PHYSICAL REVIEW D VOLUME 35, NUMBER 12 15 JUNE 1987

Kaluza-Klein ansatz for quadratic-curvature theory. A geometrical way to mass hierarchy

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(Received 5 March 1987)

Compactification of the N-dimensional gravitational action with $R\cdot R$ terms in a manifold which is a product of homogeneous compact spaces is considered. The total action of the resulting scalar-tensor-vector effective theory is deduced; in particular, the explicit formulas are derived for the gauge coupling constant and the third- and fourth-order noncanonical terms in the Kaluza-Klein gauge field strength. In pure curvature-squared theory "no-scale" double compactification provides a mass scale (m) independent of the Planck mass $(G^{-1/2})$. There is a model where the physical cosmological term is of the order Gm^6.

I. INTRODUCTION

In the Kaluza-Klein type theories the four-dimensional matter gauge fields acquire geometrical interpretation as off-diagonal elements of a higher-dimensional space metric.[1] The dynamics of higher-dimensional gravitation is conventionally described by the Einstein-Hilbert action. Wetterich[2] was the first who generalized it by introducing terms quadratic in the curvature. The action considered in that work was

$$S=l^{-(N-4)}\int (a_1R^{(N)ABCD}R^{(N)}_{ABCD}+a_2R^{(N)AB}R^{(N)}_{AB}+a_3R^{(N)2}+a_4l^{-2}R^{(N)}+a_5l^{-4})(\,|g^{(N)}|\,)^{1/2}d^Nx\ , \quad (1)$$

where l is a length parameter, a_m, $m=1,\ldots,5$, are dimensionless constants, $R^{(N)}_{ABCD}$ is the Riemann tensor for the N-dimensional space; $g^{(N)}$ is a determinant of the metric form. Later on this theory was studied in Ref. 3. The fact that terms quadratic in the curvature (and higher-order terms) arise in the field-theory limit of the anomaly-free superstring theory[4-6] initiated a number of works on the subject (Ref. 7 and references therein). In all those papers, however, including the pioneer work by Wetterich, the N-dimensional metric is supposed to have a block structure, and the dynamics of the Kaluza-Klein gauge fields in the theory with $R\cdot R$ terms was not considered. Some qualitative remarks about the dependence of the gauge coupling constant on the compact space dimension D were given in Ref. 2, but no explicit formulas were derived so far, to the author's knowledge. Moreover, the noncanonical terms in the effective action of third and fourth order in the gauge field strength have not been calculated. A standard excuse was that those terms are too small, of a Planck-length order, as compared with the canonical Yang-Mills action. This is true indeed, if the compact-space radius is comparable to the Planck length, but models are possible where the situation is different (see below). The knowledge of an exact action for the Kaluza-Klein gauge field would enable us to construct spontaneous compactification models with a Kaluza-Klein field of one subspace having nonzero components on the other compact subspace (Secs. IV and V). Besides, it may be useful in studying the stability properties of the models.

In Sec. III the generalized Kaluza-Klein ansatz is reviewed briefly and the Riemann tensor components are calculated in a noncoordinate basis. These formulas are used to derive the effective action resulting from integrating out the "internal" coordinates in the action (1). For the case where the compact space is a D-dimensional sphere, an explicit formula is obtained for the gauge coupling constant g; it turns out that if $a_2=-4a_1$ in (1) (the case where one has no spin-2 tachyons, nor ghosts) g^2 is positive for all $D>2$; g is infinite for $D=2$. In Sec. IV the model of spontaneous compactification $M^7=M^4\times S^2\times S^1$ with a purely geometrical Abelian monopole is also considered.

The case where $a_4=a_5=0$ in (1) is studied particularly; this is the theory without the input Einstein term and the cosmological term in N dimensions. As shown in Sec. V, such a theory leads after compactification to an effective action, which is, in a sense, like the Bose sector of the field limit in the superstring theory. It is shown that such a model may provide a low mass scale (m); the corresponding dynamics being a Jordan-Brans-Dicke type theory with a scalar field potential. For a special compactification this potential is of the order Gm^6 (where G is Newton's constant), that is compatible with observations if m is a hadronic mass. Possible cosmological consequence of the theory are discussed in Sec. VI.

