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Strong-interaction contributions to one-loop leptonic processes

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We classify all strong-interaction contributions to all four-lepton processes through one loop in electroweak $SU_2 \times U_1$ and to all orders in strong interactions. We show that those parts which are not reliably calculated in perturbative QCD are all related to a certain integral over the *total* cross section for $e^+e^- \to$hadrons at low energies. We evaluate this integral for the most recent data and find that, for most four-lepton processes of interest, it is dominated by the timelike $|q'^2|$ region from 1 to 100 GeV^2. We show that the associated theoretical strong-interaction uncertainty is a factor of ~ 2 smaller than previously published estimates. We give the strong-interaction contribution and associated theoretical uncertainty for future SLAC, CERN, and Fermilab precision experiments and show that the theoretical uncertainty is quite small, thus allowing precision tests of the electroweak theory at the one-loop level.

I. INTRODUCTION

One of the most attractive features of the new generation of high-energy accelerators will be their ability to study leptonic processes with great precision, thus gaining access to information about new currents and one-loop electroweak radiative corrections. These corrections depend intimately on the gauge structure of the theory and, even within the context of $SU_2 \times U_1$, vary considerably depending on which representations of particles, even very heavy ones, are included in the model. Thus by studying radiative corrections to leptonic processes, we can hope to see effects of new particles, even if they are too heavy to be produced directly. For example, there are measurable corrections to the various asymmetries in $e^+e^- \to \mu^+\mu^-$, especially the initial-state longitudinal polarization asymmetry A_{LR}, on Z^0 resonance [where statistics will be high at CERN LEP and the Stanford Linear Collider (SLC)] within the context of the standard model of Glashow, Salam, and Weinberg[1] (GSW). There are also measurable shifts from new particles (extra quarks and leptons, supersymmetry, technicolor, etc.) from beyond the GSW model which enter at the one-loop level. Some generic values for the shifts due to various one-loop effects are displayed in Table I for various precision measurements.[2]

There is one problem with this scenario. At the one-loop level there are hadronic effects due to the presence of strongly interacting particles in the various vacuum-polarization amplitudes and thus there are strong-interaction contributions even to leptonic processes and to the masses and widths of the $W^\pm$ and Z^0. Any theoretical uncertainty induced by strong interactions must be understood before the one-loop effects of new physics can be deconvoluted from the leptonic data. In this paper we study the effects of familiar quarks and hadrons on all four-lepton processes to one loop. We show that the strong-interaction uncertainties induced in the various precision asymmetries and mass measurements are smaller than most contributions of new particles listed in Table I and show that by remeasuring the total cross section for $e^+e^- \to$hadrons in the timelike energy region $1 \leq |q^2| \leq 100$ GeV^2 with greater accuracy (to, say, 5%) it could be reduced much further.

Let us write down[2] the most general neutral- and charged-current four fermion matrix elements, including all one-loop electroweak corrections and strong interactions to all orders, in electroweak $SU_2 \times U_1$, where the internal-symmetry breaking is done primarily by Higgs doublets. If external fermion masses are neglected, all external fermion vertices are helicity conserving and all cross sections may be written in terms of effective matrix elements where the initial-state left-handed isospin I_3 and electric charge Q as well as the final state I'_3, Q'_3 are specified. We choose a renormalization scheme[3] where α, the muon-decay constant G_μ (i.e., the two best known electroweak constants of nature) and M_Z, the Z^0 mass (expected to be measured very precisely by SLC/LEP), are used as precise input data. Then, in Euclidean metric ($q^2 = \bar{q}^2 - q_0{}^2$), the neutral-current matrix element (normalized to α for photon exchange in $e^+e^- \to \mu^+\mu^-$ at $q^2 = 0$) is

TABLE I. Responses at one loop of various asymmetries on Z^0 resonance and the $W^{\pm}$ mass to new one-loop physics. Numbers are generic, calculated using $M_Z=94$ GeV.

One-loop physics	$\delta A_{LR}=\delta A_{\tau\text{pol}}$	δA_{FB}	$\delta A_{\perp}$	δM_W (MeV)
GSW weak				
$m_t=30$ GeV	-0.03	-0.01	.005	-180
$m_H=100$ GeV				
Heavy top quark				
$m_t\simeq 180$ GeV	0.03	0.0075	0.004	780
Heavy Higgs boson ~ 1 TeV	-0.01	-0.0045	-0.003	-160
Heavy-quark pair				
(a) Large I splitting	0.02	0.01	0.007	300
(b) Degenerate	-0.004	-0.002	-0.001	-42
Heavy-lepton pair				
(a) Large I splitting $m_\nu=0$	0.012	0.006	0.004	300
(b) Degenerate	-0.0013	-0.0006	-0.0004	-14
Heavy-scalar-quark pair				
(a) Large I splitting	0.02	0.01	0.007	300
(b) Degenerate	0	0	0	0
Heavy-scalar-lepton pair				
(a) Large I splitting	0.012	0.006	0.004	300
(b) Degenerate	0	0	0	0
W gauginos				
(a) $m_{3/2} \ll 100$ GeV	0.005	0.0025	0.001	100
(b) $m_{3/2} \gg 100$ GeV	<0.001	<0.001	$\ll 0.001$	<10
Technicolor				
$SU_8 \times SU_8$	-0.04	-0.018	-0.012	-500
O_{16}	-0.07	-0.032	-0.021	-500
Strong-interaction uncertainty	± 0.0034	± 0.0014	± 0.001	± 25

