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Calculating the weak scale in supergravity models

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A renormalization framework is presented for calculating radiative $SU(2)\times U(1)$ gauge symmetry breaking in no-scale supergravity models. In this framework one naturally incorporates so-called threshold effects due to finite particle masses, without introducing an infrared cutoff scale. In this way it is possible to calculate the weak scale directly in terms of the fundamental parameters of the theory. The conventional way of calculating radiative symmetry breaking, using a renormalization-group (RG) approach, is reviewed, and it is explained why this approach fails to go beyond a leading-logarithmic approximation. It is pointed out that for an adequate calculation of the weak scale one has to include the two-loop leading logarithms and the one-loop calculation has to be performed in the vacuum where $SU(2)\times U(1)$ is broken. The new renormalization framework is illustrated by performing a one-loop calculation in the no-scale E_6 model. Instead of using running parameters, as in the RG approach, all quantities correspond to *physical observables* in this framework. For the study of radiative symmetry breaking a coupled set of linear algebraic equations is obtained, whose solutions coincide with those of the RG approach to the order of accuracy of the latter method.

I. INTRODUCTION

In the last couple of years there have been various attempts to obtain a phenomenologically acceptable particle spectrum from the low-energy limit of $N=1$ supergravity models.[1–3] A certain class of these models, the so-called no-scale models,[4] is particularly interesting, because they resemble the effective field theory that one obtains as the low-energy limit of string theories.[5] In these models, supersymmetry is broken in a hidden sector that only couples to the observable world through gravitational interactions. The effective potential describing the light fields of the observable sector is extracted from these theories by taking the large-M_P (Planck mass) limit in a suitable way (the so-called flat limit). The resulting low-energy potential consists of a supersymmetric part, plus some soft-supersymmetry-breaking terms. The supersymmetric part is given by the superpotential of the light fields of the theory, in the same way as in nongravitational theories. The supersymmetry-breaking terms are additional scalar mass terms m_i^0 or trilinear scalar couplings A_i^0, labeled by i. In formula

$$V_{\text{eff}}(\phi)=V(\phi,\text{SUSY})+\delta V(\phi,m_i^0,A_i^0)\ . \quad (1)$$

In addition the gauginos associated with the gauge bosons acquire a mass M_i^0. Here, and throughout this paper, I will assume the no-scale hypothesis[4,6] that m_i^0 and A_i^0 are zero: supersymmetry is only broken by the gaugino masses M_i^0.

At the tree level, V_{eff} does not have a minimum that breaks $SU(2)\times U(1)$. However, as a result of one-loop contributions some of the scalar masses are driven negative and $SU(2)\times U(1)$-breaking occurs.[7] These corrections are presently calculated using the β functions obtained from the modified minimal subtraction scheme ($\overline{\text{MS}}$), which describe the scaling of running parameters $\tilde{m}_i^2(\mu)$, $\tilde{A}_i(\mu)$, and $\tilde{M}_i(\mu)$ with respect to a fictitious scale μ. $\tilde{m}_i^2(\mu)$, $\tilde{A}_i(\mu)$, and $\tilde{M}_i(\mu)$ are equated with m_i^0, A_i^0, and M_i^0 for μ equal to the unification scale $M_X(\sim 10^{15}$ GeV). The parameters m_i, A_i, and M_i, which include the one-loop corrections, are then defined as the values of $\tilde{m}_i^2(\mu)$, $\tilde{A}_i(\mu)$, and $\tilde{M}_i(\mu)$ for μ equal to the weak scale $Q\sim 250$ GeV. For instance, in Fig. 1 the typical scaling with μ is sketched for the mass m_1^2 of a non-Higgs field (i.e., a scalar quark or lepton) and for the mass m_2^2 of a Higgs field. The negative value of $m_2^2=\tilde{m}_2^2(Q)$ will lead to spontaneous symmetry breaking in the potential (1).

Because of the large hierarchy, the one-loop corrections generate large logarithms of the form

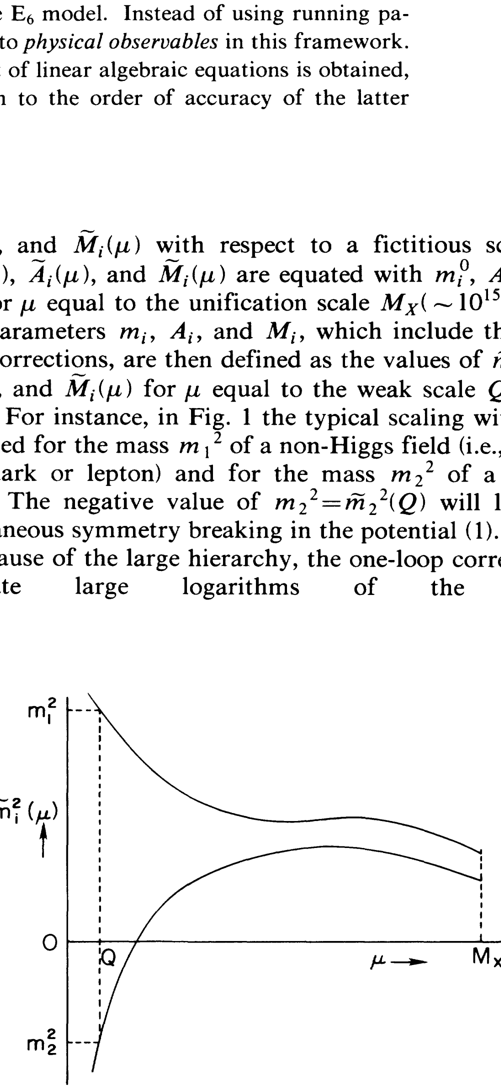


FIG. 1. Two typical behaviors of masses $\tilde{m}_i^2$ as a function of the scale μ. $\tilde{m}_1$ could describe the μ dependence of a non-Higgs-scalar mass (scalar quark or lepton), and $\tilde{m}_2$ of a Higgs scalar. Radiative corrections shift the initial values $(m_1^0)^2$ and $(m_2^0)^2$ at M_X to their final values m_1^2 and m_2^2 at Q.

$(\alpha/\pi)m_{\rm SUSY}{}^2\ln(M_X/Q)$, where $4\pi\alpha$ is a generic gauge or Yukawa $({\rm coupling})^2$, and $m_{\rm SUSY}{}^2$ is any of the supersymmetry-breaking parameters m_i^0, A_i^0, or M_i^0. The $\overline{\rm MS}$ procedure takes all these large logarithm, into account. *Although this is a one-loop calculation, I will call this an order-1 results, because* $(\alpha/\pi)\ln(M_X/Q)$ *is of order 1.*

All scalar particle masses obtained in this way are thus typically of order $m_{\rm SUSY}{}^2$. In particular, the Higgs particle masses (i.e., $m_2{}^2$ in Fig. 1) are of order $m_{\rm SUSY}{}^2$, and since they define the weak scale Q through their vacuum expectation values (VEV's), phenomenology requires $m_{\rm SUSY}$ to be of order of the weak scale. For example, in the no-scale models in the initial gaugino masses are of order Q and set the scale of all light particles in the problem.

As one can see from the steepness of $\tilde{m}_2{}^2(\mu)$ in Fig. 1 around $\mu=Q$, the value obtained for $m_2{}^2$ depends very much on the value that one assumes for Q. $\tilde{m}_2{}^2(\mu)$ depends roughly logarithmically on μ around $\mu=Q$, so that changing Q to Q' gives $\tilde{m}_2{}^2(Q')=[1-c\ln(Q'/Q)]\tilde{m}_2{}^2(Q)$, with c a number of ~ 1. This easily leads to an uncertainty in $m_2{}^2$ as big as 50%.

This question of "where to stop the running of the β functions" cannot be answered within the framework of $\overline{\rm MS}$. The steepness of $\tilde{m}_2{}^2(\mu)$ near $\mu^2\approx 0$ is caused by the familiar problem of the infrared divergences in massless field theories. $\overline{\rm MS}$ ignores all finite-mass effects of the particles in the loops. One stops running at the scale $\mu=Q$ to mimic these finite-particle-mass effects, since all particle masses are roughly of order Q. However, the introduction of the infrared cutoff Q seriously interferes with one of the most attractive features of supersymmetry: namely, its ability to explain the gauge hierarchy and to calculate the masses of the W and Z bosons in terms of more fundamental parameters of the theory (i.e., the gaugino masses).

To obtain these effects correctly, a renormalization framework, consisting of a precise definition of the parameters, has to be set up. The situation is similar to the calculation of radiative corrections to, for instance, muon decay in the standard model of the weak interactions. There are two ways of doing this calculation. One is using β functions of $\overline{\rm MS}$, which relate the muon decay constant G_μ and the electric charge e "at M_W" (where the formula $M_W{}^2=\sqrt{2}e^2/8G_\mu\sin^2\theta_W$ holds) to their values at the low-energy scales m_e or m_μ (where these couplings are measured and defined). In this way one obtains a (good) approximation to the full order α corrections, including the leading-logarithmic contributions only. The second way is setting up a full renormalization framework for this problem in the manner of Sirlin.[8] Then coupling constants are not defined as being variable with respect to a renormalization scale, but are instead fixed, well-defined, physical quantities. The radiative corrections then arise as corrections to relations between different physical quantities.

For the calculation of radiative symmetry breaking in low-energy supergravity models the situation is similar. Instead of running from m_W to m_e one runs from m_X to m_W. In this case, however, the renormalization-group equations (RGE's) of $\overline{\rm MS}$ give a much poorer approximation to the full one-loop radiative corrections, because the radiative effects are very large as a result of the large hierarchy M_X/M_W. But a more important difference is that now the scale at which one wants to stop running of the RGE's is exactly the scale that one actually wants to *calculate* in terms of the supersymmetry-breaking parameters and should therefore not be put in by hand.

For these reasons I will present a consistent renormalization framework for calculating radiative symmetry breaking in which the finite-particle-mass effects can explicitly be included. This entails a precise definition of quantities such as $(m_i^0)^2$ and $m_i{}^2$ and their interrelation. I will define the parameters in terms of physical quantities: masses will be defined as poles of propagators and coupling constants are defined on the mass shell. This will allow us to calculate the weak scale (and thus all the light-particle masses) directly in terms of more fundamental parameters of the theory, omitting the introduction of an infrared scale Q.

To study the finite-mass effects, one has to go beyond the leading-logarithmic approximation. The leading-logarithmic corrections are of order $\alpha\ln(M_X/Q)$, which is order 1, but the question whether to take Q or Q', but roughly of the weak scale, is an effect of order $\alpha\ln(Q/Q')$, i.e., order α. Therefore, to study these effects consistently one has to include all order-α effects. This will however involve two-loop effects as well. Namely, the large logarithmics that are generated at two loops will be of order $(\alpha/\pi)^2\ln(M_X/Q)$, and since $(\alpha/\pi)\ln(M_X/Q)$ is of order 1, this is an order-α effect.[9] At the order-α level there is another effect one has to worry about. One calculates the radiative correction assuming that the VEV's of the Higgs fields are equal to zero. This is motivated by the fact that in the potential (1) at the tree level $\rm SU(2)\times U(1)$ is unbroken. However, is this calculation correct, when the result of the one-loop calculation reveals that $\rm SU(2)\times U(1)$ is in fact broken so that the VEV's of the Higgs fields are actually nonzero? It will be shown using a simple example that in general this is indeed correct as far as the large logs are concerned. This means that the one-loop large logarithms (order 1 terms) and the two-loop large logarithms (order-α terms) can be correctly obtained assuming the VEV's of the Higgs fields to be zero. There is however no reason to believe that this is also true for the one-loop order-α corrections that do not involve large logarithms, i.e., the small finite terms.

The renormalization framework that I propose is conceptually quite different from the current way of thinking about the mechanism of radiative symmetry breaking. Currently, there are roughly two schools of thought. I will call them the ϕ-space approach and the p-space approach. It is not true that the authors of Refs. 4, 7, and 10 are devoted followers of either the ϕ-space or the p-space approach. The two schools are made up by myself as a possible way to understand the assumptions that underlie these calculations. Before I present the new framework, let me show that neither the p-space nor the ϕ-space approach represent an appropriate picture for carrying out a consistent renormalization framework although the order 1 leading logarithms can be correctly obtained in this way.

