Finite-temperature and -density renormalization effects in QED

K. Ahmed and Samina Saleem
Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan
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An earlier work on finite-temperature renormalization and radiative corrections in QED is generalized to incorporate finite-density effects.

Some methods of quantum field theory have their origin in many-body systems. Such theories account for finite-temperature and -density (FTD) effects in the quantum description and are known to possess nontrivial significance in heavy-ion physics and cosmology. A renewed interest in relativistic calculations in FTD recently arose from the fact that the spontaneously broken gauge symmetries can be restored at sufficiently high temperatures.[1] Other effects such as deconfinement and supersymmetry breaking, although not so well understood, have more recently been found in finite-temperature field theories.[2] Further, FTD analysis have some interesting physical effects occurring in the particle accelerators where relativistic plasma is produced. However, for a systematic understanding of FTD dynamics one must be able to solve the basic question of renormalization of FTD. In this connection, we have attempted to study analytically the electron mass-shift and wave-function renormalization and, as an illustration of these effects, have calculated the decay of the scalar Higgs boson $H\rightarrow e^+e^-$ at finite temperature.[3] The above study is meant to be a generalization of the previous works[4] on the same issues to all temperatures. In particular, as expected, it correctly reproduces all results hitherto obtained for "low-"temperature and "high-"temperature limits of this aspect of the problem belonging to finite-temperature QED. The basis of the calculations in Ref. 3 is to take into account the full and unapproximated form of the fermion density function in the presence of the heat bath. The results derived therein are however valid for zero chemical potential ($\mu=0$). But for situations where FTD effects with finite value of chemical potential are required, for instance, in neutron stars,[5] the above-mentioned work needs to be further generalized. In order to do that, the chemical potential will be included in the fermion propagator. The Feynman diagrams, however, are calculated by inserting the modified FTD propagators in the usual $\mu=T=0$ Feynman rules.

Following Levinson and Boal,[5] we proceed by introducing the FTD fermion propagator as

$$S_F^{\beta}(p)=\frac{i}{\not p-m+i\epsilon}-2\pi\delta(p^2-m^2)\times[\theta(p_0)n_F^{+}(p,\mu)+\theta(-p_0)n_F^{-}(p,\mu)]\ , \tag{1}$$

where

$$n_F^{\pm}(p,\mu)=\frac{1}{e^{\beta(|E_p|\pm\mu)}+1} \tag{1a}$$

is the μ-dependent Fermi-Dirac distribution function. A plus sign is taken for the positron and a minus sign for the electron, respectively. The photon propagator in the heat bath is taken in the Landau gauge, for convenience. Thus,

$$D_{\mu\nu}(q)=-g_{\mu\nu}\left[\frac{i}{q^2+i\epsilon}+2\pi\delta(q^2)n_B(E_q)\right], \tag{2}$$

with

$$n_B(E_q)=\frac{1}{e^{\beta|E_q|}-1}\ , \tag{2a}$$

which is the Bose-Einstein distribution.

Now we follow the calculational scheme laid down in Ref. 3 and essentially reproduce all the steps carried out there substituting the fermion propagator from Eq. (1) in place of its corresponding expression with $\mu=0$ as taken in Ref. 3. No additional ultraviolet divergence would arise in this process because of the natural "cutoff" provided by the FTD distribution functions. However, in the calculations of the vertex part, the "extra" order of infrared divergence arising from $n_B(q)$ in the limit $q_\mu\rightarrow 0$ must be nullified by the "bremsstrahlung" effect and by the spontaneous emission and absorption of photons from the heat bath. As expected, in this case, the presence of chemical potential in no way affects the result on the cancellation of "infrared" divergence[3,6] as derived for the case of $\mu=0$ and $T\neq 0$. Thus a straightforward generalization of Ref. 3 along the above lines leads to the first order in α electron FTD mass $m_{\rm phys}$ defined as

$$m_{\rm phys}=m+\delta m(T,\mu)\ , \tag{3}$$

where m is the electron mass at $\mu=T=0$ and $(\delta m/m)(T,\mu)$ can be calculated as

$$\frac{\delta m}{m}(T,\mu)\simeq\frac{\alpha\pi T^2}{3m^2}\left[1-\frac{6}{\pi}\tilde{c}(m\beta,\mu)\right]+\frac{2\alpha}{\pi}\frac{T}{m}\tilde{a}(m\beta,\mu)-\frac{3\alpha}{\pi}\tilde{b}(m\beta,\mu)\ , \tag{4}$$

