On the detection of cosmic-background neutrinos by acoustic phonon scattering

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We examine the possible detection of cosmic-background neutrinos by coherent neutrino-phonon scattering. Our conclusion is that at least for simple detector schemes the reaction rate is unobservably small.

Standard "big-bang" cosmology predicts that in addition to a relic sea of photons characterized by a blackbody temperature $T_\gamma \simeq 2.7$ K, there should also exist a relic sea of neutrinos and antineutrinos characterized by a blackbody temperature $T_\nu \simeq 1.9$ K. Observation of these cosmic-background neutrinos would provide a wealth of information about the Universe at a much earlier epoch ($t \simeq 1$ sec versus $t \simeq 4\times 10^5$ yr) and might serve as a probe of neutrino masses.[1] It is therefore unfortunate that the relevant cross sections and effects appear to be inaccessibly small. Opher[2] and Lewis[3] have proposed that for very-low-energy neutrinos coherent scattering should dominate over incoherent scattering and result in measurable forces on macroscopic bodies, the force being proportional to G_F, the Fermi coupling constant, rather than $G_F{}^2$. Subsequent analysis of this proposal by Cabibbo and Maiani[4] and Langacker, Leveille, and Shelman[5] shows, however, that in general the order-G_F effect cancels, leaving an unobservable force of order $G_F{}^2$. An exception is the nonvanishing order-G_F torque on a polarized target, although this effect is also expected to be immeasurably small.[5,6]

Lewis[3] has also commented, in passing, that the study of "coherent excitation of low-lying collective modes such as acoustic phonons" may prove fruitful. A related idea involving scattering of acoustic phonons by background neutrinos has been investigated by the present authors; here we briefly describe our, unfortunately, still discouraging result. Just as for thermal-neutron scattering, low-energy neutrino scattering by phonons can be viewed as a diffractive process arising from a periodic density fluctuation; the critical quantity to evaluate is the cross section for such scattering to occur.

The analogy with neutron scattering can be carried further since at the low energies involved the neutrino-matter interaction potential may be represented as a sum of δ functions so that many expressions[7] can be taken over with minor modifications due to the assumption of massless neutrinos; in particular the differential cross section in the Born approximation for neutrino scattering on an ideal monatomic harmonic crystal is

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\sigma_c}{4\pi}\left[\frac{E'}{E}\right]^2 \delta(E'-E+E_{\lambda'}+E_\lambda) \times \left| \sum_{\mathbf{R}} e^{-i\mathbf{q}\cdot\mathbf{R}} \langle \lambda' \mid \exp[-i\mathbf{q}\cdot\mathbf{U}(\mathbf{R})] \mid \lambda\rangle \right|^2 . \tag{1}$$

Here E (E') is the initial (final) neutrino energy and $\hbar\mathbf{q}$ is the neutrino momentum transfer. The cross section σ_c is that for coherent neutrino scattering from a single ion; with $\sin^2\theta_W = \frac{1}{4}$,

$$\sigma_c = \frac{G_F{}^2 E^2}{4\pi(\hbar c)^4}(2n_e \delta_{\nu\nu_e} + Z - A)^2 , \tag{2}$$

where n_e is the effective number of electrons in the ionic core. The $\delta_{\nu\nu_e}$ arises as electron neutrino scattering receives contributions from the charged as well as the neutral current. Differing from the usual treatment of neutron scattering, we do not sum over final states and perform a thermal average over initial states; rather, the initial and final states are fixed in terms of the phonon occupation numbers

$$|\lambda\rangle = \prod_l |n_l\rangle , \tag{3a}$$

$$|\lambda'\rangle = \prod_l |n_l'\rangle , \tag{3b}$$

with $\mathbf{q}\cdot\mathbf{U}(\mathbf{R})$ expanded as usual in terms of phonon creation and annihilation operators $a_l^\dagger$ and a_l:

$$\mathbf{q}\cdot\mathbf{U}(\mathbf{R}) = \sum_l Z_l (e^{i\mathbf{k}_l\cdot\mathbf{R}} a_l + e^{-i\mathbf{k}_l\cdot\mathbf{R}} a_l^\dagger) , \tag{4a}$$

$$Z_l = \left[\frac{\hbar}{2NM\omega_l} \right]^{1/2} \mathbf{q}\cdot\boldsymbol{\epsilon}_l \ , \tag{4b}$$

where $\mathbf{R}$ is the static lattice vector, N is the number of lattice sites, M is the effective ion mass, and $\hbar\omega_l$, $\mathbf{k}_l$, and $\boldsymbol{\epsilon}_l$ are, respectively, the energy, wave vector, and polarization of the lth mode phonon.

