Does the proton contain a pointlike diquark?

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We calculate the proton mass and the diquark radius in a previously given relativistic bound-state model. The radius of the diquark turns out to be ≈ 0.6 F. This argues against a pointlike-diquark quark structure for the proton.

Recently the split field magnet group at the CERN ISR reported data[1] on their measurement of baryon production at high transverse momenta p_T and intermediate c.m.-system (c.m.s.) polar angles at a center-of-mass energy $\sqrt{s}=62$ GeV. The fractional proton yield is not only high but also dependent on p_T and c.m.s. angle θ, whereas that for antiprotons is an order of magnitude smaller and is constant. These observations cannot be understood within the framework of lowest-order hard-quark and gluon scattering with commonly used fragmentation schemes,[2,3] or within perturbative QCD (Ref. 3). These observations are, however, well described[2,3] by a diquark scattering model.[4] In this model[4] the proton is considered to be a bound system of a quark and a diquark of *spin zero*. The high proton yield is then accounted for by hard scattering of the diquark in the incident proton, which thereafter fragments into a baryon. Detailed calculations require (i) a fragmentation scheme capable of describing diquark hadronization, (ii) quantitative formulation for the proton structure in terms of quark, gluon, and diquark structure functions, and (iii) proper construction of almost pointlike-diquark scattering amplitudes. For these and specific assumptions, we refer the reader to Refs. 2 and 3. The important point is that the model requires almost[5] pointlike spin-0 diquark structure of the proton. It is to be noted that the analysis involves dynamics of reactions and scattering rather than that of relativistic bound states. It is of interest, then, to investigate whether the above picture of the proton holds well within the framework of a relativistic dynamical model of bound states.

In an earlier paper[6] we satisfactorily described quarkonium spectra using the two-body relativistic bound-state Bethe-Salpeter equation:

$$\begin{aligned}\frac{M}{2}(4m_1m_2-4\eta_1\eta_2M^2-4\nabla_r{}^2)\chi(r)=&+\tfrac{4}{3}\lambda\left[-4\eta_1\eta_2M^2r+\frac{2}{r}+\frac{4\mathbf{r}\cdot\nabla}{r}+\frac{4r^2\nabla^2}{r}-\frac{4\mathbf{L}\cdot\mathbf{S}}{r}\right.\\&\left.+\frac{M}{m_1+m_2}\frac{m_1{}^2+m_2{}^2}{m_1m_2}\left[\frac{2}{r}+\frac{2\mathbf{r}\cdot\nabla}{r}\right]-\frac{S_{12}}{3r}-\frac{4}{3}\frac{\sigma_1\cdot\sigma_2}{r}\right]\chi(r)\\&+\tfrac{4}{3}\alpha_s\left[4\eta_1\eta_2\frac{M^2}{r}+4\pi\delta^3(\mathbf{r})\left[1-\sigma_1\cdot\sigma_2+\frac{M}{m_1m_2}\frac{m_1{}^2+m_2{}^2}{m_1m_2}\right]\right.\\&\left.-\frac{S_{12}}{r^3}-\frac{4\mathbf{L}\cdot\mathbf{S}}{r^3}-\frac{4r^2\nabla^2}{r^3}+\left[\frac{2M}{m_1+m_2}\frac{m_1{}^2+m_2{}^2}{m_1m_2}+4\right]\frac{\mathbf{r}\cdot\nabla}{r^3}\right]\chi(r)\\&+\frac{C}{m_1m_2}(-4\eta_1\eta_2M^2+4\nabla_r{}^2)\chi(r)\;. \qquad (1)\end{aligned}$$

The interaction potential used was the QCD-motivated linear plus Coulomb potential

$$V(r)=F_{12}\left[\frac{\alpha_s}{r}-\lambda r-\frac{C}{m_1m_2}\right], \qquad (2)$$

with the parameter values

$$\begin{aligned}&\alpha_s=0.6,\ \ \lambda=0.08\ (\mathrm{GeV})^2,\ \ C=-0.112\ (\mathrm{GeV})^3\ ,\\&m_u=0.3\ \mathrm{GeV},\ \ m_s=0.44\ \mathrm{GeV}\ , \qquad (3)\\&m_c=1.535\ \mathrm{GeV},\ \ m_b=4.98\ \mathrm{GeV}\ ,\end{aligned}$$

and the value of the color factor $F_{12}=-\frac{4}{3}$ for $q\bar{q}$. In another paper[7] the model was applied to calculate the

TABLE I. Proton mass as a function of the quark mass m_u when $p \equiv [(uu)_0 u]_{1/2}$ and $p \equiv [(uu)_1 u]_{1/2}$.

m_u (GeV)	Diquark $(uu)_0$[a] mass (GeV)	Proton $[(uu)_0 u]_{1/2}$[a] mass (GeV)	Diquark $(uu)_1$ mass (GeV)	Proton $[(uu)_1 u]_{1/2}$[a] mass (GeV)
0.240	0.5910	1.0588		
0.241	0.5911	1.0586		
0.242	0.5912	1.0586		
0.243	0.5923	1.0584	0.6142	1.0281
0.244	0.5928	1.0584	0.6148	1.0280
0.245	0.5933	1.0584	0.6154	1.0278
0.246	0.5937	1.0584	0.6160	1.0278
0.247	0.5943	1.0585	0.6167	1.0278
0.248	0.5948	1.0586	0.6174	1.0278
0.249	0.5952	1.0586	0.6180	1.0278
0.250	0.5958	1.0587	0.6187	1.0279

[a]In our model we do not distinguish between u and d quarks, and hence have suppressed the distinction in the notation. To be very clear, $(uu)_0$ means either a $(uu)_0$ or a $(ud)_0$ diquark, and $[(uu)_0 u]_{1/2}$ means either a $[(uu)_0 d]_{1/2}$ or $[(ud)_0 u]_{1/2}$ proton.

