Inflation with generalized initial conditions

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In many current models of the early Universe a scalar field ϕ which is only very weakly coupled to other quantum fields is used to generate inflation. In such models there are no forces which could thermalize the scalar field, and previous assumptions about its preinflation "initial" conditions must be abandoned. In this paper the onset of inflation is studied classically for more general initial conditions of the scalar field configuration. In particular, initial conditions with a nonvanishing spatial average of ϕ, with ϕ chosen at random in each initial horizon volume, and with random initial momenta are considered. We identify and discuss several mechanisms that can drive these more general initial conditions toward an inflationary state. The analysis is done in one spatial dimension.

I. INTRODUCTION

Many current cosmological models are based on considering quantum fields as the matter source for classical general relativity. As realized by Guth[1] the quantum fields may be trapped in a quasistable state in which the potential energy dominates the stress-energy tensor $T_{\mu\nu}$ (Ref. 2). In typical models of "new inflation" this "inflationary" state has a particular field (the "inflaton") which is homogeneous and localized (in field space) near a local maximum of its potential. A simple model might have a scalar inflaton ϕ with a potential (Fig. 1)

$$V(\phi)=\lambda(\phi^2-\sigma^2)^2 . \tag{1}$$

The inflationary state would have ϕ localized near $\phi=0$. Such a state remains inflationary until the inflaton slips off into regions of lower potential energy, at which point the potential energy ceases to dominate $T_{\mu\nu}$. The time scale for slipping off the potential maximum depends on the shape of the potential which in turn depends on how the inflaton is coupled to itself and other fields.

How the inflaton arrives in an inflationary state has been the topic of much discussion.[3–6] Mazenko, Unruh, and Wald[3] have stressed that the inflaton field may never enter a homogeneous inflationary state. Instead, the field would settle directly into the minima of the potential without ever becoming localized near a local maximum. Originally, it was assumed that effects due to thermal equilibration with other fields could play an important role in generating a homogeneous initial scalar field configuration. However, it has been realized that consistency with current observations demands that the inflaton is very weakly coupled to itself and other fields. The most severe constraint comes from demanding that the energy-density perturbations produced during inflation[7] be consistent with the observed isotropy of the microwave background radiation. This weak coupling makes it unreasonable to expect equilibrium with other fields to occur.

In earlier work[5,6] we have addressed the issue of how the inflaton can acquire the homogeneity necessary for the onset of inflation. We studied the classical evolution of a scalar field in an expanding flat Friedmann-Robertson-Walker background. We found that inhomogeneities are damped due to the expansion of the Universe, but they are enhanced by nonlinear effects—the field is pulled toward different minima of the potential at different locations in space. If the parameters of the potential are chosen correctly (here less fine-tuning is required than is needed to produce an acceptable amplitude of energy-density perturbations) there is a period in which the damping of the inhomogeneities dominates over the nonlinear effects and an inflationary state is entered.

It is not only important that the inhomogeneities are damped, but that the resulting homogeneous value of the classical field is close to a local maximum of the potential. In our earlier work we considered potentials symmetric under $\phi\to-\phi$ and with a local maximum at $\phi=0$. We chose initial conditions with spatial average of ϕ equal to

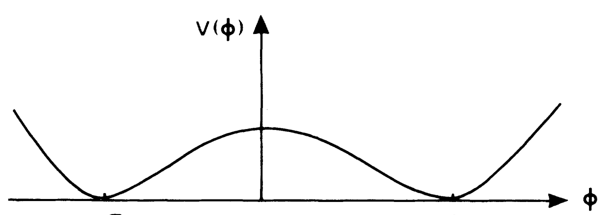

FIG. 1. Sketch of the potential $V(\phi)$ in a model of new inflation.

zero. This choice was not well justified. In this paper we extend our previous work to include initial conditions with a nonvanishing spatial average of ϕ. In particular we consider a homogeneous scalar field with a nonvanishing initial value (Sec. II) and an offset $\phi(\mathbf{x})$ chosen at random in each horizon volume at the initial time (Sec. III). We also study the effect of including random initial momenta $\dot{\phi}(\mathbf{x})$ (Sec. IV).

Our entire analysis is classical, with the idea that the mechanisms we study at the classical level should appear in some form in the full quantum system. Our analytical estimates are backed up with numerical work in one space dimension.