We now restrict our considerations to the case where the neutrino scatters Δn_j phonons out of the jth mode and Δn_i phonons into the ith mode, all other modes being spectators. The required matrix element is then evaluated as, apart from a phase

$$\sum_{\mathbf{R}} e^{-i\mathbf{q}\cdot\mathbf{R}} \langle \lambda' \mid \exp[-i\mathbf{q}\cdot\mathbf{U}(\mathbf{R})] \mid \lambda \rangle = N\delta_{\mathbf{Q},\mathbf{K}} Z_i^{\Delta n_i} Z_j^{\Delta n_j} \frac{1}{(\Delta n_i)!(\Delta n_j)!} \left[\frac{n_i'!}{n_i!} \frac{n_j!}{n_j'!} \right]^{1/2} f \ , \tag{5a}$$

$$f = e^{Z_i^2/2} \Phi(n_i'+1, \Delta n_i+1; -Z_i^2) e^{Z_j^2/2} \Phi(n_j+1, \Delta n_j+1; -Z_j^2) \prod_{l \neq i,j} e^{Z_l^2/2} \Phi(n_l+1, 1; -Z_l^2) \tag{5b}$$

with

$$\mathbf{Q} = \Delta n_j \mathbf{k}_j - \Delta n_i \mathbf{k}_i - \mathbf{q} \ , \tag{5c}$$

where $\mathbf{K}$ is a reciprocal lattice vector which we take to vanish and $\Phi(\alpha,\gamma;z)$ is the degenerate hypergeometric function. As the Z_l are small $f \simeq 1$ and will be neglected. Identification of the phonon modes i and j as acoustic is made via

$$\omega_l = c_l \mid \mathbf{k} \mid , \quad l = i,j \ . \tag{6}$$

We further specialize to transverse phonons such that the polarizations satisfy

$$\mid \mathbf{k}_j\cdot\epsilon_i \mid = \mid \mathbf{k}_j \mid \sin\theta_{ij} \ , \tag{7a}$$

$$\mid \mathbf{k}_i\cdot\epsilon_j \mid = \mid \mathbf{k}_i \mid \sin\theta_{ij} \ , \tag{7b}$$

where θ_{ij} is the angle between $\mathbf{k}_i$ and $\mathbf{k}_j$. Finally we assume n_i, Δn_i, n_j, and Δn_j are large, as is clearly a reasonable condition for observation, so by use of Stirling's formula we obtain the differential cross section

$$\frac{d\sigma}{d\Omega\, dE'} \simeq N^2 \frac{\sigma_c}{4\pi} \left[\frac{E'}{E} \right]^2 \delta \left[E' - E + h \left[\frac{c_i \Delta ni}{\lambda_i} - \frac{c_j \Delta n_j}{\lambda_j} \right] \right] \times \left[\frac{\lambda_c^i}{2\lambda_i} \frac{n_i}{N} \left[\frac{\lambda_i \Delta n_j}{\lambda_j \Delta n_i} e \sin\Theta_{ij} \right]^2 \right]^{\Delta n_i} \left[\frac{\lambda_c^j}{2\lambda_j} \frac{n_j}{N} \left[\frac{\lambda_j \Delta n_i}{\lambda_i \Delta n_j} e \sin\Theta_{ij} \right]^2 \right]^{\Delta n_j} . \tag{8}$$

