Unstable compactification in ten-dimensional theories

Y. S. Myung
Physics Department, Pohang Institute of Science and Technology, P. O. Box 125 Pohang, 680 Korea

B. H. Cho
Physics Department, Korea Advanced Institute of Science and Technology, P.O. Box 150 Chongryang, Seoul, Korea

Young-Jai Park
Physics Department, Kangwon National University, Chunchon 200, Korea

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We discuss the stability of ten-dimensional Einstein-Maxwell and Einstein-Maxwell-scalar theories with $M_4 \times S^2 \times S^2 \times S^2$ compactification. Although these theories are able to accommodate chiral fermions, it is found that this compactification contains tachyonic modes. As a result the $M_4 \times S^2 \times S^2 \times S^2$ does not exist as a true background state.

I. INTRODUCTION

Some possibilities have appeared to unify all four interactions in a consistent way by superstring theories.[1] These superstring theories can compactify to four dimensions with Calabi-Yau space, so that this compactification provides a phenomenologically acceptable model.[2] Note that Calabi-Yau space exists only as an approximate ground state.[3] In order that this space exists as an exact ground state, two difficulties should be evident: one is the lack of an apparent dynamical reason for the particular spontaneous compactification $M_{10} \rightarrow M_4 \times M^6$ and the other is the lack of a dynamical explanation for singling out a Calabi-Yau space among all possible Ricci-flat compact manifolds.[4]

On the other hand, the stability of compactification of higher-dimensional theories is crucial for the true physical ground state.[5,6] The reason is that the flat-space-time procedure for obtaining the true vacuum state must be modified, since in the presence of gravity there is no universal conserved energy functional which allows us to compare the energies of two field configurations.[7] To investigate the classical stability of the nonlinear system means to assess the reliability of the linearized approximation. Stability refers to the persistence of some properties under certain perturbations. It is required that classical stability of the vacuum under small perturbations be a necessary condition for stability in the complete quantum theory.

It seems that the full analysis of the stability for $M_4 \times$(Calabi-Yau space) is almost impossible, due to the unknown metric of Calabi-Yau space and the complexity of superstring theories. In this paper, we wish to study thoroughly the stability of $M_4 \times S^2 \times S^2 \times S^2$ compactification in ten-dimensional Einstein-Maxwell and Einstein-Maxwell-scalar theories. We note that these models do not have the same field content as in the case of superstring theories. Further, $M_4 \times S^2 \times S^2 \times S^2$ does not exist as the exact ground-state solution. However, this compactification is phenomenologically acceptable, since this is able to accommodate chiral fermions.[8] It is thus important to analyze completely the stability of $M_4 \times S^2 \times S^2 \times S^2$ compactification within the restricted models, which gives $M_4 \times S^2 \times S^2 \times S^2$ compactification and provides then the complete analysis of classical stability.

Before we proceed, it is useful to review the general procedure of stability analysis in higher-dimensional theories.[9-11] We look first for a solution to classical equations. We expand fields around their background values and then express the Lagrangian in terms of the fluctuation fields. From the bilinear parts of the Lagrangian we obtain the linearized equations, which govern the propagation of the fluctuation fields. Also these linearized equations can be obtained by linearizing the classical equations. These are solved to express the perturbation fields in terms of external sources. Finally these isolated fields are substituted into the bilinear Lagrangian to obtain the fluctuation Green's functions. If the classical background state is stable, it requires that the Green's functions all do not contain a tachyon (negative mass square) or ghost (negative-norm state).

In Sec. II we describe the criterion on the classical stability: the tachyon and ghost. Section III is devoted to the analysis of the stability of the Einstein-Maxwell theory. In Sec. IV the Einstein-Maxwell-scalar system is introduced to see the effects of a scalar field on the stability of the Einstein-Maxwell system. Finally we discuss the stability of monopole compactification in Sec. V.

II. CRITERION ON STABILITY: TACHYON AND GHOST

The constraints for not having a ghost and tachyon follow from the requirement of having a real mass and positive-definite residue at the pole. Although the condition of real mass is obvious, the requirement of a positive-definite state needs some explanations. The existence of a ghost appears to cause theory to either violate unitarity requirements or to allow negative-energy states. Unlike the gauge ghost, which cannot appear on the external legs in a Feynman diagram, the ghost can in fact propagate. To distinguish it from the gauge ghost, it is sometimes called a "poltergeist," a noisy and mischievous ghost.[12] For simplicity, let us consider for a moment quantum electrodynamics (QED), where the same situation as quantum gravity exists. We require the unitarity for the photon-photon scattering S matrix to maintain the physical significance. Using the Ward identity for photon-photon scattering, the sum over the scalar photon A_0 and longitudinal-polarized photon A_3 contributes precisely zero. To be more specific, the negative probability state of the scalar photon which is the longitudinal-polarized state has canceled against the positive longitudinal probability state. Then this leaves two transverse states which have a positive probability. In other words, the restriction of the S matrix to the space of transversely polarized photons provides us with a unitary operator. The unitarity gets rid of the unphysical components A_0 and A_3 in QED. Since the S-matrix element is just the Green's function with the external legs removed and with the external momenta put on the mass shell, all Green's functions should be positive definite at the pole.

On the other hand, the unitarity of QED is achieved in a nonperturbative way (canonical formalism).[13] It is already known that the Ward identities guarantee unitarity in QED. For an explicit example, let us consider the four-dimensional QED Lagrangian as

$$\mathcal{L}_{\rm QED} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + B\partial_\mu A^\mu + \tfrac{1}{2}\xi B^2 \,, \tag{2.1}$$

where B is an auxiliary scalar field. Note that the $\xi=1$ case corresponds to the Feynman gauge. The equation of motion leads to

$$\Box A^\mu - \partial^\mu\partial_\nu A^\nu = \partial^\mu B \,, \tag{2.2}$$

$$\partial_\mu A^\mu + \xi B = 0 \,. \tag{2.3}$$

Eliminating B from (2.2) by means of (2.3), we obtain

$$\left[\Box\eta^{\mu\nu} + \left[\frac{1}{\xi} - 1\right]\partial^\mu\partial^\nu\right]A_\nu = 0 \,. \tag{2.4}$$