The ϕ-space approach calculates radiative corrections to the shape of the effective potential (1) in a Coleman-Weinberg[11] kind of fashion. One obtains here large negative contributions to the potential of the form $m^2\phi^2\ln(\phi/M_X)$ with ϕ of order of the weak scale. In Appendix A I will show that the definition of m implicitly used in this scheme $\partial^2 V/\partial\phi^2|_{\phi=M_X}=m^2$ does not correspond to the physical mass of a particle, and furthermore, the relation of m, defined in this way, to m_i^0, that appears in the effective potential (1) is obscure. I will show that all physical quantities should be defined at the minimum of the potential. The one-loop corrections to the relations between parameters thus defined are of order α and do not involve large logarithms.

In the p-space approach, one assumes that the parameters m_i^0, A_i^0, and M_i^0 in (1) are defined at very large Euclidean momentum $p^2=-M_X{}^2$. This approach is very similar to the Georgi-Quinn-Weinberg mechanism.[12] Here one relates the gauge couplings $g_2(m_W)$, $g_2(m_W)$, and $g_1(m_W)$ of SU(3), SU(2), and U(1), respectively, to a single gauge coupling $g(M_X)$ defined at M_X through the renormalization-group equation (RGE's). In the potential (1) one now assumes that m_i^0 and A_i^0 are defined at $p^2=-M_X{}^2$ and one obtains their values at $p^2=-m_W{}^2$ by running the RGE's down to low-energy scales. The resulting parameters are then substituted in the potential (1) and spontaneous symmetry breaking occurs. I will show that the interpretation of a running mass as an effective, i.e., physical mass at some high-energy scale is incorrect. The statement that "the mass at $p^2=-M_X{}^2$ is equal to m_i^0" is ambiguous in the sense that the momentum dependence of $\tilde{m}_i{}^2(p)$ for p between M_X and Q is not well defined. Running masses can be used as a tool for the study of the RGE's of Green's functions,[13] but unlike running couplings, they themselves do not have a physical interpretation. Their momentum dependence is renormalization prescription dependent.

In practice, both the p-space and the ϕ-space approach yield the same result. Both method ultimately approximate their calculation by using the RGE's obtained from $\overline{\text{MS}}$, where the running parameter $t=\ln(\phi/M_X)$ in the ϕ-space approach, and $t=\ln(p/M_X)$ in the p-space approach. In the ϕ-space approach one stops running the RGE's at $\phi\approx m_W$ because that is where one expects the VEV of ϕ to be. In the p-space approach one stops running at $p^2=-m_W{}^2$ because one expects threshold effects due to the finite mass of the particles in the loop to effectively make the right-hand side of the RGE's zero at the point. With the scaling arguments of the p- and ϕ-space approaches one obtains the correct leading logarithms. These approaches however resist a clear definition of their parameters and are therefore not suitable for a study of the full order α effects.

I propose to introduce the following framework for calculating the radiatively corrected quantities m_i, A_i, and M_i. As we saw, m_i^0, A_i^0, and M_i^0 are defined as the numerical values that one obtains for the physical scalar masses m_i, the trilinear scalar couplings A_i, and for the gaugino masses M_i by taking the flat limit in the full supergravity theory. The flat limit is an algebraic procedure to eliminate all the heavy fields. The resulting theory is an effective theory in the sense that it is only valid up to momenta somewhere well below the heavy scale (M_X or M_P). The values m_i^0, A_i^0, and M_i^0 are only approximations to the correct values m_i, A_i, and M_i because all radiative corrections have been ignored. These parameters describe the low-energy physics and since we will assume that the heavy particles will decouple in the radiative corrections, one only has to calculate the radiative corrections to these parameters due to the light particles. In the no-scale models the low-energy theory is supersymmetric except for the presence of the gaugino masses. m_i^0 and A_i^0 are zero and supersymmetry would also prevent these parameters from being generated radiatively. However, when the supersymmetry is broken by the gaugino masses M_i, m_i, and A_i will be generated radiatively and receive contributions proportional to $\alpha M_i{}^2\ln(\Lambda/Q)$, with Q of order of M_i. Λ can be as big as the momentum scale up to which we believe that the effective theory is valid, i.e., $\Lambda\approx M_X$. No renormalization, i.e., subtraction of counterterms, on these parameters is being performed.

The idea here resembles somewhat the calculation of the electromagnetic radiative corrections to nuclear β decay in the old Fermi theory.[14] When one attempts to relate the β-decay constant to the muon-decay constant G_μ on the one-loop level, one encounters an infinite term of order $\ln(\Lambda/m_p)$, where m_p is the proton mass. One does not renormalize this term away by, for instance, introducing a counterterm for G_μ at the tree level. Instead the cutoff should be equated to the energy scale where one expects the Fermi theory to break down. Indeed, the calculation in the renormalizable standard model revealed later that this term was genuine with Λ replaced by m_Z. (In retrospect, a precise measurement of both muon decay and nuclear β decay could in principle have enabled people to measure $\ln(\Lambda/m_p)$, and thus m_Z, by the late 1950s). The presence of these large radiative effects is due to the existence of nonconserved axial-vector currents, which in the standard model arise through the breakdown of SU(2)×U(1) gauge symmetry. If the SU(2)×U(1) symmetry were unbroken these large logarithms would not be present. This fact is also reflected by the fact that the SU(2)×U(1) symmetry forbids a counterterm to cancel this infinity.

For the supersymmetric potential (1) the situation is similar. Because of supersymmetry and chiral-symmetry scalar masses m_i^0 and trilinear couplings A_i^0 are forbidden in (1). These symmetries also prevent these parameters from being generated radiatively. However, when the supersymmetry is broken by the gaugino masses M_i, m_i, and A_i will be generated radiatively and receive contributions proportional to $\alpha M_i{}^2\ln(\Lambda/Q)$. Λ can be as big as the momentum scale up to which we believe that the effective theory is valid, i.e., $\Lambda\approx M_X$.

The outlined approach gives the correct interpretation of the parameters in the theory, and results in an unambiguous procedure for calculating the order-α (or any order) corrections to the supersymmetry-breaking parameters in effective low-energy supergravity theories. For lack of a better name, I will call this the Λ-space approach. All parameters in this approach are defined on the mass shell and therefore represent physical quantities. Since the corrections to the soft-supersymmetry-breaking terms are simple one-loop integrals with a cutoff Λ, one can calculate them explicitly, including finite-mass effects in the loops, so that one does not have the infrared problems that one usually encounters using the $\overline{\text{MS}}$ β functions. The full threshold, i.e., the full order-α, corrections can be calculated in this approach. They require, however, a two-loop calculation of the leading logarithms and a better understanding of the question whether one can obtain the one-loop order-α terms (i.e., the terms not involving leading logarithms) while calculating in the wrong vacuum. I have recalculated the order 1 corrections in this framework for the no-scale model based on the gauge group E_6 and the results agree with those obtained using the $\overline{\text{MS}}$ method.

This paper is organized as follows. In Sec. II I will start with a brief outline of the derivation of the low-energy potential[1] in supergravity theories, that describes the light fields of the observable sector. In Sec. III I will introduce the Λ-space approach, which is a method for calculating radiative symmetry breaking to arbitrary accuracy. I will illustrate this approach by calculating the order 1 contributions in the no-scale model based on the gauge group E_6. Then I will show in general that at the order 1 level the Λ-space solutions agree with renormalization-group approaches. In Sec. IV I will address the question whether one can perform these calculations in the wrong vacuum. In Sec. V I will summarize the results and I will outline a program for studying threshold effects using the Λ-space approach. In Appendix A I will discuss the ϕ-space and p-space approaches as mechanisms for radiative symmetry breaking. I will review how physical parameters are defined in the language of effective potentials which will explain why the ϕ-space approach is inappropriate when one wants to calculate beyond the leading logarithms. I discuss the p-space approach and show that the concept of a running mass cannot be used for describing radiative symmetry breaking. I will remind the reader what originally motivated running masses for the study of asymptotic behavior of Green's functions, and I will explain the difference of these two applications. Finally, in Appendix B, I will summarize the order 1 results of the Λ-space approach, applied to the no-scale model based on the gauge group E_6.

II. THE EFFECTIVE POTENTIAL OF THE LIGHT FIELDS IN THE OBSERVABLE SECTOR

In the class of supergravity models I consider unextended ($N=1$) supersymmetry is broken by very large scalar-field VEV's of order 10^{19} GeV. The scalars that have these large VEV's form a "hidden sector" that does not interact directly with the ordinary fields (quarks, leptons, gauge and Higgs bosons, and their superpartners) of the "observable sector." That is, the superpotential of the theory breaks up into a sum of two terms:

$$f_{\text{total}}(S,\tilde{S})=f(S)+\tilde{f}(\tilde{S}) , \tag{2}$$

where S^a and $\tilde{S}^h$ are the left-chiral fields of the observable and hidden sector, respectively. With a minimal kinematic term and no other interactions, the potential of the scalar (nonauxiliary) components z^a, $\tilde{z}^h$ of S^a, $\tilde{S}^h$ would take the form

$$V(z,\tilde{z})=\sum_{\text{all } z}\left|\frac{\partial f_{\text{total}}}{\partial z}\right|^2 =\sum_a\left|\frac{\partial f(z)}{\partial z}\right|^2+\sum_h\left|\frac{\partial \tilde{f}(\tilde{z})}{\partial \tilde{z}}\right|^2 \tag{3}$$

and the spontaneous breakdown of supersymmetry in the hidden sector could have no effect on the observable sector. When coupled to supergravity, the news that supersymmetry is broken by the $\tilde{z}^h$ VEV's is carried over to the observable superfields by gravity and its superpartners, which interact with both sectors.

This "news" appears in the following two ways. (1) The spontaneous supersymmetry breakdown in the hidden sector produces a massless Goldstone fermion, which gets absorbed by the gravitino, giving the latter a mass $m_{3/2}$. $m_{3/2}$ is determined in terms of VEV's of the fields in the hidden sector. The minimum usually contains a flat direction, so that $m_{3/2}$ is undetermined at this stage. The finite gravitino mass will generate gaugino masses at the one-loop level through graphs in Fig. 2. The presence of the gaugino masses explicitly breaks supersymmetry in the observable sector. The size of the gaugino masses is undetermined, but has to be chosen to be of the order of the weak scale in order to lead to acceptable phenomenology.

(2) The effective potential describing the light fields which transforms under a subgroup G_1 of G [$G=\text{SU}(5),E_6$, etc., $G_1=\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)^n$, $n=1,2,\ldots$] is obtained by replacing all the heavy fields by their VEV and taking the flat limit. For instance, in the earlier supergravity models, where the supersymmetry is broken in the hidden sector with superpotential $\tilde{f}$ using the O'Raifeartaigh mechanism, the total supergravity potential is given by

$$V(z,\tilde{z})=\exp\left[8\pi G\left[\sum_a|z_a|^2+\sum_h|\tilde{z}_h|^2\right]\right]\times\left[\sum_a\left|\frac{\partial f(z)}{\partial z}+8\pi G z_a^*[f(z)+\tilde{f}(\tilde{z})]\right|^2\times\sum_h\left|\frac{\partial\tilde{f}(\tilde{z})}{\partial\tilde{z}}+8\pi G\tilde{z}_h^*[f(z)+\tilde{f}(\tilde{z})]\right|^2-24\pi G\,|f(z)+\tilde{f}(\tilde{z})|^2\right]+\sum_k D_k{}^2 , \tag{4}$$

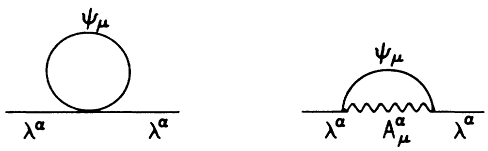


FIG. 2. Contributions to the gaugino masses from the spin-$\frac{3}{2}$ gravitino ψ_μ. A^a is the gauge boson associated with λ^a.

where $G=1/M_P{}^2$ is Newton's constant. The last term in (4) consists of the sum over D terms for the different simple sectors of the gauge group. The potential $V_{\rm eff}(z_a)$ in terms of the light fields z_a only, with $|z_a|=O(Q)$, is obtained from the full supergravity potential $V(z_a,\tilde{z}_h)$ by replacing all the heavy scalars $\tilde{z}_h$ by their VEV's: $\tilde{z}_h \to \langle \tilde{z}_h \rangle = \mu + |\hat{z}_h|$, where $|\hat{z}_h|$ is the deviation of $\langle \tilde{z}_h \rangle$ from the common scale μ. $V_{\rm eff}(z_a)$ is then obtained by taking the flat limit

$$V_{\rm eff}(z_a) = \lim_{\frac{m_W}{\mu},\frac{m_W}{M_P}\to 0} V(\mu+|\hat{z}_h|,z_a)\ .$$

For instance, for the potential (4) μ is the scale of the VEV's in the hidden sector that break supersymmetry, and one obtains[15]

$$V_{\rm eff} = \sum_a \left| \frac{\partial f(z)}{\partial z} \right|^2 + D \text{ terms}$$
$$+ m_{3/2}{}^2 \sum_a |z_a|^2 + O(1) m_{3/2} f(z)$$
$$+ m_{3/4}{}^4 O\left[\frac{m_{3/2}}{M_P}\right] \tag{5}$$

with $m_{3/2}=\mu^3/M_P{}^2$, $\tilde{f}(\tilde{z}_0)=O(\mu^3)$. f is the superpotential containing the light fields only, which transforms under G_1. The first two terms in (5) are exactly the ones one would obtain in a model just based on supersymmetric matter coupled to G_1. The last two terms break supersymmetry explicitly.