$J=\frac{1}{2}$ and $\frac{3}{2}$ baryon masses, considering them as bound states of a quark and a diquark of *spin one*. With the quark masses $m_u=0.3$ GeV and $m_s=0.44$ GeV, the theoretically calculated mass values 1.151, 1.258, 1.368, and 1.529 GeV for Δ, Σ^*, Ξ^*, and Ω respectively, were slightly lower than their experimental counterparts 1.232, 1.385, 1.532, and 1.670 GeV; whereas the calculated value of 1.058 GeV for the proton was slightly higher than its observed mass of 0.940 GeV. The calculated value of Δ could be brought up to 1.232 GeV by raising the quark mass m_u to 0.345 GeV; but it also increased the calculated value of the proton mass to 1.127 GeV, thus worsening the situation. In view of the discussion in the opening paragraph, the question has, therefore, been raised whether the situation regarding the proton can be improved by considering it a bound state of a quark and a diquark of spin zero instead of spin one.[8] Consequently we solved our two-body Bethe-Salpeter equation (1) first to find the mass of the $(uu)_0$ diquark, and then that of the proton. We did this as a function of the quark mass m_u. The results are shown in columns 1–3 of Table I.

We find a minimum in the proton mass which occurs at $m_p=1.058$ GeV for $m_u=0.244$ GeV; and note that this does not improve the situation. In view of this we then calculated the mass of the $(uu)_1$ diquark of spin one and the proton mass as a function of the quark mass m_u. The numbers are cited in columns 4 and 5 of Table I. We find a minimum in the proton mass in this case also, which occurs at $m_p=1.028$ GeV for $m_u=0.247$ GeV. Note that in the two cases the minimum value of the proton mass is almost the same and that it occurs for about the same value of the quark mass m_u.

It is to be noted that in our calculation of the proton mass implicit is the *assumption* that the diquark is a *point* particle. If this assumption were true, then on the basis of our result we would conclude that whereas the CERN ISR data on high fractional proton yield at high p_T suggests a quark—spin-zero-diquark structure for the proton as opposed to a quark—spin-one-diquark structure, our quark-diquark relativistic bound-state model of the proton does not distinguish between the two possibilities. For verification, therefore, of the pointlike nature of the diquark, we computed the rms radii of the two diquarks $(uu)_0$ and $(uu)_1$ of masses 0.5928 and 0.6167 GeV, respectively. They turn out to be 0.58 and 0.612 F, respectively,[9] showing thereby that the diquark of either spin (zero or one) is an extended object and not a pointlike particle, even if one assumes a defining radius equal to 0.2 F for a pointlike particle, the size required by the scattering model of Ref. 4. (Further verification is provided in Table II where we have given our computed radii for some other diquarks and mesons.) Consequently, the scattering model of Ref. 4 is at odds with our relativistic bound-state model.[6] In other words, if our relativistic bound-state model,[6] which is based on the Bethe-Salpeter

TABLE II. rms radii for some diquarks and mesons of mass M with parameters in Eq. (3).

Diquark	M (GeV)	rms radius (F)
$(uu)_0$	0.631	0.586
$(uu)_1$	0.662	0.636
$(ss)_0$	0.834	0.730
$(ss)_1$	0.898	0.801
$(cc)_0$	3.123	0.507
$(cc)_1$	3.167	0.571
$(ub)_0$	5.568	0.836
$(ub)_1$	5.584	0.854
Meson		
$(uu)_1$	0.866	0.488
$(cc)_1$	3.107	0.435
$(ub)_0$	5.768	0.633
$(ub)_1$	5.798	0.646

equation and the QCD-motivated interquark potential, is correct, then the data on high fractional proton yield at high p_T would have to be accounted for with some mechanism other than the assumption of an almost pointlike diquark in the proton. It is evident, in view of the large diquark radius ≈ 0.6 F, that our calculation of the proton and other baryon masses[7] is *a priori* internally inconsistent. It seems that the problem of baryon masses should best be described as a three-body problem.[10]

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[5]Based on the M^2 value in their diquark form factor, the authors of Ref. 3 find the diquark radius to be 0.2 F or less.
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[8]We wish to acknowledge receipt of a letter of inquiry from Professor S. Fredriksson, which initiated the present investigation.
[9]A simple caluclation based on the uncertainty principle gives the values 0.58 and 0.53 F, respectively; and a naive string model in which one assumes the mass of the quark to be zero, gives the values 0.93 and 0.96 F for the string length.
[10]This is also indicated by the low value of 0.5 F for the proton rms radius which we obtained in our quark-diquark model.