Violation of Einstein causality in a model quantum system

Morton H. Rubin
Department of Physics, University of Maryland Baltimore County, Catonsville, Maryland 21228
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A series of theorems have been proved showing that localization in quantum mechanics is inconsistent with Einstein causality. In this paper a simple model of a localized source and a localized detector is studied. This model does not satisfy the assumptions of the rigorous theorems; nonetheless, it is found that the model violates Einstein causality. The effect is very small and resides in the nonresonant part of the transition probability.

I. INTRODUCTION

In a series of interesting papers,[1–4] a set of theorems have proved that the localization of relativistic quantum systems leads to violation of Einstein causality. Einstein causality is taken to mean no signal can propagate faster than the speed of light. These papers show that under the condition of translation invariance of the Hamiltonian and a weak condition on the energy-momentum spectrum, a wave function vanishing outside a finite open region of space-time must vanish identically. Hegerfeldt[4] has recently strengthened this result to show that if the wave function is exponentially bounded outside a finite open spatial region at $t=0$, for $t>0$ the wave function cannot be exponentially bounded anywhere in space.

An experimental test of the violation of Einstein causality presents many difficulties. For multiparticle systems the theorems predict that the center-of-mass coordinate spreads over all space instantaneously. It is not clear what meaning can be applied to the notion of the detection of the center of mass since technically the theorem should apply to the entire system including the detector, because it is the entire Hamiltonian that is translationally invariant. Hegerfeldt and Ruijssenaars have also shown that scattering states also may spread instantaneously.[3]

In this paper we examine the question of whether noncausal signals can be detected by considering a model that does not satisfy the technical requirements of Refs. 1–4, but has the advantage of being calculable and manifestly noncausal. The source of the noncausality is the Feynman propagator of the photon. It has been known for a long time that this propagator is nonzero outside the light cone determined by the argument of the propagator.[5] It can be shown that for infinite times the noncausal part of the propagator vanishes; however, as we shall see, this is not true for finite times. If these signals can be detected nothing very surprising follows since no conflict between relativity and quantum mechanics is implied by this result, if by special relativity we understand a set of invariance principles and not Einstein causality. However, there are implications for the precise measurement of time. If such signals are undetectable because of some fundamental quantum principle, then Einstein causality is compatible with quantum mechanics.

In the next section we define the model system in detail. After presenting our analysis of the noncausal effect, we briefly examine the question of whether this system is too idealized to provide a test of the detectability of the noncausal signal.

II. MODEL

The system is composed of a source and a detector each containing two-level atoms coupled to the electromagnetic field. The atoms in the source and detector are assumed to be localized in two disjoint spatial regions V_S and V_D. At the start of the experiment all the atoms in the source are in their excited states, all the atoms in the detector are in their ground states, and no photons are present. If the minimum separation of V_S and V_D is L then noncausality manifests itself by the appearance of an excited atom in the detector at a time $t<L/c$ where c is the speed of light. As stated in the Introduction this occurs because of the nonlocal nature of the Feynman propagator.

The relation between the spreading of the wave function in Refs. 1–4 and the noncausal nature of the Feynmann propagator is not entirely clear to me. However, if the model system described above is treated as a coherent quantum system so that it is described by a wave function, then the fact that, in principle, for $t>0$ a photon can be emitted and detected instantaneously anywhere leads to the instantaneous spreading of the center of mass of the system.

One more remark might be in order before turning to the computation. The Feynman propagator is noncausal for all types of particles and so the possibility of detecting noncausal photons does not seem to be directly related to the nonexistance of an observable position operator for the photon.[6]

The Hamiltonian for the model is

$$H=H_0+H_I=H_0+H_S+H_D\ , \tag{2.1}$$

where H_0 is the unperturbed Hamiltonian for the atoms and the electromagnetic field, H_S and H_D describe the interaction between the electromagnetic field and the source and detector atoms, respectively. For simplicity the atoms are assumed to have only two levels. For $N=S$ or D

$$H_N=\sum_{j\in N}\mathbf{p}_j\cdot\mathbf{E}_j(b_j^\dagger+b_j)\ , \tag{2.2}$$

where $\mathbf{p}_j$ is the electric dipole moment of the jth atom, $\mathbf{E}_j$ is the electric field operator evaluated at the position of the jth atom, and $b_j^\dagger$ and b_j are the operators that correspond to the excitation and deexcitation of the jth atom. This Hamiltonian has been exhaustively analyzed.[7] In particular it is the leading order in a multipole expansion of the minimal coupling Hamiltonian and is manifestly gauge invariant.[8,9]

The initial state of the system is taken to be

$$\Psi(0)=\Phi_D\chi_S\Omega_\gamma \;, \tag{2.3}$$

where Φ_D is the state of the detector with all atoms in the ground state, χ_S is the state of the system with all atoms in their excited state, and Ω_γ is the vacuum state of the electromagnetic field.