$$\mathcal{M}_{pp'}=Q\frac{\alpha}{1-\Delta_\alpha(q^2)-i\,\mathrm{Im}\Pi'_{AA}(q^2)}\left[\frac{-s}{q^2}\right]Q'+\alpha\left[\frac{I_3-Q[s_\theta{}^2+\Delta_p(q^2)-is_\theta c_\theta \mathrm{Im}\Pi'_{ZA}(q^2)]}{s_\theta c_\theta}\right]_p$$
$$\times\frac{-s}{(q^2+M_Z{}^2)[1-\Delta_\rho(q^2)-0.06]-i\,\mathrm{Im}\Pi_{ZZ}(q^2)}$$
$$\times\left[\frac{I'_3-Q'[s_\theta{}^2+\Delta_p(q^2)-is_\theta c_\theta \mathrm{Im}\Pi'_{ZA}(q^2)]}{s_\theta c_\theta}\right]_{p'}+X^{\mathrm{NC}} \quad (1)$$

while the charged-current matrix element is

$$\mathcal{M}^{\mathrm{CC}}(q^2)=\frac{\alpha}{2\sin^2\theta_W}(-s)((1-0.06)\{(q^2+\cos^2\theta_W M_Z{}^2)[1-\Delta_W(0)]+\cos^2\theta_W M_Z{}^2\Delta_W(q^2)\}-i\,\mathrm{Im}\Pi_{WW}(q^2))^{-1}+X^{\mathrm{CC}}\,. \quad (2)$$

Here s is the Mandelstam variable ($q^2=-s$ in the s channel) while Δ_α, Δ_ρ, Δ_p, and Δ_W are certain finite combinations of the one-loop vacuum-polarization amplitudes Π_{ij} and Π'_{ij} in Fig. 1 to be discussed later. Here, the quantities X^{NC} and X^{CC} represent the one-loop one-particle-irreducible (1PI) vertex, box, and fermion self-energy contributions, the so-called "direct" coupling corrections. These do not suffer strong-interaction effects for leptonic processes and we drop them from further consideration. We define the weak mixing angle used throughout the calculation

$$\sin^2\theta_W\equiv s_\theta{}^2=\frac{1}{2}-\frac{1}{2}\left[1-\frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z{}^2(1-0.06)}\right]^{1/2} \quad (3)$$

to include the largest part of the QED corrections to the renormalization of α from $q^2=0$ to $M_Z{}^2$ by light quarks and leptons. *There is no strong-interaction uncertainty in the Born terms;* the constant 0.06 in the definition is chosen to establish a convention in which $s_\theta{}^2$ is directly calculable from α, G_μ, and M_Z.

The "oblique correction" functions are finite combinations of electroweak one-loop 1PI vector-boson self-energies Π_{ij} and Π'_{ij} as defined in Fig. 1 with $i,j=W,Z,A$ (photon) or SU_2 and QED currents $i,j=1,2,3,Q$, and are to contain strong interactions to all orders. We have from Ref. 2

$$
\begin{aligned}
\Delta_\alpha &= \mathrm{Re}[\Pi'_{AA}(q^2)-\Pi'_{AA}(0)]\,, \\
\Delta_\rho &= \mathrm{Re}\left[\frac{\Pi_{ZZ}(-M_Z{}^2)}{M_Z{}^2}-\frac{\Pi_{WW}(0)}{M_W{}^2}+\frac{\Pi_{ZZ}(q^2)-\Pi_{ZZ}(-M_Z{}^2)}{q^2+M_Z{}^2}\right], \\
\Delta_p &= \mathrm{Re}\left[-s_\theta c_\theta[\Pi'_{ZA}(q^2)-\Pi'_{ZA}(-M_Z{}^2)]+\frac{s_\theta{}^2 c_\theta{}^2}{1-2s_\theta{}^2}[\Delta_\alpha(-M_Z{}^2)-0.06]\right. \\
&\qquad \left.+\frac{s_\theta{}^2 c_\theta{}^2}{1-2s_\theta{}^2}\left[-\Pi'_{AA}(-M_Z{}^2)+\frac{\Pi_{WW}(0)}{M_W{}^2}-\frac{\Pi_{ZZ}(-M_Z{}^2)}{M_Z{}^2}\right]-s_\theta c_\theta \Pi'_{ZA}(-M_Z{}^2)\right], \\
\Delta_W &= \mathrm{Re}\left[\frac{s_\theta{}^2}{1-2s_\theta{}^2}[\Pi'_{AA}(0)+0.06]-\frac{\Pi_{WW}(q^2)}{M_W{}^2}+\frac{c_\theta{}^2}{1-2s_\theta{}^2}\frac{\Pi_{ZZ}(-M_Z{}^2)}{M_Z{}^2}-\frac{s_\theta{}^2}{1-2s_\theta{}^2}\frac{\Pi_{WW}(0)}{M_W{}^2}\right].
\end{aligned}
\tag{4}
$$

It is easy to see that there can only be four such functions; in $SU_2 \times U_1$, there are only four vector self-energies, Z-Z, A-A, Z-A, and W-W, and these self-energies will of course appear at one loop. The quantities $\Pi'_{AA}(0)$, $\Pi_{ZZ}(-M_Z{}^2)$, and $\Pi_{WW}(0)$ appear because we used α, M_Z, and G_μ, respectively, as physical input data and the experimental values of these quantities already include some radiative corrections.

Now it is a simple task to track down the strong-interaction contributions in the $\mathrm{Im}\Pi_{ij}$ and the Δ_i. Let us assume that the influence of strong interactions is entirely due to the presence of quarks in the various vector self-energies. Then, forgetting for the moment the top quark, we assume that we may calculate $\Pi_{ZZ}(q^2)/M_Z{}^2$, $\Pi_{WW}(q^2)/M_W{}^2$, $\Pi'_{ZA}(q^2)$, and $\Pi'_{AA}(q^2)$ for $|q^2| \gg m_q{}^2$ (where $m_q{}^2$ is a generic quark mass) dropping terms of order $\alpha_{\mathrm{EM}} m_q{}^2/q^2$ and using perturbative QCD. For example,

$$
\mathrm{Im}\Pi'_{AA}(q^2) \simeq \mathrm{Im}\Pi'_{AA}(q^2)\,|_{\text{free-field theory}} \times \left[1+\frac{\alpha_{\mathrm{QCD}}(q^2)}{\pi}+c\frac{\alpha_{\mathrm{QCD}}{}^2(q^2)}{\pi^2}\right]. \tag{5}
$$