We show that the qualitative analysis of Refs. 5 and 6 carries over to the more general initial conditions considered here. The basic idea of our method is simple. The equation of motion of the scalar field configuration is

$$\ddot{\phi}(\mathbf{x},t)+3H(t)\dot{\phi}(\mathbf{x},t)-a^{-2}(t)\nabla^2\phi(\mathbf{x},t)=-V'(\phi(\mathbf{x},t)) \ , \quad (2)$$

where $a(t)$ is the scale factor of the Universe and $H(t)=\dot{a}(t)/a(t)$ is the Hubble "constant." $H(t)$ is positive in an expanding universe. Because of the Hubble expansion of the Universe, all excitation modes of $\phi(\mathbf{x})$ lose energy. As long as the Hubble damping force exceeds the nonlinear force $V'(\phi)$ driving $\phi(\mathbf{x})$ towards a minimum of $V(\phi)$, $\phi(\mathbf{x})$ will become homogeneous. If the initial spatial average ϕ_0 of the scalar field is small, $|\phi_0| \ll \sigma$, then $T_{\mu\nu}$ is dominated by the (almost) constant potential $V(\phi)$ and the Universe begins to inflate.

In Ref. 5 we derived a simple condition under which inflation is realized

$$\lambda < N\sigma^2 \ , \quad (3)$$

where N is the number of spin degrees of freedom of particles in thermal equilibrium at the critical temperature T_c. This was confirmed numerically in Ref. 6. In numerical codes which include gravitational back reaction, similar results were found,[8] with a smaller value of the coefficient on the right-hand side (RHS). Here and in the following we use units in which $\hbar=c=G=1$.

Once spatial fluctuations of ϕ become unimportant, the spatial average ϕ_0 evolves according to

$$\begin{aligned} &\ddot{\phi}_0(t)+3H\dot{\phi}_0(t)=-V'(\phi_0) \ , \\ &H^2=\frac{8\pi}{3}\lambda\sigma^4 \ . \end{aligned} \quad (4)$$

It can be seen that unless $\phi_0(t_0) << H$ (t_0 is the initial time) the period τ of inflation for the potential $V(\phi)$ in (1) will be too short to give a cosmologically interesting amount of inflation. This problem does not arise for a potential of Coleman-Weinberg shape:[9]

$$V(\phi)=\lambda\phi^4\left[\ln\left[\frac{\phi^2}{\sigma^2}\right]-\tfrac{1}{2}\right]+\tfrac{1}{2}\lambda\sigma^4 \ . \quad (5)$$

However, since in this paper we are interested only in the onset of inflation, we will mostly work with the potential of Eq. (1) for which the analytical analysis is easier.

II. CONSTANT OFFSET

In this section we consider the effect of adding a constant offset $\phi_0(t_0)$ to the initial scalar field configuration. To simplify the analysis we include only one inhomogeneous mode:

$$\phi(\mathbf{x},t_0)=\phi_0(t_0)+\phi_k(t_0)\cos\mathbf{k}\cdot\mathbf{x} \ . \quad (6)$$

The intuitive notion that ϕ_0 must always fall away from the local maximum of $V(\phi)$ is shown to be incorrect.

For small values of the coupling constant λ and for $|\phi_0(t_0)| \ll |\phi_k(t_0)|$ we can approximate the evolution of $\phi_0(t)$ by first solving the free equation of motion for ϕ_k obtained by replacing $V(\phi)$ by a mass term $\frac{1}{12}R\phi^2$, where R is the Ricci scalar:[5]

$$\phi_k(t)\approx\phi_k(t_0)\frac{a(t_0)}{a(t)} \ . \quad (7)$$

Similar results are obtained by a WKB method for the high-frequency modes.[10] Here and in the following ϕ_k will denote the amplitude of the mode. The equation of motion for ϕ_0 becomes

$$\begin{aligned} \ddot{\phi}_0+3H(t)\dot{\phi}_0+[12\lambda{\phi_k}^2(t)-4\lambda\sigma^2]\phi_0 \\ =-4\lambda{\phi_0}^3-12\lambda\phi_k{\phi_0}^2 \ . \end{aligned} \quad (8)$$

For

$${\phi_k}^2(t)>\tfrac{1}{3}\sigma^2 \quad (9)$$

there is a small effective positive-mass term in the equation of motion for ϕ_0. As long as $\phi_k(t)>\phi_0(t)$, the mass term will dominate over the nonlinear terms on the RHS of (8). Hence, $\phi_0(t)$ will decrease until (9) ceases to be satisfied.