In the more recent no-scale or string inspired models these supersymmetry-breaking terms in the potential (5) are absent. In that case the only signal of supersymmetry breaking is the finite gaugino masses, which in those models are given by $m_{1/2} \sim m_{3/2}{}^3/M_P{}^2$. Since the supersymmetry-breaking parameters set the scale of all particle masses and in particular of the Higgs-boson mass, they are chosen so as to give the right weak scale. The main point I want to make here is that in all of these models there is a scale μ in the hidden sector, which is either a free parameter, or the VEV of a field whose size is not determined by the shape of the potential in the hidden sector. The supersymmetry-breaking terms in the observable sector are given in terms of μ and are, as far as the calculation of weak symmetry breaking is concerned, free parameters.

For the study of the low-energy physics in no-scale models it is sufficient to consider the supersymmetric potential V_1 based on the group G_1, augmented by soft-supersymmetric-breaking terms V_2, which contain mass terms m_i^0 and trilinear terms A_i^0:

$$V_{\rm eff}(z) = V_1(z;\text{SUSY}) + V_2(z;m_i^0,A_i^0) \tag{6}$$

m_i^0 and A_i^0 are functions of μ and $|\hat{z}_h|$ and have to be taken of $O(Q)$ because of phenomenology.

In the particular case (5) this means $m_{3/2}=O(Q)$, so that $\mu=O((QM_P{}^2)^{1/3})$. In the no-scale models m_i^0 and A_i^0 are zero at the tree level, and supersymmetry is only broken by the presence of nonzero gaugino masses $M_i^0 (i=3,2,1,\dots)$. The tree-level values m_i^0, A_i^0, and M_i^0 can now be taken as boundary conditions for the subsequent study of $SU(2)\times U(1)$ breaking.

III. A RENORMALIZATION FRAMEWORK FOR LOW-ENERGY SUPERGRAVITY MODELS: THE Λ-SPACE APPROACH

The current way of calculating radiative symmetry breaking uses a renormalization-group approach, which relates the input parameters "defined at a large scale M_X" to "low-energy parameters defined at the weak scale." This method correctly gives the leading-logarithms radiative corrections, but it fails to give information about threshold effects, which are crucial for the determination of the weak scale, as I explained in the Introduction.

To treat these threshold effects correctly, a consistent renormalization framework has to be introduced in which all the parameters are defined as physical quantities. The running mass picture of the ϕ-space and p-space approaches does not lend itself for this purpose as I explain in Appendix A.

I propose to introduce the following framework for calculating the radiatively corrected quantities m_i, A_i, and M_i. As we saw in Sec. II, m_i^0, A_i^0, and M_i^0 are defined as the numerical values that one obtains for the physical scalar masses m_i, the trilinear scalar couplings A_i, and for the gaugino masses M_i by taking the flat limit in the full supergravity theory. The flat limit is an algebraic procedure to eliminate all the heavy fields. The resulting theory is an effective theory in the sense that it is only valid up to momenta somewhere well below the heavy scale (M_X or M_P). The values m_i^0, A_i^0, and M_i^0 are only approximations to the correct values m_i, A_i, and M_i because all radiative corrections have been ignored. These parameters describe the low-energy physics and since we will assume that the heavy particles will decouple in the radiative corrections, one only has to calculate the radiative corrections to these parameters due to the light particles. In the no-scale models the low-energy theory is supersymmetric except for the presence of the gaugino masses. m_i^0 and A_i^0 are zero and supersymmetry would also prevent these parameters from being generated relatively. However, when the supersymmetry is broken by the gaugino masses M_i, m_i, and A_i will be generated radiatively and receive contributions proportional to $\alpha M_i{}^2 \ln(\Lambda/Q)$, with Q of order of M_i. Λ can be as big as the momentum scale up to which we believe that the effective theory is valid, i.e., $\Lambda \approx M_X$. No renormalization, i.e., subtraction of counterterms, on these parameters is being performed.

The important idea underlying this approach is that radiative corrections are finite and calculable when they are the result of the breakdown of a symmetry, in this case supersymmetry. In the Introduction I mentioned how large radiative corrections to nuclear β decay are related to the breakdown of $SU(2)\times U(1)$ gauge symmetry. With $SU(2)\times U(1)$ unbroken, there are no large radiative effects. In the case at hand, the scalar potential V is supersymmetric when m_i and A_i are zero. These parameters would remain zero also in the presence of radiative corrections, due to supersymmetry. Supersymmetry forbids counterterms for these parameters, so when supersymmetry is broken by the gaugino masses the corrections to these parameters are finite.

As an example, consider, for instance, a theory which is chiral invariant and describes a massless electron ψ_e. Chiral invariance forbids a mass counterterm for the electron and so radiative corrections to the electron mass have to be finite. In fact they are zero. Let us also assume that there is a Yukawa coupling with strength h between some massless scalar ϕ, the electron ψ_e, and another massless fermion χ:

$$h\bar{\psi}_e\chi\phi+\text{H.c.} \tag{7}$$

Equation (7) does not break chiral invariance if we let ϕ transform with an appropriate phase.

Suppose chiral invariance is broken in this theory by the sudden appearance of a mass term for χ:

$$m_\chi\bar{\chi}\chi \ . \tag{8}$$

We do not know what mechanism caused the appearance of this chiral-symmetry-breaking term, but I will assume it to be the result of some more fundamental theory which sets in at an energy scale M_X. So now our theory consisting of ϕ, ψ_e, and χ is only an effective theory valid up to the scale M_X.

Because of (8) a finite electron mass will be generated radiatively (see Fig. 3):

$$m_e\approx\frac{h^2}{4\pi}m_\chi\ln\frac{M_X}{m_\chi} \ . \tag{9}$$

In (9) an ultraviolet cutoff $\Lambda=M_X$ has been used to evaluate the integral and m_χ in the denominator of the logarithm is the result of the finite χ mass in the graph. The electron mass receives no renormalization because the chiral symmetry of the "electron sector" forbids an electron mass counterterm.

In the effective low-energy supergravity theory we have

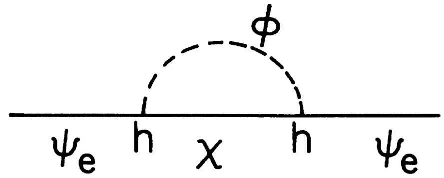


FIG. 3. Chiral symmetry breaking in the χ sector radiatively induces a finite electron mass.

exactly the same situation. It is the no-scale model that explains why the tree-level values of the supersymmetry-breaking parameters m_i^0 and A_i^0 are zero, and they would remain zero in the presence of radiative corrections if supersymmetry were unbroken. The supersymmetry breaking, in the form of gaugino masses, leads to deviations for the scalar masses and trilinear couplings from zero. They are proportional to $\ln\Lambda$, and Λ can be as big as the energy scale up to which one believes theory to be valid.

This approach results in an unambiguous procedure for calculating the order-α (or any order) corrections to the supersymmetry breaking parameters in effective low-energy supergravity theories. For lack of a better name, I will call this the Λ-space approach. All parameters in this approach are defined on the mass shell and therefore represent physical quantities. Since the corrections to the soft-supersymmetry-breaking terms are simple one-loop integrals with a cutoff Λ, one can calculate them explicitly, including finite-mass effects in the loops, so that one does not have the infrared problems that one usually encounters using the $\overline{\text{MS}}$ β functions. The full threshold, i.e., the full order-α, corrections can be calculated in this approach. They require, however, a two-loop calculation of the leading logarithms and a better understanding of the question whether one can obtain the one-loop order-α (i.e., the terms not involving leading logarithms) while calculating in the wrong vacuum. For a summary of all the order-α effects that have to be incorporated to study the threshold effects, see Sec. V.

Here I will restrict myself to demonstrating that the Λ-space approach agrees with the conventional $\overline{\text{MS}}$ procedure at the one-loop order 1 level. As an example of this method, I will present the equations for the supersymmetry-breaking parameters m_i, A_i, and M_i in the no-scale model based on the gauge group E_6 as presented in Ref. 6. It is beyond the scope of this discussion to justify an explain all the arguments that lead to the construction of the effective low-energy theory in this model. For a review I refer to Ref. 6 the references cited there.

The model consists of $N=1$ supergravity with 3 generations of 27-dimensional chiral superfield representations of E_6. In addition, there is a hidden sector which only couples to the observable world through gravity effects. Some mechanism of supersymmetry breaking in the hidden sector is assumed,[16] which leads to nonzero gravitino masses through the super-Higgs effect. Gravity-loop effects (Fig. 2) will then induce nonzero gaugino masses M_i^0, $i=3,2,1,E$, thus breaking supersymmetry in the observable sector. E_6 is broken down to $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$ either through a Higgs mechanism, where the VEV's of the Higgs fields $\tilde{z}$ are of order M_X, or through Wilson-loop effects,[17,18] if the model is the result of a Calabi-Yau compactification of a string theory. After taking the flat limit (see Sec. II) the model consists of supersymmetric E_6 with the 3 matter **27**'s plus soft-supersymmetry-breaking terms.

$SU(2)\times U(1)$ is broken because loop effects will drive some of the scalar masses negative. It is well known that in order to obtain negative (masses)^2 for the Higgs fields one needs reasonably large Yukawa couplings for the quarks. The top quark is a natural candidate for this, and since we ignore all generation mixing we can study the patterns of $SU(2)\times U(1)$ breaking using the particles of the third generation only. The transformation properties of the **27**'s under the unbroken gauge group $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$ are given in Table I.

The superpotential for the third generation is given by

$$f = hQu^cH + \lambda H\bar{H}N + \kappa DD^cN \ . \tag{10}$$

(We also neglect the bottom-quark Yukawa coupling.) Given this superpotential, the soft-supersymmetry-breaking terms in the low-energy Lagrangian are given by

$$-L_{\rm soft} = \sum_j (m_j^0)^2 |z_j|^2 + (hA_h^0 Qu^cH + \lambda A_\lambda^0 \bar{H}HN + \kappa A_\kappa^0 DD^cN + \text{H.c.}) + \tfrac{1}{2}\sum_i M_i^0(\lambda_i\lambda_i + \text{H.c}) \ . \tag{11}$$

The sum over j runs over all scalar particles in the observable sector. λ_i, $i=3,2,1,E$, are the gaugino's associated with $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$. As one can see from Table I, only the scalar components of H, $\bar{H}$, and N are candidates for acquiring VEV's. VEV's of other particles in Table I would either break color or lepton number. Assuming the VEV's of all scalars except N, H, and $\bar{H}$ to be zero, the potential relevant for the study of $SU(2)\times U(1)$ breaking is

TABLE I. Transformation properties under $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$ of the chiral matter superfields contained in the **27** of E_6. Group and generation indices are understood. The properly normalized quantites $\hat{Y}$ and $\hat{Y}_E$, such that $\mathrm{Tr}\,T_3 = \mathrm{Tr}\,\hat{Y}^2 = \mathrm{Tr}\,\hat{Y}_E^2$, are given by $\hat{Y}=\sqrt{3/5}\,Y$ and $\hat{Y}_E=\sqrt{3/5}\,Y_E$.