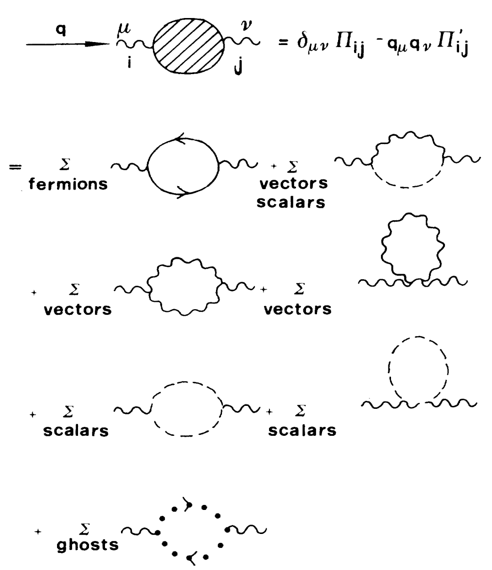

FIG. 1. Vector self-energies.

and analogously for $\Pi'_{ZA}, \Pi_{ZZ}, \Pi_{WW}$. We will, in fact, be interested in low-$|q^2|$ neutral-current neutrino scattering and so would have to evaluate $\mathrm{Im}\Pi'_{ZA}(q^2)$ but this is in the t channel and so vanishes.

Similarly, we assume perturbative QCD to be valid for the calculation of

$$
\frac{\Pi_{WW}(0)}{M_W{}^2}\,. \tag{6}
$$

Strictly speaking, $q^2=0$ is not *a priori* a suitable point for asymptotic freedom to obtain. However, we can still exploit the theoretical framework of perturbative QCD if we divide the quark contributions into "light" (u,d,s) and "heavy" $(c,b,t,\ldots)$ ones. For the latter we trust the argument, supported from several years of QCD-sum-rules phenomenology, that $q^2=0$ is still a good point for asymptotic freedom. This is probably not true for the light quarks. But for these the free-field-theory (FFT) contribution, which is of order $m_q{}^2/M_Z$, is so small that even increasing it by more than one order of magnitude (and we do not believe that the FFT approximate result can be so bad) it would still be completely negligible. Note that this argument would not apply for the quantities $\Pi'_{AA}(0), \Pi'_{ZA}(0)$ for which the dependence on the quark masses is quite different, so that the light quarks in FFT would actually give the dominant contribution.

Thus we shall rely on perturbative QCD to evaluate the terms of Eqs. (5) and (6). We stress that Π_{WW}, Π_{ZZ} *cannot* be directly determined from experimental data and thus must necessarily be evaluated by some theoretical model. The reasons why we feel that perturbative QCD should be a reasonable approach are of both theoretical *and* practical nature, since we know that this model has been able to describe rather satisfactorily the *photon* vacuum polarization both at $|q^2| \gg m_q{}^2$, as it can be directly seen by looking at the value of the total cross section for $e^+e^- \rightarrow$hadrons, and at $q^2 \simeq 0$, as it can be at least qualitatively inferred from the consequences of this assumption for QCD sum rules. We do not claim that this is the only, or the best, possibility. But we believe that it can be considered as a very reasonable one.

Having so chosen the theoretical model to evaluate Eqs. (5) and (6), we have to provide an estimate of the related theoretical uncertainty. This will be consistently considered by us as that coming from the experimental uncertainty which affects the value of $\alpha_s(q^2)$. For the latter we shall assume, following a rather conservative approach, an indetermination $\Delta\alpha_s/\alpha_s \simeq 0.3$. Typically, this will produce a theoretical uncertainty at $q^2=-M_Z{}^2$ of the order of 2% of the overall FFT term, which we shall have to take into account when writing the various theoretical predictions. As we shall see, this error will be generally much smaller than that coming from the quantities which we shall be able to compute using true experimental data, an error which in turn will be sufficiently small not to disturb the theoretical predictions that we shall finally write down. Thus we believe that the problem of the theoretical uncertainty to be attached to those quantities Eqs. (5) and (6) which cannot be directly determined from experimental data can be considered as satisfactorily controlled.

There are, though, two warnings to be made here. The first is the possibility of $t\bar{t}$ resonances which could destroy our ability to use perturbative QCD for $|q^2| \sim m_{t\bar{t}}{}^2$. If $t\bar{t}$ has some substantial mixing with the Z^0 this effect would need to be included in the analysis. The second warning is that in order to calculate absolute cross sections near the Z^0 or $W^{\pm}$ poles or the widths of these particles to 1% accuracy we should properly include the two-loop contributions to their respective propagators' imaginary parts. This we regard as beyond the scope of this paper. Note, however, that it is still safe to form asymmetries to one loop near Z^0 resonance because Z^0 propagator effects (and also the luminosity) cancel there. Either of these two effects could give strong-interaction contributions to experiments at SLC and LEP and will be included in a further analysis.[4] The hadronic contributions which are neither suppressed by powers of $m_q{}^2/M_Z{}^2$ nor calculable in perturbative QCD, enter via the two finite combinations $\Delta\alpha(q^2)$ and

$$\Delta_{ZA}(-M_Z{}^2,q^2)=\mathrm{Re}[\Pi'_{ZA}(-M_Z{}^2)-\Pi'_{ZA}(q^2)] \qquad (7)$$

for low q^2. These two combinations, then, give all of the nonperturbative strong-interaction effects for four-lepton processes at one loop and we will concentrate on these for the remainder of this paper.

We will show in Sec. II that strong-interaction effects in Δ_{ZA} can be related to those in Δ_α, which in turn can be related to low-energy data in $e^+e^-\to$hadrons. We will use the most recent available data to evaluate the hadronic contribution to Δ_α. In Sec. III we will use this to give the hadronic contributions to and bounds on the hadronic uncertainties in various precision measurements to be performed by the CHARM II Collaboration and by experimental groups at SLC, LEP, and Fermilab in the near future.

