of neutral- and charged-current processes such as $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ to give constraints on, e.g., $M_{Z'}$. Note that we will *not* use the charged-current masses M_W, $M_{W'}$ as input parameters but rather *calculate* them also as functions of α, G_μ, M_Z, $M_{Z'}, \ldots$. This will allow a precise experimental determination of M_W to separately constrain the extended gauge theory.

III. $SU(2)_L \times U(1)_Y$ GAUGE STRUCTURE

The purpose of this section is primarily to orient the reader to our method and notation so that our treatment of enlarged gauge structures will be more transparent. In the $SU(2)_L \times U(1)_Y$ model the interaction of the gauge bosons with fermions is given by the interaction Lagrangian (we suppress Lorentz four-vector indices μ in the currents J_μ):

$$\mathcal{L} = g_L J_{+L} W^- + g_L J_{-L} W^+ + g_L J_{3L} W_{3L} + g_Y J_Y B \tag{3.1}$$

with g_L, $W^\pm$, W_3 the $SU(2)_L$ coupling constant and gauge fields and g_Y and B those for the $U(1)_Y$ hypercharge group. The currents are

$$J_{+L} = \frac{1}{\sqrt{2}} \bar{\psi}\gamma_\mu I_{+L} \psi \; , \tag{3.2}$$

$$J_{3L} = \bar{\psi}\gamma_\mu I_{3L} \psi \; , \tag{3.3}$$

$$J_Y = \bar{\psi}\gamma_\mu \frac{Y}{2} \psi \; , \tag{3.4}$$

and $J_{-L} = J_{+L}^\dagger$. Fermions ψ have a definite helicity; $I_{\pm L}$ are the isospin-raising and -lowering operators; I_{3L} and Y are the operators for the third component of isospin and hypercharge, respectively. Following the notation of Sec. II we write

$$(J_{3L})_L^b = -\tfrac{1}{2}, \quad (J_{3L})_R^b = 0 , \tag{3.5}$$

$$(J_Y)_L^b = \tfrac{1}{6}, \quad (J_Y)_R^b = -\tfrac{1}{3} , \tag{3.6}$$

with obvious extension to other fermions $e, \mu, c, \ldots$. In order to completely define the matrix elements arising from (3.1) we are missing only the $W^{\pm}$ and Z masses. These of course come from the Higgs gauge-boson coupling sector in which the ith scalar develops a vacuum expectation value (VEV) $\langle \phi_i \rangle$:

$$\begin{aligned}\mathscr{L} &\sim \sum_i \langle \, | \mathscr{D}_\mu \phi_i |^2 \rangle \\ &= \sum_i \left\langle \left| \left[g_L I_{3L} W_{3L} + g_Y \frac{Y}{2} B \right] \phi_i \right|^2 \right\rangle \\ &\quad + \sum_i g_L{}^2 \langle \phi_i (\mathbf{I}_L{}^2 - I_{3L}{}^2) \phi_i \rangle W^+ W^- .\end{aligned} \tag{3.7}$$

The identity of the photon is supplied by the equations

$$Q = I_{3L} + \frac{Y}{2} , \tag{3.8}$$

$$Q \, | \phi_i \rangle = 0 \quad \text{for } \langle \phi_i \rangle \neq 0 , \tag{3.9}$$

and so clearly

$$M_W{}^2 = g_L{}^2 (\langle \mathbf{I}_L{}^2 \rangle - I_{3L}{}^2 \rangle) , \tag{3.10}$$

$$M_Z{}^2 = 2 (g_L{}^2 + g_Y{}^2) \langle I_{3L}{}^2 \rangle , \tag{3.11}$$

with definitions

$$\langle I_{3L}{}^2 \rangle = \sum_i \langle \phi_i I_{3L}{}^2 \phi_i \rangle , \tag{3.12}$$

$$\begin{aligned}\langle \mathbf{I}_L{}^2 \rangle &= \sum_i \langle \phi_i \mathbf{I}_L{}^2 \phi_i \rangle \\ &= \sum_i \langle \phi_i I_L (I_L + 1) \phi_i \rangle .\end{aligned} \tag{3.13}$$

Clearly, then, all fermion—gauge-boson processes can be written in terms of the four-parameter set (besides fermion masses and mixing angles)

$$g_L, \; g_Y, \; \langle \mathbf{I}_L{}^2 \rangle, \; \langle I_{3L}{}^2 \rangle . \tag{3.14}$$

These must be written in terms of experimentally measured quantities in order to define the model. We choose the set

$$\alpha, \; G_\mu, \; M_Z, \; \rho_L . \tag{3.15}$$

α and G_μ are the best-known electroweak parameters of nature. M_Z will be measured to $\pm 0.1\%$ by LEP/SLC. The parameter

$$\rho_L = 1 + \frac{\langle \bar{I}_L{}^2 \rangle - 3 \langle I_{3L}{}^2 \rangle}{2 \langle I_{3L}{}^2 \rangle} \tag{3.16}$$

is different from 1 at the tree level only if Higgs fields which are not $SU(2)_L$ doublets develop a VEV. In the case where only Higgs doublets get VEV's, there is an additional *global* $SU(2)_L \times SU(2)_R$ custodial isospin symmetry at the tree level in the effective low-energy Lagrangian for fermion—gauge-boson interactions, and so $\rho_L = 1$. It is known experimentally that $\rho_L \simeq 1$ to ± 0.05 and so we will treat $\rho_L - 1$ as a small parameter from now on.

It is now a simple matter to write the currents in terms of the set (3.15). We have

$$\mathscr{L} = J_W W_+ + J_W^\dagger W_- + J_{\rm EM} A + J_Z Z \tag{3.17}$$

with $J_{\rm EM}$ as before and

$$J_Z = e \frac{M_Z \rho_L{}^{1/2}}{A_0} \left[J_{3L} - \frac{e^2}{g_L{}^2} J_Q \right] , \tag{3.18}$$

$$\frac{e^2}{g_L{}^2} = \frac{1}{2} \left[1 - \left[1 - \frac{4 A_0{}^2}{M_Z{}^2 \rho_L} \right]^{1/2} \right] , \tag{3.19}$$

$$A_0{}^2 = \frac{\pi \alpha}{\sqrt{2} G_\mu (1 - 0.06)} \simeq (38.7 \text{ GeV})^2 , \tag{3.20}$$

$$J_W = g_L J_+ , \tag{3.21}$$

and

$$\frac{e^2}{g_Y{}^2} = \left[1 - \frac{e^2}{g_L{}^2} \right] . \tag{3.22}$$

Note that as $\rho_L \to 1$, $e^2/g_L{}^2$ goes to the GWS value of $\sin^2\theta_W$ in Eq. (2.19) (a number which can be calculated knowing only α, G_μ, and M_Z) and, of course, $e^2/g_Y{}^2$ goes to $\cos^2\theta_W$.

We now discuss the factor of 1—0.06 appearing in Eqs. (3.20) and (2.19) which comes from one-loop radiative corrections. This large correction is due to the renormalization of $\alpha_{\rm EM}$ from $q^2 = 0$ to $q^2 = -M_Z{}^2$ (where experiments are to be done) from the QED vacuum-polarization graphs of Fig. 3. This is a universal one-loop quantum correction in any unified electroweak gauge theory containing QED. We therefore define our Born terms (2.2) to include it.

In order to understand experimentally the small effects due to new gauge structures considered in this paper, we must understand all effects of $O(1\%)$ which might affect the asymmetries. The GWS one-loop radiative corrections to these asymmetries have been calculated,[13] but what about shifts in the asymmetries from their GWS value due to the existence of new particles [mirror fermions, supersymmetric (SUSY) particle content, etc.]

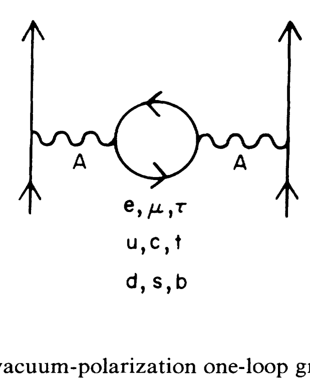

FIG. 3. The vacuum-polarization one-loop graphs for QED.

transform under $SU(2)_L \times U(1)_Y$ with quantum numbers I_{3L} and Q^2. These effects might be mistaken for the existence of new gauge structures when in fact only $SU(2)_L \times U(1)_Y$ is operative. These corrections have also been calculated[2] and are divided into two classes.

(i) Oblique corrections in which the new scalars and fermions couple only to vector particles A, Z, and $W^\pm$ self-energies as in Fig. 2. It has been shown that the effects of oblique corrections on neutral- and charged-current processes can all be thought of as renormalizing the various coupling constants. In particular, for SLC/LEP physics their effect is to change the Z current:

$$J_Z = c\left[J_{3L} - \frac{e^2}{g_L{}^2}J_Q\right] \Longrightarrow (c+\delta c)\left\{J_{3L} - \left[\frac{e^2}{g_L{}^2} + \delta\left[\frac{e^2}{g_L{}^2}\right]\right]J_Q\right\} . \quad (3.23)$$

The effects of δc will cancel in SLC/LEP asymmetries which of course are *ratios* of cross sections so the entire effect of oblique corrections for SLC/LEP physics is contained in $\delta(e^2/g_L{}^2)$. The asymmetries on Z resonance (2.13)—(2.15) will thus be shifted by small amounts

$$\delta\mathscr{A}^f = a_f \delta\left[\frac{e^2}{g_L{}^2}\right] \quad (3.24)$$

with a_f calculated in (2.20). Thus shifts in SLC/LEP asymmetries due to oblique $SU(2)_L \times U(1)_Y$ corrections will all be proportional to each other no matter what representations of scalars and fermions are responsible. For example, the small shifts

$$\delta\mathscr{A}^b \simeq \frac{a_b}{a_e}\delta\mathscr{A}^e , \quad (3.25)$$

so that the quantity is

$$\Delta^b = \delta A_{LR}^{e^+e^- \to \mu^+\mu^-}(-M_Z{}^2) - \frac{4}{3}\frac{a_e}{a_b}\delta A_{FB}^{e^+e_L^- \to b\bar{b}}(-M_Z{}^2) = 0 \quad (3.26)$$

for all oblique radiative corrections due to any new imagined scalar or fermion particles in $SU(2)_L \times U(1)_Y$. Similarly Δ^c defined in (2.18) with final-state c quarks is insensitive to oblique corrections. Oblique one-loop quantum corrections tend to be very small ($<\frac{1}{2}\%$) unless they break the global $SU(2)_L \times SU(2)_R$ isospin symmetry (which kept $\rho_L = 1$ at the tree level for Higgs doublets) and thus feed into the ρ_L parameter at the one-loop level. This can occur, e.g., via a new fermion doublet $\binom{u}{d}$ whose Yukawa couplings generate a large mass splitting $m_u \gg m_d$ after local symmetry breaking. When this doublet is included in the one-loop vector particle self-energies the effects can blow up quadratically like $\sim\alpha(m_u{}^2 - m_d{}^2)/M_Z{}^2$. In fact, large mass splitting within any representation of $SU(2)_L$ can lead to large corrections; otherwise quantum corrections tend to be small. All of these effects have been analyzed for $SU(2)_L \times U(1)_Y$ (Ref. 2). We will need this intuition about global-symmetry breaking and the size of radiative corrections in the next sections when we comment on quantum loop corrections in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

(ii) Direct corrections in which new particles couple directly to e, μ, b, c fermions such as in Fig. 3. Examples are corrections due to SUSY scalar electrons, and gauginos. Of course these cannot all be absorbed into $\delta(e^2/g_L{}^2)$ and so the combinations $\Delta^{c,b}$ will not be zero for direct corrections although they tend to be small since they do not diverge as the masses of new particle in internal loops $m^2 \gg |q^2|$ and they do not break global isospin badly. We will show in the next section that $\Delta^{c,b}$ are also nonzero for corrections due to new gauge structures and that they can be large in that case.