In Appendix A formulas for connection coefficients, Riemann and Ricci tensors' components, and scalar curvature are presented. Appendix B displays terms of the effective Lagrangian.

II. NOTATIONS AND CONDITIONS

(1) The metric signature is $(+--\ldots)$; $R^A_{BCD}=\Gamma^A_{BD,C}-\cdots$; $R_{AB}=R^C_{ACB}$; $\hbar=c=1$; $X^A=(x^\alpha,y^i)$ are the coordinates of the N-dimensional space $M^N=M^n\times C^D$ ($n\geq 4$, C^D is a compact manifold); $A,B,C,\ldots=0,1,\ldots,(N-1)$; $\alpha,\beta,\gamma,\ldots=0,1,\ldots,(n-1)$; $i,j,k,l=1,2,\ldots,D$; $g_{\alpha\beta},g_{ij}$ are inner metrics in M^n and C^D, respectively; $\tilde{C}^D$ is the compact manifold with the unit scale ("radius"); and $\tilde{g}_{ij}$ is its metric.

(2) In Sec. V for the double-compactification model $M^N = M^n \times C^D = M^4 \times K^{n-4} \times C^D$ notations are slightly changed: coordinates of M^n are denoted by carets, i.e., $x^A = (x^{\hat\alpha}, y^i) = (x^\alpha, y^p, y^i)$, where

$$\hat\alpha, \hat\beta, \hat\gamma, \ldots = 0, 1, \ldots, (n-1),$$
$$\alpha, \beta, \gamma = 0, 1, 2, 3, \quad p, q, r = 1, 2, \ldots, (n-4).$$

(3) A comma denotes an ordinary derivative; a semicolon denotes a covariant derivative. For indices $A, B, \ldots$ the total covariant derivatives act in M^N, for $\alpha, \beta, \ldots$ and for $i, j, \ldots$ the covariant derivatives are in M^n and C^D, respectively, constructed by $g_{\alpha\beta}, g_{ij}$, i.e., by the ordinary Christoffel symbols (A1), (A7) of these spaces. $R^{(n)}_{\alpha\beta\gamma\delta}, R^{(n)}_{\alpha\beta}, R^{(n)}, R^{(D)}_{ijkl}, R^{(D)}_{ij}, R^{(D)}$ are the inner Riemann and Ricci tensors and the scalar curvatures in M^n and C^D. The d'Alembertian in M^n is $\Box r = g^{\alpha\beta} r_{;\alpha\beta}$;

$$V = l^{-D} \int (|g^{(D)}|)^{1/2} d^D y$$

is the volume of C^D normalized to l;

$$\langle (\cdots) \rangle = l^{-D} V^{-1} \int (\cdots)(|g^{(D)}|)^{1/2} d^D y$$

denotes the average over the y coordinates in C^D.

(4) a, b, c, d, e, f are the indices of the C^D isometry generators; f^c_{ab} are the structure constants of the corresponding group; ξ^i_a are the Killing vectors in C^D; A^a_α and $F^a_{\alpha\beta}$ are the gauge potential and the gauge field strength. We also have

$$\nabla_\alpha F^a_{\alpha\beta} \equiv F^a_{\alpha\beta;\gamma} + f^a_{bc} F^b_{\alpha\beta} A^c_\gamma,$$
$$\phi^{abcd} \equiv F^{a\beta}_\alpha F^{b\gamma}_\beta F^{c\delta}_\gamma F^{d\alpha}_\delta.$$

(5) The following dimensional convention is adopted. Coordinates X^α have the ordinary dimension of length: $[x^d] = [l]$, whereas y^i are dimensionless angle coordinates: $[y^i] = 1$. Hence $[g_{\alpha\beta}] = 1$, $[g_{ij}] = [l^2]$, $[g^{ij}] = [l^{-2}]$, $[\tilde g_{ij}] = 1$, $[\xi^i_a] = 1$, $[A^i_\alpha] = [A^a_\alpha] = [l^{-1}]$. If a gauge field has nonzero components in a compact manifold, then its potential with lower extra-space index is dimensionless: $[A^a_p] = 1$, $[F^a_{pq}] = 1$, but $[A^{ap}] = [l^{-2}]$, $[F^{apq}] = [l^{-4}]$.

