Asymptotic flavor symmetry and its implication on $\tau \to \rho\nu_\tau$ and $K^*\nu_\tau$ branching ratio and ground-state 1^{--} meson multiplet

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A remark is made on the overall consistency between the $\tau \to \rho\nu_\tau$ and $K^*\nu_\tau$ decays and other phenomenology of the ground-state 1^{--} mesons obtained from the sole use of asymptotic flavor symmetry in the theoretical framework of a QCD Lagrangian with quark-mass terms.

The $\tau^- \to \rho^-\nu_\tau$ turned out to be the largest one-charged-prong decay mode of the τ and its branching fraction is $(22.3\pm0.6\pm1.4)\%$, according to the latest result[1] which is also compatible with previous measurements.[2] In Ref. 1 the branching fraction of $\tau^- \to K^{*-}\nu_\tau$ is found to be $(1.3\pm0.3\pm0.3)\%$ and the ratio of branching fractions $R \equiv B(\tau^- \to K^{*-}\nu_\tau)/B(\tau^- \to \rho^-\nu_\tau)$ is measured to be $0.058\pm0.013\pm0.013$. If the error involved is further reduced, the value of R provides another relatively clean test of the validity of asymptotic flavor symmetry in the well-established ground-state 1^{--}-meson multiplet. These decays were, in fact, discussed[3] quite some time ago even before the discovery of τ and ν_τ and the results were compared[4] recently with the experiments now available. Let us write down the relevant parts of weak interactions as follows in obvious notations: i.e.,

$$\mathscr{L}_l = \frac{g}{2\sqrt{2}}\bar{\tau}\gamma_\alpha(1+\gamma_5)\nu_\tau W^\alpha + \text{H.c.}$$

and

$$\mathscr{L}_q = \frac{g}{2\sqrt{2}}(\cos\theta V_\alpha^{\pi^+} + \sin\theta V_\alpha^{K^+})W^\alpha + \text{H.c.}$$

Here $V_\alpha^{\pi^+} = \bar{u}\gamma_\alpha d$ and $V_\alpha^{K^+} = \bar{u}\gamma_\alpha s$, respectively. W_α is the charged weak vector boson and θ denotes the Cabibbo angle. The ρ-W and K^*-W coupling constants G_ρ and G_{K^*} are defined by

$$(2q_0)^{1/2}\langle 0 \,|\, V_\mu^{\pi^+}(0) \,|\, \rho^-(\mathbf{q})\rangle = G_\rho \epsilon_\mu^\rho(\mathbf{q})$$

and

$$(2q_0)^{1/2}\langle 0 \,|\, V_\mu^{K^+}(0) \,|\, K^{*-}(\mathbf{q})\rangle = G_{K^*} \epsilon_\mu^{K^*}(\mathbf{q}) ,$$

respectively, where $\epsilon_\mu^\rho(\mathbf{q})$ is the polarization four-vector of the ρ meson. In flavor-$SU_f(3)$-symmetry limit $(m_u = m_d = m_s)$, $G_\rho = G_{K^*}$. The rates of $\tau^- \to \rho\nu_\tau$ and $K^*\nu_\tau$ are then given[5] by $(G_W = g/2\sqrt{2})$

$$\Gamma(\tau \to \rho\nu_\tau) = \frac{1}{16\pi}\left[\cos\theta \frac{G_W{}^2}{m_W{}^2} G_\rho\right]^2 (m_\tau{}^{-3} m_\rho{}^{-2}) \times (m_\tau{}^2 - m_\rho{}^2)^2 (m_\tau{}^2 + 2m_\rho{}^2) , \quad (1)$$

$$\Gamma(\tau \to K^*\nu_\tau) = \frac{1}{16\pi}\left[\sin\theta \frac{G_W{}^2}{m_W{}^2} G_{K^*}\right]^2 (m_\tau{}^{-3} m_{K^*}{}^{-2}) \times (m_\tau{}^2 - m_{K^*}{}^2)^2 (m_\tau{}^2 + 2m_{K^*}{}^2) . \quad (2)$$

For simplicity, we have chosen a zero mass for ν_τ and used the narrow-width approximation for ρ and K^*. What is the relation between G_ρ and G_{K^*}?