Chiral superfield	$SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$ quantum numbers
$Q=\begin{bmatrix}u\\ d\end{bmatrix}$	$(3,2,\frac{1}{6},\frac{1}{3})$
u^c	$(\bar{3},1-\frac{2}{3},\frac{1}{3})$
d^c	$(\bar{3},1,\frac{1}{3},-\frac{1}{6})$
$L=\begin{bmatrix}\nu\\ e\end{bmatrix}$	$(1,2-\frac{1}{2},-\frac{1}{6})$
e^c	$(1,1,1,\frac{1}{3})$
ν^c	$(1,1,0,\frac{5}{6})$
$H=\begin{bmatrix}H^+\\ H^0\end{bmatrix}$	$(1,2,\frac{1}{2},-\frac{2}{3})$
$\bar{H}=\begin{bmatrix}\bar{H}^0\\ \bar{H}^-\end{bmatrix}$	$(1,2,-\frac{1}{2},-\frac{1}{6})$
N	$(1,1,0,\frac{5}{6})$
D	$(3,1,-\frac{1}{3},-\frac{2}{3})$
D^c	$(\bar{3},1,\frac{1}{3},-\frac{1}{6})$

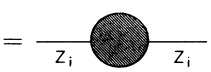
$$V_{\rm Higgs} = (m_H^0)^2|H|^2 + (m_{\bar{H}}^0)^2|\bar{H}|^2 + (m_N^0)^2|N|^2 + (\lambda A_\lambda^0 H\bar{H}N + \text{H.c.}) + \lambda^2(|H|^2|N|^2 + |\bar{H}|^2|N|^2 + |\bar{H}H|^2) + \frac{g_2{}^2}{2}\left[H^\dagger\frac{\tau}{2}H + \bar{H}^\dagger\frac{\tau}{2}\bar{H}\right]^2 + \frac{3g_1{}^2}{40}(|H|^2 - |\bar{H}|^2)^2 + \frac{g_E{}^2}{120}(5|N|^2 - 2|H|^2 - |\bar{H}|^2)^2 \ . \tag{12}$$

The first four terms in (12) are the supersymmetry-breaking terms from (11). The other terms come from the superpotential (10) and the SU(2), $U(1)_Y$, and $U(1)_E$ D terms:

$$\sum_{z_i=H,\bar{H},N}\left|\frac{\partial f}{\partial z_i}\right|^2 + D \text{ terms} \ . \tag{13}$$

In the no-scale scenario all soft-supersymmetry-breaking parameters in (11) and (12), except for the gaugino masses, are equal to zero. We will proceed with the calculation of the one-loop contributions to the supersymmetry-breaking parameters with this assumption.

I calculate the corrections to the scalar masses m_i^0 in the following way. Let $i\Pi_{z_i}(\Lambda,p)$ denote the one-loop contribution to the inverse z_i propagator. z_i can be any scalar field with tree-level mass m_i^0. I include Λ in Π_z to remind us that all the integrals in Π_z are evaluated using an ultraviolet cutoff Λ which I eventually will equate with M_X. The inverse propagator is given by

$\Gamma_{z_i}^{(2)} =$


$$= i[p^2 - (m_i^0)^2 + \Pi_{z_i}(\Lambda,p)] \ . \tag{14}$$

The one-loop mass m_i is defined to be the solution to the equation

$$[p^2 - (m_i^0)^2 + \Pi_{z_i}(\Lambda,p)]\,|_{p^2=m_i{}^2} = 0 \ . \tag{15}$$

A mass defined in this way, as the pole of the propagator, is gauge invariant, as has been explicitly verified using R_ξ gauges. (This is not true for the running masses used in the p-space approach.)

In this way one obtains, for instance, for $m_H{}^2$,

$$m_H{}^2=\frac{1}{16\pi^2}\left\{3M_2{}^2g_2{}^2\ln\left[\frac{\Lambda^2}{M_2{}^2}\right]+\tfrac{3}{5}M_1{}^2g_1{}^2\ln\left[\frac{\Lambda^2}{M_1{}^2}\right]+\tfrac{16}{15}M_E{}^2g_E{}^2\ln\left[\frac{\Lambda^2}{M_E{}^2}\right]\right.$$
$$-3h^2\left[m_Q{}^2\ln\left[\frac{\Lambda^2}{m_Q{}^2}\right]+m_u{}^2\ln\left[\frac{\Lambda^2}{m_u{}^2}\right]+m_H{}^2\ln\left[\frac{\Lambda^2}{m_H{}^2}\right]+A_h{}^2\ln\left[\frac{\Lambda^2}{m_H{}^2}\right]\right]$$
$$\left.-\lambda^2\left[m_H{}^2\ln\left[\frac{\Lambda^2}{m_H{}^2}\right]+m_{\bar H}{}^2\ln\left[\frac{\Lambda^2}{m_{\bar H}{}^2}\right]+m_N{}^2\ln\left[\frac{\Lambda^2}{m_N{}^2}\right]+A_\lambda{}^2\ln\left[\frac{\Lambda^2}{m_H{}^2}\right]\right]+O(Q^2\alpha)\right\}, \tag{16}$$

where $O(Q^2\alpha)$ contains small calculable order α corrections. In (16) the different terms between the brackets are multiplied by different logarithmic factors, because the particle masses in the loops are different for each loop graph. Notice that in (16) m_H appears on both sides of the equation. This equation, together with similar equations for the other scalar masses, has to be solved self-consistently for all scalar masses and trilinear couplings in terms of the supersymmetry-breaking parameters, i.e., the gaugino masses M_i.

Equation (16) displays the full *one-loop* corrections up to order α (i.e., leading logarithms plus threshold effect). However, order-α effects coming from two-loop leading logs (see Introduction) and order-α effects associated with the fact that (16) is calculated in the wrong vacuum (see Sec. IV) are expected to interfere with this order-α result. Therefore, this one-loop calculation is only consistent at the order 1 level. Since all scalar and gaugino masses are of the same order of magnitude (i.e., the weak scale Q), the difference between, for instance, $\ln(\Lambda^2/M_2{}^2)$ and $\ln(\Lambda^2/m_H{}^2)$ is a small order-α effect and should be ignored in this order 1 calculation. I will therefore put all logarithms equal to a common $\ln(\Lambda^2/Q^2)$:

$$m_H{}^2=(3M_2{}^2g_2{}^2+\tfrac{3}{5}M_1{}^2g_1{}^2+\tfrac{16}{15}M_E{}^2g_E{}^2-3h^2F_h-\lambda^2F_\lambda)t\ ,$$
$$F_h=m_Q{}^2+m_u{}^2+m_H{}^2+A_h{}^2\ , \tag{17}$$
$$F_\lambda=m_H{}^2+m_{\bar H}{}^2+m_N{}^2+A_\lambda{}^2\ ,$$

with $t=(\frac{1}{16}\pi^2)\ln(\Lambda^2/Q^2)$. I want to stress however that the only reason that I have not included these order-α effects is for the reasons stated above and is not a limitation of the Λ-space approach.

The trilinear scalar couplings A_i are obtained in the following way. Let $\Gamma^{(3)}_{\text{one loop}}(\Lambda,p)$ denote the one-loop contributions to $\Gamma^{(3)}$. I now define

$$A_i\equiv\Gamma^{(3)}=A_i^0+\Gamma^{(3)}_{\text{one loop}}(\Lambda,p^2=Q^2)\ , \tag{18}$$

i.e., A_i is defined roughly on mass shell for the scalar particles. Notice that with definition (18) the color coupling constant g_3 which appears in $\Gamma^{(3)}_{\text{one loop}}$ is in the perturbative region $[g_3{}^2(Q^2)/4\pi<1]$. With (18) one obtains, for instance, for A_h ($A_h^0=0$),

$$A_h=-(\tfrac{16}{3}M_3g_3{}^2+3M_2g_2{}^2+\tfrac{13}{15}M_1g_1{}^2+\tfrac{4}{3}M_Eg_E{}^2+6A_hh^2+A_\lambda\lambda^2)t\ , \tag{19}$$

where, for the same reasons as above, I have put all threshold masses equal to Q. The full set of algebraic equations for A_i and $m_i{}^2$ is given in Appendix B.

Equations (16) and (19) are to be compared with the RGE's for the running mass $m_H{}^2(t)$ and $A_h(t)$ that one obtains, in the ϕ-space and p-space approaches,

$$\frac{\partial m_H{}^2}{\partial t}=\frac{1}{16\pi^2}(-3M_2{}^2g_2{}^2-\tfrac{3}{5}M_1{}^2g_1{}^2-\tfrac{16}{15}M_E{}^2g_E{}^2+3h^2F_h+\lambda^2F_\lambda)\ , \tag{20}$$

$$\frac{\partial A_h}{\partial t}=\frac{1}{16\pi^2}(\tfrac{16}{3}M_3g_3{}^2+3M_2g_2{}^2+\tfrac{13}{15}M_1g_1{}^2+\tfrac{4}{3}M_Eg_E{}^2+6A_hh^2+A_\lambda\lambda^2)\ . \tag{21}$$

t here is now the renormalization scale, $t=\ln(M^2/M_X{}^2)$, and M runs form M_X to Q. The differences between (17), (19) and (20), (21) are the following. In Eqs. (17) and (19) the scalar masses $m_i{}^2$ and gaugino masses M_i are defined as the poles of their propagators. The dimensionless coupling constants g_i, $i=3,2,1,E,h,\lambda$ and the dimensionfull trilinear scalar couplings A_i are defined at the weak scale Q. All the parameters in Eqs. (B1)—(B14) are constants, defined at the weak scale. The solution is obtained by solving this set of coupled linear algebraic equations. Equations (20) and (21) are two of a set of coupled linear differential equations. All the parameters that appear here are functions of the renormalization scale t. The initial condition for these RGE's is the vanishing of m_i^0 and A_i^0 at a scale $M=M_X$. In the Λ-space equations parameters are not defined at a scale M_X. Instead we require that m_i^0 and A_i^0 vanish when the (low-energy) effective potential is calculated as the flat limit of the full supergravity potential.

The dimensionless parameters (gauge couplings and Yukawa couplings), are not generated as a result of supersymmetry breaking. Their values have to be put in by hand, either by specifying them directly at the weak scale [i.e., $h^2(Q^2)$] or, if one wishes, one can use RGE's to relate them to their values at M_X [i.e., $h^2(M_X{}^2)$] and use those as input parameters. The RGE's for h, λ, κ, and g_i are also given in Appendix B. It is important to realize, however, that the dimensionless parameters that appear in (B1)—(B14) are defined at the weak scale.

The solutions to the order 1 algebraic equations if the Λ-space approach that I derived in this section are as general as those of the differential equations of $\overline{\text{MS}}$. Both equations sum over the leading logarithms. The latter because the derivatives of the running parameters are expressed in terms of the running parameters themselves, and the former because the constant physical quantities appear on both sides of the equations and have to be solved self-consistently. However, the Λ-space approach lends itself to include all threshold effects, whereas the $\overline{\text{MS}}$ method does not.

Let me explain these statements in more detail. Let the supersymmetry breaking parameters be given by $x_i=(m_i,A_i)$. Suppose we have performed a full order-α calculation including the one-loop threshold effects and the two-loop leading logarithms according to the Λ-space approach. We arrive at the coupled set of algebraic equations:

$$x_i=F_i(x_j,M_k,\Lambda^2) \; . \tag{22}$$

In (22) the dependence of F_i on the supersymmetry-breaking gaugino masses and the cutoff $\Lambda(=M_X)$ is displayed, and its dependence on gauge and Yukawa couplings is suppressed. Let $\bar{x}_i(M_j,\Lambda^2)$ by a self-consistent solution to these equations.

Differentiating both sides of (22) with respect to Λ^2 yields

$$\Lambda^2\frac{dx_i}{d\Lambda^2}=\Lambda^2\frac{\partial F_i}{\partial\Lambda^2}+\Lambda^2\frac{dx_i}{d\Lambda^2}\frac{\partial F_i}{\partial x_j}+\Lambda^2\frac{dM_i}{d\Lambda^2}\frac{\partial F_i}{\partial M_j} \; . \tag{23}$$

The RGE's of $\overline{\text{MS}}$ are given by (23) if one ignores the last two terms on the right-hand side, which are of order α^2. In a full order-α treatment these terms should however not be ignored because they contain large logs. It is then obvious that the Λ-space solution $\bar{x}_i$ is also a solution to the RGE's (23). However, $\bar{x}_j$ is a solution that takes the threshold effects into account. These effects are lost when one differentiates (22) with respect to Λ^2 and this can only be patched up by stopping the running of the differential equations t a common scale Q. It is in this way that the Λ-space approach answers the question: "Where do we stop the running of the masses?"

IV. COMPARISON OF THE RADIATIVE CORRECTIONS TO THE HIGGS-BOSON MASS EVALUATED IN THE SYMMETRIC AND ASYMMETRIC VACUA

In this section I will address an issue that has been ignored in the preceding discussions. At the tree-level SU(2)$\times$U(1) is unbroken. One calculates the one-loop corrections to the Higgs-boson masses assuming that the VEV's of the Higgs fields are equal to zero. The question is whether this calculation is correct when the one-loop calculation reveals that SU(2)$\times$U(1) is in fact broken and $\langle\phi\rangle\neq0$.