We checked the above analysis by numerically integrating the original equation of motion (2) for $\phi(\mathbf{x},t)$ in one space dimension given the initial conditions (6). The analysis was performed for a radiation-dominated universe, i.e.,

$$a(t)\sim t^{1/2} \ . \quad (10)$$

The starting time was taken to be the Planck time $t_{\rm Pl}$. $k=T_{\rm Pl}=1$ corresponds to the wave number of a typical excitation at this temperature. The corresponding amplitude is constrained by requiring the energy density of $\phi_k(\mathbf{x})$ to be of the same order of magnitude as the radiation energy density. Hence we chose $\phi_k(t_0)=1$. A value of σ close to 1 will decrease the number of integration steps required. Our choice was $\sigma=0.1$.

In Fig. 2 the time evolution of $|\phi_0(t)|$ is shown for a run with $\lambda=10^{-4}$ and initial offset $\phi_0(t_{\rm Pl})=0.02$. $|\phi_0(t)|$ decreases rapidly until $t\approx 600$, then starts to increase again. The agreement with the predictions from our above analysis is good. From (9) and using (10) we expect the decrease of $\phi_0(t)$ to stop when $t\approx 300$.

The situation just described is similar to that which occurs due to thermal effects in the potential for ϕ_0, the effective potential.[11] The point is that any excitations, thermal or of different origin, contribute effective terms which can drive ϕ_0 towards zero. In more realistic models with many fields the excitations in the other fields can

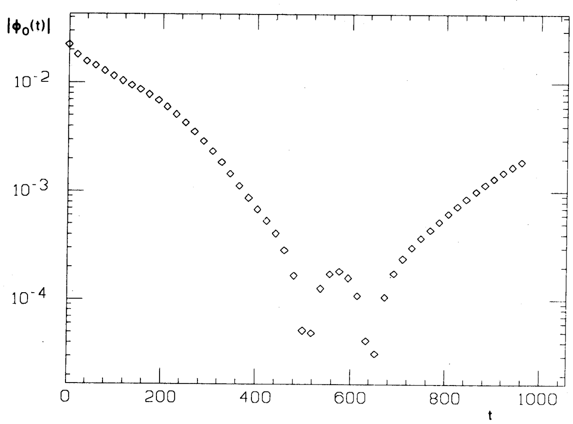

FIG. 2. The time evolution of the spatial average of $\phi(\mathbf{x},t)$, $\phi_0(t)$ in a radiation-dominated universe. The initial offset at the Planck time $t_{\rm Pl}$ is $\phi_0(t_{\rm Pl})=0.02$. The parameters in the potential are $\sigma=10^{-1}$ and $\lambda=10^{-4}$.

play a similar role. Still, the weak coupling required in inflationary models prevents the effect from being strong. It could reduce an offset which is already small, but it could not keep a large offset $|\phi_0(t_0)| \geq |\phi_k(t_0)|$ from evolving towards a minimum of $V(\phi)$. Notice that for this simple model the condition for excitations in other modes to decrease the amplitude of the zero mode is independent of λ, although the strength of this effect does depend on λ.

We conclude that a constant offset of similar magnitude to that of the nonhomogeneous excitations would prevent inflation, at least in simple models of new inflation. A smaller offset, however, would be reduced further by the stabilizing mechanism described above. Once the stabilizing period is concluded due to the Hubble damping of the inhomogeneous modes, the traditional analysis[12] can then be applied to determine the further evolution of ϕ_0 and in particular the duration of the inflationary phase.

III. RANDOM INITIAL OFFSET

The inclusion of an overall constant offset which we discussed in the previous section leads to a more general choice of initial conditions than we considered earlier.[5,6] However, there are grounds for criticism of such a choice. To choose a constant offset over all space amounts to setting up correlations between fields in causally disconnected regions of space. A type of initial condition which avoids the above problem is considered in this section.

We take the starting time to be the Planck time $t_{\rm Pl}$, and assign to ϕ a constant value randomly chosen in each horizon volume. The constraint

$$\phi \in [-\lambda^{-1/4}, \lambda^{-1/4}] \tag{11}$$

is imposed to keep the energy density in ϕ smaller than the Planck density.

In effect we are letting the horizon provide a lower cutoff for the wavelength of initial excitations. Of course, we are making a particular choice of a constant offset—as the sample volume increases the net offset decreases. Our choice is motivated by the fact that in a system which is symmetric under $\phi \to -\phi$ there is no reason for ϕ to take on positive values more often than negative values.