II. PHOTON AND Z-A MIXING VACUUM POLARIZATION

As we saw in the previous section, all complicated strong-interaction effects in one-loop leptonic processes are contained in the two quantities $\Delta_\alpha(q^2)$ and Δ_{ZA}. Here, q^2 represents a four-momentum square which is typically small, $|q^2| \ll M_Z{}^2$ which prevents us from relying on FFT particularly when light-quark contributions are involved.

In the case of $\Delta_\alpha(q^2)$, the problem can be circumvented since this quantity is directly related to $e^+e^-\to$hadrons data. The situation is less simple in the case of Δ_{ZA}, where a more detailed analysis of flavor-dependent effects is required. Π'_{ZA} is defined from the vacuum expectation value of the product of $J^{\mathrm{EM}}=eJ^Q$ and

$$J^{Z^0}_{\text{vector}}=\frac{e}{s_\theta c_\theta}(J^3_{\text{vector}}-s_\theta{}^2J^Q) ,$$

where J^3_{vector} is the vector part of the third component of the weak-isospin current. Thus Δ_{ZA} contains the flavor-dependent term $\langle J^3_{\text{vector}}J^Q\rangle$. Now write

$$J^3_{\text{vector}}=\tfrac{1}{2}J^Q+(J^3_{\text{vector}}-\tfrac{1}{2}J^Q)$$

and note that the hadronic part of the last term can be written

$$\begin{aligned} J^3_{\text{vector}}-\tfrac{1}{2}J^Q &= -\tfrac{1}{12}(\bar{d}\gamma_\mu d+\bar{u}\gamma_\mu d+\bar{s}\gamma_\mu s+\bar{c}\gamma_\mu c \\ &\quad +\bar{b}\gamma_\mu b+\bar{t}\gamma_\mu t+\cdots) \\ &= -\tfrac{1}{2}J^\omega+\tfrac{1}{4}J^\phi+\text{heavy quarks} , \end{aligned}$$

$$\begin{aligned} &J^\omega=\tfrac{1}{6}(\bar{d}\gamma_\mu d+\bar{u}\gamma_\mu u) , \\ &J^\phi=-\tfrac{1}{3}\bar{s}\gamma_\mu s , \qquad (8) \\ &J^\rho=\tfrac{1}{2}(\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d) , \\ &J_Q=J^\rho+J^\omega+J^\phi+\text{heavy quarks} . \end{aligned}$$

Thus $J^3_{\text{vector}}-\frac{1}{2}J_Q$ does not contain the (dominant) ρ-meson current and is entirely of weak isospin $\mathbf{I}=0$. Taking care with the various isospin components we have

$$\begin{aligned} \Pi'_{ZA} &= \frac{1}{s_\theta c_\theta}(\tfrac{1}{2}-s_\theta{}^2)\Pi'_{AA}+\frac{1}{4s_\theta c_\theta}(\Pi'^{(\phi\phi)}_{AA}-2\Pi'^{(\omega\omega)}_{AA}) \\ &\quad +\frac{1}{4s_\theta c_\theta}(\Pi'^{(\rho\phi)}_{AA}-2\Pi'^{(\rho\omega)}_{AA}-\Pi'^{(\omega\phi)}_{AA}) \\ &\quad +\text{heavy-quark terms} , \qquad (9) \end{aligned}$$

where $\langle J^a_\mu J^b_\nu\rangle=\delta_{\mu\nu}q^2\Pi'^{(ab)}_{AA}$ gives the relevant flavor contribution to the photon vacuum polarization with *vector currents* only.

Let us start our examination of (9) with the heavy quark components of Π'_{AA}. We will assume that all flavor-mixing terms involving heavy quarks vanish. If we are in the spacelike q^2 region or even at $q^2=0$ we know from QCD sum rules[5] that FFT plus calculable QCD should be a reliable approximation for the remainder of the heavy-quark contribution. So this contribution can be straightforwardly evaluated; in fact, we find it to be very small.

The second term in the parentheses contains all possible nonvanishing interference terms $(\rho\omega,\omega\phi,\rho\phi)$, e.g., $\sim\langle J^{(\rho)}_\mu J^{(\omega)}_\nu\rangle$. These are familiar quantities,[5] their expression is well known and given by nonperturbative QCD condensates, isospin-conservation-breaking terms or flavor-mixing terms suppressed by factors $\alpha_{EM}(m_u^2 - m_d^2)/\,\mathrm{GeV}^2$ or α_{EM}^2 or suppressed via Zweig's flavor-mixing rule. These contributions are orders of magnitude smaller than the leading one in Eq. (9). Thus we see that for those q^2 values which are relevant for $\bar{\nu}_\mu e, \nu_\mu e$ scattering, we can safely "reduce" $\Delta_{ZA}(Q_0^2,q^2) \equiv \mathrm{Re}[\Pi'_{ZA}(Q_0^2)-\Pi'_{ZA}(q^2)]$ (where Q_0^2 is a suitable subtraction point where FFT can be used) to a sum:

$$\Delta_{ZA}(Q_0^2,q^2)=\frac{\frac{1}{2}-s_\theta^2}{s_\theta c_\theta}[\Delta_\alpha(Q_0^2)-\Delta_\alpha(q^2)]+\frac{1}{4s_\theta c_\theta}\{[\Delta_\alpha^{(\phi\phi)}(Q_0^2)-\Delta_\alpha^{(\phi\phi)}(q^2)]-2[\Delta_\alpha^{(\omega\omega)}(Q_0^2)-\Delta_\alpha^{(\omega\omega)}(q^2)]\}$$
$$+\text{"small" terms with negligible errors .} \qquad (10)$$