It is easy to *calculate* the $W^\pm$ mass in terms of the set (3.15). The result is, of course,

$$M_W{}^2 = \rho_L\left[1 - \frac{e^2}{g_L{}^2}\right]M_Z{}^2 = \frac{\rho_L}{2}\left[1 + \left[1 - \frac{4A_0{}^2}{M_Z{}^2\rho_L}\right]^{1/2}\right]M_Z{}^2 , \quad (3.27)$$

with ρ_L given in Eq. (3.16). Note that M_W is not a free parameter of the theory. In (3.27) we have included the largest radiative correction in those from Fig. 3.

IV. $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ GAUGE SYMMETRY

A. Currents

In this section we will study a theory with an extra $U(1)_{Y'}$ gauge symmetry and with the symmetry-breaking pattern which preserves the charge relation $Q = I_{3L} + Y/2$ of the Weinberg-Salam theory. This gauge symmetry is interesting, since it can arise from string theories[6] as an effective low-energy symmetry.[5] The extension to other symmetry-breaking patterns with different relations for the charge within this gauge group is obvious.

The charged currents are the same as in Sec. III. However, the neutral currents have a new form. The part of the Lagrangian which includes neutral currents has the form

$$\mathscr{L} = g_L J_{3L} W_{3L} + g_Y J_Y B + g_{Y'} J_{Y'} B' , \quad (4.1)$$

where $g_{Y'}$, $J_{Y'}$, and B' are the coupling constant, current, and gauge field of the new $U(1)_{Y'}$. The current

$$J_{Y'} = \bar{\psi}\gamma_\mu \frac{Y'}{2}\psi \quad (4.2)$$

includes the new hypercharge operator $Y'/2$. A simple extension of our notation in analogy with Eq. (3.6) would have us write $(J_{Y'})_L^e = \frac{1}{2}Y'_{e_L}$ with Y'_{e_L} the hypercharge of left-handed electrons under the new $U(1)_{Y'}$. In the string model, $Y'_{e_L} = \frac{1}{3}$. There are now three neutral gauge bosons and their masses are obtained by studying the Higgs-boson—gauge-boson coupling with Higgs VEV's $\langle\phi_i\rangle$:

$$\sum_i \langle |\mathscr{D}_\mu \phi_i|^2 \rangle = \left\langle \left| \left[g_L I_{3L} W_{3L} + g_Y \frac{Y}{2} B + g_{Y'} \frac{Y'}{2} B' \right] \phi_i \right|^2 \right\rangle . \tag{4.3}$$

Now use

$$Q = I_{3L} + \frac{Y}{2} , \tag{4.4}$$

$$Q \,|\phi_i\rangle = 0 \quad \text{for } \langle \phi_i \rangle \neq 0 . \tag{4.5}$$

The first relation identifies the photon while the second ensures that $U(1)_{EM}$ will be unbroken and the photon

$$A = (g_L{}^2 + g_Y{}^2)^{-1/2} (g_Y W_{3L} + g_L B) \tag{4.6}$$

remains massless. Then we have

$$\sum_i \langle |\mathscr{D}_\mu \phi_i|^2 \rangle = \left\langle \left| \left[I_{3L}(g_L W_{3L} - g_Y B) + g_{Y'} \frac{Y'}{2} B' \right] \phi_i \right|^2 \right\rangle , \tag{4.7}$$

so that in the basis $(g_L{}^2+g_Y{}^2)^{-1/2}(g_L W_{3L} - g_Y B)$ and B' the neutral mass matrix is

$$M^2 = 2 \begin{bmatrix} (g_L{}^2+g_Y{}^2)\langle I_{3L}{}^2 \rangle & (g_L{}^2+g_Y{}^2)^{1/2} g_{Y'} \left\langle I_{3L} \frac{Y'}{2} \right\rangle \\ (g_L{}^2+g_Y{}^2)^{1/2} g_{Y'} \left\langle I_{3L} \frac{Y'}{2} \right\rangle & g_{Y'}{}^2 \left\langle \frac{Y'^2}{4} \right\rangle \end{bmatrix} . \tag{4.8}$$

The two physical eigenstates Z and Z' and masses M_Z, $M_{Z'}$ are obtained by diagonalizing (4.8). The Z' is a new massive neutral gauge boson which we take heavier than the Z: $M_{Z'} > M_Z$. In analogy with (3.12) and (3.13) we have defined

$$\left\langle I_{3L} \frac{Y'}{2} \right\rangle = \sum_i \left\langle \phi_i I_{3L} \frac{Y'}{2} \phi_i \right\rangle , \tag{4.9}$$

$$\left\langle \frac{Y'^2}{4} \right\rangle = \sum_i \left\langle \phi_i \frac{Y'^2}{4} \phi_i \right\rangle , \tag{4.10}$$

where the summation is over all Higgs fields with nonvanishing VEV's $\langle \phi_i \rangle$. It is clear that all the interactions of fermions and gauge bosons for *any* $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ theory with any Higgs structure are given in terms of *seven* parameters. [Here we assume zero at the tree level a possible $U(1)_Y \times U(1)_{Y'}$ mixing term $F_{\mu\nu} F'_{\mu\nu}$ with $F_{\mu\nu}$ and $F'_{\mu\nu}$ the field strengths of the B and B'. The coefficient of this, if included, would be the eighth parameter. Such a term would of course appear at one-loop unless there was imposed some global symmetry to prevent it.] The seven parameters are

$$g_L,\ g_Y,\ g_{Y'},\ \rho_L,\ \langle I_{3L}{}^2 \rangle ,\quad \left\langle I_{3L} \frac{Y'}{2} \right\rangle,\ \left\langle \frac{Y'^2}{4} \right\rangle . \tag{4.11}$$

Basically, these are the three gauge couplings, $W^\pm$ mass, and three entries in the 2×2 Z-Z' mass matrix. We replace these by the seven-parameter set

$$\alpha,\ G_\mu,\ M_Z,\ \rho_L ,\quad \epsilon = \frac{M_Z{}^2}{M_{Z'}{}^2},\ \frac{g_{Y'}}{g_Y},\ \rho_{Y'} , \tag{4.12}$$

with ρ_L as in Sec. III and $\rho_{Y'}$ being

$$\rho_{Y'} = \frac{\langle I_{3L} Y'/2 \rangle}{\langle I_{3L}{}^2 \rangle} , \tag{4.13}$$

a measure of the Z-Z' mixing; this parameter will be very important for seeing effects of the heavy Z' while doing experiments on Z resonance. Once the fermion representation under the gauge group is chosen the theory is completely determined by the four quantities (3.15) which determine the $SU(2)_L \times U(1)_Y$ model and the new *three* parameters:

$$\epsilon,\ \frac{g_{Y'}}{g_Y},\ \rho_{Y'} . \tag{4.14}$$

We now rewrite all of the neutral and charged currents (and matrix elements) in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ in terms of this set of parameters. The 2×2 Z-Z' mass matrix is diagonalized by the unitary matrix

$$U = \begin{bmatrix} \cos\theta_N & -\sin\theta_N \\ \sin\theta_N & \cos\theta_N \end{bmatrix}$$

with

$$\tan\theta_N = \frac{-2\gamma_Z}{(\beta_Z - 1) + [(\beta_Z - 1)^2 + 4\gamma_Z{}^2]^{1/2}} , \tag{4.15}$$

with

$$\gamma_Z = -\frac{g_{Y'}}{g_Y} \frac{e}{g_L} \rho_{Y'} , \tag{4.16}$$

$$\beta_Z = \frac{1}{2} \left\{ \frac{1}{\epsilon} + \epsilon + \left[\left[\frac{1}{\epsilon} - \epsilon \right]^2 - 4\gamma_Z{}^2 \frac{(\epsilon+1)^2}{\epsilon} \right]^{1/2} \right\} . \tag{4.17}$$

We get the ratio e/g_L by solving the algebraic equation

$$\left[\frac{e^2}{g_L{}^2} \right] \left[1 - \frac{e^2}{g_L{}^2} \right] = \frac{A_0{}^2}{2 M_Z{}^2 \rho_L} \{ \beta_Z + 1 - [(\beta_Z - 1)^2 + 4\gamma_Z{}^2]^{1/2} \} . \tag{4.18}$$

The currents are

$$J_Z = c\left[J_{3L} - \frac{e^2}{g_L{}^2}J_Q + \frac{g_{Y'}}{g_Y}\frac{e}{g_L}\tan\theta_N J_{Y'}\right] , \quad (4.19)$$

$$J_{Z'} = c\left[-\tan\theta_N\left[J_{3L} - \frac{e^2}{g_L{}^2}J_Q\right] + \frac{g_{Y'}}{g_Y}\frac{e}{g_L}J_{Y'}\right] , \quad (4.20)$$

with the overall constant

$$c = \frac{e\cos\theta_N}{\frac{e}{g_L}\left[1 - \frac{e^2}{g_L{}^2}\right]^{1/2}} \quad (4.21)$$

and $e^2/g_L{}^2$ is the solution of Eq. (4.18).