We briefly recapitulate the derivation of the following prediction based on asymptotic flavor-$SU_f(3)$ symmetry, which was derived[6] quite some time ago without having direct confrontation with experiment:

$$\frac{G_\rho}{m_\rho} = \frac{G_{K^*}}{m_{K^*}} . \quad (3)$$

Consider an annihilation operator of a *physical* (i.e., "in" or "out") hadron $a_\alpha(\mathbf{k},\lambda)$ with a physical $SU_f(N)$ index α, momentum $\mathbf{k}$, and helicity λ. The transformation of $a_\alpha(\mathbf{k},\lambda)$ under $SU_f(N)$ can be expressed as

$$[V_i, a_\alpha(\mathbf{k},\lambda)] = i\sum_\beta u_{i\alpha\beta}(\mathbf{k},\lambda)a_\beta(\mathbf{k},\lambda) + \delta u_{i\alpha\lambda}(\mathbf{k}) . \quad (4)$$

In exact symmetry, δu vanishes for *any* $\mathbf{k}$ and the indices α and β belong to the *same* $SU_f(N)$ multiplet. However, in broken symmetry the δu term is present and the transformation becomes nonlinear. However, if dynamics permits, we may still hope to possess linear transformation in the limit $\mathbf{k} \to \infty$, where multiplet masses do not play a role. However, particle mixing which inevitably takes place does not disappear even in this limit. Therefore, on the right-hand side of Eq. (4), $\sum_\beta$ is extended over *all* possible particles β which have the same J^{PC} or J^P as α but belong to a different $SU_f(N)$ representation. With this modification, the asymptotic symmetry proposed[7] requires

$$\delta u_{i\alpha\lambda}(\mathbf{k}) \to \frac{1}{|\mathbf{k}|^{1+\epsilon}}, \quad \epsilon > 0 \text{ for } \mathbf{k} \to \infty . \quad (5)$$

The usual linear $SU_f(N)$ transformation is expressed in terms of a (hypothetical) representation operator $a_j(\mathbf{k},\lambda)$ which satisfies, for example,

$$[V_i, a_j(\mathbf{k},\lambda)] = i f_{ijl}\, a_l(\mathbf{k},\lambda) .$$

Therefore, for *only* $\mathbf{k}\to\infty$, the physical operator $a_\alpha(\mathbf{k},\lambda)$ can be related *linearly* to $a_j(\mathbf{k},\lambda)$ by

$$a_\alpha(\mathbf{k},\lambda)=\sum_j C_{\alpha j}(\lambda)a_j(\mathbf{k},\lambda),\quad \mathbf{k}\to\infty\ . \tag{6}$$

In Eq. (6), j thus includes, in principle, all possible relevant representations. Therefore, the matrix $C_{\alpha j}$ involves mixing parameters. Although mixing parameters appearing in $C_{\alpha j}$ play exactly the same role as the conventional mixing parameters in mass formulas, there is a subtle but potentially important difference. That is, our $C_{\alpha j}$'s are defined, for the creation and annihilation operators of the "in" or "out" fields, in the limit $\mathbf{k}\to\infty$ where the $SU_f(N)$ transformation may restore its linearity in *broken* symmetry. Usual mixing (defined for field variables) does not make such a discrimination and the procedure is more approximate. Moreover, in the asymptotic symmetry formulated above, vacuum annihilation by $SU_f(N)$ charges is used *only* among the states with infinite momenta (a process called vacuum annihilation by lightlike charges). Physical on-mass-shell states $|\alpha,\mathbf{k},\lambda\rangle$ can then be expressed *linearly* in terms of the states $|j,\mathbf{k},\lambda\rangle$ if $\mathbf{k}\to\infty$. Therefore, the charge V_α can act in our asymptotic limit as if it were a "perfect generator" of $SU_f(N)$. This yields an enormous simplification in broken symmetry. Asymptotic flavor-$SU_f(N)$ symmetry provides a powerful tool when it is applied in conjunction with the variety of equal-time commutators involving the *generators* of (broken) flavor symmetry, which exist and are *valid* in the theoretical framework of a QCD Lagrangian with a quark-mass term. For a detailed recent review, see Ref. 8. For the present problem we pick out the charge-charge-density commutator (V_{K^0} is the generator of $SU_f(3)$, $V_{K^0}=V_6-iV_7$), $[V_0^{\pi^+}(0),V_{K^0}]=V_0^{K^+}(0)$. Sandwich this commutator between the states $\langle 0|$ and $|K^{*-}(\mathbf{q})\rangle$ with $\mathbf{q}\to\infty$ and use the asymptotic $SU_f(3)$ symmetry described above. We obtain