III. CALCULATION OF THE RIEMANN TENSOR COMPONENTS AND DERIVATION OF THE EFFECTIVE ACTION

The first step of generalized Kaluza-Klein theory is to split the coordinates x^A of the N-dimensional space into two groups: $X^A = (x^\alpha, y^i)$ $(x^\alpha \in M^n,\ y^i \in C^D,\ N = n + D)$ and to parametrize metric g_{AB} as

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + A^k_\alpha A^l_\beta g_{kl} & A^k_\alpha g_{ki} \\ A^k_\alpha g_{ki} & g_{ij} \end{bmatrix}. \tag{2}$$

It is well known[1] that all the calculations simplify if metric (2) is transformed to the noncoordinate (nonholonomic) basis:

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} & 0 \\ 0 & g_{ij} \end{bmatrix},$$
$$\frac{\partial}{\partial x^\alpha} \to D_\alpha = \frac{\partial}{\partial x^\alpha} - A^i_\alpha \frac{\partial}{\partial y^i}, \quad \frac{\partial}{\partial y^i} \to \frac{\partial}{\partial y^i}. \tag{3}$$

In the definition (3) of the gauge-covariant derivative D_α the gauge coupling constant is absorbed in A^i_α; the absence of any dimensional constant before A^i_α in (2) and (3) is dictated by the accepted dimensional condition (see Sec. II, item 5). In the basis (3) the nonzero nonholonomic symbols C^M_{AB}, defined by $(D_A, D_B) = C^M_{AB} D_M$, are

$$C^i_{\alpha\beta} = A^i_{\alpha,\beta} - A^i_{\beta,\alpha} + A^j_\alpha A^i_{\beta,j} - A^j_\beta A^i_{\alpha,j} \equiv F^i_{\alpha\beta}, \tag{4a}$$

$$C^j_{\alpha i} = -C^j_{i\alpha} = A^j_{\alpha,i}, \tag{4b}$$

and connection coefficients are calculated by[8]

$$\Gamma^M_{AB} = \tfrac{1}{2} g^{MN}(D_A g_{BN} + D_B g_{AN} - D_N g_{AB} - C^E_{AN} g_{BE} - C^E_{BN} g_{AE}) - \tfrac{1}{2} C^M_{AB}. \tag{5}$$

Results are presented in Appendix A, Eqs. (A1)—(A7).

Parametrization (2) is possible for any N-dimensional Riemannian space if $g_{\alpha\beta}, g_{ij}, A^i_\alpha$ are arbitrary functions of x^A. It was assumed in Eq. (5) that $g_{\alpha\beta}$ is independent of y^i. As usual, we impose further restrictions on g_{AB}: let C^D be a homogeneous manifold with an isometry group and the Killing vectors ξ^i_a satisfying

$$\xi_{ai;j} + \xi_{aj;i} = 0, \tag{6}$$

$$\xi^j_a \xi^i_{b,j} - \xi^j_b \xi^i_{a,j} = f^c_{ab} \xi^i_c, \tag{7}$$

and let A^i_α, g_{ij} have the following representation:

$$A^i_\alpha(x,y) = A^a_\alpha(x) \xi^i_a(y), \tag{8}$$

$$g_{ij}(x,y) = r^2(x) \tilde g_{ij}(y), \tag{9}$$

where r is a "radius" of the compact space depending in general on x^α. Setting (8) into (4a), taking account of (7), gives the familiar result

$$F^i_{\alpha\beta} = \xi^i_a (A^a_{\alpha,\beta} - A^a_{\beta,\alpha} + f^a_{bC} A^b_\alpha A^C_\beta) \equiv \xi^i_a(y) F^a_{\alpha\beta}(x). \tag{10}$$

Components of the N-dimensional Riemann tensor in the noncoordinate basis are given by the general expression[8]

$$R^{(N)A}{}_{BCD} = D_C \Gamma^A_{BD} - D_D \Gamma^A_{BC} + \Gamma^A_{MC} \Gamma^M_{BD} - \Gamma^A_{MD} \Gamma^M_{BC} - \Gamma^A_{BM} C^M_{CD}. \tag{11}$$