To be more specific, consider, for instance, complex massive ϕ^4 theory with a potential

$$V=\mu^2\phi^2+\lambda\phi^4 \; . \tag{24}$$

In addition ϕ may couple to other particles and the specific form of (24) is purely for the sake of argument. When $\mu^2>0$, the VEV of ϕ is zero and μ^2 corresponds to the mass of ϕ. The ϕ propagator can be evaluated at one loop and this will result in a certain infinite contribution $\delta\mu^2(\Lambda,p)$ to μ^2. In the previously discussed supergravity models the contribution $\delta\mu^2(\Lambda,p)$ are in fact such that μ^2 becomes negative. When $\mu^2<0$, μ^2 does no longer correspond to the mass of a particle. If η and χ are the real and imaginary part of ϕ, $\phi=1/\sqrt{2}(\eta+i\chi)$, then the masses of η and χ are given by

$$m_\eta{}^2=\mu^2+3\lambda v^2 \; ,$$
$$\mu_\chi{}^2=\mu^2+\lambda v^2 \; , \tag{25}$$

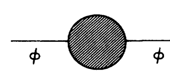
and $\lambda v^2=-\mu^2$. One can calculate the one-loop corrections to $m_\eta{}^2$ and $m_\chi{}^2$,

$$m_\eta{}^2\to m_\eta{}^2+\delta m_\eta{}^2(\Lambda,p) \; ,$$
$$m_\chi{}^2\to m_\chi{}^2+\delta m_\chi{}^2(\Lambda,p) \; , \tag{26}$$

by calculating the propagators for η and χ, but what we are really interested in are the corrections to μ^2, i.e., corrections to the coefficient of ϕ^2, evaluated in the broken phase. The question can thus be rephrased as: are the corrections to μ^2 that are induced in (25) as a result of the corrections in (26) equal to the corrections $\delta\mu^2(\Lambda,p)$ obtained in the symmetric case?

For the calculation of the threshold effects we are interested in the full order 1 + order-α effects. The order 1 terms come from the infinities of the one-loop graphs. The order-α effects come from two sources: two-loop large logarithms $[\alpha^2\ln\Lambda^2=O(\alpha)]$, and small one-loop terms of the form $\alpha\ln(x_i{}^2/Q^2)$, with x_i any of the supersymmetry-breaking parameters m_i, A_i, or M_i. In this section I will show that the infinite contributions to μ^2 are the same whether they are evaluated in the symmetric phase (i.e., $\langle\phi\rangle=0$) or in the correct vacuum with $\langle\phi\rangle=v\neq0$. Consequently, the leading logarithms (from one loop, two loops, or any loop) are correctly obtained in the symmetry phase. However, there is no reason to believe that this is also true for the small one-loop order-α corrections.

I will first consider the unbroken ($\mu^2>0$) case: Let the tree level plus one-loop contributions to the inverse ϕ propagator be given by



$$=\Gamma^{(2)}_{\text{one loop}}$$
$$=p^2-\mu_0{}^2+C_1(\Lambda,p)-p^2\delta Z(\lambda,p) \; . \tag{27}$$

The subscript zero denotes an unrenormalized quantity (μ_0^2, λ_0, ϕ_0, etc.). C_1 and δZ are functions of momentum and of various masses and coupling constants that appear in the loops. The integrals have been evaluated using an ultraviolet cutoff Λ. From (27) we see that the corrections to μ^2 are given by

$$\delta\mu^2(\Lambda,p)=\mu^2-\mu_0{}^2=-C_1(\Lambda,p)+\mu^2\delta Z(\Lambda,p) \tag{28}$$

and the field ϕ acquires a wave-function renormalization:

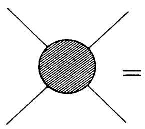
$$\phi=Z^{-1/2}(\Lambda,p)\phi_0=[1-\tfrac{1}{2}\delta Z(\Lambda,p)]\phi_0 \; . \tag{29}$$

We can define a counterterm $\delta\hat{\mu}^2$ by specifying (28) at a renormalization point:

$$\delta\hat{\mu}^2=[-C_1(\Lambda,p)+\mu^2\delta Z(\Lambda,p)]\,|_{p=\text{renormalization point}}\ . \tag{30}$$

(Quantities with carets will denote counterterms.) I will show that in the broken phase $\delta\hat{\mu}^2$ is given by the same expression (30). This implies that large $\ln\Lambda$ contributions to μ^2 are the same whether they are being evaluated in the symmetric phase or in the broken phase.

$\delta\lambda(\Lambda,p)$ is defined as the one-loop contribution to the connected four-point function (i.e., without the wave function contributions from the external legs):



$$=\lambda_0+\delta\lambda(\Lambda,p)\ . \tag{31}$$

In the broken phase, $\mu^2<0$, the Lagrangian for the scalar field ϕ is given by

$$L=|\partial_\mu\phi|^2-\mu_0{}^2\phi_0{}^2-\lambda_0\phi_0{}^4 \tag{32}$$

with

$$\phi_0=\frac{1}{\sqrt{2}}(\eta_0+i\chi_0+v_0)\ . \tag{33}$$

I will now rewrite L in terms of renormalized parameters and fields. Define

$$v_0=v+\delta\hat{v},\quad \lambda_0=\lambda+\delta\hat{\lambda},\quad \mu_0{}^2=\mu^2+\delta\hat{\mu}^2\ ,$$
$$\phi=\hat{Z}^{-1/2}\phi_0=(1-\tfrac{1}{2}\delta\hat{Z})\phi_0\ , \tag{34}$$

L becomes

$$L=\hat{Z}[\,|\partial_\mu\phi|^2-\tfrac{1}{2}(m_\eta{}^2+\delta\hat{m}_\eta{}^2)\eta^2-\tfrac{1}{2}\delta\hat{\tau}\chi^2]$$
$$-(\lambda v+\tfrac{3}{2}\lambda v\delta\hat{Z}+v\delta\hat{\lambda}+\lambda\delta\hat{v})\eta(\eta^2+\chi^2)$$
$$-\tfrac{1}{4}(\lambda+\delta\hat{\lambda}+2\lambda\delta\hat{Z})(\eta^2+\chi^2)^2-v\delta\hat{\tau}\eta\ . \tag{35}$$

v is in (35) chosen so as to cancel the tree-level term linear in η: $v^2=-\mu^2/\lambda$. The η mass counterterm $\delta\hat{m}_\eta{}^2$ and the combined tadpole and χ mass counterterm $\delta\hat{\tau}$ are given by

$$\delta\hat{m}_\eta{}^2=\delta\hat{\mu}^2+3\lambda\delta\hat{v}^2+3v^2\delta\hat{\lambda}\ ,$$
$$\delta\hat{\tau}=\delta\hat{\mu}^2+\lambda\delta\hat{v}^2+v^2\delta\hat{\lambda} \tag{36}$$

with $\delta\hat{v}^2=2v\delta\hat{v}$. Since $v_0=\langle\phi_0\rangle$, $v=\langle\phi\rangle$, and $\phi=\hat{Z}^{-1/2}\phi_0$ we obtain the useful identity:

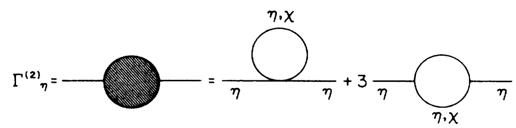
$$\delta\hat{v}^2=v^2\delta\hat{Z}\ . \tag{37}$$

[In addition, the presence of $\delta\hat{v}$ should not prompt anyone to think of the idea of some sort of RGE that expresses the behavior of v as a function of the renormalization scale. v is a constant defined by the shape of the potential at zero momentum. The presence of $\delta\hat{v}$ is a remnant of the fact that v is defined as a space-time-independent part of the field ϕ. The latter is subject to wave-function renormalization. This is clearly expressed by Eq. (37).]

Similar to (28), let the one-loop correction to $m_\eta{}^2$ be given by

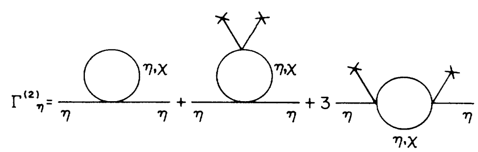
$$\delta m_\eta{}^2(\Lambda,p)=-C_2(\Lambda,p)+m_\eta{}^2\delta Z(\Lambda,p)\ . \tag{38}$$

$\delta m_\eta{}^2$ can be evaluated using the vertices and propagators of the *unbroken* theory, if we add the following rule: Every external η line can disappear into the vacuum with an amplitude v. So, for instance, the η propagator in the case of pure complex ϕ^4 theory is in the unbroken phase given by



(39)

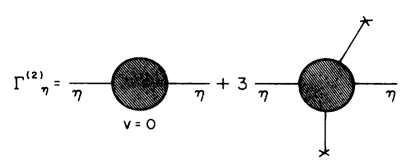
This can be written in terms of operators of the unbroken theory as



(40)

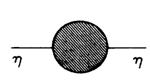
The first plus the second term and the third term in (40) equal the first and the second term in (39), respectively. The first term in (40) is independent of v and the second and third terms are proportional to v^2. Note that a term linear in v is absent because of the absence of a η^3 coupling in the unbroken potential (24). The terms are higher powers of v lead to convergent graphs and are of no interest for us. From this expansion we see clearly that infinite contributions to the wave-function renormalization are the same in (28) and (38): any insertion of v in δZ leads only to finite contributions. For the same reason, the infinite contributions to $\delta\lambda$, as defined in (31), are the same whether they are evaluated in the symmetric or the broken phase.

So in the general case, when ϕ also couples to other fermions and gauge fields, we can write



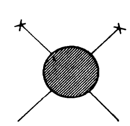
(41)

The first term in (41) is equal to the corrections to $\Gamma_\phi^{(2)}$ in (27):



$$=p^2-\mu_0{}^2+C_1(\Lambda,p)-p^2\delta Z(\Lambda,p)\ . \tag{42}$$

The second term in (41) is equal to v^2 times the one-loop corrections to λ [see (31)]:



$$=v^2[\lambda_0+\delta\lambda(\Lambda,p)]\ . \tag{43}$$

So that we obtain, for $\delta m_\eta{}^2(\Lambda,p)$,

$$\delta m_\eta{}^2(\Lambda,p) = -C_1(\Lambda,p) + \mu^2 \delta Z(\Lambda,p) - 3\lambda v^2 \delta Z(\Lambda,p) - 3v^2 \delta\lambda(\Lambda,p) \ . \quad (44)$$

This is to be compared with the expressions for the counterterms (36) and (37):

$$\delta \hat{m}_\eta{}^2 = \delta \hat{\mu}^2 + 3\lambda v^2 \delta \hat{Z} + 3v^2 \delta \hat{\lambda} \ . \quad (45)$$

From (44) and (45) we clearly see that

$$\delta \hat{\mu}^2 = [-C_1(\Lambda,p) + \mu^2 \delta Z(\Lambda,p)]\,|_{p=\text{renormalization point}} \ ; \quad (46)$$

i.e., the infinite contributions to μ^2 in the broken phase are the same as in the unbroken phase [see (28)].

This is of course not really surprising. The unbroken theory is renormalizable, which means that the parameters in the theory (μ, λ, and ϕ) can be consistently redefined to absorb all infinities. The broken theory is different in the sense that, for instance, the particles have different masses, and trilinear couplings appear. But the renormalizability of the theory is still controlled by the counterterms of the original parameters μ^2, λ, and ϕ through the relations (36). It is therefore not surprising that in the broken theory these parameters absorb the same infinities as in the unbroken theory. It is clear that this result is not restricted to one loop only, but should hold at any loop.

To restate the results of this section: To any order in perturbation theory, the leading logs of the Λ-space approach (or $\overline{\text{MS}}$ β functions) can be correctly obtained in the symmetric vacuum ($\langle\phi\rangle=0$), which is not the ground state of the theory. This facilitates calculations enormously. However, there is no reason to believe that this result also holds for the (small) finite order-α corrections that have to be included to calculate the threshold effects correctly.

V. A RECIPE FOR CALCULATING THRESHOLD EFFECTS: SUMMARY OF RESULTS

This paper was motivated by the observation that threshold effects can play an important role in the study of radiative symmetry breaking. Low-energy supergravity models have the potential to calculate the weak scale in terms of more fundamental parameters (i.e., the Planck scale and the supersymmetry-breaking scale μ of that hidden sector). However, in the conventional $\overline{\text{MS}}$ methods this feature is spoiled by the explicit introduction of a weak cutoff scale Q at which one stops the running of the RGE's. In addition, lack of knowledge about the precise value of Q can lead to inaccuracies in the results of the order of 50%.