where $\tilde{a}(m\beta,\mu)$, $\tilde{b}(m\beta,\mu)$, and $\tilde{c}(m\beta,\mu)$ are symmetrized

functions over the fermionic charge defined as

$$\tilde{a}(m\beta,\mu)=\tfrac{1}{2}[a(m\beta,\mu)+a(m\beta,-\mu)]\ , \tag{5a}$$

$$\tilde{b}(m\beta,\mu)=\tfrac{1}{2}[b(m\beta,\mu)+b(m\beta,-\mu)]\ , \tag{5b}$$

$$\tilde{c}(m\beta,\mu)=\tfrac{1}{2}[c(m\beta,\mu)+c(m\beta,-\mu)]\ , \tag{5c}$$

with

$$a(m\beta,\pm\mu)=\ln(1+e^{-\beta(m\pm\mu)})\ , \tag{6a}$$

$$b(m\beta,\pm\mu)=\sum_{n=1}^{\infty}(-1)^n e^{\mp n\beta\mu}\mathrm{Ei}(-nm\beta)\ , \tag{6b}$$

$$c(m\beta,\pm\mu)=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-n\beta(m\pm\mu)}\ , \tag{6c}$$

as used in Ref. 3. A comparison of Eq. (4) with its analogous Eq. (2.6b) of Ref. 3 suggests that $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ replace a,b,c in the presence of finite chemical potential. Further, as in Ref. 3, Eq. (4) here is obtained by retaining the leading terms in the relevant Green's-function expansion in powers of m/E_l, where E_l is the loop energy in a single-loop Feynman amplitude. Also, the symmetrized functions make the physical mass and the self-energy independent of charge.

It is trivial to check that in the limit $\mu\to 0$ and $T>0$, we can recover the previously obtained result on self-mass as the functions $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ in Eqs. (5) simply yield the corresponding functions a, b, and c [Eq. (2.5) of Ref. 3]. Combining Eqs. (3) and (4)

$$m_{\mathrm{phys}}{}^2\simeq m^2\left[1-\frac{6\alpha}{\pi}\tilde{b}(m\beta,\mu)\right]+\frac{4\alpha}{\pi}mT\tilde{a}(m\beta,\mu)+\tfrac{2}{3}\alpha\pi T^2\left[1-\frac{6}{\pi}\tilde{c}(m\beta,\mu)\right]\ . \tag{7}$$

Similarly, following the procedure for calculating the wave-function renormalization constant Z_2 as given in Refs. 6 and 7 and using Eqs. (1)—(7), we obtain

$$Z_2{}^{-1}(m\beta,\mu)\simeq Z_2{}^{-1}(T=\mu=0)-\frac{2\alpha}{\pi}\int\frac{dk}{k}n_B(k)-\frac{5\alpha}{\pi}\tilde{b}(m\beta,\mu)+\frac{\alpha}{\pi}\frac{T^2}{E^2}\frac{1}{v}\ln\frac{1-v}{1+v}\left[\tilde{c}(m\beta,\mu)-\frac{\pi^2}{6}-m\beta\tilde{a}(m\beta,\mu)\right]\ . \tag{8}$$

Next, we proceed to calculate the FTD corrections to the decay rate of the scalar Higgs boson $H\to e^+e^-$. Here again, we use the symmetrized decay amplitude over the electronic charge because in the final state for this process, there is an equal probability of e^+e^- pairs produced. Thus the two-body FTD corrections to the decay rate can simply be calculated as

$$\Delta\Gamma_2\simeq 2\Gamma_0\left\{\left[\frac{2\alpha}{\pi}+\frac{\alpha}{\pi}\frac{1+v^2}{v}\ln\frac{1-v}{1+v}\right]\int\frac{dk}{k}n_B(k)+\frac{\alpha}{\pi v^2}\left[3v^2-v\ln\frac{1-v}{1+v}-\frac{m^2}{E^2}-\frac{1}{2}\right]\tilde{b}(m\beta,\mu)\right.$$

$$\left.+\frac{\alpha}{\pi v^3}\frac{T}{E^2}\left[2mv+\left[m-\frac{1+v^2}{2}E\right]\ln\frac{1-v}{1+v}\right]\tilde{a}(m\beta,\mu)+\frac{\alpha\pi}{6v^3}\frac{T^2}{E^2}\left[\ln\frac{1-v}{1+v}+2v\right]\left[1-\frac{6}{\pi^2}\tilde{c}(m\beta,\mu)\right]\right\} \tag{9}$$