$$\langle 0|V_0^{\pi^+}(0)|\rho^-\rangle\langle\rho^-|V_{K^0}|K^{*-}(\mathbf{q})\rangle+\sum_{n=\rho',\ldots}\langle 0|V_0^{\pi^+}(0)|n\rangle\langle n|V_{K^0}|K^{*-}(\mathbf{q})\rangle=\langle 0|V_0^{K^+}(0)|K^{*-}(\mathbf{q})\rangle,\quad \mathbf{q}\to\infty\ . \tag{7}$$

The second term of the left-hand side of Eq. (7) contributes *only* if ground-state 1^{--} mesons mix *appreciably* with the radially or orbitally excited 1^{--} states or with exotic 1^{--} $q\bar{q}q\bar{q}$ mesons, etc., and can be neglected to the precision which will be remarked upon later. Then asymptotic $SU_f(3)$ implies $\langle\rho^-(\mathbf{q}')|V_{K^0}|K^{*-}(\mathbf{q})\rangle=1$ [apart from the trivial factor $(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')$] but *only* for $\mathbf{q}\to\infty$. In the frame $q_\mu=(0,0,|\mathbf{q}|,E_V(q))$ with $\mathbf{q}\to\infty$ and $E_V(q)=(m_V{}^2+\mathbf{q}^2)^{1/2}$, Eq. (7) then produces Eq. (3) for the polarization vector $\epsilon_\mu^V(\mathbf{q})=(1/m_V)(0,0,E_V(q),|\mathbf{q}|)$ where $V=\rho$ and K^*. For comparison, we remark here that the same procedure applied for similar commutators involving an electromagnetic current yields[6,8] similar sum rules, which now involves the ω-ϕ mixing angle $\theta_{\omega\phi}$ defined by Eq. (6), for the ρ-γ, ω-γ, and ϕ-γ couplings, i.e.,

$$(G_\phi/m_\phi)=(1/\sqrt{2})\cos\theta_{\omega\phi}(G_\rho/m_\rho)$$

and

$$(G_\omega/m_\omega)=(-1/\sqrt{2})\sin\theta_{\omega\phi}(G_\rho/m_\rho)\ .$$

They yield,[6] upon eliminating $\theta_{\omega\phi}$, a broken-$SU_f(3)$ sum rule for the rates of ρ, ω, and $\phi\to e^+e^-$ decays,

$$\tfrac{1}{3}m_\rho\Gamma_\rho=m_\omega\Gamma_\omega+m_\phi\Gamma_\phi\ , \tag{8}$$

which is well satisfied experimentally, i.e., 1.82 $(\text{MeV})^2=1.85$ $(\text{MeV})^2$. Therefore, we expect that Eq. (3) explains the ratio R to similar accuracy. Equation (3) predicts, using Eqs. (1) and (2),

$$R\equiv\frac{B(\tau^-\to K^{*-}\nu_\tau)}{B(\tau^-\to\rho^-\nu_\tau)}=\frac{(m_\tau{}^2-m_{K^*}{}^2)^2(m_\tau{}^2+2m_{K^*}{}^2)}{(m_\tau{}^2-m_\rho{}^2)^2(m_\tau{}^2+2m_\rho{}^2)}\tan^2\theta\simeq 0.053 \tag{9}$$

for the value of $\sin\theta\simeq 0.231$ given[9] recently. If we instead take the exact symmetry value $G_\rho=G_{K^*}$ we obtain $R=0.039$, while experiment gives[1] $R\simeq 0.058$ with still appreciable error. Equation (9), however, seems to indicate at least that the effect of symmetry breaking given by Eq. (3) is in the right direction. If the mass of ν_τ is large ($\simeq 100$ MeV), its effect has to be included in addition to the correction due to the narrow-width approximation used for the vector mesons.

