Magnetic moments of composite W bosons

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We have calculated the anomalous magnetic moment of the W boson (κ_W) in a class of nonrelativistic composite models. We have found, for example, that in a model with only scalar preons, $\kappa_W \geq 3$. For the case of two spin-$\frac{1}{2}$ preons with $L=0$ and equal masses we find the very surprising result that $\kappa_W=1$, the standard gauge-theory value. There are four possible preon spin states. We have found a general expression for κ_W for any linear combination of these states. A range of allowed values for κ_W has been obtained. We also discuss the quadrupole moment for a composite W.

The idea that quarks and leptons may be composite objects has received much attention recently.[1] The possibility exists that gauge bosons, such as the W and Z, are also composite objects.[2] One signal for a composite W boson would be the observation of an anomalous magnetic moment (κ_W) which differs from the standard gauge-theory value[3] $\kappa_W=1$. A method of determining the value of κ_W is to make use of the phenomenon of radiation amplitude zeros.[4] These could be observed in the process $\bar{p}p \to W^{\pm}\gamma + X$ at the CERN Collider or the Fermilab Tevatron. There is also a dramatic change in the cross section for $e^+e^- \to W^+W^-$ at high energies[5] when $\kappa_W \neq 1$.

In this paper we calculate the anomalous magnetic moment of the W boson in a class of nonrelativistic composite models. The general expression for the Z component of the W^+ magnetic moment is given in a nonrelativistic model by

$$\mu_z^W = \frac{e}{2M_W}(1+\kappa_W)J_z^W = \frac{e}{2m_1} g q S_{1z} + \frac{e}{2m_2} g(1-q) S_{2z} + \frac{e}{2\mu} L_z \,, \qquad (1)$$

where m_1 and m_2 are the masses of the W constituents, which have charges q and $(1-q)$, respectively, and gyromagnetic ratio g. μ is the reduced mass of the system: $\mu = m_1 m_2/(m_1+m_2)$.

We first consider the simple use of two spinless preons which require $L=1$ to form a spin-1 W^+. In this case $S_{1z}=S_{2z}=0$ and we obtain

$$\kappa_W = x^{-1}(1+x)^2 - 1 \,, \qquad (2)$$

where $x = m_1/m_2$. It is easy to see from this expression that $\kappa_W \geq 3$ and thus must be different from the gauge-theory value $\kappa_W=1$. The minimum value ($\kappa_W=3$) occurs when $x=1$; i.e., the preons have the same mass. There are, however, renormalizable models[6] of the weak interactions where the W is a bound state of spinless preons and $\kappa_W=1$.

We now turn to the more complex case of two spin-$\frac{1}{2}$ preons with $L=0$. In this case we find that

$$\kappa_W = \frac{g}{2}[x(1-q) + q/x + 1] - 1 \,. \qquad (3)$$

Note that if $x=1$, κ_W is independent of q, $\kappa_W = g-1$, and for $g=2$, $\kappa_W=1$, the gauge-theory value. (If preons are elementary we may expect that their gyromagnetic ratio should be that for a point Dirac particle, i.e., $g=2$.) Examining κ_W as a function of x [Eq. (3)] we find that a minimum value of κ_W exists provided q is in the range $0<q<1$ for which we obtain (for $g \geq 0$)

$$x^2 = q(1-q)^{-1} \,, \qquad (4)$$

$$\kappa_W^{\min} = g q^{1/2}(1-q)^{1/2} + \left[\frac{g}{2} - 1\right] . \qquad (5)$$

Note that for q in the above range, the preons have the same-sign charge. From (5) we see that $\kappa_W^{\min}$ lies in the range ($g \neq 2$), $g/2 - 1 \leq \kappa_W^{\min} \leq g-1$. For $g=2$, however, $\kappa_W^{\min}$ must lie in the more limited region $0 \leq \kappa_W^{\min} \leq 1$. If q lies outside of the above range (opposite-sign preon charges) then all values of κ_W are possible and are given by (3). For the special case of $g=2$, if we demand $\kappa_W=1$ we find $q = x(1+x)^{-1}$ for $x \neq 1$; for $x=1$ any value of q will produce $\kappa_W=1$.