To be more precise, let us consider the specific value $q^2=0$. We find, in this case,

$$\Delta_{ZA}(Q_0^2,0)=\frac{\frac{1}{2}-s_\theta^2}{s_\theta c_\theta}[\Pi'_{AA}(Q_0^2)-\Pi'_{AA}(0)]-\frac{1}{4s_\theta c_\theta}\left[\frac{\alpha}{9\pi}\left[\ln\frac{|Q_0^2|}{m_c^2}-\frac{5}{3}\right]\right]$$
$$+\frac{1}{4s_\theta c_\theta}\{[\Pi'^{(\phi\phi)}_{AA}(Q_0^2)-\Pi'^{(\phi\phi)}_{AA}(0)]-2[\Pi'^{(\omega\omega)}_{AA}(Q_0^2)-\Pi'^{(\omega\omega)}_{AA}(0)]\}+\text{calculable small terms .} \qquad (11)$$

Q_0^2 must be such that we can safely use FFT for $\Pi'_{AA}(Q_0^2),\Pi'_{ZA}(Q_0^2)$.

The evaluation of the last square-bracketed term in the right-hand side (RHS) of Eq. (11) could, in principle, be performed if precise flavor-isospin tagging data in $e^+e^-\to$hadrons existed; we could compute it phenomenologically from its definition:

$$\Delta_\alpha^{(\phi\phi)}(Q_0^2)-2\Delta_\alpha^{(\omega\omega)}(Q_0^2)=\frac{\alpha Q_0^2}{3\pi}\int_{-\infty}^{0}\frac{dq'^2}{q'^2(q'^2-Q_0^2)}[R^{(\phi)}(q'^2)-2R^{(\omega)}(q'^2)] \qquad (12)$$

for $|Q_0^2| \ll M_Z^2$, where

$$R^{(\phi)}=\frac{\sigma_{e^+e^-\to\phi,\phi',\ldots}}{\sigma_{e^+e^-\to\mu^+\mu^-}}\,,$$
$$R^{(\omega)}=\frac{\sigma_{e^+e^-\to\omega,\omega',\ldots}}{\sigma_{e^+e^-\to\mu^+\mu^-}}\,. \qquad (13)$$

In practice, the data are not to our knowledge available at the moment. We will still be able to give a reasonable estimate because the asymptotic part ($|q'^2| \gg m_s^2$) of the numerator of (12) vanishes exactly in FFT. Thus it is only the region of small q'^2 values, $|q'^2| \lesssim 1\ \mathrm{GeV}^2$, which can effectively contribute. For this region it is certainly a good approximation to consider the ω,ϕ contribution as due to the dominant resonances treated in the narrow width representation. Thus we can write this contribution as

$$\frac{\alpha Q_0^2}{3\pi}\int_{-\infty}^{0}\frac{dq'^2}{q'^2(q'^2-Q_0^2)}[R^{(\phi)}(q'^2)-2R^{(\omega)}(q'^2)]\simeq\frac{3Q_0^2}{\alpha}\left[\frac{\Gamma_{\phi\to e^+e^-}}{m_\phi}\frac{1}{(Q_0^2+m_\phi^2)}-\frac{2\Gamma_{\omega\to e^+e^-}}{m_\omega}\frac{1}{(Q_0^2+m_\omega^2)}\right]. \qquad (14)$$

Numerically, this turns out to be $\simeq -0.000\,25$ at the spacelike point $Q_0^2=79\ \mathrm{GeV}^2$; we shall assume that the possible error on this estimate is equal to the estimate itself; although based on QCD sum rules we feel that our approximation should not be that bad. A final comment on this $(\phi-2\omega)$ term is that if we had used the (*a priori* unjustified) FFT evaluation of the LHS of Eq. (12), we would have obtained a result $\simeq -(2\alpha/9\pi)\ln(m_s/m)$ (where m denotes the common value of $m_{u,d}$) which for any reasonable choice of the m_s/m ratio turns out to be numerically very close to our estimate in Eq. (14). A similar narrow width estimate can be given when q^2 increases from zero to spacelike values, with minor modification, and we shall not discuss it further.

We now give an explicit evaluation of Δ_α from the most recent available $e^+e^-\to$hadrons data and discuss in some detail the related experimental errors. Figure 2 shows the results of our evaluation of the relevant expression (writ-

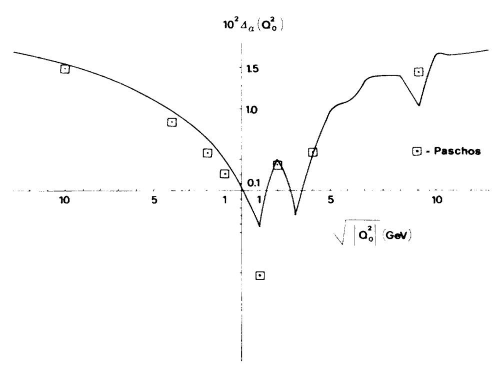

FIG. 2. The graph of $\mathrm{Re}10^2\Delta_\alpha(Q_0^2)$ at timelike (right-hand axis) and spacelike (left-hand axis) Q_0^2 values $|Q_0^2| \leq 200\ \mathrm{GeV}^2$. The squares represent the older evaluation by Paschos (Ref. 6).

TABLE II. Contributions to the quantity $10^2\Delta_\alpha(79\ \mathrm{GeV}^2)$ in Eq. (16) coming from the different regions a–f and related error.