It was argued that the RGE's of $\overline{\text{MS}}$, whether interpreted in the ϕ-space or the p-space approach, do not provide a clear definition of the supersymmetry-breaking parameters and therefore fail to provide a framework for calculating threshold effects. For this reason, the Λ-space approach was introduced in which threshold effects can naturally be studied. An additional advantage of this method is that all parameters are well-defined, physical (i.e., measurable) quantities.

It was then observed that the study of threshold effects requires a full order-α calculation, because the difference between two thresholds $\alpha\ln(\Lambda^2/Q^2)$ and $\alpha\ln(\Lambda^2/Q'^2)$ is an order-α effect (i.e., not order 1). A full order-α calculation in the Λ-space consists of the following steps.

(1) Calculate the full one-loop corrections to the supersymmetry-breaking parameters (Sec. III) in the *broken vacuum* ($\langle\phi\rangle\neq0$). This will yield the same order 1 terms as in the unbroken phase, but has to be performed in the broken phase to obtain the correct threshold effects (see Sec. IV).

(2) Calculate the two-loop leading logarithms. Since $\alpha\ln\Lambda^2$ is order 1, $\alpha^2\ln\Lambda^2$ contributes to the order-α corrections. This part of the calculation can be performed in the symmetric ($\langle\phi\rangle=0$) vacuum (see Sec. IV).

(3) Solve the set of coupled algebraic equations that result from steps (1) and (2) for the supersymmetry-breaking parameters $m_i{}^2$ and A_i. Since these equations are self-consistency equations (such as, for instance, Schwinger-Dyson equations) they do not just take the leading logs into account, but sum over all next to leading logs, $\alpha^n\ln^n$, $n>1$, as well (see Sec. III).

(4) Perform a Coleman-Weinberg-type calculation on the low-energy scalar potential to obtain the (small) order α corrections to the relations but express $\langle\phi\rangle$ in terms of the parameters of the potential (see, for instance, Appendix A).

(5) Substitute the supersymmetry-breaking parameters m_i and A_i obtained in step (3) plus the low-energy values for the gauge and Yukawa couplings into the equations of step (4) to study the patterns of symmetry breaking and the particle spectrum. It was demonstrated in Sec. III that the solutions to the order 1 approximation in the Λ-space approach agree with the solutions obtained from the RGE's using $\overline{\text{MS}}$.

ACKNOWLEDGMENTS

I am very grateful to Professor A. I. Sanda for the many fruitful discussions that we had about this work. His observation that the weak scale should in principle be calculable in supergravity models motivated much of this research. Also, I would like to thank Professor W. J. Marciano and Professor K. Enoue for their useful suggestions.

APPENDIX A

In this appendix I will review two current ways of thinking about the mechanism of radiative symmetry breaking. I will call them the ϕ-space approach and the p-space approach. It is not true that the authors of Refs. 4, 7, and 10 are devoted followers of either the ϕ-space or the p-space approach. The two schools are made up by myself as a possible way to understand the assumption that underlie these calculations. Nevertheless, I hope that the following discussion will be a fruitful contribution to a better understanding of the radiative symmetry breaking.

In both these approaches parameters defined at a high-energy scale are related to those at a low-energy scale through renormalization-group equations and produce the correct leading-logarithmic results. However, I will argue that in these approaches it is unclear how to define the parameters in terms of physical quantities. Since threshold effects are caused by nonzero physical masses of particles, it is clear that these approaches are inappropriate for obtaining these effects.

1. The one-loop corrections to the effective potential: the ϕ-space approach

The ϕ-approach calculates radiative corrections to the shape of the potential (6) in a Coleman-Weinberg-type fashion. One obtains here large negative contributions to the potential of the form $m^2\phi^2\ln(\phi/M_X)$. In this section I would like to explain that when a physical renormalization prescription is adopted, these large logarithms for the mass terms and the other couplings in the potential do not occur. I will do this by reviewing the original Coleman-Weinberg calculation[11] in massless scalar electrodynamics. The purpose of this will be demystify the often quoted dimensional-transmutation mechanism. Subsequently, I will consider the massive case. Before this, let me explain what an effective action is, and how it relates to physical observables. This will be useful for choosing a physical renormalization prescription.

For simplicity, in explaining the formalism, I will restrict myself to the theory of a single scalar field ϕ, whose dynamics are described by a Lagrange density $L(\phi,\partial_\mu\phi)$. The generalization to more complicated cases is trivial. Let us consider the effect of adding to the Lagrangian density a linear coupling of ϕ to an external source $J(x)$, a c-number function of space and time:

$$L(\phi,\partial_\mu\phi)\rightarrow L+J(x)\phi(x)\ . \tag{A1}$$

The connected generating functional $W(J)$ is defined in terms of the transition amplitude from the vacuum state in the far past to the vacuum state in the far future, in the presence of the source $J(x)$:

$$e^{iW(J)}=\langle 0^+ | 0^- \rangle\ . \tag{A2}$$

We can expand W in a functional Taylor series

$$W=\sum_n \frac{1}{n!}\int d^4x_1\cdots d^4x_n G^{(n)}(x_1\cdots x_n)\times J(x_1)\cdots J(x_n)\ . \tag{A3}$$

It is well known that the successive coefficients in this series are the connected Green's functions; $G^{(n)}$ is the sum of all connected Feynman diagrams with n external lines.

The classical field ϕ_c is defined by

$$\phi_c(x)=\frac{\delta W}{\delta J(x)}=\left[\frac{\langle 0^+|\phi(x)|0^-\rangle}{\langle 0^+|0^-\rangle}\right]_J\ . \tag{A4}$$

The effective action $\Gamma(\phi_c)$, is defined by a functional Legendre transformation:

$$\Gamma(\phi_c)=W(J)-\int d^4x\, J(x)\phi_c(x)\ . \tag{A5}$$

From this definition, it follows directly that

$$\frac{\delta\Gamma}{\delta\phi_c(x)}=-J(x)\ . \tag{A6}$$

The effective action may be expanded in a manner similar to that of (A3):

$$\Gamma=\sum_n \frac{1}{n!}\int d^4x_1\cdots d^4x_n \Gamma^{(n)}(x_1\cdots x_n)\times\phi_c(x_1)\cdots\phi_c(x_n)\ . \tag{A7}$$

It is possible to show that the successive coefficients in this series are the one-particle-irreducible (1PI) Green's functions (sometimes called proper vertices); $\Gamma^{(n)}$ is the sum of all 1PI Feynman diagrams with n external lines. (A 1PI Feynman diagram is a connected diagram that cannot be disconnected by cutting a single internal line. By convention, 1PI diagrams are evaluated with no propagators on the external lines.) There is an alternative way to expand the effective action: Instead of expanding in powers of ϕ_c, we can expand in powers of momentum (about the point where all external momenta vanish). In position space, such an expansion looks like

$$\Gamma=\int d^4x[-V(\phi_c)+\tfrac{1}{2}(\partial_\mu\phi_c)^2Z(\phi_c)+\cdots]\ , \tag{A8}$$

where the ellipsis indicates higher-derivative terms. $V(\phi_c)$ is called the effective potential. By comparing the expansions (A7) and (A8), it is easy to see that the nth derivative of V is the sum of all 1PI graphs with n vanishing external momenta. In tree approximation (that is to say, neglecting all diagrams with closed loops), V is just the ordinary potential.

The usual renormalization conditions of perturbation theory can be expressed in terms of the functions that occur in (A8). For example, if one defines the squared mass of ϕ as the value of the inverse propagator at zero momentum, then

$$\mu^2=\left.\frac{d^2V}{d\phi_c{}^2}\right|_{\phi=\langle V\rangle}\ , \tag{A9}$$

where ϕ has to be evaluated at the minimum of $\Gamma:\phi=\langle\phi\rangle$ because of (A6). Likewise, if we define the four-point function at zero external momenta to be the coupling constant λ, then

$$\lambda=\left.\frac{d^4V}{d\phi_c{}^4}\right|_{\phi=\langle\phi\rangle}\ . \tag{A10}$$

In massless (complex) scalar electrodynamics the one-loop potential is given by

$$V=\frac{\lambda}{4!}\phi^4+\tfrac{1}{2}\delta m^2\phi^2+\frac{\delta\lambda}{4!}\phi^4+\tfrac{1}{2}c_1\Lambda^2\phi^2+\frac{1}{4!}c_2\phi^4\left[\ln\left[\frac{\lambda\phi^2}{2\Lambda^2}\right]-\frac{1}{2}\right]\ , \tag{A11}$$

where $\delta\lambda$ and δm^2 are counterterms to absorb the Λ-dependent parts of the one-loop contributions. c_1 and c_2 are calculable constants of order α. At the tree level, the fact that the theory is massless and interacts with a coupling strength λ is expressed by

$$\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi=\langle\phi\rangle=0}=0,\quad \left.\frac{d^4V}{d\phi_c^4}\right|_{\phi=\langle\phi\rangle=0}=\lambda\ . \tag{A12}$$

When $\langle\phi\rangle=0$, the coefficient of ϕ^2 is equal to the mass of the particle. On the one-loop level $\langle\phi\rangle\neq0$ and this is no longer true. Now we have two choices for our mass renormalization prescription. We can demand the coefficient of ϕ^2 to be equal to zero, i.e., $\partial^2V/\partial\phi^2\,|_{\phi=0}$. This corresponds to a slight positive curvature at $\phi=\langle\phi\rangle$ and thus to a massive theory (this is the original Coleman-Weinberg choice). Alternatively we can demand the theory to be massless, i.e., $\partial^2V/\partial\phi^2\,|_{\phi=\langle\phi\rangle}=0$. This leads to a nonzero coefficient of ϕ^2 in the potential. In the latter case,

$$\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi=\langle\phi\rangle}=0,\quad \left.\frac{d^4V}{d\phi_c^4}\right|_{\phi=\langle\phi\rangle}=\lambda\ , \tag{A13}$$

the potential (A11) becomes

$$V=\frac{\lambda}{4!}\phi^4+\frac{1}{2}m^2\phi^2+\frac{1}{4!}c_2\phi^4\left[\ln\left[\frac{\phi^2}{\langle\phi\rangle^2}\right]-\frac{25}{6}\right] \tag{A14}$$

with

$$m^2=(3c_2-\lambda)\langle\phi\rangle^2\ . \tag{A15}$$

Requiring the minimum of the potential to be obtained at $\langle\phi\rangle$ gives

$$0=\left.\frac{dV}{d\phi_c}\right|_{\phi=\langle\phi\rangle}=\tfrac{1}{3}(\tfrac{8}{3}c_2-\lambda)\langle\phi^2\rangle\ . \tag{A16}$$

Note that for this minimum to exist, λ has to be chosen a value of order α and no large logarithms appear. $\langle\phi\rangle$ is not determined by (A16). This reflects the scale invariance of massless ϕ^4 theory. We did not put a mass scale into the problem, so we can hardly expect to get one out.

In the massive case, calculations get fairly complicated. If one adopts however a physical renormalization scheme, i.e., one defines all masses and couplings at the minimum of the potential, the one-loop corrections to these parameters are small.

For instance, in real massive scalar ϕ^4 theory:

$$V=\tfrac{1}{2}m_0^2\phi^2+\frac{1}{4!}\lambda_0\phi^4 \tag{A17}$$

with $m_0^2<0$ we can define, as one-loop renormalized parameters,

$$\begin{aligned}m_0^2&=\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi=0},\quad \lambda_0=\left.\frac{d^4V}{d\phi_c^4}\right|_{\phi=0},\\ m^2&=\left.\frac{d^2V}{d\phi_c^2}\right|_{\phi=\langle\phi\rangle},\quad \lambda=\left.\frac{d^4V}{d\phi_c^4}\right|_{\phi=\langle\phi\rangle}.\end{aligned} \tag{A18}$$

The one-loop corrections to the tree-level relations, $m^2=\lambda/3\langle\phi\rangle^2$ and $m^2=-2m_0^2$, are small:

$$\begin{aligned}m^2&=\left[1+\frac{9}{32\pi^2}\lambda\right]=\frac{1}{3}\lambda\langle\phi\rangle^2\ ,\\ m^2&=-2m_0^2+\frac{\lambda}{32\pi^{32}}\left[O(\langle\phi\rangle^2,m^2)\right.\\ &\qquad\left.-2\lambda m^2\ln\frac{m^2+\frac{1}{2}\lambda\langle\phi\rangle^2}{m^2}\right].\end{aligned} \tag{A19}$$

(The question of whether or not radiative effects should be calculated in the shifted vacuum $\phi\to\phi+\langle\phi\rangle$ is addressed in Sec. IV.) The first term in large parentheses is a complicated expression in $\langle\phi\rangle^2$ and m^2 that does not contain any logarithms. Also since the corrections to m_0^2 in (A18) are small, the Coleman-Weinberg mechanism does not provide a way to drive masses negative.