In general for a system of two spin-$\frac{1}{2}$ particles a total angular-momentum state $J=1$ can be obtained from four spin combinations:

$$|S=0,L=1\rangle = {}^1P_1, \quad |S=1,L=0\rangle = {}^3S_1 \,, \quad |S=1,L=1\rangle = {}^3P_1, \quad |S=1,L=2\rangle = {}^3D_1 \,. \qquad (6)$$

We now consider the possibility that the W^+ is a general linear combination of these four states and calculate the corresponding value of κ_W. To be explicit we calculate the matrix element $\langle J^W=1, J_z^W=1 | \mu_W | J^W=1, J_z^W=1\rangle$ using Eq. (1) and the decomposition

$$
\begin{aligned}
|J^W=1,J_z^W=1\rangle = {} & \alpha\,|s=1,s_z=1\rangle\,|L=L_z=0\rangle+\sqrt{3/5}\beta\,|s=1,s_z=-1\rangle\,|L=2,L_z=2\rangle \\
& -\sqrt{3/10}\beta\,|s=1,s_z=0\rangle\,|L=2,L_z=1\rangle+\sqrt{1/10}\beta\,|s=1,s_z=1\rangle\,|L=2,L_z=0\rangle \\
& +\frac{\gamma}{\sqrt{2}}\,|s=1,s_z=1\rangle\,|L=1,L_z=0\rangle-\frac{\gamma}{\sqrt{2}}\,|s=1,s_z=0\rangle\,|L=1,L_z=1\rangle \\
& +\delta\,|s=0,s_z=0\rangle\,|L=1,L_z=1\rangle \ . \qquad (7)
\end{aligned}
$$

We find that

$$
\begin{aligned}
\kappa_W = {} & g(1+x)\left[\frac{q}{x}+(1-q)\right]\left[\frac{\alpha^2}{2}-\frac{\beta^2}{4}+\frac{\gamma^2}{4}\right] \\
& +\frac{(1+x)^2}{x}\left(\tfrac{3}{2}\beta^2-\sqrt{2}\gamma\delta\right) \\
& -g(1+x)[q/x-(1-q)](\gamma\delta/\sqrt{2})-1 \ . \qquad (8)
\end{aligned}
$$

We now consider the following special cases assuming $g=2$ and $x=1$ in which case (8) simplifies to

$$\kappa_W=2\alpha^2+5\beta^2+\gamma^2-\frac{4\gamma\delta}{\sqrt{2}}(1+2q)-1 \ . \qquad (9)$$

First, consider the mixing of only the two negative-parity states ${}^3S_1$ and ${}^3D_1$ implying $\gamma=\delta=0$ and $\alpha^2=1-\beta^2$ so that $\kappa_W=1+3\beta^2$. Thus κ_W is confined to the range $1\le\kappa_W\le 4$; note as $\beta\to 0$ we recover our previous result $\kappa_W=1$. Next, consider mixing only the two positive-parity states ${}^1P_1$ and ${}^3P_1$ for which $\alpha=\beta=0$ and $\delta^2=1-\gamma^2$. We find that

$$\kappa_W=(\gamma^2-1)-\frac{4\gamma}{\sqrt{2}}(1-\gamma^2)^{1/2}(1+2q) \ . \qquad (10)$$

The next possibility to consider is the mixing of the three states which are odd under charge conjugation (C): ${}^1P_1$, ${}^3S_1$, ${}^3D_1$. In this case $\gamma=0$ and $\delta^2=1-\alpha^2-\beta^2$ so that $\kappa_W=2\alpha^2+5\beta^2-1$; thus κ_W is in the range $-1\le\kappa_W\le 4$. There is only a single state which is C even — ${}^3P_1$. If the W^+ corresponds to this state then $\alpha=\beta=\delta=0$ and $\gamma=1$ so that $\kappa_W=0$ uniquely.

If CP is a good classification symmetry then we must consider both the CP-even and CP-odd possibilities as well. The CP-even states are the ${}^3S_1$, ${}^3P_1$, and ${}^3D_1$ for which $\delta=0$ and $\kappa_W=\alpha^2+4\beta^2$. Thus in this case $0\le\kappa_W\le 4$. In the CP-odd case the W^+ must be the ${}^1P_1$ state so that $\alpha=\beta=\gamma=0$ and $\delta=1$. Here we find that $\kappa_W=-1$ uniquely.

We now turn to a calculation of the quadrupole moment of a composite W. We make use of Eq. (7) again. We obtain

$$
\begin{aligned}
\langle J=1,J_z=1\,|\,Q\,|\,J=1,J_z=1\rangle & = \int \langle J,J\,|\,r^2(3\cos^2\theta-1)\,|\,J,J\rangle d^3r \\
& =\alpha^2C_{00}^0(0)+\tfrac{3}{5}\beta^2C_{22}^2(-\tfrac{4}{7})+\tfrac{3}{10}\beta^2C_{21}^2(\tfrac{2}{7}) \\
& \quad +\tfrac{1}{10}\beta^2C_{20}^2(\tfrac{4}{7})+\tfrac{1}{2}\gamma^2C_{10}^1(\tfrac{4}{5})+\tfrac{1}{2}\gamma^2C_{11}^1(-\tfrac{2}{5})+\delta^2C_{11}^1(-\tfrac{2}{5}) \\
& \quad \text{with } C_{lm}^l=C_l=\langle L=l,\ L_z=m\,|\,r^2\,|\,L=l,\ L_z=m\rangle\ge 0 \ . \qquad (11)
\end{aligned}
$$