Region	$10^2\Delta_\alpha(79\ \mathrm{GeV}^2)$
a	0.29 ± 0.01
b	0.43 ± 0.05
c	0.53 ± 0.05
d	0.19 ± 0.02
e	0.007
f	0.004
Total	1.45 ± 0.13

ten in Euclidean metric):

$$\Delta_\alpha(Q_0{}^2)=\frac{\alpha}{3\pi}Q_0{}^2\int_{-\infty}^{0}\frac{dq'^2R(q'^2)}{q'^2(q'^2-Q_0{}^2)}\ ,$$
$$R=\frac{\sigma_{e^+e^-\to\mathrm{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} \qquad (15)$$

when $Q_0{}^2$ varies in the range $-200\le Q_0{}^2\le 200\ \mathrm{GeV}^2$. For comparison, we have also included an older estimate by Paschos.[6] One notices that the two tend to differ somewhat in the very-low-q^2 region, i.e., in that dominated by the very-low-energy e^+e^- data, for which we have taken the most recent results, quoting an accurate estimate of the systematic error.[7] This was not available for the earlier estimate. Assuming for the remaining higher-energy data[8] a realistic systematic error of 10% (see later), we have computed what we consider to be a realistic error for $\Delta_\alpha(Q_0{}^2)$ in the $Q_0{}^2$ range above. To get a deeper understanding of the details of our evaluation, we have divided the integration region of Eq. (14) into six parts; i.e., region a: $|q'^2|\le(0.8\ \mathrm{GeV})^2$, where one expects[7] the systematic error to be no larger than $\sim4\%$; region b: $(0.8\ \mathrm{GeV})^2\le|q'^2|\le m_\psi{}^2$, where the systematic error[8] is expected to range from $\sim4\%$ to 15%; region c: $(m_\psi)^2\le|q'^2|\le(m_\Upsilon)^2$; region d: $(m_\Upsilon)^2\le|q'^2|\le(46\ \mathrm{GeV})^2$; region e: $(46\ \mathrm{GeV})^2\le|q'^2|\le(80\ \mathrm{GeV})^2$; region f: $(80\ \mathrm{GeV})^2\le|q'^2|$. In the last four regions, we accepted the quoted[8] systematic error of 10%.

Having divided the integration range in this way, we can now see how much of the overall error, at variable q^2, comes from the different regions. Considering, e.g., the specific spacelike value $Q_0{}^2=79\ \mathrm{GeV}^2$ which corresponds to the "optimal" subtraction point to be discussed later, we have listed in Table II the individual contributions coming from the six regions a–f. As one sees, the overall result is

$$\Delta_\alpha(79\ \mathrm{GeV}^2)=0.0145\pm0.0013\ . \qquad (16)$$

Note that the error is ~2 times smaller than the previous estimate ±0.002 by Sirlin[3] and, consequently will lead to smaller errors in A_{LR} than previously estimated.[1] Of the ±0.0013 error, only ±0.0001 comes from region a, while ±0.0010 comes from the two regions b and c. Thus our result Eq. (16) is *not* dominated by the very-low-energy e^+e^- data, but rather from those data approximately in the timelike $|q'^2|$ region from 1 to 100 GeV^2. More precise measurements of the threshold region would consequently not be of great help for $Q_0{}^2=79\ \mathrm{GeV}^2$. Note however that if the experimental error in the region $1\le|q'^2|\le100\ \mathrm{GeV}^2$ were reduced to 5%, the error on Δ_α (79 GeV^2) would be ±0.0008, a substantial reduction. We urge experimentalists to reexamine this region in order to make the hadronic uncertainties in future SLAC, CERN, and Fermilab precision experiments completely negligible.

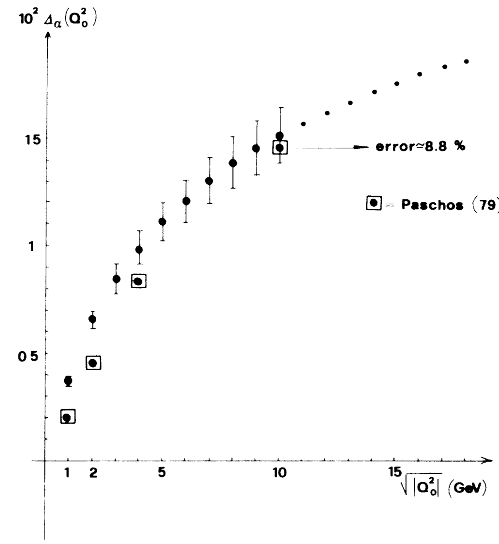

FIG. 3. Same as in Fig. 2 for spacelike $Q_0{}^2$ values $|Q_0{}^2|\le400\ \mathrm{GeV}^2$. A few error bars have been computed for some low $Q_0{}^2$ points, according to the prescription given in Sec. II.

These same conclusions would in general apply to the range of spacelike $Q_0{}^2$ investigated, i.e., $|Q_0{}^2|\le400\ \mathrm{GeV}^2$. In Fig. 3 we have shown in more detail the values of $\Delta_\alpha(Q_0{}^2)$ in this range, together with some error bars. Table III contains the contributions of the different regions to the overall results at a number of $Q_0{}^2$ values. As one sees, the contribution from the threshold region becomes less and less relevant as $Q_0{}^2$ increases. As a general rule, in the whole range $|Q_0{}^2|\gtrsim1\ \mathrm{GeV}^2$ the main contribution, giving the largest fraction of the error, to the relevant quantities comes from the region of the data $(1\ \mathrm{GeV})^2\le|q'^2|\lesssim(10\ \mathrm{GeV})^2$.

These same conclusions apply for timelike $|Q_0{}^2|$ larger than approximately 150 GeV^2. For smaller timelike $Q_0{}^2$ values, as can be seen in Fig. 1, the quantity $\Delta_\alpha(Q_0{}^2)$ is subject to oscillations due to contributions from regions a–f of various signs. As a consequence, the overall error, which is of the order of $\sim8\%$ in the more favorable spacelike $Q_0{}^2$ or large timelike $Q_0{}^2$ cases, becomes somewhat larger ($\sim20\%$).

As a final comment to motivate our choice of the optimal spacelike subtaction point $\tilde{Q}_0{}^2=79\ \mathrm{GeV}^2$ for Eq. (16), we would like to point out that it is possible to derive bounds on the quantity $\Delta_\alpha(Q_0{}^2)$ in the spacelike region[9] which are a consequence of the experimental value of the muon anomaly and of the assumptions that QED is correct and that QCD gives respectable predictions for the

TABLE III. Contributions to the quantity $10^2\Delta_\alpha(Q_0{}^2)$ coming from the different regions a–f at several spacelike $Q_0{}^2$ values. The related error can be easily worked out and is pictured in Fig. 3.