The computation is straightforward, and consists of computing the probability of finding an atom in the detector in the excited state, an atom in the source in the ground state, and no photons. This is done using perturbation theory after transforming to the interaction picture. This computation has been done any number of times; however, in general only the dominant resonant term is retained and this term is causal.[8] It is of interest to note that if the electric field operator due to radiation by a source atom is calculated to leading order with the Hamiltonian used in this paper, it is found to be causal.[8] This may seem contradictory, but it is not. The measurement of the electric field operator requires a different type of experiment than is described here.

III. CALCULATION

Since the computation is standard we summarize it. The quantity of interest is the probability of finding an excited atom in the detector and a source atom in the ground state. The amplitude for this process is

$$(\Psi_f,U_I(t)\Psi_0)=(b_D^\dagger b_S\Psi_0,U_I(t)\Psi_0) \;, \tag{3.1}$$

where Ψ_0 is defined in (2.3), the D denotes a detector atom, the S denotes a source atom,

$$U_I(t)=1+\sum_{n=1}^{\infty}\left[\frac{-i}{\hbar}\right]^n\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}}dt_n H_I(t_1)\cdots H_I(t_n) \tag{3.2}$$

and

$$H_I(t)=e^{(i/\hbar)H_0t}H_Ie^{-(i/\hbar)H_0t} \tag{3.3}$$

with H_0 and H_I defined in (2.1). The leading nonzero term for the process of interest is the $n=2$ term which can be written as

$$(\Psi_f,U_I^{(2)}(t)\Psi_0)=\frac{1}{4\pi^2\epsilon_0\hbar}(\mathbf{P}_S\cdot\mathbf{p}_D\nabla_R{}^2-\mathbf{P}_S\cdot\nabla_R\mathbf{P}_D\cdot\nabla_R)\frac{I}{R}\frac{e^{i(\omega_d-\omega_S)t/2}}{i(\omega_D-\omega_S)} \;, \tag{3.4}$$

where $\hbar\omega_D$ and $\hbar\omega_S$ are the excitation energies of the atoms and $\mathbf{R}$ is the displacement of the detector atom from the source atom. Finally,

$$I=J(\omega_S,\omega_D)-J(\omega_D,\omega_S) \;, \tag{3.5}$$

where

$$J(x,y)=\int_0^\infty d\omega[G(x-\omega,t)e^{-i(x-y)t/2} - G(-x-\omega,t)e^{i(x-y)t/2}]\sin\frac{\omega R}{c} \;, \tag{3.6}$$

$G(2\Omega,t)=e^{i\Omega t}\sin\Omega t/\Omega$, and c is the speed of light.

The first term in (3.6) contains the resonant term for which $\omega=x$. In order to analyze J for $t<R/c$, it is useful to isolate this term. Starting with

$$\int_{-\infty}^{\infty}d\omega\, G(x-\omega,t)\sin\frac{\omega R}{c}=-i\pi\Theta(t-R/c)e^{ixR/c} \;,$$

where Θ is the Heaviside step function, it is a simple matter to show

$$\int_0^\infty d\omega\, G(x-\omega,t)\sin\frac{\omega R}{c}=-i\pi\Theta(t-R/c)e^{ixR/c} + \int_0^\infty d\omega\, G(x+\omega,t)\sin\frac{\omega R}{c} \;.$$

Then, using $G(\Omega,t)^*=G(-\Omega,t)$

$$J(x,y)=-i\pi\Theta(t-R/c)e^{ixR/c} +2i\,\mathrm{Im}\int_0^\infty d\omega\, G(x+\omega,t)e^{-i(x-y)t/2}\sin\frac{\omega R}{c} \;. \tag{3.7}$$

In the standard calculation only the first term, which is causal, is retained. The second term is much smaller than the first term, but is nonzero for all $t>0$.