We will show that with these initial conditions the inhomogeneities can damp out leading to the start of an inflationary period.

Two factors contribute to the decrease in the spatial average of ϕ inside a horizon volume. In a radiation-dominated universe the horizon radius increases in comoving coordinates as $a(t)$. Thus the number of initial Planck-time horizon volumes contained in a horizon volume at time t grows as $a(t)^d$, where d is the number of spatial dimensions. Since the initial value of ϕ was random, the average value of $\phi(x)$ in a horizon volume, $\bar\phi(t)$, will decrease as $a(t)^{-d}$. In our case $d=1$.

$\bar\phi(t)$ will also evolve dynamically. Here we set up a simple approximate model for the dynamics. The random fluctuations per initial horizon volume can be roughly approximated by a plane wave with initial physical wavelength $t_0=t_{\rm Pl}$. For weak coupling a reasonable approximation will be to neglect the nonlinear forces. Then, as discussed in the Introduction and in more detail in Ref. 5, the evolution of $\phi(\mathbf{x},t)$ is given by a damped oscillation (in conformal time τ). The amplitude is proportional to $a(t)^{-1}$:

$$\begin{gathered}\phi(\mathbf{x},t)=\phi(\mathbf{x},t_0)\frac{a(t_0)}{a(t)}\sin(\mathbf{k}\cdot\mathbf{x})\cos(k\tau),\\ k=|\mathbf{k}|=2\pi,\quad \tau(t)=2t^{1/2}t_0^{1/2}.\end{gathered} \tag{12}$$

Combining the two effects (dynamical evolution and increase in the number of Hubble volumes) we see that $\bar\phi(t)$ will decay linearly in time

$$\bar\phi(t) \sim a(t)^{-2} \sim t^{-1}. \tag{13}$$

The decay of $\bar\phi(t)$ will continue as long as the damping force $H\dot\phi$ in the equation of motion for ϕ is larger than the nonlinear force $V'(\phi)$ which tends to drive ϕ away from the symmetric state $\phi=0$. t_c will denote the time

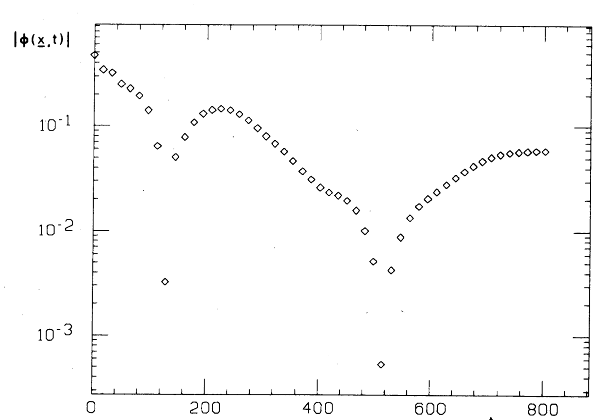

FIG. 3. The time evolution of the spatial average $|\bar\phi(t)|$ of $\phi(\mathbf{x},t)$ in a horizon volume at time t for a run with random initial offset as described in the text. The parameters in the potential are $\sigma=10^{-1}$ and $\lambda=10^{-3}$.

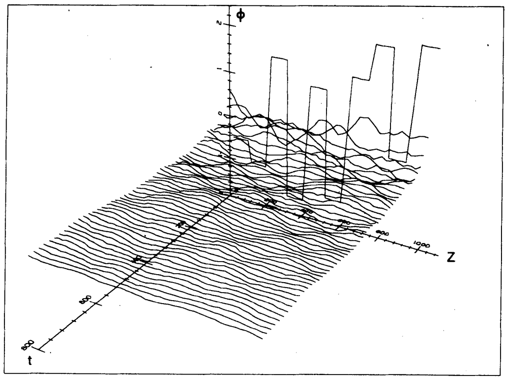

FIG. 4. Time evolution of $\phi(t,z)$ in the run of Fig. 2.

when the Hubble damping force ceases to dominate. A rough criterion for t_c is

$$|2\lambda(\phi^2-\sigma^2)\phi| = H(t_c)\dot{\phi}(t_c) . \tag{14}$$

For small values of ϕ and assuming $\dot{\phi} \sim H\phi$ this becomes

$$2\lambda\sigma^2 = H^2(t_c) = \frac{1}{4t_c^{\,2}} \tag{15}$$

or

$$t_c = \frac{1}{2\sqrt{2}}\lambda^{-1/2}\sigma^{-1} . \tag{16}$$

The qualitative analysis which leads to the predictions (13) and (16) can be confirmed by numerical analysis. We numerically integrated the equation of motion (3) for the double-well potential (1). The initial scalar field configuration at the Planck time $t_{\rm Pl}$ was generated by a random-number-generating routine which set $\phi(\mathbf{x})$ equal to a value chosen at random in the energetically allowed range (11). $\phi(\mathbf{x})$ is constant within each horizon volume at $t_{\rm Pl}$. Again the dimension of space is 1.