In the ϕ-space approach, i.e., in the approach that one *does* get large corrections using the Coleman-Weinberg mechanism, one defines λ not as in (A13) or (A18) but rather

$$\lambda_M=\left.\frac{d^4V}{d\phi_c^4}\right|_{\phi=M}, \tag{A20}$$

where M is an arbitrary scale, usually taken to be the grand-unified-theory (GUT) scale. For the massless potential (A14) λ_M is related to λ through

$$\lambda_M=\lambda+c_2\ln\frac{M^2}{\langle\phi\rangle^2}\ . \tag{A21}$$

When expressed in terms of λ_M, the condition for the minimum (A16) becomes

$$\lambda_M=c_2\left[\frac{8}{3}-\ln\frac{M^2}{\langle\phi\rangle^2}\right]. \tag{A22}$$

The assumption in the ϕ-space approach is that λ_M is the parameter that is obtained from the supergravity calculation, i.e., one of the parameters that appear in (6). Then a renormalization-group equation (RGE) is set up to obtain the "low-energy" value λ, which is in our case essentially achieved by Eq. (A21). With λ_M and M given ($M=M_X$), a prediction for $\langle\phi\rangle$ follows from (A16) or (A22). In this way the RGE obtained from an equation like (A21) leads to the correct leading-logarithmic corrections to λ.

I find this scenario insufficiently rigorous for the following reason. The input value λ_M at the scale $\phi=M$, although a legitimate parameter to describe the theory, does not correspond to the value of the coefficient of ϕ^4 in the effective potential that is obtained from the supergravity model, after integrating out the heavy fields. Recall that in obtaining (6) the heavy fields $\tilde{z}$ were expanded as $\tilde{z}=M_X+\phi$, with ϕ of order of the weak scale, so that, heuristically,

$$\lambda=\left.\frac{d^4V}{d\tilde{z}^4}\right|_{\tilde{z}\approx M_X}=\left.\frac{d^4V}{d\phi^4}\right|_{\phi\approx m_W}, \tag{A23}$$

which agrees very much with our first definition for λ in (A13), and disagrees very much with the definition (A20) with $M=M_X$. The latter definition is used in the ϕ-space approach, and causes large logarithms. So the parameters in (6) are most accurately interpreted as derivatives of V with respect to ϕ, evaluated at roughly the weak scale, and we should not expect large logarithms to come form the Coleman-Weinberg-type one-loop potential.

The dimensional-transmutation mechanism does essentially the following. Let λ be defined at the minimum of V through Eq. (A13). Fix λ so as to satisfy some minimalization equation such as (A16), i.e., λ has to be some small order α number. Then use (A21) to express λ in terms of a λ_M which is defined through (A20) a distance $M-\langle\phi\rangle$ away from the physically relevant minimum. In this way large logarithms are generated. It is clear that there are a lot of pairs (λ_M, M) that will satisfy (A21), but they have no physical interpretation as I explained.

For the scalar mass $m_M{}^2$ the situation is analogous to λ_M. $m_M{}^2$ is defined far away from the minimum of the potential through

$$\left.\frac{d^2V}{d\phi_c{}^2}\right|_{\phi_c=M} = m_M{}^2 \ . \tag{A24}$$

This will then again generate large logarithms of the form $\alpha m^2\phi^2\ln(M^2/\langle\phi\rangle^2)$, and in particular this will drive some of the scalar masses negative. Like λ_M, $m_M{}^2$ is as correct a parameter to describe the theory with as any other. But it is also of as little physical significance as λ_M and there is no apparent reason to equate $m_M{}^2$, as defined in (A24), with the input parameter $(m_i^0)^2$ in (6).

So the ϕ-space approach does not provide an adequate understanding of radiative symmetry breaking, although it yields the correct leading-logarithmic corrections. When masses and couplings are defined so that they correspond to physical quantities, the one-loop corrections are small and do not involve large logarithms. Of course when we are interested in the full order α corrections in order to understand threshold effects, these Coleman-Weinberg-type corrections should nevertheless be included.

2. Running masses and coupling constants: the p-space approach

The p-space approach argues in the following way. It is assumed that the fields in the effective potential of the observable sector are of the order of the weak scale Q. This is a point of view that I defended in the last section, and led to the conclusion that in the ϕ-space approach large logarithms are absent. However, the p-space now assumes that the parameters in the potential (6), m_i^0, A_i^0, and the gauge and Yukawa couplings, are defined at *momenta* $p^2=-M_X{}^2$. Such a definition is usually motivated by the statement that this is a natural consequence of eliminating the heavy fields.

I do not agree with this point of view for the following reasons. First of all, the effective potential (6) is obtained from a tree-level calculation and parameters in a tree-level potential are not defined at any scale in particular. Different values at different momentum scales only come in as a result of loop effects. Secondly, the full supergravity potential can be interpreted as the zero-momentum term of the full supergravity action in a momentum expansion of the form (A8):

$$T_{\rm sugra} = \int d^4x\left[-V_{\rm sugra}(z^a,\tilde{z}^h) + \tfrac{1}{2}(\partial_\mu z^A)^2 Z_{\rm sugra}(z^a,\tilde{z}^h)+\cdots\right],$$
$$A=a,h \ . \tag{A25}$$

In this way the parameters in the full supergravity potential $V_{\rm sugra}$ are defined at zero momenta and tree-level manipulations of integrating out the heavy fields result in a potential (6) whose parameters are still defined at zero momenta.

A different version of the p-space approach reasons as follows. We agree that the parameters in the potential (6) are defined at $p^2=0$ and we do no really know their numerical value. We can however obtain information about their values by relating them to high-momentum values in very much the same way as the three gauge couplings g_3, g_2, and g_1 of $SU(3)\times SU(2)\times U(1)$ in a GUT scenario are explained in terms of one gauge coupling $g_{\rm GUT}$ of SU(5) at scale $p^2=-M_X{}^2$. For the gauge couplings g_i one can define running couplings $g_i(p)$, which represent the interaction strength of particles interacting with a momentum transfer $-p^2$ (see Fig. 4). One can do this consistently for any trilinear or quartic couplings in (6). The RGE's that one obtains for these parameters in the approximation that one only takes the large $\ln p^2$ terms into account are the same as the ones obtained from $\overline{\rm MS}$. The momentum dependence of trilinear and quartic couplings is unambiguous because they are equal to the renormalized three- and four-point functions, and the momentum dependence of the latter is well defined.

The momentum dependence of the renormalized two-point function is also well defined, but in general its momentum dependence is parametrized in terms of two finite momentum-dependent functions:

$$\Gamma^{(2)} = Z(p)[p^2-m^2(p)] \ . \tag{A26}$$

The functions $Z(p)$ and $m^2(p)$ are for a particular momentum (the renormalization point) fixed by the renormalization prescription, but the momentum dependence away from this point can be distributed arbitrarily over $Z(p)$ and $m^2(p)$. So the momentum dependence of the propagator is uniquely defined, but the momentum dependence of the mass is not. Therefore, specifying "the mass at M_X" $[m^2(M_X)]$ and calculating "the mass at the weak scale" $[m^2(Q)]$, through the momentum dependence of $m^2(p)$ is an ambiguous procedure.

To be more specific, let the one-loop two-point function

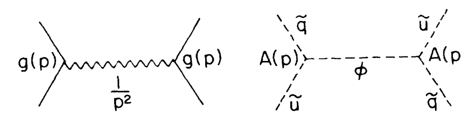


FIG. 4. The running couplings $g_i(p)$ and $A_i(p)$ as obtained from $\overline{\rm MS}$ represent the effective interaction strength of particles interacting with momentum transfer $-p^2$. $\tilde{u}$ and $\tilde{q}$ are scalar quarks.

$\Gamma^{(2)}_\phi$ of a scalar field ϕ with mass m be given by

$$\Gamma^{(2)}_\phi = p^2 - m^2 + F(p, m_i, \Lambda) + p^2 \delta Z + \delta m^2 \ . \quad \text{(A27)}$$

I have explicitly introduced the momentum independent wave function and mass counterterms δZ and δm^2. All the other parameters in (A27) are renormalized quantities. F is an order α quantity, which depends on momentum, various masses m_i, among which the mass of the scalar field ϕ itself, and couplings g_i. The integrals in F are evaluated using a UV cutoff. We will assume the absence of quadratic divergences in F, as is the case in (softly broken) supersymmetric theories.

Observe that there are two counterterms in (A27) to remove the infinities in F. The requirement that m is the pole of the ϕ propagator and that the residue equals 1 fixes these two counterterms uniquely:

$$F(m, m_i, \Lambda) + m^2 \delta Z + \delta m^2 = 0 \ , \quad \text{(A28)}$$

$$\left. \frac{\partial F}{\partial p^2} \right|_{p^2 = m^2} + \delta Z = 0 \ , \quad \text{(A29)}$$

and $\Gamma^{(2)}$ becomes

$$\Gamma^{(2)}_\phi = p^2 - m^2 + F(p) \ . \quad \text{(A30)}$$

The finite function $\widetilde{F}$ and $\widetilde{F}'$ are defined through the Taylor expansion of F around the mass shell:

$$\begin{aligned} F(p) &= F(m) + (p^2 - m^2) \left. \frac{\partial F}{\partial p^2} \right|_{p^2 = m^2} + \widetilde{F}(p) \ , \\ \widetilde{F}(p) &= (p^2 - m^2) \widetilde{F}'(p), \quad \widetilde{F}(m) = \widetilde{F}'(m) = 0 \ , \end{aligned} \quad \text{(A31)}$$

so that we can write (A30) either as

$$\begin{aligned} &\Gamma^{(2)}_\phi = p^2 - m^2(p), \quad m^2(p) = m^2 - \widetilde{F}(p) \ , \\ &m^2(m) = m^2 \ , \end{aligned} \quad \text{(A32)}$$

or

$$\begin{aligned} &\Gamma^{(2)}_\phi = Z(p)(p^2 - m^2), \quad Z(p) = 1 - \widetilde{F}'(p) \ , \\ &Z(m) = 1 \ . \end{aligned} \quad \text{(A33)}$$

In (A32) $\Gamma^{(2)}$ is parametrized in terms of a "running mass" and in (A33) in terms of a "running wave-function parameter," which shows the ambiguity of the notion of a running mass explicitly.

So the assumption used in the calculation of the RGE's for the scalar masses in the p-space approach that the momentum dependence in the leading-logarithmic approximation corresponds to the RGE's obtained from $\overline{\text{MS}}$ is not correct, simply because a there does not exist a unique momentum dependence for the masses.

At this point it is perhaps clarifying to review the renormalization prescription that is used in $\overline{\text{MS}}$, and why the renormalized parameters obtained in this renormalization scheme are useful for the study of the asymptotic (i.e., large p^2) behavior of the Green's functions,[13] and not for the study of radiative symmetry breaking following the p-space approach.

In the conventional renormalization prescriptions the dependence of the masses, couplings, and wave-function renormalization parameters on momentum is usually very complicated due to the presence of masses in the loops. The resulting RGE's for the n-point functions are hard to solve. $\overline{\text{MS}}$ solves this problem in the following way. A new, unphysical, scale parameter μ is introduced into the problem, and all parameters are renormalized at momenta $p = \mu$. So after renormalization the parameters m^2, g_j, and Z are not only a function of momentum, but also of the renormalization scale μ: $m^2(p,\mu)$, $g_j(p,\mu)$, $Z(p,\mu)$. Notice that the dependence of these parameters on μ arises only through the counterterms. The counterterms are now chosen so that the dependence of m^2, g_i, and Z on μ is as simple as possible, with of course the constraint that all infinities are canceled. This is where $\overline{\text{MS}}$ owes its name to.