The parameter ϵ is always smaller than 1 and actually has a strict upper bound which is determined by noticing that the diagonal elements of the Hermitian Z-Z' mass matrix are real. Thus

$$\beta_Z = \frac{g_{Y'}{}^2}{g_L{}^2 + g_Y{}^2}\frac{\langle Y'^2/4\rangle}{\langle I_{3L}{}^2\rangle} \quad (4.22)$$

is real. One can show that this bound is always stronger than the bound which arises from the constraint that $M_Z{}^2 M_{Z'}{}^2 > 0$ and is of the form

$$\epsilon \equiv \frac{M_Z{}^2}{M_{Z'}{}^2} \le [(1+\gamma_Z{}^2)^{1/2} - |\gamma_Z|]^2 \le 1 , \quad (4.23)$$

with γ_Z defined by Eq. (4.16). Therefore Eq. (4.23) has an interesting feature that for each particular model there is a lower bound on the value of $M_{Z'}$ arising simply from the self-consistency of the model. Note that if $\langle I_3 Y'\rangle = 0$ (for example, if the Higgs fields with nonvanishing VEV's have at least one of the quantum numbers I_3, Y' zero) $\gamma_Z = 0$ and the constraint (4.23) becomes trivial.

In order to complete the discussion of all four-fermion charged- and neutral-current processes, we need to calculate the $W^{\pm}$ mass:

$$M_W{}^2 = \rho_L\left[1 - \frac{e^2}{g_L{}^2}\right](M_Z{}^2\cos^2\theta_N + M_{Z'}{}^2\sin^2\theta_N) , \quad (4.24)$$

with $e^2/g_L{}^2$ from Eq. (4.18) and $\tan\theta_N$ from Eq. (4.15). Note that we have *calculated* M_W as a function of the parameters in (4.12); it is *not* a free parameter. Furthermore, the $W^{\pm}$ current J_W is still given by (3.21) with g_L given by (4.18) so all charged-current processes are now calculable.

For $\epsilon \ll 1$ the theory reduces to the $SU(2)_L \times U(1)_Y$ theory with corrections of order ϵ. In this case θ_N and $e^2/g_L{}^2$ are determined through

$$\tan\theta_N = -\gamma_Z\epsilon + O(\epsilon^2) , \quad (4.25)$$

$$\frac{e^2}{g_L{}^2}\left[1 - \frac{e^2}{g_L{}^2}\right] = \frac{A_0{}^2}{M_Z{}^2\rho_L}[1 - \gamma_Z{}^2\epsilon + O(\epsilon^2)] . \quad (4.26)$$

Thus, as $\epsilon \to 0$, $\theta_N \to 0$ and the $SU(2)_L \times U(1)_Y$ model is recovered.

The value of $g_{Y'}/g_Y$ is undetermined in general. However, in string theories with the grand unified gauge group E_6 the relationship between the coupling constants determines $g_{Y'} = g_Y$ at some mass scale.

The value of the parameter $\rho_{Y'}$ depends on the particular representations and magnitudes of the vacuum expectation values of the Higgs fields. In general, $\rho_{Y'}$ is of order 1. In particular, for the model based on the string theory $\rho_{Y'}$ can assume a range of values from $-\frac{4}{3}$ to $+\frac{1}{3}$. In this theory with quantum numbers (I, Y, Y') the two doublets $H \sim (\frac{1}{2}, -1, \frac{1}{3})$ and $H' \sim (\frac{1}{2}, 1, \frac{4}{3})$ contribute to σ_Y in a way that $\rho_Y \to \frac{1}{3}$ when $\langle H'\rangle \to 0$ and $\rho_Y \to -\frac{4}{3}$ when $\langle H\rangle \to 0$ (Ref. 14).

In the following subsection, we will be studying the response of the various asymmetries to the deviation from $SU(2)_L \times U(1)_Y$. These will be quite small and the reader may worry that we should properly include one-loop quantum corrections in the full $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ theory in order to fully understand the response to the new gauge group at the $\sim 1\%$ level. We now address the question of radiative corrections in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$.

We will consider here only oblique corrections. Imagine that we want to write down the effect of some new fermions and scalars in the extended gauge group which enter as oblique quantum corrections as in Fig. 3. These particles have quantum numbers I_{3L}, Q, Y' and couple via the parameters discussed in (4.12). There is, however, a decoupling theorem, good at the tree and one-loop levels,[15,16] which says

$$SU(2)_L \times U(1)_Y \times U(1)_{Y'} \underset{\frac{M_Z}{M_{Z'}} \to 0}{\longrightarrow} SU(2)_L \times U(1)_Y . \quad (4.27)$$

Thus, the oblique quantum corrections to the deviation from GWS of some asymmetry A at LEP/SLC which is evaluated at low energy $q^2 \simeq -M_Z{}^2$ can be separated into two parts

$$\delta A\,|_{\text{oblique}/SU(2)_L \times U(1)_Y \times U(1)_{Y'}} = \delta A\,|_{\text{oblique}/SU(2)_L \times U(1)_Y} + O\left[\frac{\alpha}{\pi}\frac{M_Z{}^2}{M_{Z'}{}^2}\right] . \quad (4.28)$$

If we are willing to drop the $O((\alpha/\pi)M_Z{}^2/M_{Z'}{}^2)$ terms (as we will in this paper; they will be studied later[16]) we may compute all oblique quantum corrections by studying the transformation properties of the new and old scalars and fermions under $SU(2)_L \times U(1)_Y$. To compute these, we need only the parameters listed in Sec. III [in (3.15)] and the particles' quantum numbers I_L, Q. No knowledge of the quantum number Y' is necessary.

Having reduced the calculation of oblique quantum corrections to $SU(2)_L \times U(1)_Y$, we wonder whether such corrections can be large for the particles which enter naturally in the extended gauge group theory. These corrections have been studied extensively elsewhere.[2,8,15] As discussed in Sec. III such quantum corrections are large when they contribute to ρ_L at the one-loop level by breaking the custodial global $SU(2)_L \times SU(2)_R$ symmetry. This occurs when there is large mass splitting within a local $SU(2)_L$ representation of scalars or fermions. Clearly, we must introduce new particles (at least new scalars) into a theory with an extended gauge group. The question is will these have large mass splitting within the representations? We might naively expect so since there are two very different scales M_Z and $M_{Z'}$ in the problem; will, for example, the Higgs fields which break the local symmetry at the large scale $M_{Z'}$ transform under the custodial global symmetry into those which break the local symmetry at the lower scale M_Z? We see immediately that if they are to avoid a gauge hierarchy problem they cannot since the new Higgs structure must be engineered such that $SU(2)_L \times U(1)_Y$ is a good local symmetry from the scale $M_{Z'}$ all the way down to M_Z where, of course, it breaks. Thus, a solution to the gauge hierarchy problem in the scalar sector will simultaneously give Higgs representations whose masses respect the custodial global $SU(2)_L \times SU(2)_R$ symmetry and thus quantum corrections from the new Higgs scalars will be small in the extended gauge group. All of this discussion applies to the charged currents and M_W as well.

If the gauge hierarchy problem is unsolved in the extended theory, LEP/SLC asymmetries (or the $W^{\pm}$ mass) could receive oblique quantum corrections of $O((\alpha/\pi)M_{Z'}^2/M_Z^2)$ (Refs. 15 and 16). We will assume in the rest of this paper that the gauge hierarchy problem in the scalar sector for the mass scales M_Z, $M_{Z'}$ has been solved by some means (fine-tuning, supersymmetry) and thus that oblique quantum corrections from the Higgs sector are small. We will therefore display results in this paper for extended gauge groups considering only tree-level effects.

B. Physical implications

The experimental values of the Z width and total cross section, A_{LR} (left-right polarization asymmetries) and A_{FB}'s (forward-backward asymmetries) can be determined from SLC and LEP measurements.[7] The deviation of these values from the GWS theory can thus indicate new gauge structure, i.e., the existence of new currents such as $J_{Y'}$, and can impose a lower bound on $M_{Z'}^2$, for any particular model. The various cross sections, widths and asymmetries can be evaluated by using the definitions in Sec. II and expressions (4.19) and (4.20) for the currents. The asymmetries are studied for a range of parameter space and are presented in Figs. 4–8. The calculations are *exact* at the tree level. Note that all asymmetries go to their GWS values as $M_{Z'} \to \infty$.

Figure 4 represents $A_{LR}^{e^+e^- \to \mu^+\mu^-}$ evaluated on the Z resonance as a function of $1/\sqrt{\epsilon} = M_{Z'}/M_Z$. The fermion representations are chosen as suggested by string theories[14] to be those of the 27 of E_6 with quantum numbers (I_{3L}, Y, Y'): $Q \sim (\frac{1}{2}, \frac{1}{3}, -\frac{2}{3})$, $u_R \sim (0, -\frac{4}{3}, -\frac{2}{3})$, $d_R \sim (0, \frac{2}{3}, \frac{1}{3})$, $L \sim (\frac{1}{2}, 1, \frac{1}{3})$, and $e_R \sim (0, -2, -\frac{2}{3})$. The numerical results are given for $M_Z = 94$ GeV, $\rho_L = 1$ and $g_{Y'} = g_Y$, while $\rho_{Y'}$ is chosen for two extreme values $\rho_{Y'} = -\frac{4}{3}$ and $\rho_{Y'} = \frac{1}{3}$ as also suggested from the string theory. The consistency bound (4.16) implies that the theory is defined for $M_{Z'} \gtrsim 2M_Z$ for a wide range of models. One observes that by measuring $A_{LR}^{e^+e^- \to \mu^+\mu^-}$ to within 1% the effects of new gauge structures can either

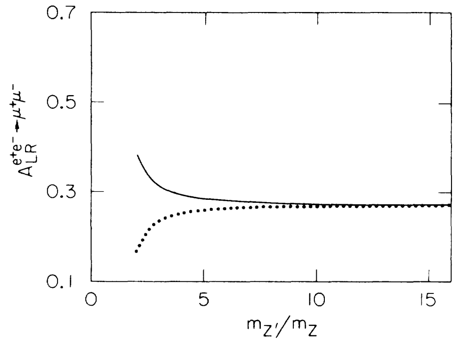

FIG. 4. $A_{LR}^{e^+e^- \to \mu^+\mu^-}$ (and $A_{LR}^{e^+e^- \to \text{hadrons}}$) evaluated on Z resonance as a function of $M_{Z'}/M_Z$ is given for $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ gauge structure with $M_Z = 94$ GeV, $\rho_L = 1$ and for two typical values of $\rho_{Y'} = 0.33$ (solid line) and $\rho_{Y'} = -1.33$ (dotted line).

be seen or the lower limit $M_{Z'}/M_Z \gtrsim 10$ can be imposed for a wide class of models. Even a 10% determination of A_{LR} would set interesting bounds on a new Z' mass $M_{Z'}/M_Z \gtrsim 3$–4 for some models. Note that since $A_{LR}^{e^+e^- \to f\bar{f}}$ (with $f \neq e, \nu_e$) is independent of final states[10] on Z resonance SLC/LEP data including final-state hadrons could be used to study these shifts thus making full use of the increased statistics. These effects would then be visible with relatively few ($\sim 10^4$) Z's when e^- polarization is available at SLC (Ref. 7). Note further that comparison of A_{LR} with $A_{\tau\text{pol}}$ [see Eq. (2.11)] would yield information about the universality of the coupling of new gauge structures to e and τ.