The grid points of the spatial lattice are at constant comoving coordinates. The size of the box was set by demanding that it contain at least one horizon volume at the critical temperature T_c, the temperature when the potential energy density $V(0)$ equals the radiation energy density:

$$T_c \approx \lambda^{1/4} N^{-1/4} \sigma . \tag{17}$$

The corresponding time is larger than t_c of Eq. (16) provided $\sigma < 1$ and N, the number of spin degrees of freedom, is not very big.

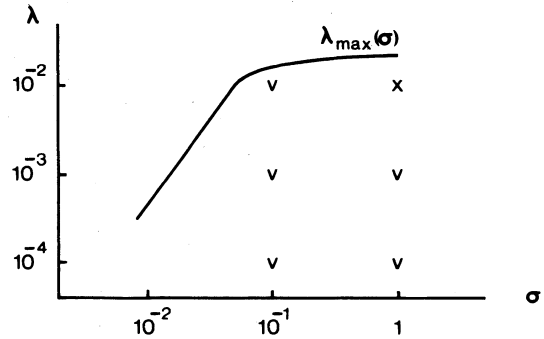

FIG. 5. Runs with random initial velocities $\dot{\phi}(t_0,z) \in [-\sigma^2,\sigma^2]$ which yield (checks) and do not yield (crosses) inflation. The analysis is for the double-well potential of Eq. (1). For comparison the values of $\lambda_{\max}(\sigma)$ from Ref. 6 are shown. For $\lambda < \lambda_{\max}(\sigma)$ the Universe enters an inflationary period given no initial velocities.

Numerical runs were performed for different values of λ and σ. Figure 3 shows the time evolution of $|\bar{\phi}(t)|$, the absolute value of the spatial average of $\phi(\mathbf{x},t)$ in a horizon volume at time t, for a run with $\sigma = 10^{-1}$ and $\lambda = 10^{-3}$. $|\bar{\phi}(t)|$ decays until $t \approx 500$, compared to the theoretical estimate $t_c \approx 100$ from (20). In the interval $40 < t < 400$ the slope of $\phi(t)$ is roughly -1, in good agreement with our prediction (17). Thus the crude model discussed here seems to reproduce the right qualitative features of the evolution of $\bar{\phi}(t)$. We do not expect it to explain the fine points, e.g., the dip in $|\bar{\phi}(t)|$ at $t \approx 150$. Figure 4 shows the evolution of $\phi(x,t)$ in the above run. The degree to which the initial fluctuating scalar field configuration becomes homogeneous is rather spectacular.

The following table (Table I) presents the results from various runs. $\phi(\mathbf{x})$ was chosen at random in the region $[-\lambda^{-1/4},\lambda^{-1/4}]$, but was taken to be constant in any initial Hubble volume. It compares the theoretical predictions for t_c and for the slope of $\bar{\phi}(t)$ during the time period $t < t_c$ with the numerical results. We conclude that while our model gives the correct qualitative behavior, it only agrees quantitatively to within an order of magnitude. In Table II the results for the same value of the parameters λ and σ but for different random choices for $\phi(\mathbf{x})$ are shown. The order of magnitude of t_c

TABLE I. Comparison between the simple model and numerical integrations: rate of decrease of $\bar{\phi}(t)$ and duration of the period of decrease. Initial data are $\phi(\mathbf{x}) \in [-\lambda^{-1/4},\lambda^{-1/4}]$ random, but constant in regions of initial Hubble volume.

σ	λ	Slope (model)	Slope (numerical)	t_c (model)	t_c (numerical)
1	10^{-3}	1	0.2	10	5
0.33	10^{-2}	1	1.4	10	30
0.33	10^{-3}	1	0.6	30	40
0.33	10^{-4}	1	1.6	100	200
0.1	10^{-3}	1	1	100	500

TABLE II. Variation of the result for the same choice of the parameters in the potential $\sigma=0.33$ and $\lambda=10^{-2}$ but different random initial conditions.