The integral in (3.7) is easily evaluated in terms of the sine and cosine integrals.[10]

$$J(x,y)=-i\pi\Theta(t-R/c)e^{ixR/c} +i\,\mathrm{Im}\{e^{i(x+y)t/2}[F(U_+)-F(U_-)] - e^{-i(x-y)t/2}K(R/c)\} \;, \tag{3.8}$$

where

$$F(z)=e^{-ixz}\{\mathrm{Ci}(x\,|z|)+i\,\mathrm{sgn}(z)[\mathrm{Si}(x\,|z|)-\pi/2]\} \;,$$

$$K(z)=-i\{\sin xz\,\mathrm{Ci}(xz)-\cos xz[\mathrm{Si}(xz)-\pi/2]\} \;,$$

$U_\pm=t\pm R/c$, and $\mathrm{sgn}(x)=x/|x|$ is the signum function. For $0<t<R/c$, the term containing $F(U_-)$ is the dominant term. Taking $|\omega_S-\omega_D|\ll\omega_S$, $1\ll\omega_S|U_-|\ll\omega_S R/c$ we finally find, for I in (3.4),

$$I\simeq i\frac{\omega_D-\omega_S}{\omega_S\omega_D}\frac{1}{U_-}\cos\frac{\omega_S+\omega_D}{2}t\ , \tag{3.9}$$

where the terms that have been dropped are of order cU_-/R. This term is rapidly oscillating; nontheless, there is a nonzero probability of finding the detector atom excited.

Substituting Eq. (3.9) into Eq. (3.4) and retaining the dominant term we find

$$(\psi_f,U_I^{(2)}(t)\psi_0)=\frac{1}{4\pi^2\epsilon_0\hbar}(\mathbf{p}_S\cdot\mathbf{p}_D-\mathbf{p}_S\cdot\hat{\mathbf{R}}\mathbf{p}_D\cdot\hat{\mathbf{R}})\frac{1}{R}\frac{e^{i(\omega_S-\omega_S)t/2}}{\omega_S\omega_D}\frac{2}{c^2}\frac{1}{U_-}\cos\frac{\omega_S+\omega_D}{2}t\ .$$

The probability of finding a detector atom excited, a source atom in the ground state, and no photons is

$$P=\frac{1}{R^2}\frac{{p_S}^2{p_D}^2}{9\pi^2{\epsilon_0}^2\hbar^2c^4}\frac{1}{{\omega_S}^4}\frac{1}{{U_-}^6}$$

$$=\frac{1}{\omega_S\tau_S}\frac{1}{\omega_S\tau_D}\frac{1}{{k_S}^8R^2(cU_-)^6}\ , \tag{3.10}$$

where we have summed over the final states of the detector atoms, averaged over the orientation of the source dipoles, and replaced the square of the rapidly oscillating term by its average. In addition, it has been assumed that the shuttering of the source and detector from one another takes place adiabatically so $\omega_S=\omega_D$ by energy conservation.[11] In the last term $k=\omega/c$ and the radiative lifetimes of the source and detector atoms has been inserted:

$$\frac{1}{\tau}=\frac{p^2\omega^3}{3\pi\epsilon_0hc^3}\ .$$

In averaging over the orientation of the dipoles it has been assumed that R is constant. This places a condition on the size of the source and detector. Finally, the counting rate is given by

$$r=N_S\frac{dp}{dt}=N_S\frac{1}{\tau_S}\frac{1}{\omega_S\tau_D}\frac{6}{{k_S}^9R^2(R-ct)^7} \tag{3.11}$$

when N_S is the number of source atoms. For comparison the steady-state counting rate for $t>R/c$ is

$$r_+=N_S\frac{1}{\tau_S}\frac{1}{\omega_S\tau_D}\frac{1}{{k_S}^2R^2}\frac{4\pi\omega_S}{\Gamma_S}\ ,$$

where Γ is the fullwidth at half-maximum of the spectral line in radians per sec.[12] The ratio of the counting rates is

$$r/r_+=3\left[\frac{1}{2\pi}\right]^8\left[\frac{\lambda_S}{R-ct}\right]^7\frac{\Gamma_S}{\omega_S}\ . \tag{3.12}$$

Putting in some typical numbers, $\lambda_S=600$ nm, $R-ct=3\times10^{-4}$ m, and $\Gamma_S/\omega_S=2\times10^{-7}$, we find $r/r_+\simeq3\times10^{-32}$. This corresponds to allowing the source and detector to interact for a time up to 1 psec before the causal radiation reaches the detector. Because of the seventh power appearing in (3.12) if timing could be improved to 1 fsec, then $r/r_+\simeq1.5\times10^{-11}$. In any case, the effect while very small is nonetheless present. If it is detectable in principle, then Einstein causality and localization in quantum mechanics are incompatible. Alternatively, it may be argued that such a small signal cannot in principle be detected. We now turn to this question.

IV. DISCUSSION

In trying to determine whether the effect computed in the preceding section is detectable, several questions arise naturally which are related to the general problem of whether the system described is overidealized. First there are questions of whether limitations on sensitivity or due to noise make it impossible to recognize the signal. Second, it is necessary to determine whether any shuttering system can be constructed which is compatible with conditions required for operating the system. Third, there is the question of whether the Hamiltonian used describes a physical system at the low level of probability being considered or whether higher-order terms in the multipole expansion will contribute in such a way to render the present analysis useless. Finally there is the question of whether the semiclassical assumption of the localizability of the source and detector make sense in the context of the problem being discussed.