$Q_0{}^2/\text{GeV}^2$	a	b	c	d	e	f	Total
1	0.20	0.14	0.03				0.37
4	0.26	0.27	0.11	0.01			0.65
9	0.27	0.34	0.19	0.03			0.83
16	0.28	0.37	0.28	0.05			0.98
25	0.28	0.40	0.35	0.08			1.11
36	0.29	0.41	0.41	0.10			1.21
49	0.29	0.42	0.46	0.13	0.005	0.002	1.31
64	0.29	0.43	0.50	0.16	0.006	0.003	1.39
81	0.29	0.43	0.53	0.20	0.007	0.003	1.46
100	0.29	0.43	0.56	0.23	0.009	0.005	1.52

photon vacuum polarization in the spacelike region, in the spirit suggested by Shifman, Vainshtein, and Zakharov.[5] In particular, it was shown in a previous paper[10] that these bounds become optimal, i.e., most strict, at the point $Q_0{}^2=79\ \text{GeV}^2$, where one obtains the general result

$$0.0115\leq\Delta_\alpha(79\ \text{GeV}^2)\leq0.0157\ . \qquad (17)$$

As one notices, the upper limit of this general bound (coming from theoretical considerations of a vastly different sort) is exactly saturated by the purely phenomenological evaluation based on e^+e^- data, Eqs. (15) and (16). Actually, the two different estimates are consistent over the whole spacelike region $|Q_0{}^2|\leq150\ \text{GeV}^2$ where the general bounds can be derived. This strengthens our belief in the correctness of the result Eq. (16) to be used in what follows.

Inserting Eqs. (16) and (14) in Eq. (11) we obtain (assuming $s_\theta{}^2=0.215$, i.e., $M_Z=94$ GeV):

$$\begin{aligned}\Delta_{ZA}(79\ \text{GeV}^2,0)&=(0.0101\pm0.0009)-(0.0003)\\&\quad-(0.0002\pm0.0002)\\&=0.0096\pm0.0011\ ,\end{aligned} \qquad (18)$$

where the numbers on the RHS represent the contributions to the overall quantity coming from the three pieces in Eq. (11). Thus, we see that the bulk of the result and of its error comes from the same e^+e^- data which determined the photon vacuum polarization Eq. (16), and the same considerations of that case still apply.

Having completed our numerical analysis of the two quantities, Eqs. (16) and (18), which are affected by strong-interaction uncertainties, we are now ready to discuss what the effects of these uncertainties on a number of measurable quantities will be. This will be done in Sec. III.

III. MEASURABLES

Let us begin our analysis with a discussion of the contribution due to strong interactions to the theoretical prediction for the initial-state polarization asymmetry A_{LR} for the process $e^+e^-_{L,R}\to\mu^+\mu^-$ at the Z_0 resonance, which will soon be measured at SLC. This is given by the expression[1,2]

$$A_{LR}(-M_Z{}^2)\equiv\left[\frac{\sigma(e^+e_L^-\to\mu^+\mu^-)-\sigma(e^+e_R^-\to\mu^+\mu^-)}{\sigma(e^+e_L^-\to\mu^+\mu^-)+\sigma(e^+e_R^-\to\mu^+\mu^-)}\right]_{q^2=-M_Z{}^2}\ . \qquad (19)$$

The contribution to this quantity due to u, d, s, c, b, and t quarks is easily written from Eqs. (1) and (4):

$$\begin{aligned}\delta A_{LR}^{(u\to t)}(-M_Z{}^2)&=\frac{-64c_\theta{}^2s_\theta{}^4}{(1+v_\theta{}^2)^2}\Delta_\alpha^{(u\to t)}(-M_Z{}^2)+\text{const}\\&=\frac{-64c_\theta{}^2s_\theta{}^4}{(1+v_\theta{}^2)^2}\{[\Delta_\alpha(-m_Z{}^2)-\Delta_\alpha(79\ \text{GeV}^2)]+\Delta_\alpha(79\ \text{GeV}^2)\}^{(u\to t)}+\text{const}\ ,\end{aligned} \qquad (20)$$

with $v_\theta=4s_\theta{}^2-1$.

The first term in the curly brackets is evaluated using perturbative QCD while the second can be gotten from Table II. Collecting the various terms [remember that since $m_t\gg m_b$ there is a contribution from the ρ parameter $\Delta\rho(0)$] we have for $M_Z=94$ GeV, $m_b=3m_c=4.5$ GeV and $m_t=30$ GeV

$$\delta A_{LR}^{(u\to t)}=-0.0615\pm0.0028\pm0.0006\ , \qquad (21)$$

where the larger error comes from that of e^+e^- data of Table II, while the smaller one has been obtained under the assumption that the experimental value of α_s appearing in the QCD corrections to the first term in the curly brackets is plagued by a relative error of approximately thirty per cent, $\Delta\alpha_s/\alpha_s\simeq0.3$.