Run number	Slope (model)	Slope (numerical)	t_c (model)	t_c (numerical)
1	1	1.4	10	30
2	1	7	10	25
3	1	1.7	10	100
4	1	4	10	150

and of the slope of $\bar{\phi}(t)$ agree, but in quantitative terms there is a large variation.

IV. RANDOM VELOCITIES

An exact analysis of the effect of the nonlinear force $V'(\phi)$ in the equation of motion (2) for $\phi(\mathbf{x},t)$ is difficult since it mixes Fourier modes of ϕ. In Ref. 5 the maximal effect of this force was estimated by a perturbative Green's-function method. To a limited extent this was checked in Ref. 6 in which the evolution of $\phi(\mathbf{x},t)$ was studied numerically for initial conditions with one or two Fourier modes excited. Here we extend the analysis of mode mixing by considering initial conditions with

$$\phi(\mathbf{x},t)=0 \quad \text{or} \quad \phi(\mathbf{x},t)=A\,\sin\mathbf{k}\cdot\mathbf{x} \tag{18}$$

and

$$\dot{\phi}(\mathbf{x},t)\in[-\sigma^2,\sigma^2] \tag{19}$$

chosen at random at each point of the grid. These initial conditions are consistent with the energy constraint

$$\rho(\phi(\mathbf{x},t))<\sigma^4 \tag{20}$$

and amount to adding higher-frequency modes excited to similar energies.

This problem was analyzed numerically in the case of the double-well potential (1). Again, the analysis was performed for one space dimension. The temperature of the Universe in the beginning of the runs was taken to be $T=\sigma$.

The numerical results for $A=k=\sigma$ are summarized in Fig. 5. Checks mark runs for which an inflationary state [$\phi(\mathbf{x})$ localized near zero] arises, crosses mark runs for which the Universe fails to enter an inflationary phase. For comparison the maximal value $\lambda_{\max}(\sigma)$ of the coupling constant for which inflation occurs for plane-wave initial conditions with vanishing initial velocities is shown (see Ref. 6). We conclude that modifying the initial conditions by adding random velocities does not significantly alter the parameter space region for which the Universe enters an inflationary period.

V. CONCLUSIONS

The onset of inflation in models in which a single scalar field is coupled to gravity has been studied analytically and numerically for generalized initial scalar field configuration. The scalar field is not required to start off in thermal equilibrium, nor must it be at a constant value in space.

Three particular sets of initial scalar field configurations were considered. The evolution of $\phi(\mathbf{x},t)$ in the presence of a constant offset $\phi_0(t)$ with $|\phi_0(t)|<\sigma$ was discussed. Random initial scalar field velocities $\dot{\phi}(\mathbf{x},t_0)$ were considered, and in the least restrictive set of initial conditions $\phi(\mathbf{x},t)$ was allowed to take on random values in the energetically allowed region $\phi\in[-\lambda^{-1/4},\lambda^{-1/4}]$, ϕ being constant in domains of the size of the initial Hubble volume.

It was found that for all the above initial conditions, inhomogeneities in $\phi(\mathbf{x},t)$ are damped out by the Hubble expansion. Since the number of initial-time horizon volumes contained in a horizon volume at time t increases, the spatial average of $\phi(\mathbf{x})$ decreases. For models in which the self-coupling λ is weak the decrease continues long enough for the Universe to enter a period of inflation, provided the initial overall offset is not too large. A mechanism by which the presence of inhomogeneities acts to reduce an initial offset was demonstrated. This mechanism is, however, unimportant for weakly coupled theories.

The numerical analysis was performed in 1 space dimension. We do not expect the qualitative features to change in three dimensions. Naturally, since the number of initial horizon volumes contained in a horizon volume at time t scales at $a(t)^{-d}\approx t^{-d/2}$, the decrease in $\bar{\phi}(t)$ in Sec. III will be more pronounced in 3 than in 1 spatial dimensions. The analysis has recently been extended to 3 spatial dimensions.[13] The analysis was done in a radiation-dominated background metric. Metric perturbations were neglected. Including such perturbations is obviously very important. Results in this direction have recently been reported.[8,14] In the full back-reacting analyses performed so far[8] qualitative agreement is found, but the actual parameter space for which inflation occurs is somewhat smaller (when the Coleman-Weinberg potential is used). For the double-well $\lambda\phi^4$ model there is never enough inflation, as mentioned already in Sec. I.

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