We assume central forces so that the C_{lm}^l are m_l independent and

$$C_{1i}^1=C_1>0,\quad C_{2i}^2=C_2>0 \ . \qquad (12)$$

Thus we obtain the result

$$Q=\tfrac{1}{5}[C_1(\gamma^2-2\delta^2)-C_2\beta^2] \ . \qquad (13)$$

For negative-parity $P=-1$ states $\gamma=\delta=0$ and $Q=-\frac{1}{5}\beta^2C_2<0$. For positive parity $P=+1$, $\alpha=\beta=0$ and $Q=\frac{1}{5}C_1(\gamma^2-2\delta^2)$ and the sign is undetermined.

If we have positive charge conjugation $C=+1$, $\alpha=\beta=\delta=0$ and $\gamma=1$ and, hence, $Q=\frac{1}{5}C_1>0$. For negative charge conjugation $C=-1$, $\gamma=0$ and $Q=-\frac{1}{5}(2C_1\delta^2+C_2\beta^2)<0$. If we have $CP=+1$, $\delta=0$ and $Q=\frac{1}{5}(C_1\gamma^2-C_2\beta^2)$ and the sign is undetermined. For $CP=-1$, $\alpha=\beta=\gamma=0$ and $\delta=1$, $Q=-\frac{2}{5}C_1<0$. The magnetic moment μ and the quadrupole moment Q are given, in terms of the parameters κ and γ by

$$\mu=\frac{e}{2M_W}(1+\kappa+\gamma)$$

and

$$Q=\frac{e}{{M_W}^2}2(\lambda-\kappa) \ . \qquad (14)$$

Thus $Q>0\to\lambda>\kappa$ and $Q<0\to\lambda<\kappa$.

In conclusion, we have shown, in a class of nonrelativistic composite models, how the anomalous magnetic moment of the W boson (κ_W) can serve as a probe of compositeness. In general, for a composite W one will obtain a value which differs from the gauge-theory value, $\kappa_W=1$. A surprising exception to this occurs for the case of two spin-$\frac{1}{2}$ preons with $L=0$ and equal masses for which we obtain $\kappa_W=1$. There are four possible preon spin states. We have found a general expression for κ_W for any linear combination of these states. A range of allowed values for κ_W has been obtained. We have also determined the quadrupole moment for a composite W.

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[1]For recent reviews on composite models see, for example, H. Terazawa, lectures given at Mexican School on Particles and Fields, Oaxtepec, Mexico (1984) (unpublished); R. D. Peccei, in *Proceedings of the International Europhysics Conference on High Energy Physics, Brighton, 1983,* edited by J. Guy and C. Costain (Rutherford, Appleton Laboratory, Chilton, Didcot, United Kingdom, 1984); D. Schildknecht, in *Electroweak Effects at High Energies,* proceedings of the 21st Course of the "Ettore Majorana" International School of Subnuclear Physics, Erice, 1983, edited by H. B. Newman (Plenum, New York, 1985).

[2]This possibility has been discussed by a large number of authors; see, for example, A. Grau and J. A. Grifols, Phys. Lett. **148B**, 358 (1984); M. Kuroda and D. Schildknecht, *ibid.* **121B**, 173 (1983); P. D. B. Collins and N. A. Speirs, J. Phys. G **11**, L115 (1985); G. Girardi, S. Narison, and M. Perrottet, Phys. Lett. **133B**, 234 (1983); S. R. Chowdhury, R. G. Ellis, and G. C. Joshi, Phys. Rev. D **32**, 1250 (1985).

[3]S. Weinberg, Phys. Rev. Lett. **19**, 1364 (1967); Phys. Rev. D **5**, 1412 (1972); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).

[4]K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. **43**, 746 (1979); S. J. Brodsky and R. W. Brown, *ibid.* **49**, 966 (1982); M. A. Samuel, Phys. Rev. D **27**, 2724 (1983); R. W. Brown, K. Kowalski, and S. J. Brodsky, *ibid.* **28**, 624 (1983); M. A. Samuel, in *Neutrino Mass and Gauge Structure of Weak Interactions,* proceedings of the Miniconference, Telemark, Wisconsin, 1982, edited by V. Barger and D. Cline (AIP, New York, 1983); R. W. Brown, in *Electroweak Effects at High Energies* (Ref. 1).

[5]K. Hagiwara, R. D. Peccei, and D. Zeppenfeld, University of Wisconsin Report No. MAD/PH/279, 1986 (unpublished).

[6]M. Claudson *et al.*, Phys. Rev. D **34**, 873 (1986); L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981); Nucl. Phys. **B189**, 547 (1981).