Further from these equations we learn that $\Box B = 0$ and $\Box\partial\cdot A = 0$, so that both B and $\partial\cdot A$ are free fields. In this case, the subsidiary condition for getting rid of the unphysical states A_0 and A_3 is

$$B^+(x)\,|\,0\rangle_{\rm phys} = 0 \,, \tag{2.5}$$

where B^+ is the positive-frequency (annihilating) part of B as

$$B = B^+ + B^- = \int \frac{d^3\bar{k}}{[(2\pi)^3 2\omega]^{1/2}}[a(\bar{k})e^{-ik\cdot x} + a^\dagger(\bar{k})e^{ik\cdot x}] \tag{2.6}$$

with $\omega = (\bar{k}^2)^{1/2}$. Here $a(\mathbf{k})$ and $a^\dagger(\bar{k})$ satisfy the following commutation rules:

$$\begin{aligned} &[a(\bar{k}), a^\dagger(\bar{k}')] = \delta^3(\bar{k} - \bar{k}') \,, \\ &[a(\bar{k}), a(\bar{k}')] = [a^\dagger(\bar{k}), a^\dagger(\bar{k}')] = 0 \,. \end{aligned} \tag{2.7}$$

Note that the Becchi-Rouet-Stora (BRS) symmetry is all that is required to guarantee the Ward identity in QED. Adding the $-\eta\Box\omega$ term, which comes from the gauge invariance of (2.3) to (2.1), this Lagrangian is invariant under the infinitesimal BRS transformation:

$$\begin{aligned} &A'_\mu = A_\mu + \epsilon\partial_\mu\omega, \quad \eta' = \eta + \epsilon B \,, \\ &\omega' = \omega, \quad B' = B \,, \end{aligned} \tag{2.8}$$

where η is the Lagrangian multiplier and ω comes from the U(1) gauge transformation ($A'_\mu = A_\mu + \partial_\mu\omega$). Via the Noether procedure we also can obtain the generator Q_B of BRS symmetry. Then the physical vacuum can be defined by

$$Q_B\,|\,0\rangle_{\rm phys} = 0 \,. \tag{2.9}$$

Kugo and Ojima have shown that (2.9) reproduces (2.5). However, in path-integral quantization, the maintenance of unitarity is not as easily achieved as in the canonical formalism. Now we wish to show an explicit example for this purpose. In order to include both the effects of higher dimensions and curvature-squared terms, let us consider the five-dimensional pure gravity with squared terms as[14]

$$\mathcal{L} = -\sqrt{-g}\,(-\delta R + \alpha R^2 + \beta R_{MN}R^{MN} + \gamma R_{MNPQ}R^{MNPQ}) \,, \tag{2.10}$$

where δ is positive constant with dimension of $(\text{mass})^3$ and α,β,γ are constants with dimension of $(\text{mass})^1$. From the Lagrangian we obtain the field equation

$$R^0_{MN} = 0 \tag{2.11}$$

which corresponds to $M_4 \times S^1$ background geometry. To see whether or not this background really exists, we need the small fluctuation analysis. According to the general procedure of stability analysis, we obtain the massive pole terms as

$$I_{\rm massive} \propto \tfrac{1}{4}\left[\frac{1}{p^2+M^2} - \frac{\beta+4\gamma}{(\beta+4\gamma)(p^2+M^2)+\delta}\right] \times |\,T_{ij} - \tfrac{1}{3}\delta_{ij}T_{kk}\,|^2 \,,$$

where $M^2 = (n/2\pi R)^2$ with the radius of extra circle R. We observe from (2.12) that the only $\beta = -4\gamma$ and $\alpha = \gamma$ case avoids the negative-norm state (ghost). Here the relation $\alpha = \gamma$ comes from the Bianchi identity for graviton.

III. EINSTEIN-MAXWELL THEORY

The D-dimensional Einstein-Maxwell theory with cosmological constants λ is

$$-\int d^D x\sqrt{-g}\left[\frac{1}{\kappa^2}R + \tfrac{1}{4}F_{MN}F^{MN} + \lambda\right] \,, \tag{3.1}$$

where R is the curvature scalar with $R_{MN} = R_{PMN}{}^P$, the metric is $(-+++\cdots+)$. The index M is split into $\mu=0,1,2,3$; $m_1=4,5$; $m_2=6.7$; $m_3=8,9$. Here κ^2 is the ten-dimensional gravitational coupling. The field equations for graviton and Maxwell parts are

$$R_{MN} = -\frac{\kappa^2}{2}\left[F_{MP}F_N{}^P + \frac{F^2}{2(2-D)}g_{MN} - \frac{2\lambda}{2-d}g_{MN}\right] \,, \tag{3.2}$$

$$\nabla_M F^{MN}=0 \; . \tag{3.3}$$

In this paper dimensions D is chosen to be 10 and this space is compactified into $M_4\times S^2\times S^2\times S^2$ by giving the following vacuum expectation values:

$$F^0_{m_1n_1}=\frac{n_1}{2ea_1{}^2}\epsilon_{m_1n_1} \; ,$$
$$F^0_{m_2n_2}=\frac{n_2}{2ea_2{}^2}\epsilon_{m_2n_2} \; , \tag{3.4}$$
$$F^0_{m_3n_3}=\frac{n_3}{2ea_3{}^2}\epsilon_{m_3n_3} \; .$$

The lower latin indices starting from the middle with subscripts 1, 2, and 3 refer to the world indices of $S^2\times S^2\times S^2$, respectively. Here n_i and a_i are the monopole charges and radii of each S^2, and e is the gauge coupling constant of U(1) interactions. To give Minkowski-space compactification, the cosmological constant is adjusted as

$$\lambda=\tfrac{1}{4}F^{02}=\tfrac{1}{2}\sum_{i=1}^{3}\left[\frac{n_i}{2ea_i{}^2}\right]^2 . \tag{3.5}$$

The radii of $S^2\times S^2\times S^2$ are related to the monopole charges as

$$a_i{}^2=\kappa^2\frac{n_i{}^2}{8e^2} \; . \tag{3.6}$$

Then the vacuum expectation values of F^0_{AB} can be written in the tangent frame as

$$F^0_{45}=\frac{\sqrt{2}}{\kappa a_1}, \quad F^0_{67}=\frac{\sqrt{2}}{\kappa a_2}, \quad F^0_{89}=\frac{\sqrt{2}}{\kappa a_3} \; . \tag{3.7}$$