The detection process has been idealized in the first instance by assuming every photon that excites a detector atom is recorded. Since the detector is open for such a short time, if a detector atom is excited, the probability of the excited atom reradiating the photon so it can escape is negligible. Therefore, the loss of counts will be due to the internal structure of the detector. It seems that this effect can be made quite negligible so that every atom that is excited will generate a signal in the detector.

The detection process has also been idealized by neglecting false counts. This should be a more serious problem. The effect of thermal photons can be made smaller than the signal if the entire experiment is isolated and run at low temperature. For example, if the temperature is of order 10^{-3} K, then with the detector parameters given in Sec. III, the thermal photon counting rate is less than 10% of the noncausal counting rate. This temperature gives us a general idea of the noise suppression necessary to avoid a false count. In addition to thermal photons, excitations of detector atoms due to radioactive background and cosmic rays may not be negligible without special consideration. These effects cannot be estimated without a more specific model of the system.

Perhaps the most likely cause of false counts arises from the switching necessary to perform the experiment.

The opening and closing of shutters on the time scale required most likely requires some form of optical switching. This leads to the possibility of photons from the switching beams being scattered into the detector.

It seems unlikely that the effect considered here is eliminated by higher-order effects. First, the higher-order multipoles which have been omitted will make much smaller contributions to the noncausal signal. The sixth power of $(k_S cU_-)$ in Eq. (3.10) is characteristic of the electric dipole interaction, and the higher-order multipole would contain at least the eighth power of this factor. Therefore, in the example given following Eq. (3.12), the higher-order multipoles lead to contributions at least 10^{-8} times smaller than the electric dipole. Furthermore, it might be imagined that the transition of interest in the source and detector atoms have selection rules that suppress as many of the higher-order multipoles as desired.

Second, it might be imagined that higher orders in the electromagnetic coupling might cancel the noncausal effect computed above. This also seems unlikely since these effects are suppressed because they are higher order in the electromagnetic coupling constant, i.e., the fine-structure constant. In addition, the noncausality arises from the fact that the photon propagator in Eq. (3.6) contains only non-negative frequencies; consequently, its source is closely related to the analogous effect found in Refs. 1—4. It is difficult to see how higher-order perturbation terms could alter this fundamental fact and the consequent analytic behavior of the propagator. Of course, in the absence of a calculation to all orders, it is impossible to rule out the possibility altogether.

It is evident that several technical problems make the detection of the noncausal signal difficult. However, none of the difficulties mentioned above seem to be due to a fundamental limitation imposed by the laws of quantum mechanics. This may not be true of the basic assumption that the source and detector are localized and that there are no photons present initially. If it were possible to overcome the technical problems of searching for the nonacausal signal and it were not found, then the assumption of localization would be suspect. This would require that a set of conditions be placed on initial conditions which are more stringent than those presently imposed.

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[8]For a recent detailed discussion, see E. A. Power and T. Thirumanachandran, Phys. Rev. A **28**, 2649 (1983); **28**, 2663 (1983); **28**, 2671 (1983).

[9]For a discussion of the gauge properties of this Hamiltonian see *Quantum Electrodynamics and Quantum Optics,* edited by A. O. Barut (Plenum, New York, 1984), in particular the paper of R. R. Schlicher, W. Becker, J. Bergon, and M. O. Scully.

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[11]The adiabatic theorem in this instance takes the form

$$\begin{aligned}(E_f-E_i)(\psi_f,U_I(t)\psi_I) &= (\psi_f,[H_0,U_I(t)]\psi_I) \\ &= (\psi_f,[-H_I(t)U_I(t) \\ &\quad + U_I(t)H_I(0)]\psi_i) \\ &\quad + \int_{t_0}^{t} dt_1\, U_I(t,t_1)\dot{H}_I(t_1) \\ &\quad \times U_I(t_1,t_0)\ ,\end{aligned}$$

where

$$\dot{H}_I(t_1)\equiv e^{(+iH_0/\hbar)t}\dot{H}_1(t)e^{(-iH_0/\hbar)t}\ .$$

$\dot{H}_I(t)=H_I(0)=0$ and $\dot{H}_1(t)\neq 0$ only when the shutters are being open and closed. To the order considered the right-hand side vanishes.

[12]It may be noted that the effect of the finite lifetime of the excited states of the source has not been included in the computation. This effect is negligible over the short time interval of interest in the computation.