In Fig. 5 we give results for the forward-backward asymmetry without observation of longitudinal polarization in $e^+e^- \to f\bar{f}$ for $\rho_L = 1$, $M_Z = 94$ GeV, $g_{Y'} = g_Y$, $\rho_{Y'} = \frac{1}{3}$ as a function of $M_{Z'}$. The solid lines are for final-state muons, the dashed lines are for final state c quarks, the dotted lines are for final-state b quarks. Note that $A_{FB}^{e^+e^- \to \mu^+\mu^-}$ is much less sensitive to new gauge structures than A_{LR}. This can be remedied in part by

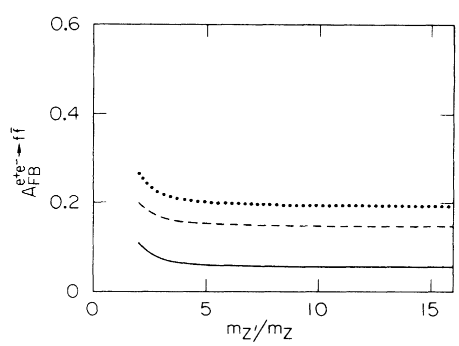

FIG. 5. $A_{FB}^{e^+e^- \to f\bar{f}}$ on Z resonance as a function of $M_{Z'}/M_Z$ for $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ gauge group is given for $f = \mu$ (solid line), $f = c$ (dashed line) and $f = b$ (dotted line). We chose $M_Z = 94$ GeV, $\rho_L = 1$, $\rho_{Y'} = 0.33$.

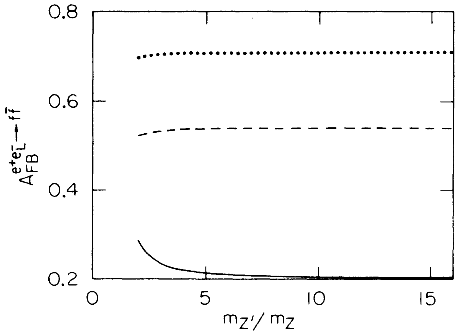

FIG. 6. $A_{FB}^{e^+e^-\to f\bar{f}}$ on Z resonance as a function of $M_{Z'}/M_Z$ for $SU(2)_L\times U(1)_Y\times U(1)_{Y'}$ gauge group is given for $f=\mu$ (solid line), $f=c$ (dashed line) and $f=b$ (dotted line). We chose $M_Z=94$ GeV, $\rho_L=1$, $\rho_{Y'}=0.33$.

forming $A_{FB}^{e^+e_L^-\to\mu^+\mu^-}$ with electron beam polarization. These are displayed in Fig. 6 for final state μ, c, b with the same set of parameters and conventions as in Fig. 5.

Another possibility for seeing effects of the new gauge structure would be in studying the s dependence of the various asymmetries and, in particular, the slope near $s\simeq M_Z{}^2$. This is plotted for $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ in Fig. 7 with dotted, dotted-dashed, dashed, and solid lines corresponding to $M_{Z'}/M_Z=2.5$, 3.0, 3.5, ∞, respectively; the solid line is clearly the GWS tree-level prediction. Here we have used $\rho_{Y'}=\frac{1}{3}$, $M_Z=94$ GeV, $\rho_L=1$ and $g_{Y'}=g_Y$. Note that the slope depends substantially on the presence of the new currents via their interference with the photon-exchange diagrams because the $SU(2)_L\times U(1)_Y$ vector couplings of e, μ to the Z is suppressed by the factor

$$4e^2/g_L{}^2-1\simeq 4\sin^2\theta_W-1\ .$$

In Fig. 8 we plot

$$A_{LR}^{e^+e^-\to\Sigma f\bar{f}}\ ,$$

$f\neq e$, ν_e,t as a function of $\sqrt{s}$ including the leading QCD corrections for final-state hadrons. The dependence of the

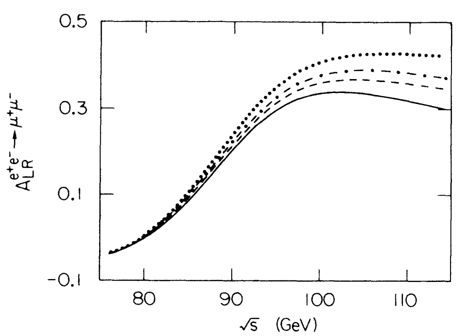

FIG. 7. $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ as a function of $\sqrt{s}$ for $SU(2)_L\times U(1)_Y\times U(1)_{Y'}$ gauge group is given for $M_{Z'}/M_Z=2.5$ (dotted line), 3.0 (dotted-dashed line), 3.5 (dashed line), ∞ (solid line—standard model). We choose $M_Z=94$ GeV, $\rho_L=1$, $\rho_{Y'}=0.33$.

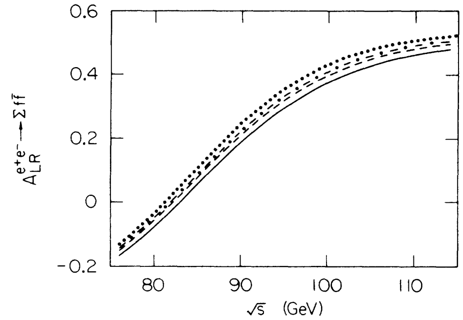

FIG. 8. $A_{LR}^{e^+e^-\to\Sigma f\bar{f}}$ with $f\neq e,\nu_e,t$ as a function of $\sqrt{s}$ for $SU(2)_L\times U(1)_Y\times U(1)_{Y'}$ is given presented for $M_{Z'}/M_Z=2.5$ (dotted line), 3.0 (dotted-dashed line), 3.5 (dashed line), ∞ (solid line). $M_Z=94$ GeV, $\rho_L=1$, $\rho_{Y'}=0.33$. The leading corrections from QCD are taken into account.

slope near Z resonance is somewhat washed out here because the final-state quark vector coupling to the Z is not suppressed. We have also studied the $\sqrt{s}$ dependence of forward-backward asymmetries for individual final-state fermions $A_{FB}^{e^+e^-\to f\bar{f}}$ and $A_{FB}^{e^+e_L^-\to f\bar{f}}$ but did not display it here because the dependence of the slope near Z resonance on new gauge structures is not very pronounced. The most interesting quantity then turns out to be $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ because its slope changes significantly as the value of $M_{Z'}/M_Z$ changes. Therefore the measurement of the initial-state polarization asymmetry into μ pairs around the Z resonance would be a sensitive test of new currents, especially when the mixing angle θ_N is relatively small. Thus, even when $\delta A_{LR}^{\mu}(-M_Z{}^2)\ll 1$, the $\sqrt{s}$ dependence of $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ can be significantly changed due to new contributions from the Y' currents and the Z'-boson exchange.

Finally, we calculate M_W in the $SU(2)_L\times U(1)_Y\times U(1)_{Y'}$ theory and display the results in Fig. 9 for the choice of parameters above. Note that, with a projected experimental error of $\Delta M_W=\pm 50$ MeV it will be possible to either set very strict bounds on $M_{Z'}\lesssim 10M_Z$ (and the other parameters) or see the effects of new gauge structures. Less accurate measurements will be interesting once the precise Z mass is known.[7]

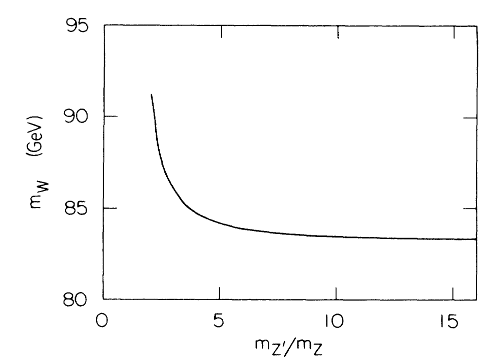

FIG. 9. M_W as a function of $M_{Z'}/M_Z$ with the parameters $M_Z=94$ GeV, $\rho_L=1$, $\rho_{Y'}=-1.33$.

Note that the behavior of, e.g., M_W in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ in Fig. 9 is not the most general; Eq. (4.24) is. It should be remembered that for $\langle I_3 Y' \rangle = 0$ the two neutral heavy boson sectors decouple and we are left with the $SU(2)_L \times U(1)_Y$ results. Thus Figs. 1–10 indicate only a possible outcome of experiments although the most general outcome can be easily extracted from this section.

All of the above calculations were done exactly at the tree level. We now want to study the particular combinations of shifts in asymmetries from their GWS values Δ^b, Δ^c in the approximation that $M_Z^2/M_{Z'}^2 = \epsilon \ll 1$ keeping only the leading terms in $M_Z^2/M_{Z'}^2$ and dropping terms of $O((\alpha/\pi)M_Z^2/M_{Z'}^2)$. The Z current is then

$$J_Z = c\left\{ J_{3L} - \left[\left.\frac{e^2}{g_L^2}\right|_{2\times 1} + \delta\left[\frac{e^2}{g_L^2}\right]\right] J_Q + \frac{M_Z^2}{M_{Z'}^2}\lambda J_{Y'} \right\}, \tag{4.29}$$

where $(e^2/g_L^2)\,|\,2\times 1$ is the value of e^2/g_L^2 computed in $SU(2)_L \times U(1)_Y$ at the tree level, $\delta(e^2/g_L^2)$ includes oblique quantum corrections in $SU(2)_L \times U(1)_Y$ as well as $O(M_Z^2/M_{Z'}^2)$ corrections to e^2/g_L^2 in $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$. λ is a model dependent $O(1)$ parameter of the extended gauge group. In the theory with an extra $U(1)$ it is

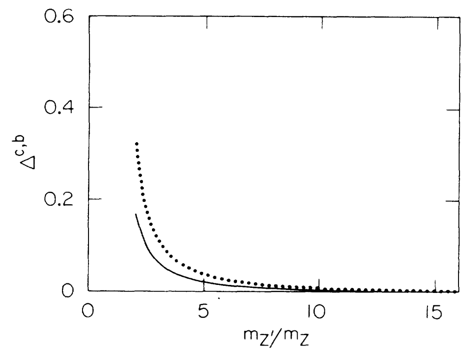

FIG. 10. $\Delta^{c,b}$ (solid line, dotted line) is plotted as a function of $M_{Z'}/M_Z$ for $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ gauge groups. We chose $M_Z = 94$ GeV, $\rho_L = 1$, and $\rho_{Y'} = 0.33$.