The stability of this compactification will be determined by examining the tachyonic modes in the linearized level. The fields are expanded around the background state:

$$g_{MN}=g^0_{MN}+\kappa h_{MN} \; , \tag{3.8}$$

$$A_M=A^0_M+a_M \; , \tag{3.9}$$

where a superscript zero denotes the background values. The backgrounds for the Ricci tensor in tangent frames are read from (3.2) as

$$R^0_{44}=R^0_{44}=-\frac{1}{a_1{}^2}, \quad R^0_{66}=R^0_{77}=-\frac{1}{a_2{}^2} \; ,$$
$$R^0_{88}=R^0_{99}=-\frac{1}{a_3{}^2} \; , \tag{3.10}$$

and other terms vanish. The small fluctuation of R_{MN} is

$$\delta R_{MN}=\kappa(\tfrac{1}{2}\hat{\Delta}h_{MN}+\nabla_{(M}\nabla^Q h_{N)Q}-\tfrac{1}{2}\nabla_{(M}\nabla_{N)}h_Q{}^Q) \tag{3.11}$$

with the Lichnerowicz operator $\hat{\Delta}h_{MN}$:

$$\hat{\Delta}h_{MN}=-\Box h_{MN}-2R^0_{MPNQ}h^{PQ}+2R^0_{(MP}h_{N)}{}^P \; . \tag{3.12}$$

The symmetrization (antisymmetrization) with unit strength is shown by round (square) brackets. The gauge is fixed to eliminate dependent fields corresponding to the invariance of general coordinate and Abelian gauge transformations as

$$\nabla_A h^A{}_B=\tfrac{1}{2}\nabla_B h_A{}^A, \quad \nabla_A a^A=0 \; . \tag{3.13}$$

The linearized equations of (3.2) and (3.3) in tangent frames with external sources T_{AB} and J_A are

$$\Box(h_{AB}-\tfrac{1}{2}\eta_{AB}h_C{}^C)+2R^0_{(AC}h^C{}_{B)}$$
$$+2\kappa F^0_{(BC}f_{A)}{}^C-\tfrac{1}{2}\kappa\eta_{AB}F^{0CD}f_{CD}=-T_{AB} \; , \tag{3.14}$$

$$\nabla^A f_{AB}+\kappa\nabla^A(\tfrac{1}{2}h^C{}_C F^0_{AB}-2h_{[A}{}^C F^0_{CB]}=-J_B \; . \tag{3.15}$$

Here we have introduced external sources to obtain the fluctuation Green's functions. As a result of the Bianchi identity for a graviton, these satisfy the source conservation law:

$$\nabla_A T^{AB}=\kappa F^{0BC}J_C \; , \tag{3.16}$$

$$\nabla_A J^A=0 \; . \tag{3.17}$$

For a ten-dimensional massless graviton, there are 35 independent components.[15] Considering both the ten-dimensional symmetric tensor and the gauge condition in (3.13), we find that there exist 45 components. The remaining ten dependent components can be eliminated from the above conservation law.

In practical calculations, the computation would be rather difficult because of the curved background associated with the three two-spheres. To cope with this it is useful to expand all fields and sources in spherical harmonics on each two-sphere. The symmetry group of the background includes rotations of three two-spheres coupled to appropriate frame rotations and three U(1) gauge transformations. For an explicit example, let us choose the two-sphere as a SU(2)/U(1). The frame rotations are determined by the invariance $e^{\alpha}(y)$, $y=x_4,x_5$, $\alpha=+,-$, and the U(1) transformations by the invariance of $e^3(y)$. A left translation g of the quotient space SU(2)/U(1) (parametrized by the representative elements L_y) induces a U(1) rotation h, such that $gL_y=L_yh$. On writing $h=\exp(\zeta Q_3)$, one finds

$$e^{\pm}(y')=e^{\pm}(y)\exp(\pm i\zeta)\ ,$$
$$e^{3}(y')=e^{3}(y)-d\zeta\ . \qquad (3.18)$$

Invariance of the background under rotation of the two-sphere is obtained if the frames $e^{\pm}$ are rotated through ζ and the Maxwell field A_y^0 undergoes a gauge transformation with the parameter $\Lambda=n_1\zeta/2$ as

$$A_y^0\rightarrow A_y^0+\frac{1}{e}\partial_y\Lambda\ . \qquad (3.19)$$

This is quite enough to give invariance under x-independent left translations. In other words, rotations of the two-sphere must be associated with tangent space rotations in the 4-5 plane in order to preserve the form of the background. Therefore, the fluctuation fields to be expanded $(h_{44},h_{55},h_{45},\ldots)$ must first be rearranged into irreducible pieces of SO(2), the tangent space group of the two-sphere. In general, the tangent space group of D-dimensional space is SO(D). The reason is that the principle of equivalence requires that the special relativity should apply in a locally inertial frame, and in particular, that it should make no difference which locally inertial frame we choose at each point. Note that the background state is invariant under the transformations of Poincaré $\times(\mathrm{SU}(2)\times\mathrm{U}(1))_1\times(\mathrm{SU}(2)\times\mathrm{U}(1))_2\times(\mathrm{SU}(2)\times\mathrm{U}(1))_3$ where the subscripts 1,2,3 denote each two-sphere, respectively.