For instance, for $\Gamma^{(2)}$ in (A27) this can be achieved in the following way. Since F has dimension of (mass)^2, we can write F as a sum of terms, proportional to p^2 and $m_i{}^2$:

$$F(p, m_i, \Lambda) = p^2 G_0(p, m_i, \Lambda) + \sum_i m_i{}^2 G_i(p, m_i, \Lambda) \ . \quad \text{(A34)}$$

The functions G_0 and G_i are dimensionless, and consist of sums of logarithms with numerical coefficients (i.e., no $(p^2/m^2)^n$ or $(m^2/p^2)^n$, $n = 1, 2, 3, \ldots$, coefficients). In $\overline{\text{MS}}$, δZ and δm^2 are defined as

$$\delta Z = -G_0(\mu^2, 0, \Lambda) \ , \quad \text{(A35)}$$

$$\delta m^2 = -m_i{}^2 G_i(\mu^2, 0, \Lambda) \ . \quad \text{(A36)}$$

So that

$$\Gamma^{(2)}_\phi(p, m_i) = Z_{\overline{\text{MS}}}(p, \mu, m_i)[p^2 - m_{\overline{\text{MS}}}{}^2(p, \mu, m_i)] \quad \text{(A37)}$$

with

$$\begin{aligned} m_{\overline{\text{MS}}}{}^2(p, \mu, m_i) = m^2 &- m_i{}^2 G_i(p, m_i, \Lambda) \\ &+ m_i{}^2 G_i(\mu, 0, \Lambda) - m^2 G_0(p, m_i, \Lambda) \\ &+ m^2 G_0(\mu, 0, \Lambda) \end{aligned} \quad \text{(A38)}$$

and

$$Z_{\overline{\text{MS}}}(p, \mu, m_i) = 1 + G_0(p, m_i, \Lambda) - G_0(\mu, 0, \Lambda) \ . \quad \text{(A39)}$$

$\Gamma^{(2)}$ in (A37) is a physical parameter and is therefore of course independent of the renormalization scale μ. The dependence of $m_{\overline{\text{MS}}}{}^2$ and $Z_{\overline{\text{MS}}}$ on μ is very simple because $G_0(\mu, 0, \Lambda)$ and $G_i(\mu, 0, \Lambda)$ can only be a function of $\ln(\Lambda/\mu)$. So

$$\mu \frac{d}{d\mu} \Gamma^{(2)} = \left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \gamma_{m_i} \frac{\partial}{\partial m_i} + \gamma_Z \right] \Gamma^{(2)} = 0$$

with

$$\beta = \mu \frac{\partial}{\partial \mu} g_{\overline{\text{MS}}}, \quad \gamma_{m_i} = \mu \frac{\partial}{\partial \mu} m_{i\overline{\text{MS}}}, \quad \gamma_Z = \mu \frac{\partial}{\partial \mu} \ln Z_{\overline{\text{MS}}} \ . \quad \text{(A40)}$$

Because of $\overline{\text{MS}}$, β, γ_{m_i}, and γ_Z are simple polynomials in the coupling constants and are independent of m_i.

Of course, we are interested in the momentum dependence of $\Gamma^{(2)}$, and not in the μ dependence. This can however easily be established through the relation

$$\left[\mu\frac{\partial}{\partial\mu}+m_i\frac{\partial}{\partial m_i}+p\frac{\partial}{\partial p}\right]\Gamma^{(2)}=2\Gamma^{(2)} . \tag{A41}$$

So the running masses obtained in MS, together with the anomalous dimensions, are calculational tools to solve for the momentum dependence of the Green's functions. Through a judicious choice of the counterterms, their dependence on the renormalization scale μ can be chosen to be very simple. The momentum dependence of the Green's functions is then a result of (A40) and (A41). Observe that the momentum dependence of the masses, as explicitly displayed in (A38), is never addressed.

A nice illustration of the fact that scaling of a running mass with respect to μ is something very different from the momentum dependence of the mass is given by the following example. Consider the contribution to the one-loop scalar propagator coming from the difference of two tadpole diagrams as in Fig. 5 with masses m_1 and m_2 in the loop. This contribution is proportional to $(m_1{}^2-m_2{}^2)\ln(\Lambda)$ [or $(m_1{}^2-m_2{}^2)(1/\epsilon+\ln\mu)$ when dimensional regularization is used], and contributes to the RGE's of $\overline{\text{MS}}$, but clearly does not contribute to the momentum dependence of the mass of ϕ, $m^2(p)$.

A second problem with this latter approach of running masses is that with $p^2=-M_X{}^2$, the physics at that scale does not really care about these masses at all. With $|p|\approx 10^{15}$ GeV and masses of the order of the weak scale the physics at the scale should be completely insensitive to whether the masses are m_W or, for instance, $10m_W$, or exactly zero. On the other hand, in the p-space approach the weak symmetry breaking *is* sensitive to what values one assumes for these masses at high energy.

The main result of this section is that specifying scalar masses m at $p^2=-M_X{}^2$ is an ambiguous procedure, because the momentum dependence of m is not uniquely defined. The Georgi-Quinn-Weinberg mechanism[12] *does* work for trilinear and quartic couplings because these couplings are defined as physical n-point functions.

APPENDIX B

In this appendix I write down explicitly the supersymmetric one-loop equations for the supersymmetry-breaking parameters of the E_6 model considered in Sec. III. An unbroken $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_E$ gauge symmetry is assumed, with "light" matter content given by a **27** of E_6. These will be assumed to be the third generation of the standard model.

The scalar masses.

$$m_H{}^2=(3M_2{}^2g_2{}^2+\tfrac{3}{5}M_1{}^2g_1{}^2+\tfrac{16}{15}M_E{}^2g_E{}^2-3h^2F_h-\lambda^2F_\lambda)t , \tag{B1}$$

$$m_{\overline{H}}{}^2=(3M_2{}^2g_2{}^2+\tfrac{3}{5}M_1{}^2g_1{}^2+\tfrac{1}{15}M_E{}^2g_E{}^2-\lambda^2F_\lambda)t , \tag{B2}$$

$$m_N{}^2=(\tfrac{5}{3}M_E{}^2g_E{}^2-2\lambda^2F_\lambda-3\kappa^2F_\kappa)t , \tag{B3}$$

$$m_D{}^2=(\tfrac{16}{3}M_3{}^2g_3{}^2+\tfrac{4}{16}M_1{}^2g_1{}^2+\tfrac{16}{15}M_E{}^2g_E{}^2-\kappa^2F_\kappa)t , \tag{B4}$$

$$m_{D_c}{}^2=(\tfrac{16}{3}M_3{}^2g_3{}^2+\tfrac{4}{15}M_1{}^2g_1{}^2+\tfrac{1}{15}M_E{}^2g_E{}^2-\kappa^2F_\kappa)t , \tag{B5}$$

$$m_Q{}^2=(\tfrac{16}{3}M_3{}^2g_3{}^2+3M_2{}^2g_2{}^2+\tfrac{1}{15}M_1{}^2g_1{}^2+\tfrac{4}{15}M_E{}^2g_E{}^2-h^2F_h)t , \tag{B6}$$

$$m_u{}^2=(\tfrac{16}{3}M_3{}^2g_3{}^2+\tfrac{16}{15}M_1{}^2g_1{}^2-\tfrac{4}{14}M_E{}^2g_E{}^2-2h^2F_h)t , \tag{B7}$$

$$m_d{}^2=(\tfrac{16}{3}M_3{}^2g_3{}^2+\tfrac{4}{15}M_1{}^2g_1{}^2+\tfrac{1}{15}M_E{}^2g_E{}^2)t , \tag{B8}$$

$$m_L{}^2=(3M_2{}^2g_2{}^2+\tfrac{3}{5}M_1{}^2g_1{}^2+\tfrac{1}{15}M_E{}^2g_E{}^2)t , \tag{B9}$$

$$m_e{}^2=(\tfrac{12}{5}M_1{}^2g_1{}^2+\tfrac{4}{15}M_E{}^2g_E{}^2)t , \tag{B10}$$

$$m_\nu{}^2=(\tfrac{5}{3}M_E{}^2g_E{}^2)t . \tag{B11}$$

The trilinear couplings.

$$A_h=-(\tfrac{16}{3}M_3g_3{}^2+3M_2g_2{}^2+\tfrac{13}{15}M_1g_1{}^2+\tfrac{4}{5}M_Eg_E{}^2+6h^2A_h+\lambda^2A_\lambda)t , \tag{B12}$$

$$A_\lambda=-(3M_2g_2{}^2+\tfrac{3}{5}M_1g_1{}^2+\tfrac{7}{5}M_Eg_E{}^2+3h^2A_h+4\lambda^2A_\lambda+3\kappa^2A_\kappa)t , \tag{B13}$$

$$A_\kappa=-(\tfrac{16}{3}M_3g_3{}^2+\tfrac{4}{15}M_1g_1{}^2+\tfrac{7}{5}M_Eg_E{}^2+2\lambda^2A_\lambda+5\kappa^2A_\kappa)t \tag{B14}$$

with

$$F_h=m_Q{}^2+m_u{}^2+m_H{}^2+A_h{}^2 ,\quad F_\lambda=m_H{}^2+m_{\overline{H}}{}^2+m_N{}^2+A_\lambda{}^2 ,\quad F_\kappa=m_D{}^2+m_{D_c}{}^2+m_N{}^2+A_\kappa{}^2 . \tag{B15}$$

In (B1)–(B14) $t=(\frac{1}{16}\pi^2)\ln(\Lambda^2/Q^2)$, Λ is the UV cutoff, and Q is of the order of the weak scale: 100 GeV $\lesssim Q\lesssim 500$ GeV. Equations (B1)–(B11) have been obtained using the fact that the quantities

$$S_Y=m_e{}^2-m_L{}^2+m_H{}^2-m_{\overline{H}}{}^2-m_D{}^2+m_{D_c}{}^2+m_d{}^2-2m_u{}^2+m_Q{}^2 \tag{B16}$$

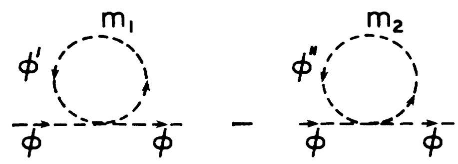


FIG. 5. The difference of two tadpoles diagrams with masses m_1 and m_2 in the loop is proportional to $(m_1{}^2-m_2{}^2)\ln(\Lambda)$. These graphs contribute to the RGE's of $\overline{\text{MS}}$, but clearly do not contribute to the momentum dependence of the mass of ϕ, $m^2(p)$.

and

$$S_E = 2m_Q{}^2 + m_u{}^2 - \tfrac{1}{2}m_d{}^2 - \tfrac{1}{3}m_L{}^2 + \tfrac{1}{3}m_e{}^2 + \tfrac{5}{6}m_\nu{}^2 - \tfrac{4}{3}m_H{}^2 - \tfrac{1}{3}m_{\overline{H}}{}^2 + \tfrac{5}{6}m_N{}^2 - 2m_D{}^2 - \tfrac{1}{2}m_{D_c}{}^2 \tag{B17}$$

are solutions to (B1)—(B14) with

$$S_Y = S_E = 0 \ .$$

All terms in (B1)—(B14) that are proportional to S_Y and S_E have been put equal to zero. The vanishing of these quantities is related to the vanishing of the $U(1)_Y$ and $U(1)_E$ trace anomalies in the **27** of E_6.

The RGE's for the gaugino masses and the gauge couplings are given by

$$\frac{dg_a{}^2}{dt} = \frac{1}{8\pi^2} b_a g_a{}^4 \ , \tag{B18}$$

$$\frac{dM_a}{dt} = \frac{1}{8\pi^2} b_a g_a{}^2 M_a \tag{B19}$$

with $t = \ln(M/M_X)$, M is the renormalization scale. $b_3 = 0$, $b_2 = 3$, $b_1 = b_E = 9$.

The RGE's for the Yukawa couplings h, λ, and κ are given by

$$\frac{dh}{dt} = \frac{h}{8\pi^2}\left(-\tfrac{8}{3}g_3{}^2 - \tfrac{3}{2}g_2{}^2 - \tfrac{13}{30}g_1{}^2 - \tfrac{2}{5}g_E{}^2 + 3h^2 + \tfrac{1}{2}\lambda^2\right) , \tag{B20}$$

$$\frac{d\lambda}{dt} = \frac{\lambda}{8\pi^2}\left(-\tfrac{3}{2}g_2{}^2 - \tfrac{3}{10}g_1{}^2 - \tfrac{7}{10}g_E{}^2 + \tfrac{3}{2}h^2 + 2\lambda^2 + \tfrac{3}{2}\kappa^2\right) , \tag{B21}$$

$$\frac{d\kappa}{dt} = \frac{\kappa}{8\pi^2}\left(-\tfrac{8}{3}g_3{}^2 - \tfrac{2}{15}g_1{}^2 - \tfrac{7}{10}g_E{}^2 + \tfrac{5}{2}\kappa^2\right) . \tag{B22}$$

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