$$\lambda = \frac{g_{Y'}}{g_Y}\frac{e}{g_L}\rho_{Y'} . \tag{4.30}$$

Clearly, asymmetries on Z resonance are insensitive to the model-dependent constant c. If we calculate the combinations of shifts in asymmetries Δ^c and Δ^b in Eq. (2.18), these will be insensitive to $\delta(e^2/g_L^2)$ as proved in Sec. III. Thus, neglecting terms of $O(M_Z^4/M_{Z'}^4)$ and $O((\alpha/\pi)M_Z^4/M_{Z'}^4)$ a simple calculation yields

$$\Delta^b = -a_\mu \frac{M_Z^2}{M_{Z'}^2}\lambda \left\{ \sin^2\theta_W \frac{(J_{Y'})_R^b - (J_{Y'})_L^b}{(J_{3L})_L^b} - \frac{(J_{Y'})_R^b}{(J_Q)_R^b} - \sin^2\theta_W \frac{(J_{Y'})_R^e - (J_{Y'})_L^e}{(J_{3L})_L^e} + \frac{(J_{Y'})_R^e}{(J_Q)_R^e} \right\} \tag{4.31}$$

with a similar expression for Δ^c with substitution $b \to c$ in Eq. (4.31). Note that the expression in large curly brackets depends only on the *quantum numbers* Y' of the b quark and electron under the new $U(1)_{Y'}$ gauge group; $\sin^2\theta_W$ and a_μ are calculated in terms of α, G_μ, M_Z alone in Eqs. (2.19) and (2.20) and the J_{3L} and J_Q quantum numbers are known. The only model dependence is in the parameter $M_Z^2/M_{Z'}^2\lambda$. Furthermore, Δ^b is zero unless b or e have Y' quantum numbers. Thus Δ^b is directly sensitive to the new gauge current. [Remember though that we saw in Sec. III that it is also sensitive to the direct quantum corrections of Fig. 3 of $SU(2)_L \times U(1)_Y$.] Thus $\Delta^b \neq 0$ is a clear, unambiguous experimental signal that e^- and/or b couples directly to some new as-yet undiscovered particle.

We plot in Fig. 10 (dotted line) Δ^b from Eq. (4.31) as a function of $M_{Z'}/M_Z$ for $\rho_L = 1$, $\rho_{Y'} = \frac{1}{3}$, $g_{Y'} = g_Y$, and $J_{Y'}$ quantum numbers obtained by requiring that e,b appear in the 27 of E_6 as suggested by string theories. We also plot Δ^c (solid line) there although we expect this to be experimentally more difficult to measure. Those shifts can be huge for $M_{Z'} \simeq 3M_Z$ which is not ruled out by other low-energy experiments. We expect[15] direct $SU(2)_L \times U(1)_Y$ quantum corrections to be small ($\lesssim \frac{1}{2}\%$). Nevertheless, they will be studied elsewhere.[16] If so, observation of such a large Δ^b or Δ^c would probably indicate the existence of a Z' just above LEP/SLC energies. Note that one can easily form Δ^t for the top quark by taking final phase space into account.[17] We expect that for $2M_{\rm top} \lesssim M_Z - 10$ GeV there is enough phase space left so that the results of this section for the various asymmetries to c quarks should be qualitatively good for t quarks as well.

It is easy to form a similar quantity for muons

$$\Delta^\mu = \delta A_{LR} - \tfrac{2}{3}\delta A_{FB}^{e^+e^-_L \to \mu^+\mu^-} . \tag{4.32}$$

The expression for Δ^μ is obtained from Eq. (4.31) by substituting $b \to \mu$ so this would be zero if e-μ universality held for the extended gauge group. The "direct" quantum corrections in $SU(2)_L \times U(1)_Y$ would also largely cancel[15,16] (except small quantum correction "box" diagrams with new heavy particles in virtual states) if e-μ universality held so observation of $\Delta^\mu \neq 0$ would be spectacular, indeed, probably signaling a breakdown of e-μ universality coupling to a new Z'. Remember that there is already a check on such physics: the comparison of A_{LR} and $A_{\tau\rm pol}$ on Z resonance.

It is amusing to imagine that both Δ^b and $\Delta^c \neq 0$ experimentally. The ratio is insensitive to the parameters of the $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ model because the factor $\lambda M_Z^2/M_{Z'}^2$ cancels in the ratio. Thus,

$$\frac{\Delta^b}{\Delta^c}=\frac{\{\ \}_{b\to b}}{\{\ \}_{b\to c}} \tag{4.33}$$

with the curly brackets written in Eq. (4.31). This depends *only* on the quantum numbers of b, c, e under $J_{Y'}$. It is also independent of the symmetry-breaking pattern and the relation $Q=I_3+Y/2$ could also be changed without affecting it. Once the quantum numbers of b, c, e under $J_{Y'}$ are known, it can be calculated with no other information from beyond GWS. For the **27** of E_6 we get

$$\left.\frac{\Delta^b}{\Delta^c}\right|_{27\text{ of }E_6}\simeq 0.57 \ . \tag{4.34}$$

For $M_Z=94$ GeV. Thus, this ratio allows us to probe at SLC/LEP directly for the *quantum numbers* of b, c, e under new gauge groups even if all the new structure is too heavy to produce directly.

We have used e^- beam polarization in Δ^b, Δ^c, Δ^μ in order to avoid factors of $4e^2/{g_L}^2-1\simeq 4\sin^2\theta_W-1$. It is easy to see that we can form similar quantities without beam polarization, all of which will be proportional to Δ^b, Δ^c, or Δ^μ. For example, the following combination of unpolarized forward-backward asymmetries:

$$\begin{aligned}\Delta^f_{\text{unpolarized}}&=\delta A_{FB}^{e^+e^-\to f\bar f}(-{M_Z}^2)\\&\quad-\frac{1}{2}\left[\frac{\mathscr{A}^f}{\mathscr{A}^e}+\frac{a_f}{a_e}\right]\delta A_{FB}^{e^+e^-\to\mu^+\mu^-}(-{M_Z}^2)\\&=-\frac{3}{4}\frac{a_f}{a_\mu}\mathscr{A}^e\Delta^f\end{aligned} \tag{4.35}$$

for $f=b,c,\mu$ with $\mathscr{A}^f,\mathscr{A}^e$ calculated at the tree level in $SU(2)_2\times U(1)$. Unfortunately, a_b/a_μ is a small number (~ 0.1) as is $\mathscr{A}^e$ (~ 0.3). So $\Delta^f_{\text{unpolarized}}$ is quite insensitive to this new physics. We note from the figures that asymmetries without observation of longitudinal polarization are also less sensitive to new physics. *Thus, longitudinal e^- beam polarization is crucial to observation of effects which could reveal the existence of new gauge structures beyond $SU(2)_L\times U(1)_Y$ at SLC/LEP.*

V. $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ GAUGE SYMMETRY

A. Currents

Here, we repeat the study of Sec. IV for left-right-symmetric theories[3,4] with spontaneous-symmetry-breaking patterns which determine the electric charge as $Q=I_{3L}+I_{3R}+(B-L)/2$; the so-called standard one[4,18] with certain interesting phenomenological consequences. Extension to theories with breaking patterns which determine Q in a different way is obvious.

Because of this extended gauge symmetry the charged and neutral currents are changed. The part of the Lagrangian with charged- and neutral-current coupling of fermions to gauge bosons has the form

$$\begin{aligned}\mathscr{L}&=g_LJ_{+L}W_L^-+g_LJ_{-L}W_L^++g_RJ_{+R}W_R^-+g_RJ_{-R}W_R^+\\&\quad+g_LJ_{3L}W_{3L}+g_RJ_{3R}W_{3R}+g_{B-L}J_{B-L}B\ ,\end{aligned} \tag{5.1}$$

where $g_{L,R}$ and $W^\pm_{L,R}$, $W_{3L,3R}$ are the $SU(2)_{L,R}$ gauge coupling constant and gauge fields while g_{B-L} and B are the coupling constant and the gauge field for $U(1)_{B-L}$. There are new neutral and charged currents defined as

$$J_{+L,+R}=\frac{1}{\sqrt{2}}\bar\psi\gamma_\mu I_{+L,+R}\psi\ , \tag{5.2}$$

$$J_{3L,3R}=\bar\psi\gamma_\mu I_{3L,3R}\psi\ , \tag{5.3}$$

$$J_{B-L}=\bar\psi\gamma_\mu\frac{B-L}{2}\psi\ , \tag{5.4}$$

and $J_{-L,-R}=J^\dagger_{+L,+R}$. Here $I_{+L,+R}$, $I_{3L,3R}$, and $B-L$ refer to the isospin-raising operator for $SU(2)_{L,R}$, the third component of the isospin for $SU(2)_{L,R}$ and the quantum number of $U(1)_{B-L}$, respectively.

There are two charged and three neutral gauge bosons whose masses are obtained by studying again the Higgs—gauge-boson coupling with a Higgs-field VEV $\langle\phi_i\rangle$. The relations

$$Q=I_{3L}+I_{3R}+\left[\frac{B-L}{2}\right]\ , \tag{5.5}$$

$$Q\,|\,\phi_i\rangle=0\quad\text{for }\langle\phi_i\rangle\neq 0\ , \tag{5.6}$$

ensure again that $U(1)_{\text{EM}}$ is preserved with photons remaining massless:

$$A=e\left[\frac{W_{3L}}{g_L}+\frac{W_{3R}}{g_R}+\frac{B}{g_{B-L}}\right]\ . \tag{5.7}$$

Here, the electric charge is

$$e=g_{B-L}g_Rg_L[{g_L}^2{g_R}^2+{g_{B-L}}^2({g_L}^2+{g_R}^2)]^{-1/2}\ . \tag{5.8}$$

Using (5.6) and (5.5) one obtains

$$\begin{aligned}\sum_i\langle\,|\mathscr{D}_\mu\phi_i|^2\rangle&=\sum_i\langle\,|[I_{3L}(g_LW_{3L}-g_{B-L}B)+I_{3R}(g_RW_{3R}-g_{B-L}B)]\phi_i|^2\rangle\\&\quad+\sum_i\langle\phi_i({g_L}^2I_{-L}I_{+L}W_L^+W_L^-+g_Lg_RI_{-R}I_{+L}W_R^+W_L^-\\&\qquad+g_Rg_LI_{-L}I_{+R}W_L^+W_R^-+{g_R}^2I_{-R}I_{+R}W_R^+W_R^-)\phi_i\rangle\ .\end{aligned} \tag{5.9}$$