After the harmonic expansions are performed, the four-dimensional scalars are given by

$$\begin{aligned}
&h_{+1-1}=\tfrac{1}{2}(h_{44}+h_{55}),\quad h_{+2-2}=\tfrac{1}{2}(h_{66}+h_{77})\ ,\\
&h_{+3-3}=\tfrac{1}{2}(h_{88}+h_{99})\ ,\\
&h_{\pm1\pm1}=\tfrac{1}{2}(h_{44}-h_{55}\mp 2ih_{45})\ ,\\
&h_{\pm2\pm2}=\tfrac{1}{2}(h_{66}-h_{77}\mp 2ih_{67})\ ,\\
&h_{\pm3\pm3}=\tfrac{1}{2}(h_{88}-h_{99}\mp 2ih_{89})\ ,\\
&h_{\pm1\pm2}=\tfrac{1}{2}(h_{46}-h_{57}\mp ih_{65}\mp ih_{47})\ .\\
&h_{\pm1\pm3}=\tfrac{1}{2}(h_{48}-h_{59}\mp ih_{58}\mp ih_{49})\ ,\\
&h_{\pm2\pm3}=\tfrac{1}{2}(h_{68}-h_{57}\mp ih_{78}\mp ih_{69})\ ,\\
&h_{\pm1\mp2}=\tfrac{1}{2}(h_{46}+h_{57}\mp ih_{56}\pm ih_{47})\ ,\\
&h_{\pm1\mp3}=\tfrac{1}{2}(h_{48}+h_{59}\mp ih_{58}\pm ih_{49})\ ,\\
&h_{\pm2\mp3}=\tfrac{1}{2}(h_{68}+h_{57}\mp ih_{78}\pm ih_{69})\ ,\\
&a_{\pm1}=\frac{1}{\sqrt{2}}(a_4\mp ia_5),\quad a_{\pm2}=\frac{1}{\sqrt{2}}(a_6\mp ia_7)\ ,\\
&a_{\pm3}=\frac{1}{\sqrt{2}}(a_8\mp ia_9)\ .
\end{aligned} \qquad (3.20)$$

Labeling the isohelicity of each two-sphere as $(\lambda_1,\lambda_2,\lambda_3)$, then h_{-1+2} has $(-1,1,0)$. The four-dimensional graviton h_{ab}, its trace $h_a{}^a$, vector a_a, and the above scalars satisfy the following linearized equations of motion:

$$\left[P^2+\sum_{i=1}^{3}\frac{L_i}{a_i^2}\right]\left[h_a{}^a+4\sum_i h_{+i-i}\right]+4\sum_i\frac{\sqrt{L_i}}{a_i^2}(a_{+i}-a_{-i})=-T_a{}^a\ , \qquad (3.21)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]\left[h^a{}_a+2\left[\sum_i h_{+i-i}-h_{+j-j}\right]\right]-\frac{4}{a_j^2}h_{+j-j}$$
$$-2\left[2\frac{\sqrt{L_j}}{a_j^2}(a_{+j}-a_{-j})-\sum_i\frac{\sqrt{L_i}}{a_i^2}(a_{+i}-a_{-i})\right]=-2T_{+j-j}\ , \qquad (3.22)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]h_{a\pm j}\pm\frac{\sqrt{2}P_a}{a_j}a_{\pm j}\mp\frac{\sqrt{L_j}}{a_j^2}a_a=T_{a\pm j}\ , \qquad (3.23)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}-\frac{2}{a_i^2}\right]h_{\pm j\pm j}=T_{\pm j\pm j}\ , \qquad (3.24)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]h_{\mp j\pm k}\mp\frac{\sqrt{L_k}}{a_k^2}\left[1+\frac{a_k}{a_j}\right]a_{\mp j}\pm\frac{\sqrt{L_j}}{a_j^2}\left[1+\frac{a_j}{a_k}\right]a_{\pm k}=T_{\mp j\pm k}\ , \qquad (3.25)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]h_{\pm j\pm k}\mp\frac{\sqrt{L_k}}{a_k^2}\left[1-\frac{a_k}{a_j}\right]a_{\pm j}\mp\frac{\sqrt{L_j}}{a_j^2}\left[1-\frac{a_j}{a_k}\right]a_{\pm k}=T_{\pm j\pm k}\ , \qquad (3.26)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]a_a-\sum_i\frac{\sqrt{L_i}}{a_i^2}(h_{a+i}-h_{a-i})=J_a\ , \qquad (3.27)$$

$$\left[P^2+\sum_i\frac{L_i}{a_i^2}\right]a_{\pm j}\pm\frac{\sqrt{L_j}}{a_j^2}\left[2h_{\pm j\mp j}-\sum_i h_{\pm i\pm i}\right]\pm\frac{\sqrt{2}P^a}{a_j}h_{a\pm j}\pm\sum_{i\neq j}\frac{\sqrt{L_i}}{a_i^2}\left[-1+\frac{a_i}{a_j}\right]h_{\pm j\pm i}$$
$$\pm\sum_{i\neq j}\frac{\sqrt{L_i}}{a_i^2}\left[1+\frac{a_i}{a_j}\right]h_{\pm j\mp i}\mp\frac{1}{2}\frac{\sqrt{L_j}}{a_j^2}h^a{}_a=J_{\pm j}\ , \qquad (3.28)$$

with $L_1 = l_i(l_i+1)$. Here all indices i,j,k run from 1 to 3. The harmonic expansions of the source conservation laws are

$$\sqrt{2}P^a T_{a\pm j} + \frac{\sqrt{L_j-2}}{a_j} T_{\pm j \pm j} + \frac{\sqrt{L_j}}{a_j} T_{+j-j} + \sum_{i\neq j} \frac{\sqrt{L_j}}{a_j}(T_{\pm j+i}+T_{\pm j-i}) = \pm\frac{2}{a_j} J_{\pm j} , \quad (3.29)$$

$$\sqrt{2}P^b T_{ab} + \sum_i \frac{\sqrt{L_i}}{a_i}(T_{a+i}+T_{a-i}) = 0 , \quad (3.30)$$

$$\sqrt{2}P^a J_a + \sum_i \frac{\sqrt{L_i}}{a_i}(J_{+i}+J_{-i}) = 0 . \quad (3.31)$$