We also emphasize here that the branching ratio R can serve as a good source for determining the value of the Cabibbo angle, since the 1^{--} nonet is the most well-understood meson multiplet from the point of view of broken flavor symmetry as will be discussed below.

It is well known that Eq. (3) and a sum rule such as Eq. (8) have also been derived[10] by Das, Mathur, and Okubo a long time ago using a version of asymptotic symmetry for the spectral functions of the currents. It is interesting to notice that the concept of asymptotic flavor symmetry formulated in two different ways produces very similar consequences. Asymptotic symmetry described here is concerned more directly with such a pertinent question: How do the creation and annihilation operators of "in" or "out" hadron fields behave under flavor transformation, when quark masses are generated?

Finally, we remark here that asymptotic flavor symmetry alone can predict the characteristic properties of the 1^{--} ground-state mesons, when it is combined with the equal-time commutators present in the usually considered theoretical framework of a QCD Lagrangian with quark-mass terms. For other mesons such as the 0^{-+} mesons, see Ref. 8. A QCD Lagrangian with quark-mass terms implies the presence of equal-time charge commutators such as

$$[V_{K^0}^{(n)}, V_{K^+}]=0, \quad V_{K^0}^{(n)}=\frac{d^n V_{K^0}}{dt^n}, \quad n=1,2,\ldots, \tag{10}$$

$$[\dot{V}_{K^0}, A_{\pi^-}]=0, \quad \dot{V}_{K^0}=\frac{dV_{K^0}}{dt}, \tag{11}$$

where A_{π^-} is the axial-vector charge A_1-iA_2. In Eq. (10), the $n=1$ commutator is generally valid as long as the SU(3)-breaking interaction belongs to an octet. However, the validities of the commutators with $n=2,3,\ldots$ given in Eq. (10) and also of Eq. (11) require more specific models of symmetry breaking.

If we sandwich Eqs. (10) with $n=1,2,\ldots$ between the states $\langle K^{*+}(\mathbf{q})|$ and $|\bar{K}^{*0}(\mathbf{q})\rangle$ with $\mathbf{q}\to\infty$ and if we assume, as we did in Eq. (7), that ground-state 1^{--} nonet mesons mix *only* among themselves (i.e., we consider only ω-ϕ mixing), asymptotic $SU_f(3)$ symmetry immediately implies[8] the ideal nonet structure for the 1^{--} mesons, i.e., $\tan\theta_{\omega\phi}=1/\sqrt{2}$ and ideal mass spectrum $m_\rho^2=m_\omega^2$ and $m_\phi^2-m_{K^*}^2=m_{K^*}^2-m_\rho^2$. Then Eq. (11) inserted between $\langle K^{*0}(\mathbf{q},\lambda=1)|$ and $|\rho^+(\mathbf{q},\lambda=1)\rangle$ with $\mathbf{q}\to\infty$ yields, under the same approximation, $\langle\rho^-|A_{\pi^-}|\phi(\mathbf{q},\lambda=1)\rangle=0$ for $\mathbf{q}\to\infty$, which implies via PCAC (partial conservation of axial-vector current) that $\phi\to\rho\pi$ decay is forbidden. We thus see that the well-known characteristic of the ground-state 1^{--} meson multiplet and the presence of dynamical selection rule (i.e., quark-line rule) can be *derived* simultaneously in this simple theoretical framework, if asymptotic flavor symmetry[11] is valid. Small deviations from the ideal structure of the 1^{--} multiplet and also the predictions made in Eqs. (3) and (8), etc., will arise from the inclusion of small further mixings which were mentioned but discarded and also of the ω-ϕ-ρ^0 mixing due to the breaking of $SU_I(2)$ symmetry.

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