We choose $W_L^\pm$ and $W_R^\pm$ as basis states for the charged sector and the charged mass matrix is

$$\mathscr{M}_W{}^2=\begin{bmatrix}{g_L}^2(\langle\mathbf{I}_L{}^2\rangle-\langle {I_{3L}}^2\rangle) & 2g_Rg_L\langle I_{+L}I_{-R}\rangle\\ 2g_Rg_L\langle I_{-L}I_{+R}\rangle & {g_R}^2(\langle\mathbf{I}_R{}^2\rangle-\langle {I_{3R}}^2\rangle)\end{bmatrix}\ . \tag{5.10}$$

In analogy with Secs. III and IV, where we have defined

$$\langle I_{+L}I_{-R}\rangle=\sum_i\langle\phi_i I_{+L}I_{-R}\phi_i\rangle=\langle I_{-L}I_{+R}\rangle\ , \quad (5.11)$$

$$\langle I_{3L,3R}{}^2\rangle=\sum_i\langle\phi_i I_{3L,3R}{}^2\phi_i\rangle\ , \quad (5.12)$$

$$\langle I_{L,R}{}^2\rangle=\sum_i\langle\phi_i I_{L,R}(I_{L,R}+1)\phi_i\rangle\ . \quad (5.13)$$

For the neutral mass-squared matrix we must choose an orthonormal basis. With basis vectors Z_1 and Z_2

$$\begin{aligned}Z_1&=N_1(g_R W_{3R}-g_{B-L}B)\ ,\\ Z_2&=N_2 g_L W_{3L}-\frac{eN_1}{g_L}(g_{B-L}W_{3R}+g_R B)\ ,\end{aligned} \quad (5.14)$$

with constants

$$\begin{aligned}N_1&=(g_R{}^2+g_{B-L}{}^2)^{-1/2}\ ,\\ N_2N_1&=\frac{e}{g_Lg_Rg_{B-L}}\ ,\end{aligned} \quad (5.15)$$

the neutral mass-squared matrix $\mathcal{M}_{ij}{}^2(i,j=1,2)$ becomes

$$\begin{aligned}M_{11}{}^2&=\frac{2}{N_2{}^2}\langle I_{3L}{}^2\rangle\ ,\\ M_{22}{}^2&=\frac{2}{N_1{}^2}\langle(I_{3R}+g_{B-L}{}^2N_1{}^2I_{3L})^2\rangle\ ,\\ M_{12}{}^2&=M_{21}{}^2\\ &=\frac{2}{N_1N_2}\langle(I_{3R}+g_{B-L}{}^2N_1{}^2I_{3L})I_{3L}\rangle\ ,\end{aligned} \quad (5.16)$$

where we define

$$\langle I_{3L}I_{3R}\rangle=\sum_i\langle\phi_i I_{3L}I_{3R}\phi_i\rangle\ . \quad (5.17)$$

The two physical charged eigenstates W and W' are obtained by diagonalizing (5.10) while the two neutral eigenstates Z and Z' are obtained by diagonalizing (5.16).

Thus, the interactions of fermions and gauge bosons in $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge theory with any Higgs structure is given in terms of *nine* parameters:

$$\begin{aligned}&g_L,\ g_R,\ g_{B-L},\langle I_L{}^2\rangle,\ \langle I_{3L}{}^2\rangle,\ \langle I_R{}^2\rangle\ ,\\ &\langle I_{3R}{}^2\rangle,\ \langle I_{3L}I_{3R}\rangle,\ \langle I_{+L}I_{-R}\rangle\ .\end{aligned} \quad (5.18)$$

Essentially, one has three gauge couplings, three entries in the W-W' mass matrix, and three entries in the Z-Z' mass matrix. We replace them by the following nine-parameter set:

$$\begin{aligned}&\alpha,\ G_\mu,\ M_Z,\ \rho_L,\ \epsilon=\frac{M_Z{}^2}{M_{Z'}{}^2}\ ,\\ &\frac{g_R}{g_L},\ \rho_R,\ \sigma_+,\sigma_3\ .\end{aligned} \quad (5.19)$$

Therefore, in addition to the four quantities (3.15) which determine the $SU(2)_L\times U(1)_Y$ model there are *five* new parameters:

$$\epsilon,\ \frac{g_R}{g_L},\ \rho_R,\ \sigma_+,\sigma_3\ , \quad (5.20)$$

where we introduce the parameters

$$\rho_R=\frac{\langle I_R{}^2\rangle-\langle I_{3R}{}^2\rangle}{2\langle I_{3R}{}^2\rangle}\ , \quad (5.21)$$

$$\sigma_+=\frac{\langle I_{+L}I_{-R}\rangle}{\langle I_{3L}{}^2\rangle}\ , \quad (5.22)$$

$$\sigma_3=\frac{\langle I_{3L}I_{3R}\rangle}{\langle I_{3L}{}^2\rangle}\ . \quad (5.23)$$

Note that all but ϵ in the set (5.20) are $O(1)$ parameters. We now reexpress all the currents (and matrix elements) in the basis of mass eigenstates.

The mixing angle θ_N for the neutral gauge boson is determined by the same equation (4.15) with β_Z and γ_Z given by

$$\gamma_Z=\left[1-\frac{e^2}{g_L{}^2}\right]\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2}\left[\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1}+\frac{g_R{}^2}{g_L{}^2}\sigma_3\right] \quad (5.24)$$

and again [compare with (4.17)]

$$\beta_Z=\frac{1}{2}\left\{\frac{1}{\epsilon}+\left[\left[\frac{1}{\epsilon}-\epsilon\right]^2-4\gamma_Z{}^2\frac{(\epsilon+1)^2}{\epsilon}\right]^{1/2}\right\}. \quad (5.25)$$

The mixing angle of the charged-gauge-boson mass matrix is

$$\tan\theta_+=\frac{-2\gamma_W}{(\beta_W-1)+[(\beta_W-1)^2+4\gamma_W{}^2]^{1/2}}$$

with

$$\beta_W=\frac{g_R{}^2}{g_L{}^2}\rho_R\sigma_R/\rho_L\ , \quad (5.26)$$

$$\gamma_W=\frac{g_R}{g_L}\sigma_+/\rho_L\ . \quad (5.27)$$

Here σ_R is not an independent parameter, but it is actually determined in terms of parameters (5.19) in the following way:

$$\sigma_R=\frac{g_L{}^4}{g_R{}^4}\left\{\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]\left[1-\frac{e^2}{g_L{}^2}\right]^{-2}\beta_Z-2\frac{g_R{}^2}{g_L{}^2}\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1}\sigma_3-\frac{e^4}{g_L{}^4}\left[1-\frac{e^2}{g_L{}^2}\right]^{-2}\right\}. \tag{5.28}$$

Now the ratio $e^2/g_L{}^2$ is determined for heavy right-handed neutrinos by the algebraic equation

$$\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]=\frac{A_0{}^2}{2M_Z{}^2\rho_L}\frac{\beta_Z+1-[(\beta_Z-1)^2+4\gamma_Z{}^2]^{1/2}}{(1-\sigma_+{}^2/4\rho_L\rho_R\sigma_R)} . \tag{5.29}$$

Then, the charged currents assume the form

$$J_W=c_W(J_{+L}+\tan\theta_+J_{+R}) , \tag{5.30}$$

$$J_{W'}=c_W(-\tan\theta_+J_{+L}+J_{+R}) , \tag{5.31}$$

and

$$c_W=g_L\cos\theta_+ . \tag{5.32}$$

Similarly, the neutral currents are of the form

$$J_Z=c_1\left[J_{3L}-\frac{e^2}{g_L{}^2}J_Q\right]+c_2\left[J_{3R}-\frac{e^2}{g_R{}^2}J_Q\right] , \tag{5.33}$$

$$J_{Z'}=c_1'\left[J_{3L}-\frac{e^2}{g_L{}^2}J_Q\right]+c_2'\left[J_{3R}-\frac{e^2}{g_R{}^2}J_Q\right] , \tag{5.34}$$

with

$$c_1=e\left[\left[\frac{e}{g_L}\right]^{-1}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1/2}\cos\theta_N+\frac{e}{g_L}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1/2}\sin\theta_N\right]\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2} , \tag{5.35}$$

$$c_2=e\left[\frac{e}{g_L}\right]^{-1}\left[1-\frac{e^2}{g_L{}^2}\right]^{1/2}\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2}\frac{g_R{}^2}{g_L{}^2}\sin\theta_N , \tag{5.36}$$

$$c_1'=e\left[\left[\frac{e}{g_L}\right]^{-1}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1/2}(-\sin\theta_N)+\frac{e}{g_L}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1/2}\cos\theta_N\right]\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2} , \tag{5.37}$$

$$c_2'=e\left[\frac{e}{g_L}\right]^{-1}\left[1-\frac{e^2}{g_L{}^2}\right]^{1/2}\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2}\frac{g_R{}^2}{g_L{}^2}\cos\theta_N . \tag{5.38}$$

The parameter ϵ is again smaller than one and could be used as an expansion parameter of the theory. By noticing that the diagonal elements of the Hermitian Z-Z' mass matrix are real one obtains the upper bound (4.23) for ϵ, with γ_Z defined in (5.24). Similarly, one can obtain an upper bound on $\epsilon_W=M_W{}^2/M_{W'}{}^2$ from the constraint that the diagonal elements of the Hermitian W-W' mass matrix are real. The bound is the same as in Eq. (4.23); however, ϵ and γ_Z are now replaced by ϵ_W and γ_W which is defined in Eq. (5.27).

One can again see that for $\epsilon \ll 1$, the theory reduces to the $SU(2)_L\times U(1)_Y$ theory with corrections of order ϵ. In this case the mixing angles θ_+ of the charged mass matrix (5.25), the mixing angle θ_N of the neutral mass matrix (4.15), assume the form

$$\tan\theta_+=\left[1-\frac{e^2}{g_L{}^2}\right]^2\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1}\frac{g_R{}^3}{g_L{}^3}\frac{\sigma_+}{\rho_R}\epsilon+O(\epsilon^2) , \tag{5.39}$$

$$\tan\theta_N=-\left[1-\frac{e^2}{g_L{}^2}\right]\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1/2}\left[\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1}+\frac{g_R{}^2}{g_L{}^2}\sigma_3\right]\epsilon+O(\epsilon^2) , \tag{5.40}$$

and the algebraic equation for $e^2/g_L{}^2$ for heavy right-handed neutrinos is of the form

$$\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]=\frac{A_0{}^2}{M_Z{}^2\rho_L}\left[1+\left[1-\frac{e^2}{g_L{}^2}\right]^2\left[\frac{g_R{}^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]-\frac{e^2}{g_L{}^2}\right]^{-1}\right.$$
$$\left.\times\left\{\frac{g_R{}^2}{g_L{}^2}\sigma_+{}^2-\left[\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1}+\frac{g_R{}^2}{g_L{}^2}\sigma_3\right]^2\right\}\epsilon+O(\epsilon^2)\right] . \tag{5.41}$$

Thus, one can again explicitly observe that as $\epsilon\to 0$, θ_+, $\theta_N\to 0$, and $e^2/g_L{}^2(1-e^2/g_L{}^2)\to A_0{}^2/M_Z{}^2\rho_L$; i.e., the standard model is recovered.