To isolate the true physical modes, we follow the procedure of Refs. 8—11. Solving the fields in terms of sources requires a 29×29 matrix. However a 29×29 matrix is split into 16×16 and 13×13 matrices by the following linear combinations of fields. The former one is composed of $h^a{}_a, h_{+j-j}$, $(a_{+j}-a_{-j})$, $(h_{a+j}-h_{a-j})$, $(h_{-j+k}+h_{+j-k})$ for $j\neq k$, and $(h_{+j+k}+h_{-j-k})$ for $j\neq k$. The latter consists of $(a_{+j}+a_{-j})$, $(h_{a+j}-h_{a-j})$, $(h_{-j+k}-h_{+j-k})$ for $j\neq k$, $(h_{+j+k}-h_{-j-k})$ for $j\neq k$, and a_a. Note that the repeated indices j and k do not mean to sum over j and k. At this stage it seems that the analytic computation for all different l_i and a_i is not feasible. However, as suggested by our previous work,[11] the analytic computations can be carried out for $a_1=a_2=a_3=a$, $l_i=l_j=1$, and $l_k=0$. We choose here $l_1=l_2=1$ and $l_3=0$. Let us first consider a 16×16 matrix. This reduces to 9×9 and 4×4 matrices with three decoupling combinations $(h_{+j+k}+h_{-j-k})$. The 4×4 matrix is composed of $(a_{+3}-a_{-3})$, $(h_{a+3}+h_{a-3})$, $(h_{-1+3}+h_{+1-3})$, and $(h_{-2+3}+h_{+2-3})$. The fields of the 4×4 form are

$$a_{+3}-a_{-3} = S_1 = \frac{1}{(P^2+M_{1+}{}^2)(P^2+M_{1-}{}^2)} \times \left[\left[P^2+\frac{2L}{a^2}\right](J_{+3}-J_{-3}) - \frac{2\sqrt{L}}{a^2}(T_{-2+3}+T_{+2-3}+T_{-1+3}+T_{+1-3}) - \frac{\sqrt{2}}{a}P^a(T_{a+3}+T_{a-3}) \right] , \quad (3.32)$$

$$h_{a+3}+h_{a-3} = \frac{1}{P^2+\frac{2L}{a^2}} \left[T_{a+3}+T_{a-3} - \frac{\sqrt{2}}{a} P_a S_1 \right] , \quad (3.33)$$

$$h_{+1-3}-h_{-1+3} = \frac{1}{P^2+\frac{2L}{a^2}} \left[T_{+1-3}+T_{-1+3} - \frac{2\sqrt{L}}{a^2} S_1 \right] , \quad (3.34)$$

$$h_{+2-3}+h_{-2+3} = \frac{1}{P^2+\frac{2L}{a^2}} \left[T_{+2-3}+T_{-2+3} - \frac{2\sqrt{L}}{a^2} S_1 \right] , \quad (3.35)$$

where

$$M_{1\pm}{}^2 = \frac{2(L\pm\sqrt{2L}\,)}{a^2} . \quad (3.36)$$

The system of the 9×9 matrix is further split into 6×6 and 3×3 matrices by the following combinations. The former is composed of $h^a{}_a$, $(h_{+1-1}+h_{+2-2})$, h_{+3-3}, $(a_{+1}-a_{-1}+a_{+2}-a_{-2})$, $(h_{a+1}+h_{a-1}+h_{a+2}+h_{a-2})$, and $(h_{+1-2}+h_{-1+2})$. The latter consists of $(h_{+1-1}-h_{+2-2})$, $(h_{a+1}+h_{a-1}-h_{a+2}-h_{a-1})$, and $(a_{+1}-a_{-1}-a_{+2}+a_{-2})$. From the fields of the 6×6 matrix, we obtain the relevant physical poles:

$$a_{+1}-a_{-1}+a_{+2}-a_{-2}=\frac{1}{\prod_{i=1}^{4}(P^2+M_{4i}^2)}\left\{\left[2P^2+\frac{1}{a^2}(4L+1)\right]\left[P^2+\frac{2}{a^2}(L+1)\right]\right.$$

$$\times\left[\left[P^2+\frac{2L}{a^2}\right](J_{+1}-J_{-1}+J_{+2}-J_{-2})-\frac{4\sqrt{L}}{a^2}(T_{+1-2}+T_{-1+2})\right.$$

$$\left.-\frac{\sqrt{2}}{a}P^a(T_{a+1}+T_{a-1}+T_{a+2}+T_{a-2})\right]$$

$$-\frac{2\sqrt{L}}{a^2}\left[3P^2+\frac{2}{a^2}(3L+2)\right]\left[P^2+\frac{2L}{a^2}\right](T_{+1-1}+T_{+2-2})$$

$$+\frac{2\sqrt{L}}{a^2}\left[P^2+\frac{2}{a^2}(L-1)\right]\left[P^2+\frac{2L}{a^2}\right]T_{+3-3}$$

$$\left.+\frac{\sqrt{L}}{a^2}\left[P^2+\frac{2L}{a^2}-\frac{2}{a^2}\right]\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]T^a{}_a\right\}, \tag{3.37}$$

where

$$\prod_{i=1}^{4}(P^2+M_{4i}^2)=2\left[P^2+\frac{2L}{a^2}\right]^4+\frac{1}{a^2}\left[P^2+\frac{2L}{a^2}\right]^3-\frac{2(7L+4)}{a^4}\left[P^2+\frac{2L}{a^2}\right]^2-\frac{4(4L+1)}{a^6}\left[P^2+\frac{2L}{a^2}\right]+\frac{8L}{a^8}\quad (L\geq 2)\,. \tag{3.38}$$

One can easily find the negative pole (tachyonic mass) for $L_1=L_2=2$ with an arbitrary value of the radius of S^2. The explicit results are given by

$$M_{4i}^2=-0.41,+3.67,+5.42,+7.83\quad \text{for } a=1\,. \tag{3.39}$$

The residue at this negative pole does not vanish, and hence this pole is not an artifact of gauge. Thus this compactification is unstable. The fields from the 3×3 matrix are

$$a_{+1}-a_{-1}-a_{+2}+a_{-2}=S_2=\frac{1}{(P^2+M_{4+}^2)(P^2+M_{40}^2)(P^2+M_{4-}^2)}$$

$$\times\left[\frac{-4\sqrt{L}}{a^2}\left[P^2+\frac{2L}{a^2}\right](T_{+1-1}-T_{+2-2})\right.$$

$$+\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]\left[P^2+\frac{2L}{a^2}\right](J_{+1}-J_{-1}-J_{+2}+J_{-2})$$

$$\left.-\frac{\sqrt{2}}{a}\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]P^a(T_{a+1}+T_{a-1}-T_{a+2}-T_{a-2})\right], \tag{3.40}$$

$$h_{+1-1}-h_{+2-2}=\frac{1}{\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]}\left[(T_{+1-1}-T_{+2-2})-\frac{2\sqrt{L}}{a^2}S_2\right], \tag{3.41}$$

$$h_{a+1}+h_{a-1}+h_{a+2}+h_{a-2}=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{a+1}+T_{a-1}+T_{a+2}+T_{a-2}-\frac{\sqrt{2}}{a}P_aS_2\right], \tag{3.42}$$

where

$$(P^2+M_{4+}{}^2)(P^2+M_{40}{}^2)(P^2+M_{4-}{}^2)=\left[P^2+\frac{2L}{a^2}\right]^3-4\left[P^2+\frac{2L}{a^2}\right]\frac{L+1}{a^4}+\frac{8L}{a^6}\quad (L\geq 2)\ . \tag{3.43}$$