(Γ_0 is the uncorrected decay rate). Whereas the correction to the phase-space contribution of the decay rate is

$$\Delta\Gamma_{\mathrm{PS}}\simeq 36\frac{\alpha}{\pi v^2}\frac{m^2}{m_H{}^2}\tilde{b}(m\beta,\mu)-24\frac{\alpha}{\pi v^2}\frac{mT}{m_H{}^2}\tilde{a}(m\beta,\mu)-\frac{4\alpha\pi}{v^2}\frac{T^2}{m_H{}^2}\left[1-\frac{6}{\pi^2}\tilde{c}(m\beta,\mu)\right]+O(T^4)\ . \tag{10}$$

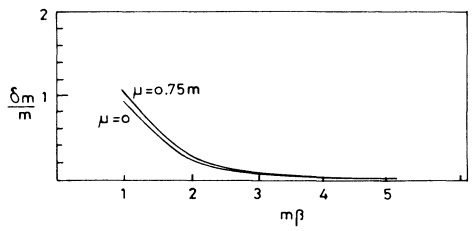

FIG. 1. $\delta m/m$ as a function of $m\beta$ is plotted for the electron for typical values $\mu=0$ and $0.75m$.

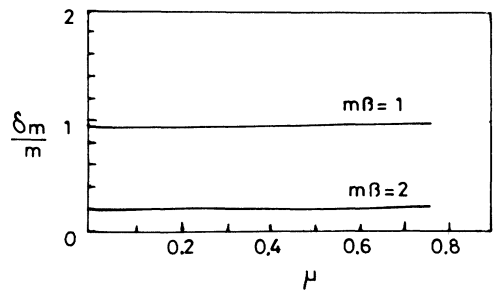

FIG. 2. $\delta m/m$ is plotted against μ for typical values of temperature $T=m$ and $T=m/2$.

TABLE I. For the decay process $H \rightarrow e^+e^-$, the ratio of the total radiative (temperature) correction $\Delta\Gamma_{\text{tot}}$ to the decay rate compared to the uncorrected rate Γ_0 is tabulated for certain allowed values of μ. The values of $m\beta$ (or T) chosen here are those where some significant correction is expected.

		$\Delta\Gamma_{\text{tot}}/\Gamma_0$			
$m\beta$	T (MeV)	$\mu=0$	$\mu=0.25m$	$\mu=0.5m$	$\mu=0.75m$
0.001	500	0.2629	0.3087	0.3142	0.3128
0.005	100	0.2275	0.2158	0.2150	0.2131
0.01	50	0.1834	0.1851	0.1860	0.2082
0.05	10	0.1147	0.1175	0.1184	0.1173
0.1	5	0.0843	0.0908	0.0908	0.0901
0.5	1	0.0326	0.0333	0.0334	0.0385
1	0.5	0.0147	0.0149	0.0158	0.0171
2	0.25	0.0037	0.0039	0.0041	0.0071

Adding phase-space contributions to the two-body and three-body decay rates,[3] the total FTD corrections to the decay rate (for $\mu < m$) becomes

$$\Delta\Gamma_{\text{tot}} \simeq 2\Gamma_0 \left\{ \frac{\alpha}{\pi v^2} \left[3v^2 - v \ln\frac{1-v}{1+v} + \frac{11}{2}\frac{m^2}{E^2} - \frac{1}{2} \right] \tilde{b}(m\beta,\mu) \right.$$
$$\left. + \frac{\alpha}{\pi v^2}\frac{T}{E^2} \left[-m + \frac{1}{v}\ln\frac{1-v}{1+v} \left[m - \frac{1+v^2}{2}E \right] \right] \tilde{a}(m\beta,\mu) - \frac{\alpha}{\pi v^3}\frac{T^2}{E^2} \left[\ln\frac{1-v}{1+v} + 3v \right] \tilde{c}(m\beta,\mu) \right\} . \quad (11)$$

It is again trivial to check that Eq. (11) reduces to Eq. (2.19) of Ref. 3 in the limit $\mu \rightarrow 0$.