The ratio of coupling constants g_R/g_L is a quantity of order 1. In manifestly left-right-symmetric theories one chooses $g_L=g_R$ at some mass scale. For the recently proposed theories with the left-right-symmetric group incorporated in a bigger gauge group, SO(10) (Ref. 19) or $SU(8)_L\times SU(8)_R$ (Ref. 20) $M_{Z'}$ is permitted to be light; i.e, $M_{Z'}\lesssim 10M_Z$. In these theories it turns out that $g_R<g_L$, and typically one has $g_R\approx 0.7g_L$.

Parameters ρ_R, σ_3, and σ_+ can assume the following range of values: $\rho_R=\{0,1\}$, $\sigma_3=\{-1,0\}$, $\sigma_+=\{0,1\}$. The particular value of these parameters depends on the pattern of the Higgs-field VEV. In the standard left-right-symmetric theory with triplet fields one has the Higgs-field multiplets with quantum numbers $(I_L,I_R,B-L)$: $\Delta_L\sim(1,0,2)$, $\Delta_R\sim(0,1,2)$, and $\phi\sim(\frac{1}{2},\frac{1}{2},0)$ with the vacuum expectation patterns

$$\langle\Delta_R\rangle\gg\langle\phi\rangle\gg\langle\Delta_L\rangle \tag{5.42}$$

with

$$\langle\phi\rangle=\begin{bmatrix}\kappa & 0\\ 0 & \kappa'\end{bmatrix},\quad \kappa\ll\kappa' . \tag{5.43}$$

Also, the quarks transform as $Q_L\sim(\frac{1}{2},0,\frac{1}{3})$, $Q_R\sim(0,\frac{1}{2},\frac{1}{3})$, and leptons transform as $L_L\sim(\frac{1}{2},0,-1)$ and $L_R\sim(0,\frac{1}{2},-1)$. In this case $\rho_R=\frac{1}{2}$, $\sigma_3\simeq-1$, and $\sigma_+\ll 1$.

In the following subsection we shall study the effects of the left-right-symmetric structure on the various asymmetries: these effects are of $O(M_Z{}^2/M_{Z'}{}^2)$ compared to the one of the $SU(2)_L\times U(1)_Y$. As already explained in the previous section radiative corrections arising from the new gauge structure at most of $O(\alpha M_Z{}^2/M_{Z'}{}^2)$ and therefore they can be neglected.

Finally we consider the $W^\pm$ mass as a function of the set (5.19) in left-right-symmetric theories. Note that neither M_W or $M_{W'}$ is to be considered a free parameter, but rather are to be calculated. In the case where all right-handed neutrino masses are larger than the muon mass we have

$$M_W{}^2=\rho_L\left[1-\frac{e^2}{g_L{}^2}\right]\times\frac{1+\beta_W-[(1-\beta_W)^2+4\gamma_W{}^2]^{1/2}}{1+\beta_Z-[(1-\beta_Z)^2+4\gamma_Z{}^2]^{1/2}}M_Z{}^2 . \tag{5.44}$$

We will display numerical results for this in the next subsection. For completeness, we display the $W^{\pm\prime}$ mass here as well

$$M_{W'}{}^2=\rho_L\left[\frac{e^2}{g_L{}^2}\right]\times\frac{1+\beta_W+[(1-\beta_W)^2+4\gamma_W{}^2]^{1/2}}{1+\beta_Z-[(1-\beta_Z)^2+4\gamma_Z{}^2]^{1/2}}M_Z{}^2 . \tag{5.45}$$

Here β_W, γ_W, β_Z, γ_Z, and $e^2/g_L{}^2$ are defined by Eqs. (5.26), (5.27), (4.17), (5.24), and (5.29), respectively.

There is a particularly simple relation among the masses

$$M_Z{}^2+(M_{Z'}{}^2-M_Z{}^2)\sin^2\theta_N=\left[1-\frac{e^2}{g_L{}^2}\right]^{-1}\rho_L{}^{-1}[M_W{}^2+(M_{W'}{}^2-M_W{}^2)\sin^2\theta_+] \tag{5.46}$$

which clearly reduces to the $SU(2)_L\times U(1)_Y$ relation (3.27) between the $W^\pm$ and Z masses as $M_{Z'}$, $M_{W'}\to\infty$ since $\sin^2\theta_N$ and $\sin^2\theta_+$ are $O(\epsilon^2)$.

B. Physical implications

We evaluated various SLC and LEP asymmetries (see Sec. II for definitions) in the case of left-right-symmetric gauge structure. They are presented in Figs. 11—16.

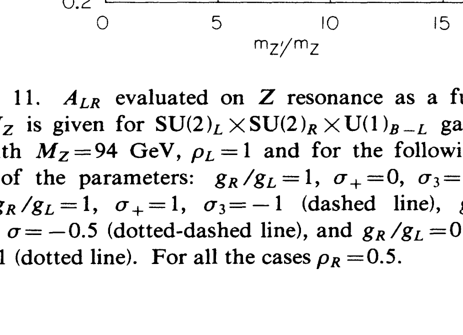

FIG. 11. A_{LR} evaluated on Z resonance as a function of $M_{Z'}/M_Z$ is given for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge structure with $M_Z=94$ GeV, $\rho_L=1$ and for the following typical values of the parameters: $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$ (solid line), $g_R/g_L=1$, $\sigma_+=1$, $\sigma_3=-1$ (dashed line), $g_R/g_L=1$, $\sigma_+=0$, $\sigma=-0.5$ (dotted-dashed line), and $g_R/g_L=0.7$, $\sigma_+=0$, $\sigma_3=-1$ (dotted line). For all the cases $\rho_R=0.5$.

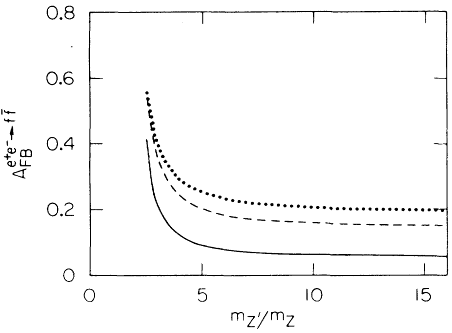

FIG. 12. $A_{FB}^{e^+e^-\to f\bar f}$ on Z resonance as a function of $M_{Z'}/M_Z$ for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge group is given for $f=\mu$ (solid line), $f=c$ (dashed line), and $f=b$ (dotted line). We choose $M_Z=94$ GeV, $\rho_L=1$, and $\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$.

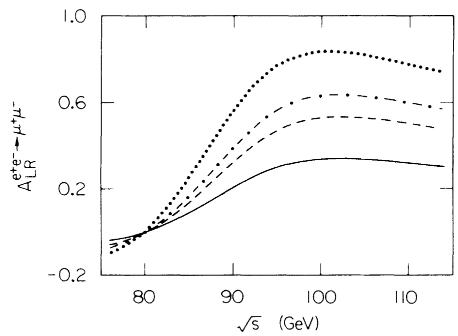

FIG. 14. $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ as a function of $\sqrt{s}$ for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge group is given for $M_{Z'}/M_Z=2.5$ (dotted line), 3.0 (dotted-dashed line), 3.5 (dashed line), ∞ (solid line). Parameters are $M_Z=94$ GeV, $\rho_L=1$, and $\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$.

Figure 11 represents $A_{LR}^{e^+e^-\to\mu^+\mu^-}$, evaluated at $s=M_Z{}^2$, as a function of $1/\sqrt{\epsilon}=M_{Z'}/M_Z$. The results are given for $M_Z=94$ GeV, $\rho_L=1$ while other choices of parameters are $g_R/g_L=1,0.7$, $\rho_R=0.5$, $\sigma_3=-1,-0.5$, and $\sigma_+=0,1$. We chose only one value of the ρ_R parameter because asymmetries do not depend significantly on ρ_R. Note that θ_N does not depend on ρ_R in the leading correction of order ϵ. The upper bound (4.23) for ϵ implies that for a wide range of models the left-right-symmetric theory is defined for $M_{Z'}\gtrsim 2.5M_Z$. From Fig. 11 we find that for $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ measured to 1% the limit $M_{Z'}/M_Z\gtrsim 10$ can be imposed for a wide class of models. Note that even for measurements of order 10% one can still set interesting bounds on the Z' mass $M_{Z'}\gtrsim(3\text{–}4)M_Z$ for most models. Furthermore, we may use the hadronic data on Z resonance in $A_{LR}^{e^+e^-\to\text{hadrons}}$ to augment the statistics. Also comparison of A_{LR} and $A_{\tau\text{pol}}$ on the resonance will provide a check on e-τ universality coupling of new gauge structures.

In the following we use for illustration a typical set of parameters $M_Z=94$ GeV, $\rho_L=1$, $g_R/g_L=1$, $\rho_R=0.5$, $\sigma_3=-1$, $\sigma_+=0$. In Fig. 12 the forward-backward asymmetry without longitudinal polarization $A_{FB}^{e^+e^-\to f\bar f}$ is given for the final fermion state $f=\mu$ (solid line), $f=c$ (dashed line), and $f=b$ (dotted line). Note again that $A_{FB}^{e^+e^-\to\mu^+\mu^-}$ is much less sensitive to the new gauge structure than A_{LR}. However, $A_{FB}^{e^+e_L^-\to\mu^+\mu^-}$ with electron beam polarization is much more sensitive to the effects of new currents than $A_{FB}^{e^+e^-\to\mu^+\mu^-}$. We present $A_{FB}^{e^+e_L^-\to\mu^+\mu^-}$ in Fig. 13 (solid line) along with $A_{FB}^{e^+e_L^-\to f\bar f}$ with $f=c$ (dashed line) and $f=b$ (dotted line).