The fields from the 3×3 matrix are found to be free from the tachyon for $L=2$.

The remaining equations of the 13×13 matrix can be reduced to 6×6 and 4×4 matrices with three decoupling fields $(h_{+j+k}-h_{-j-k})$ for $j\neq k$. The 6×6 matrix is further split into 3×3 and 3×3 forms by combining the fields. The former is given by $(a_{+1}+a_{-1}+a_{+2}+a_{-2})$, $(h_{a+1}-h_{a-1}+h_{a+2}-h_{a-2})$, and a_a. The latter consists of $(a_{+1}+a_{-1}-a_{+2}-a_{-2})$, $(h_{a+1}-h_{a-1}-h_{a+2}+h_{a-2})$, and $(h_{+1-2}-h_{-1+2})$. The explicit forms of field from the former are given by

$$a_{+1}+a_{-1}+a_{+2}+a_{-2}=S_3=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]\left[P^2+\frac{2L-1+(4L+1)^{1/2}}{a^2}\right]\left[P^2+\frac{2L-1-(4L+1)^{1/2}}{a^2}\right]}$$
$$\times\left[-\frac{\sqrt{2}}{a}\left[P^2+\frac{2L}{a^2}\right]P^a\left[T_{a+1}-T_{a-1}+T_{a+2}-T_{a-2}+\frac{4\sqrt{L}}{a^4}J_a\right]\right.$$
$$\left.+(P^2+M_{1+}{}^2)(P^2+M_{1-}{}^2)\times(J_{+1}+J_{-1}+J_{+2}+J_{-2})\right], \tag{3.44}$$

$$S_4=h_{a+}-h_{a-1}+h_{a+2}-h_{a-2}$$
$$=\frac{1}{(P^2+M_{1+}{}^2)(p^2+M_{1-}{}^2)}\left[\left[P^2+\frac{2L}{a^2}\right](T_{a+1}-T_{a-1}+T_{a+2}-T_{a-2})+\frac{4\sqrt{L}}{a^2}J_a-\frac{\sqrt{2}}{a}P_a\left[P^2+\frac{2L}{a^2}\right]S_3\right], \tag{3.45}$$

$$a_a=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[\frac{\sqrt{L}}{a^2}S_4+J_a\right]. \tag{3.46}$$

The latter of the 3×3 matrices are given by

$$a_{+1}+a_{-1}-a_{+2}-a_{-2}=S_5=\frac{1}{(P^2+M_{1+}{}^2)(P^2+M_{1-}{}^2)}$$
$$\times\left[\left[P^2+\frac{2L}{a^2}\right](J_{+1}+J_{-1}-J_{+2}-J_{-2})-\frac{4\sqrt{L}}{a^2}(T_{+1-2}-T_{-1+2})\right.$$
$$\left.-\frac{\sqrt{2}}{a^2}P^a(T_{a+1}-T_{a-1}-T_{a+2}+T_{a-2})\right], \tag{3.47}$$

$$h_{a+1}-h_{a-1}-h_{a+2}+h_{a-2}=S_6=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{a+1}-T_{a-1}-T_{a+2}+T_{a-2}-\frac{\sqrt{2}}{a}P_aS_5\right], \tag{3.48}$$

$$h_{+1-2}-h_{-1+2}=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{-1+2}-T_{+1-2}+\frac{2\sqrt{L}}{a^2}S_6\right]. \tag{3.49}$$

The final 4×4 matrix is composed of $(a_{+3}+a_{-3})$, $(h_{a+3}-h_{a-3})$, $(h_{+1-3}-h_{-1+3})$, and $(h_{+2-3}-h_{-2+3})$. The explicit forms of the fields are

$$a_{+3}-a_{-3}=S_7=\frac{1}{(P^2+M_{1+}{}^2)(P^2+M_{1-}{}^2)}$$
$$\times\left[\left[P^2+\frac{2L}{a^2}\right](J_{+3}+J_{-3})-\frac{2\sqrt{L}}{a^2}(T_{-1+3}-T_{+1-3}+T_{-2+3}-T_{+2-3})\right.$$
$$\left.-\frac{\sqrt{2}}{a}P^a(T_{a+3}-T_{a-3})\right], \tag{3.50}$$

$$h_{a+3}-h_{a-3}=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{a+3}-T_{a-3}-\frac{\sqrt{2}}{a}P_aS_7\right], \tag{3.51}$$

$$h_{+1-3}-h_{-1+3}=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{+1-3}-T_{-1+3}-\frac{2\sqrt{L}}{a^2}S_7\right], \tag{3.52}$$

$$h_{+2-3}-h_{-2+3}=\frac{1}{\left[P^2+\frac{2L}{a^2}\right]}\left[T_{+2-3}-T_{-2+3}-\frac{2\sqrt{L}}{a^2}S_7\right]. \tag{3.53}$$

We note that all these fields are free from the tachyonic modes for $L_1=L_2=2$ and $L_3=0$.