In higher-dimensional theories models are considered frequently which involve several compact subspaces. In this case expression (2) for g_{AB} is evidently generalized (nondiagonal components of g_{AB} mixing different compact subspaces are supposed to be zero) and formulas (4) and (A1)—(A7) are valid for each subspace separately. Calculations by (11) were performed for this case [see (A8)]. Expressions for Ricci tensor and the scalar curvature are also given in Appendix A. Expressions for the Riemann tensor components in Kaluza-Klein theory were also obtained in a recent paper;[9] the method and notations used there are different from ours.

To get the effective action in M^n we substitute Eqs. (A8)–(A10) into (1) where the integration with respect to y must be performed. The following averages of the Killing vectors are present in the final result [in Eqs. (12)–(15) indices a,b,c,d are related to the same compact subspace]:

$$h_{ab} \equiv \langle (-\tilde{g}_{ij}\xi^i_a \xi^j_b) \rangle = \frac{D}{W}\delta_{\alpha\beta} , \quad (12)$$

where δ_{ab} is the Kronecker symbol, W is the dimensionality of the isometry group of the C^D space. If $C^D = S^D$ then $W = D(D+1)/2$, so

$$h_{ab} = \frac{2}{D+1}\delta_{ab} . \quad (12a)$$

We have

$$K_{ab} \equiv \langle \tilde{g}_{ij}\tilde{g}^{kl}\xi^i_{a;k}\xi^j_{b;l} \rangle = \langle R^{(D)}_{ij}\xi^i_a \xi^j_b \rangle , \quad (13)$$

K_{ab} is obtained by integration by parts, since

$$\xi^j_{a;ij} - \xi^j_{a;ji} = \xi^j_a R^{(D)}_{ij}$$

for $C^D = S^D$ taking account of (12a)

$$K_{ab} = \frac{2(D-1)}{D+1}\delta_{ab} , \quad (13a)$$

$$\langle \xi^K_a \xi^i_{b;K} \xi^j_c \tilde{g}_{ij} \rangle = -\tfrac{1}{2} f^d_{ab} h_{cd} \quad (14)$$

[(14) is obtained by means of (6) and (7)],

$$E_{ab\,|\,cd} \equiv \langle \tilde{g}_{ij}\xi^i_a \xi^j_b \tilde{g}_{Kl}\xi^K_c \xi^l_d \rangle ,$$
$$E_{a(s)b(s)\,|\,c(t)d(t)} = h_{a(s)b(s)}h_{c(t)d(t)}, \quad s \neq t . \quad (15)$$

[The second equality in (15) is the average of Killing vectors belonging to different compact subspaces; see notations in Appendix A.] A general expression may exist for $E_{ab\,|\,cd}$, but the author has not succeeded in finding it. For a two-dimensional sphere the explicit calculation in the orthogonal basis of the SO(3) group ($f^C_{ab} = \xi_{abc}$) gives

$$E_{ab\,|\,cd} = \tfrac{2}{5}\delta_{ab}\delta_{Cd} + \tfrac{1}{15}(\delta_{aC}\delta_{bd} + \delta_{ad}\delta_{bC}) . \quad (15a)$$

Taking account of (12)–(15) one obtains the desired effective action:

$$S^{(n)} = l^{-(n-4)} \int V(a_1 L_1 + a_2 L_2 + a_3 L_3 + a_4 l^{-2} L_4 + a_5 l^{-4}) \times (|g^{(n)}|)^{1/2} d , \quad (16)$$

where L_1 originates from the $R^{(N)ABCD}R^{(N)}_{ABCD}$ term in (1), etc. It is suitable to split each of these four Lagrangians into three parts, e.g.,

$$L_1 = L_1^{(g,r)} + L_1^{(A)} + L_1^{(A,g,r)} , \quad (17)$$

where $L_1^{(g,r)}$ is the Lagrangian of the scalar-tensor theory; the $L_1^{(A)}$ is the gauge field Lagrangian; the $L_1^{(A,g,r)}$ terms describe the nonminimal interaction of A^a_α with gravity and its interaction with gradients of the scalar fields. These 12 terms are presented in Appendix B.