Note that this hadronic contribution is a substantial fraction of the prediction from the complete standard model of Glashow, Salam, and Weinberg[1] through one

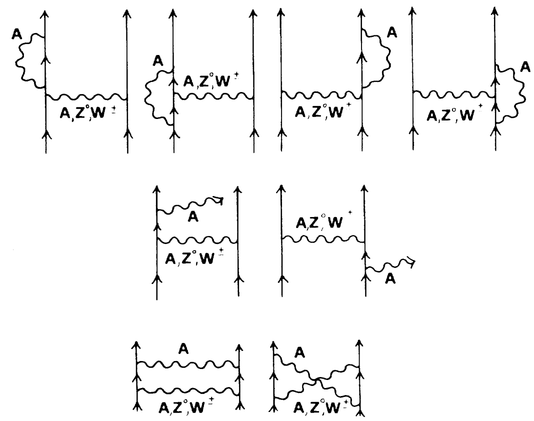

FIG. 4. Detector-dependent QED contributions.

loop (all one-loop GSW predictions quoted in this paper specifically exclude only the detector-dependent QED contributions from graphs in Fig. 4) including leptons, quarks, and vector and scalar bosons in internal loops for $M_Z=94$, $m_t=30$, and $m_{\text{Higgs}}=100$ GeV

$$A_{LR}^{\text{GSW}}(q^2=-M_Z{}^2)=0.2692 \tag{22}$$

but that the *hadronic uncertainty* is quite small compared to the total radiative correction from GSW listed in Table I or the contributions to A_{LR} from beyond the standard model. The hadronic uncertainty in (21) is a factor ~ 2 smaller than previous estimates.[1,3] We conclude that the GSW prediction Eq. (22) for A_{LR} is theoretically "clean" since the uncertainties from strong-interaction effects of light quarks can be safely controlled. Thus any shifts from this value greater than say, 0.005, must be attributed to new physics from beyond the standard model. Some candidates are listed in Table I.

We have also considered other possible asymmetries in e^+e^- annihilation. At the Z_0 resonance and including one-loop effects, their expressions and the related strong-interaction contributions and uncertainties are simply related to those of the longitudinal asymmetry A_{LR}, as has been extensively discussed elsewhere.[2] Table I contains the relevant uncertainties, which one can evaluate straightforwardly with Eq. (16).

As a next application, assuming M_Z to be very accurately measured at SLC and LEP, we consider the theoretical prediction for the W mass (to be measured at LEP II and Fermilab), related to Eq. (16) through Sirlin's formula:[3]

$$M_W{}^2\left[1-\frac{M_W{}^2}{M_Z{}^2}\right]=\frac{(37.281\text{ GeV})^2}{1-\Delta r}\,,$$
$$\Delta r=\frac{2s_\theta{}^2-1}{s_\theta{}^2}\Delta_W(-M_W{}^2)+\text{const}\,, \tag{23}$$

which can be gotten, alternatively, by examination of the pole structure of Eq. (2). The contribution from u, d, s, c, b and t quarks is for $M_Z=94$ GeV, $m_t=30$ GeV

$$\Delta r^{(u\to t)}=\Delta_\alpha^{(u\to t)}(-M_Z{}^2)+\text{const}'$$
$$=0.0333\pm0.0013\pm0.0004\,, \tag{24}$$

where, again, the large part of the error comes from that of e^+e^- data of Table II. This gives a contribution to $W^\pm$ mass

$$\delta M_W^{(u\to t)}=-484\pm19\pm6\text{ MeV}\,. \tag{25}$$

A glance at Table I shows that the strong-interaction uncertainty is smaller than the possible effects coming from physics beyond the GSW model. The $(u\to t)$ contribution is to be compared to the prediction for M_W including all one-loop standard model contributions[1-3,11] for $M_Z=94$, $m_t=30$, and $m_H=100$ GeV

$$M_W=83.33\text{ GeV}\,. \tag{26}$$

To conclude our analysis, we have considered the following two ratios, soon to be measured by the CHARM II collaboration:

$$R_{\nu\bar\nu}(-t)=\frac{\sigma_{\nu_\mu e\to\nu_\mu e}}{\sigma_{\bar\nu_\mu e\to\bar\nu_\mu e}}$$
$$\simeq\left[1-\frac{\hat v_\theta}{1+\hat v_\theta{}^2}\right]\Big/\left[1+\frac{\hat v_\theta}{1+\hat v_\theta{}^2}\right], \tag{27}$$

$$R_{\text{NC,CC}}(-t)=\frac{\sigma_{\nu_\mu e\to\nu_\mu e}}{\sigma_{\nu_\mu e\to\nu_e\mu}}$$
$$\simeq\frac{1-\hat v_\theta+\hat v_\theta{}^2}{12}[1-\Delta_\rho(-t)]^{-2}$$
$$\times\left[1-\frac{m_\mu{}^2}{2m_eE_\nu}\right]^{-2} \tag{28}$$

with $\hat v_\theta=4[s_\theta{}^2+\Delta_\rho(-t)]-1$. We evaluated the contribution from $u\to t$ quarks and the resulting theoretical uncertainties. We find for $M_Z=94$, $m_t=30$ GeV, and $E_\nu=70$ GeV

$$\delta R_{\nu\bar\nu}^{(u\to t)}=-0.163\pm0.0089\pm0.0022\,, \tag{29}$$

$$\delta R_{\text{NC,CC}}^{(u\to t)}=-0.0094\pm0.0005\pm0.0001 \tag{30}$$

(again, the smaller error comes from the QCD corrections) which are to be compared to the prediction from the complete GSW theory to one loop[2,12] for $M_Z=94$, $m_t=30$, $m_{\text{Higgs}'}=100$, and for $E_\nu=70$ GeV

$$R_{\nu\bar\nu}=1.2862\,, \tag{31}$$

$$R_{\text{NC,CC}}=0.1295\,. \tag{32}$$

Note that the $u\to t$ quarks give quite a large fraction of the $R_{\nu\bar\nu}$ and $R_{\text{NC,CC}}$ but that the theoretical uncertainties are small.

IV. CONCLUSIONS

We have classified the hadronic contributions to *all* one-loop four-lepton processes. We conclude that the hadronic corrections to most leptonic processes contain a rather small uncertainty, which is mainly due to that of the e^+e^- data in the region of timelike $|q'^2|$ from 1 to 100 GeV^2. This uncertainty could be further substantially reduced if an experimental effort in this region brought the systematic error below that of the available data (which is of approximately 10%) to, say, 5%. We have shown however that the theoretical uncertainty with present data is sufficiently small to allow a whole series of future experiments at SLAC, CERN, and Fermilab to carry on a systematic test of the theory of electroweak forces at one loop with clean theoretical predictions.

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