Here λ_l is the lth mode phonon wavelength, and

$$\lambda_c^l = \frac{h}{Mc_l} \ . \tag{9}$$

As λ_i and λ_j will be of comparable magnitude, and

$$\left[\frac{\Delta n_i}{\Delta n_j} \right]^{\Delta n_j - \Delta n_i}$$

will be small unless $\Delta n_i \simeq \Delta n_j \simeq \Delta n$, the expression for the differential rate can be expressed as

$$j_\nu \frac{d\sigma}{d\Omega\, dE'} \simeq N^2 \frac{j_\nu \sigma_c}{4\pi} \left[\frac{E'}{E} \right]^2 \delta \left[E' - E + h \left[\frac{c_i \Delta n_i}{\lambda_i} - \frac{c_j \Delta n_i}{\lambda_j} \right] \right] \left[\left[\frac{\lambda_c^i}{2\lambda_i} \frac{n_i}{N} \right] \left[\frac{\lambda_c^j}{2\lambda_j} \frac{n_j}{N} \right] (e \sin\Theta_{ij})^4 \right]^{\Delta n} . \tag{10}$$

Taking $j_\nu \simeq 10^{13}\ \text{cm}^{-2}\text{sec}^{-1}$ for the (massless) neutrino flux, and $N \gtrsim 10^{23}$ the scale of the event rate for ν_μ or ν_τ on a lead target

$$N^2 j_\nu \sigma_c \gtrsim 10^{-2}/\text{sec} \tag{11}$$

is reasonable provided that the argument of the large power Δn is of order unity; this requires that

$$\frac{n_l}{N} \gtrsim (0.27) \frac{\lambda_l}{\lambda_c^l} \tag{12}$$

for each of the two modes. Now λ_l will be of the order of 1 mm or larger for an acoustic phonon while λ_c^l, which is a measure of the ionic motion, will be of the order of 1 Å so the inequality requires $n_l \gg N$ which cannot be realized for any stable lattice. Thus the event rate will be much smaller than that suggested by Eq. (11) and so neutrino-phonon scattering will apparently be unobservable.

It is also interesting to consider the case $\Delta n_i = \Delta n_j = 1$; here we find

$$j_\nu \frac{d\sigma}{d\Omega\, dE'} \simeq (n_i+1)n_j \frac{j_\nu \sigma_c}{4\pi} \left[\frac{E'}{E}\right]^2 \times \delta\left[E'-E+h\left[\frac{c_i}{\lambda_i}-\frac{c_j}{\lambda_j}\right]\right] \times \left[\frac{\lambda_c^i}{2\lambda_i}\right]\left[\frac{\lambda_c^j}{2\lambda_j}\right]\sin^4\Theta_{ij} , \quad (13)$$

which is coherent over the number of phonons rather than the number of lattice sites. Consequently, aside from the λ_c^l/λ_l suppressions, we have, for $n_i \simeq n_j \simeq 10^{10}$,

$$n_i n_j j_\nu \sigma_c \simeq 10^{-21}/\text{yr} ,$$

so single phonon scattering is also unobservable.

In conclusion, we have shown that there exists a spatially coherent neutrino scattering amplitude over macroscopic domains. However, this is still not sufficient to cause observable effects which would allow construction of a neutrino detector in the mm wavelength range of this simple design. The reason for this is that even for very large phonon occupation numbers the density fluctuations are so small that the effective induced periodicity of the index of refraction for the propagation of the neutrinos is too small in magnitude to generate observable effects. However, while our conclusions thus far are negative, we must admit that, having juggled around with various clever detector designs, e.g., incorporating substantial imposed intrinsic structure, we are hopeful that such a detector could be conceived.

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[1]J. Bernstein, CERN Yellow Report No. 84-06 (unpublished).
[2]R. Opher, Astron. Astrophys. **37**, 135 (1974).
[3]R. R. Lewis, Phys. Rev. D **21**, 663 (1980).
[4]N. Cabibbo and L. Maiani, Phys. Lett. **114B**, 115 (1982).
[5]P. Langacker, J. P. Leveille, and J. Sheiman, Phys. Rev. D **27**, 1228 (1983).
[6]L. Stodolski, Phys. Rev. Lett. **34**, 110 (1974).
[7]G. L. Squires, *Introduction to the Theroy of Thermal Neutron Scattering* (Cambridge University Press, Cambridge, England, 1978).