IV. EINSTEIN-MAXWELL-SCALAR SYSTEM

Since the compactification of ten-dimensional Maxwell-Einstein theory into $M_4\times S^2\times S^2\times S^2$ is unstable, one may try to cure this instability by coupling some additional fields to the Maxwell-Einstein system. Most natural candidates motivated by supergravity theories are scalar, two-, and three-form potentials, or their dual fields. In this section, we only consider the case where a scalar field is coupled to the Einstein-Maxwell system as

$$-\int d^{10}x\sqrt{-g}\left[\frac{1}{\kappa^2}R+\partial_M\phi\partial^M\phi+\tfrac{1}{4}e^{\alpha\phi}F_{MN}F^{MN}+\lambda e^{-\alpha\phi}\right]. \tag{4.1}$$

The field equations are

$$R_{MN}=-\partial_M\phi\partial_N\phi-\frac{\kappa^2}{2}e^{\alpha\phi}F_{MP}F_N{}^P+\frac{1}{2\alpha}\Box\phi g_{MN}\ , \tag{4.2}$$

$$\Box\phi=\frac{\alpha\kappa^2}{8}e^{\alpha\phi}F_{MN}F^{MN}-\frac{\alpha\lambda}{2}e^{-\alpha\phi}\ , \tag{4.3}$$

$$\nabla_M(e^{\alpha\phi}F^{MN})=0\ . \tag{4.4}$$

The compactification of ten dimensions into four space-time dimensions with $S^2\times S^2\times S^2$ is achieved by the following vacuum expectation values, in addition to Eq. (3.4):

$$\phi^0=C\ . \tag{4.5}$$

The cosmological constant is adjusted as

$$\lambda=\tfrac{1}{4}e^{\alpha C}F^{02} \tag{4.6}$$

to give Minkowski-space compactification.

The stability of this compactification will be shown by the absence of tachyonic modes in the linearized equations of motion. In addition to Eqs. (3.8) and (3.9) the scalar field is expanded around the background as

$$\phi=C+\varphi\ . \tag{4.7}$$

The linearized equations of (4.2)–(4.4) in tangent frames with external sources T_{AB}, J_A, and J_φ coupled to the linearized fields h_{AB}, a_A, and φ are

$$\Box(h_{AB}-\tfrac{1}{2}h_{AB}h^C{}_C)+2R^0_{(AC}h^C{}_{B)}+2e^{\alpha C}F^0_{(BC}f_{A)}{}^C+2\alpha\varphi R^0_{AB}-\tfrac{1}{2}e^{\alpha C}\eta_{AB}F^{0CD}f_{CD}=-T_{AB}\ , \tag{4.8}$$

$$\Box\varphi-\tfrac{1}{8}\alpha e^{\alpha C}F^0_{AB}f^{AB}-\frac{\alpha}{4}R^0_{AB}h^{AB}+\frac{\alpha}{4}R^0\varphi=-J_\varphi\ , \tag{4.9}$$

$$\nabla^Af_{AB}-\alpha F^0_{BA}\nabla^A\varphi+\nabla^A(\tfrac{1}{2}h^C{}_CF^0_{AB}-2h_{[A}{}^CF^0_{CB]})=-e^{-\alpha C}J_B\ . \tag{4.10}$$

Following the same procedure as in the previous section, the linearized equations of motion are given by

$$\left[P^2+\sum_{i=1}^{3}\frac{L_i}{a_i{}^2}\right]\left[h^a{}_a+4\sum_i h_{+i-i}\right]+4\sum_i\frac{\sqrt{L_i'}}{a_i^2}(a_{+i}-a_{-i})=-T^a{}_a\ , \tag{4.11}$$

$$\left[P^2+\sum_i\frac{L_i}{a_i{}^2}\right]\left[h^a{}_a+2\left[\sum_i h_{+i-i}-h_{+j-j}\right]\right]-\frac{4}{a_j{}^2}h_{+j-j}-\frac{4\alpha}{a_j{}^2}\varphi-2\left[2\frac{\sqrt{L_j'}}{a_j{}^2}(a_{+j}-a_{-j})-\sum_i\frac{\sqrt{L_i'}}{a_i{}^2}(a_{+i}-a_{-i})\right]=-2T_{+j-j}\ , \tag{4.12}$$

$$\left[P^2+\sum_i\frac{L_i}{a_i{}^2}\right]h_{a\pm j}-\frac{(2e^{\alpha C})^{1/2}}{a_j}P_aa_{\pm j}\mp\frac{\sqrt{L_j'}}{a_j{}^2}a_a=T_{a\pm j}\ , \tag{4.13}$$

$$\left[P^2+\sum_i \frac{L_i}{a_i^2}-\frac{2}{a_j^2}\right]h_{\pm j\pm j}=T_{\pm j\pm j}\ , \tag{4.14}$$

$$\left[P^2+\sum_i \frac{L_i}{a_i^2}\right]h_{\mp j\pm k}\mp\frac{\sqrt{L_k'}}{a_k^2}\left[1+\frac{a_k}{a_j}\right]a_{\mp j}\pm\frac{\sqrt{L_j'}}{a_j^2}\left[1+\frac{a_j}{a_k}\right]a_{\pm k}=T_{\pm j\pm k}\ , \tag{4.15}$$

$$\left[P^2+\sum_i \frac{L_i}{a_i^2}\right]h_{\pm j\pm k}\mp\frac{\sqrt{L_k'}}{a_k^2}\left[1-\frac{a_k}{a_j}\right]a_{\pm j}\mp\frac{\sqrt{L_j'}}{a_j^2}\left[1-\frac{a_j}{a_k}\right]a_{\pm k}=T_{\pm j\pm k}\ , \tag{4.16}$$

$$\left[P^2+\sum \frac{L_i}{a_i^2}\right]a_a-\sum_i \frac{\sqrt{L_i}}{a_i^2}(h_{a+i}-h_{a-i})=e^{-\alpha C}J_a\ , \tag{4.17}$$