In conclusion, we make the following observations. FTD corrections to the electron self-energy are now given by a generalized formula (4). This result shows that the chemical potential enters through the functions $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ only. However, in the case $0 < \mu < m$, we find that

$$\frac{\delta m}{m} \simeq \frac{\alpha\pi T^2}{3m^2} \quad \text{for } T \ll m$$

and

$$\frac{\delta m}{m} \simeq \frac{\alpha\pi T^2}{2m^2} \quad \text{for } T \gg m .$$

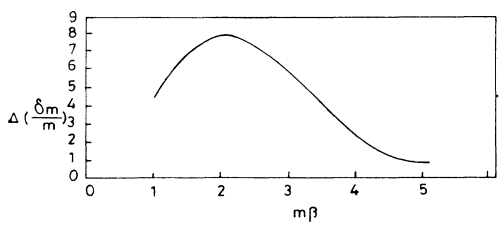

FIG. 3. Percentage deviation $\Delta\delta m/m$ in the values of $\delta m/m$ as a function of μ from the corresponding zero chemical potential value has been plotted. The plot covers values of $(\delta m/m)$ for $\mu = 0.75m$ as in Fig. 1.

A comparison of the above result with Refs. 3, 4, and 6 suggests that the chemical-potential effects for the above temperature ranges may be neglected. In the temperature range $T \sim m$, however, the chemical-potential effects are non-negligible, which is obvious from Fig. 1. After $m\beta = 3$, the values of $\delta m/m$ for $\mu = 0.75m$ and $\mu = 0$ coincide and the curves flatten out. Thus, the μ effects are again negligible for this range. In Fig. 2 we plot $\delta m/m$ as a function of μ for those values of $m\beta$ where μ dependence is significant. We notice that the $\delta m/m$ vs μ plot is almost a straight line with vanishing slope including temperatures $T \sim m$. In Fig. 3 the percentage deviations in $\delta m/m$ for typical nonvanishing μ values, say, $\mu = 0.75m$ and $\mu = 0$, i.e.,

$$\Delta\left[\frac{\delta m}{m}\right] = \frac{(\delta m/m)(\mu=0.75m) - (\delta m/m)(\mu=0)}{(\delta m/m)(\mu=0)} \times 100 ,$$

is plotted against $m\beta$. It shows that for $m\beta = 2$, $\Delta(\delta m/m)$ is maximum, nearly 8%. The same behavior in Δ values is predicted for other permissible values of μ. Equation (11) gives an illustration of a renormalized decay rate calculation for a simple process. In Table I $(\Delta\Gamma_{\text{tot}}/\Gamma_0)$ has been computed for various values of μ and $m\beta$. For values of $T \lesssim m/2$, $(\Delta\Gamma_{\text{tot}}/\Gamma_0)$ is nearly 0.4% and hence these temperature points are not included in Table I.

Finally, we note that for $\mu = 0$ and $T \ll m$ or $T \gg m$, we obtain from Eq. (11), $(\Delta\Gamma_{\text{tot}}/\Gamma_0) \simeq 0$, as the functions $\tilde{a}$, $\tilde{b}$, and $\tilde{c}$ in these limits may be neglected, thus reproducing the results of Refs. 3 and 6.

[1]L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Weinberg, *ibid.* **9**, 3357 (1974); D. A. Kirzhnitz and A. D. Linde, Phys. Lett. **42B**, 471 (1972); A. D. Linde, Rep. Prog. Phys. **42**, 389 (1979).

[2]See, e.g., Daniel Boyanovsky, Phys. Rev. D **29**, 743 (1984); Hideaki Aoyama and Daniel Boyanovsky, *ibid.* **30**, 1356 (1984).

[3]K. Ahmed and Samina Saleem, Phys. Rev. D **35**, 1861 (1987).

[4]J. F. Donoghue, B. R. Holstein, and R. W. Robinett, Ann. Phys. (N.Y.) **164**, 233 (1985), and references therein; G. Peressutti and B. S. Skagerstam, Phys. Lett. **110B**, 406 (1982).

[5]Edward J. Levinson and David H. Boal, Phys. Rev. D **31**, 3280 (1985).

[6]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **28**, 340 (1983); **29**, 3004(E) (1983).

[7]For details, see the Appendix of Ref. 3.