The s dependence of $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ is tested in Fig. 14 for $M_{Z'}/M_Z=2.5$ (dotted line), 3.0 (dotted-dashed line), 3.5 (dashed line), ∞ (solid line). The slope is very sensitive to the effects of the new currents and thus even when

$$\delta A_{LR}^{e^+e^-\to\mu^+\mu^-}(-M_Z{}^2)\ll 1\ ,$$

the $\sqrt{s}$ dependence of $A_{LR}^{e^+e^-\to\mu^+\mu^-}$ can be significantly

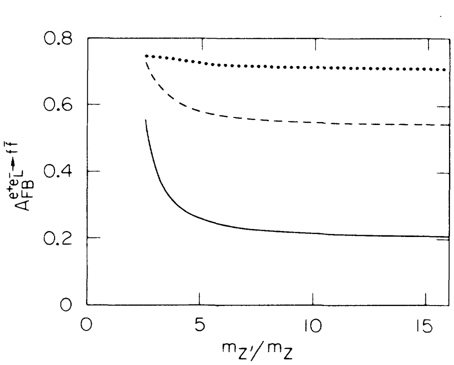

FIG. 13. $A_{FB}^{e^+e_L^-\to f\bar f}$ on Z resonance as a function of $M_{Z'}/M_Z$ for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge group is given for $f=\mu$ (solid line), $f=c$ (dashed line), and $f=b$ (dotted line). We chose $M_Z=94$ GeV, $\rho_L=1$, and $\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$.

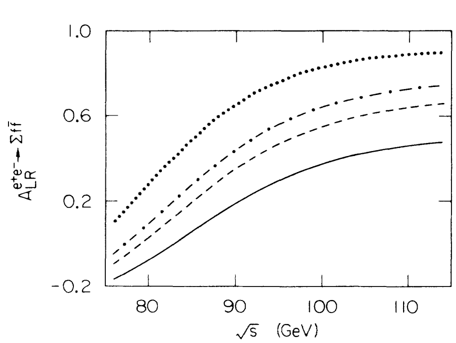

FIG. 15. $A_{LR}^{e^+e^-\to\Sigma f\bar f}$ with $f\neq e,\nu_e,t$ as a function of $\sqrt{s}$ for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ is presented for $M_{Z'}/M_Z=2.5$ (dotted line), 3.0 (dotted-dashed line), 3.5 (dashed line), ∞ (solid line). Parameters are $M_Z=94$ GeV, $\rho_L=1$, and $\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$.

changed due to new contributions from the new currents and the Z'-boson exchange. We have also studied the $\sqrt{s}$ dependence of $A_{FB}^{e^+e^-\to f\bar{f}}$ and $A_{FB}^{e^+e_L^-\to f\bar{f}}$ and note here that the dependence of the slope near Z resonance on new gauge structures is again not very pronounced.

In Fig. 15, $A_{LR}^{e^+e^-\to\Sigma f\bar{f}}$, $f\neq e,\nu_e,t$ as a function of $\sqrt{s}$ is plotted with the leading QCD corrections included. The slope changes less drastically when the ratio $M_{Z'}/M_Z$ changes because the final-state quark vector couplings to the Z are not suppressed by a factor $\simeq 4\sin^2\theta_W - 1$.

We shall now exhibit $\Delta^{b,c}$, the particular linear combinations of shifts in asymmetries from their GWS values, in the approximation $\epsilon \ll 1$, i.e., keeping only terms up to $O(M_Z{}^2/M_{Z'}{}^2)$. In this approximation J_Z is of the form

$$J_Z = \text{const}\times\left\{J_{3L} - \left[\left.\frac{e^2}{g_L{}^2}\right|_{2\times 1} + \delta\left[\frac{e^2}{g_L{}^2}\right] - \frac{M_Z{}^2}{M_{Z'}{}^2}\tilde{\lambda}\frac{e^2}{g_R{}^2}\right]J_Q + \frac{M_Z{}^2}{M_{Z'}{}^2}\tilde{\lambda}J_{3R}\right\} \tag{5.47}$$

with $\tilde{\lambda}$ being

$$\tilde{\lambda} = -\frac{e^2}{g_L{}^2}\frac{g_R}{g_L}\frac{\left[1-\dfrac{e^2}{g_L{}^2}\right]^{5/2}}{\left[1-\dfrac{e^2}{g_L{}^2}-\dfrac{e^2}{g_R{}^2}\right]^{3/2}}\left[\frac{e^2}{g_L{}^2}\left[1-\frac{e^2}{g_L{}^2}\right]^{-1} + \frac{g_R{}^2}{g_L{}^2}\sigma_3\right]. \tag{5.48}$$

A simple calculation yields a similar expression for $\Delta^{b,c}$ as in Eq. (4.30):

$$\Delta^b \simeq -a_\mu \frac{M_Z{}^2}{M_{Z'}{}^2}\tilde{\lambda}\left[\sin^2\theta_W\frac{(J_{3R})_R^b}{(J_{3L})_L^b} - \frac{(J_{3R})_R^b}{(J_Q)_R^b} - \sin^2\theta_W\frac{(J_{3R})_R^e}{(J_{3L})_L^e} + \frac{(J_{3R})_R^e}{(J_Q)_R^e}\right] \tag{5.49}$$

with obvious notation $(J_{3R})_R^b = \frac{1}{2}$ and $(J_{3R})_R^e = -\frac{1}{2}$. A similar expression for the charmed-quark (or top-quark) asymmetry Δ^c is obtained from (5.49) by the replacement $b\to c$. The large parentheses in expression (5.49) depends only on the quantum numbers of the b quark and electron under the new gauge group $SU(2)_R$.

Thus $\Delta^{b,c}$ are again directly sensitive to the new gauge currents and they are presented in Fig. 16 with dotted and solid lines, respectively. For $M_{Z'} \simeq 3M_Z$ this effect is again huge. Of course $\Delta^{b,c}$ are also sensitive to *direct* radiative corrections of Fig. 2 in $SU(2)_L\times U(1)_Y$. However, since these effects are usually small[15,16] ($\lesssim \frac{1}{2}\%$), the observation of $\Delta^{b,c} > 1\%$ would probably indicate the existence of a new gauge structure.

Another interesting observation is that if both Δ^b and $\Delta^c \neq 0$ the ratio Δ^b/Δ^c would again depend only on the quantum numbers and b, c, and e under the new gauge group $SU(2)_R$; the dependence on $(M_Z{}^2/M_{Z'}{}^2)\tilde{\lambda}$ is canceled in the ratio. Thus the value of Δ^b/Δ^c has a characteristic value for a particular gauge group. For the left-right-symmetric gauge group one has, for $M_Z = 94$ GeV,

$$\left.\frac{\Delta^b}{\Delta^c}\right|_{SU(2)_L\times SU(2)_R\times U(1)_{B-L}} = 1.24\,. \tag{5.50}$$

This is to be compared with Eq. (4.34). Thus SLC/LEP physics would allow us to probe directly the quantum numbers of b,c,e under the new gauge group, providing a clue as to the nature of the new gauge group.

Finally, we display in Fig. 17 the $W^{\pm}$ mass as a function of $M_{Z'}/M_Z$, with fixed $M_Z = 94$ GeV, $\rho_L = 1$,

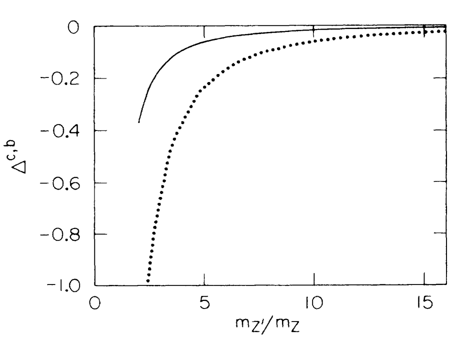

FIG. 16. $\Delta^{c,b}$ (solid line, dotted line), evaluated on Z resonance as a function of $M_{Z'}/M_Z$, is given for $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge group. We chose $M_Z=94$ GeV, $\rho_L=1$ and $\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=-1$.

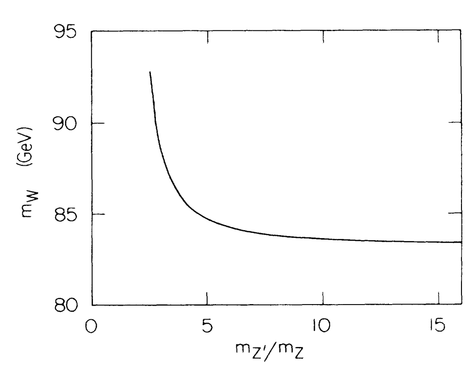

FIG. 17. M_W as a function of $M_{Z'}/M_Z$ for the same parameters as in Fig. 16.

$\rho_R=0.5$, $g_R/g_L=1$, $\sigma_+=0$, $\sigma_3=1$ and note that the effects can be large. A precise experimental determination of the $W^\pm$ mass would give very serious constraints on left-right-symmetric models.

VI. CONCLUSIONS

We analyzed the effects of extra gauge symmetries $SU(2)_L\times U(1)_Y\times Y(1)_{Y'}$ and $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ on polarization and forward-backward asymmetries as well as cross sections and Z width readily measured on and around Z^0 resonance at SLC/LEP. These theories are treated *exactly* at the tree level and depend only on a *fixed* number of parameters. A particular linear combination of the polarized forward-backward asymmetry and the polarization asymmetry is constructed. A deviation of this quantity from the standard model might be due to new currents only and shows unambiguously that some new undiscovered heavy particle couples directly to e, b, or c.

The numerical results show that the qualitative results are similar for $SU(2)_L\times U(1)_Y\times U(1)_{Y'}$ and $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge groups, thus making it difficult to distinguish between different gauge groups. However, we observe that the effect of the additional gauge symmetry *can* be significant, i.e., much larger than radiative corrections in $SU(2)_L\times U(1)_Y$, and the measurement of the asymmetries to 1% can clearly exclude a wide range of models and put a lower bound on $M_{Z'}$ to be of order $10M_Z$. Measurements to 10% accuracy also yield interesting limits on $M_{Z'}\gtrsim 3-4M_Z$. Another important observation is that $A_{LR}^{e^+e^-\to\mu^+\mu^-}(-s)$ changes slope drastically as the ratio of $M_{Z'}/M_Z$ is changed, even when the mixing angle θ_N is very small, because the contribution from the Z' propagator can be significant. Therefore studying the precise $W^\pm$ mass and the various asymmetries on and around the Z resonance would either put stronger bounds on M_Z *and* the mixing angle θ_N than bounds recently derived[12] from experiments at lower energies, or betray the existence of new undiscovered particles lying above SLC/LEP energies.

ACKNOWLEDGMENTS

The authors would like to thank the Mark II Collaboration for their kind hospitality at Asilomar, California. B.W.L. would like to thank S. Dimopoulos and L. Lochra for interesting discussions concerning this work. M.C. would like to acknowledge useful discussions with F. Gilman and P. Franzini. This work was supported by Department of Energy Contract No. DE-AC03-76SR00515.

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