$$\begin{aligned}\left[P^2+\sum_i \frac{L_i}{a_i^2}\right]a_{\pm j}&\pm\frac{\sqrt{L_j}}{a_j^2(e^{\alpha C})^{1/2}}\left[2h_{\pm j\mp j}-\sum_i h_{\pm i\pm i}\right]\pm\frac{\sqrt{2}P^a}{a_j(e^{\alpha C})^{1/2}}h_{a\pm j}\pm\sum_{i\neq j}\frac{\sqrt{L_i}}{a_i^2(e^{\alpha C})^{1/2}}\left[-1+\frac{a_i}{a_j}\right]h_{\pm j\pm i}\\ &\pm\sum_{i\neq j}\frac{\sqrt{L_i}}{a_i^2(e^{\alpha C})^{1/2}}\left[1+\frac{a_i}{a_j}\right]h_{\pm j\mp i}\mp\frac{\alpha\sqrt{L_j}}{a_j^2(e^{\alpha C})^{1/2}}\varphi\mp\frac{1}{2}\frac{\sqrt{L_j}}{a_j^2(e^{\alpha C})^{1/2}}h_a{}^a=e^{-\alpha C}J_{\pm j}\ ,\end{aligned} \tag{4.18}$$

$$\left[P^2+\sum \frac{L_i}{a_i^2}\right]\varphi-\sum_i \frac{\alpha\sqrt{L_i'}}{4a_i^2}(a_{+i}-a_{-i})-\frac{\alpha}{2}\sum_i \frac{h_{+i-i}}{a_i^2}+\frac{\alpha^2}{2}\sum_i \frac{1}{a_i^2}\varphi=J_\varphi\ , \tag{4.19}$$

with

$$L_i'=e^{\alpha C}l_i(l_i+1)\ .$$

The harmonic expansions of the source conservation laws are the same as in Eqs. (3.29) and (3.31). Solving the fields in terms of sources requires a 30×30 matrix. Using the same combinations of fields in the previous section, the 30×30 matrix is split into 17×17 and 13×13 matrices. To see whether or not this system is stable, it is enough to consider only the relevant 17×17 matrix. The analytic computations are carried out for $a_i=a$, $l_1=l_2=1$, and $l_3=0$. In this case, the relevant form reduces to 10×10 and 4×4 matrices with three decoupling combinations. Furthermore the relevant 10×10 matrix is split into 7×7 and 3×3 matrices by the recombination of fields. From the fields of the 7×7 matrix, the relevant physical poles are

$$\begin{aligned}&a_{+1}-a_{-1}+a_{+2}-a_{-2}\\ &\quad=\frac{1}{\prod_{i=1}^{4}(P^2+M_{4i}'^2)}\\ &\qquad\times\left\{\left[2P^2+\frac{4L+1+3\alpha^2}{a^2}\right]\left[P^2+\frac{2}{a^2}(L+1)\right]\right.\\ &\qquad\times\left[e^{-\alpha C}\left[P^2+\frac{2L}{a^2}\right](J_{+1}-J_{-1}+J_{+2}-J_{-2})\frac{-4\sqrt{L'}}{e^{\alpha C}a^2}(T_{-1+2}+T_{+1-2})\right.\\ &\qquad\qquad\left.-\frac{\sqrt{2}}{(e^{\alpha C})^{1/2}a}P^a(T_{a+1}+T_{a-1}+T_{a+2}+T_{a-2})\right]\\ &\qquad-\frac{2\sqrt{L'}}{e^{\alpha C}a^2}\left[3P^2+\frac{2(3L+2+3\alpha^2)}{a^2}\right]\left[P^2+\frac{2L}{a^2}\right](T_{+1-1}+T_{+2-2})\\ &\qquad+\frac{2\sqrt{L'}}{e^{\alpha C}a^2}\left[P^2+\frac{2}{a^2}(L-1)\right]\left[P^2+\frac{2L}{a^2}\right]T_{+3-3}+\frac{\sqrt{L'}}{e^{\alpha C}}\left[P^2+\frac{2L}{a^2}-\frac{2}{a^2}\right]\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]T^a{}_a\\ &\qquad\left.+\frac{8\alpha\sqrt{L'}}{e^{\alpha C}a^2}\left[P^2+\frac{2L}{a^2}-\frac{2}{a^2}\right]\left[P^2+\frac{2L}{a^2}+\frac{2}{a^2}\right]J_\varphi\right\}\ ,\end{aligned} \tag{4.20}$$

where

$$\prod_{i=1}^{4}(P^2+M_{4i}'^2)^2=2\left[P^2+\frac{2L}{a^2}\right]^4+\frac{1+3\alpha^2}{a^2}\left[P^2+\frac{2L}{a^2}\right]^3-\frac{2(7L+4+\alpha^2)}{a^4}\left[P^2+\frac{2L}{a^2}\right]^2$$
$$-\frac{4(4L+1)+4(4L+3)\alpha^2}{a^b}\left[P^2+\frac{2L}{a^2}\right]+\frac{8L}{a^8}(1-\alpha^2)\ \ (L\geq 2)\ . \tag{4.21}$$

We note that the limit $\alpha\rightarrow 0$ of the above equation recovers the Einstein-Maxwell case in the previous section. We can easily find that there exists an imaginary mass as well as real masses for $l_1=1=l_2$ and $l_3=0$ for the arbitrary real value α of the coupling of the scalar to Maxwell field strength. The residue at this pole does not vanish and thus this pole is not an artifact of the gauge. Therefore, in spite of the presence of the scalar field, this compactification is unstable.

V. DISCUSSIONS

Ten-dimensional Einstein-Maxwell theory with $M_4\times S^2\times S^2\times S^2$ compactification is meaningless in the sense that this system is suffering from the tachyon in physical poles. Thus, we do not need the next step for stability, that is, the existence of a ghost. First of all it is important to cure this tachyonic mode. Natural candidates for this purpose are to couple scalar or two- and three-form potentials to this system in the context of supergravity.[6] From these couplings the most simple case is the Einstein-Maxwell-scalar system. However, from the analysis of Einstein-Maxwell-scalar system, the scalar field does not cure the tachyonic mass of the Einstein-Maxwell system. The fact that the presence of a scalar field does not affect the stability of Einstein-Maxwell theory is well proved in several models.[10,11] Further there is no sense in studying the cosmological implications of Einstein-Maxwell or Einstein-Maxwell-scalar theories with an $M_4\times S^2\times S^2\times S^2$ background configuration, since $M_4\times S^2\times S^2\times S^2$ compactification no longer exists as a solution of these models.

Finally we note that the full analysis of $D=10$, $N=1$ supergravity (the low-energy limit of type-I or heterotic superstring theories) with $M_4\times S^2\times S^2\times S^2$ compactification will remain an open problem.

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