Terms quadratic in $F^a_{\alpha\beta}$ in (B1b)–(B4b) contribute to the gauge-coupling constant in Minkowski space (for $n=4$). The "nonminimal" terms in (B1c)–(B3c), also quadratic in $F^a_{\alpha\beta}$, "renormalize" the gauge coupling in the presence of a nonstationary cosmological background. Formulas from Appendixes A and B may be useful in different models of spontaneous compactification. They can be applied to more complicated theories with higher-order curvature terms in the primordial action. The only nontechnical difficulty would be to find averages of the products of the Killing vectors.

Let us apply the general formulas to the simplest compactification in the D-dimensional sphere of a constant radius r ($C^D = S^D$). Summarizing (16), (17), and (B1)–(B4), taking account of (12a) and (13a), we get the following action in M^n:

$$S^{(n)} = \frac{1}{l^{n-4}} \int \left[\frac{r}{l}\right]^D \Omega^{(D)} \left[\left[\frac{a_4}{l^2} - \frac{2a_3 D(D-1)}{r^2} \right] R^{(n)} - U(r) + a_1 R^{(n)\alpha\beta\gamma\delta}R^{(n)}_{\alpha\beta\gamma\delta} + a_2 R^{(n)\alpha\beta}R^{(n)}_{\alpha\beta} + a_3 R^{(n)2} + \frac{1}{D+1}\left[(D-1)(6a_1 + a_2 - Da_3) + \frac{a_4 r^2}{2l^2} \right] F^a_{\alpha\beta}F^{a\alpha\beta} + r^2 P + r^4 Q \right] (|g^{(n)}|)^{1/2} d^n x , \quad (18)$$

where

$$\Omega^{(D)} = \frac{2\pi^{(D+1)/2}}{\Gamma((D+1)/2)}$$

is the volume of the D-dimensional unit sphere;

$$U(r) = -\frac{D(D-1)}{r^4}[2a_1 + (D-1)a_2 + D(D-1)a_3] + \frac{D(D-1)a_4}{l^2 r^2} - \frac{a_s}{l^4} \quad (19)$$

is the potential whose extremum gives the vacuum solution; the terms P, Q have dimensions of $[l^{-6}], [l^{-8}]$, respectively, they are noncanonical terms in the Yang-Mills Lagrangian for the gauge group SO($D+1$) and include also a nonminimal interaction with gravity:

$$P = \frac{1}{D+1}[-3a_1 f^d_{ab}\delta_{cd}F^{a\beta}_{\alpha}F^{b\gamma}_{\beta}F^{c\alpha}_{\gamma} - 2a_1(\nabla^\gamma F^{a\alpha\beta})(\nabla_\gamma F^a_{\alpha\beta}) - a_2(\nabla_\gamma F^{a\alpha\gamma})(\nabla_\delta F^{a\delta}_{\alpha}) - a_1 R^{(n)\alpha\beta\gamma\delta}(F^a_{\beta\delta}F^a_{\gamma\alpha} - F^a_{\beta\gamma}F^a_{\delta d} - 2F^a_{\delta\gamma}F^a_{\beta\alpha}) + 2a_2 R^{(n)\alpha\beta}F^a_{\alpha\gamma}F^{a\gamma}_{\beta} + a_3 R^{(n)}F^a_{\alpha\beta}F^{a\alpha\beta}] , \quad (20)$$

$$Q=\tfrac{1}{16}E_{ab\,|\,cd}[(6a_1+a_2)F^a_{\alpha\beta}F^{c\alpha\beta}F^b_{\gamma\delta}F^{d\gamma\delta}+a_3F^a_{\alpha\beta}F^{b\alpha\beta}F^c_{\gamma\delta}F^{d\gamma\delta}+4(2a_1+a_2)\phi^{abcd}+2a_1\phi^{abcd}]\ . \tag{21}$$

For $D=2$, $E_{ab\,|\,cd}$ in (21) is given by (15a); for ϕ^{abcd}, see Sec. II, item 4. For the Abelian reduction ($D=1, E_{11\,|\,11}=1$) the expressions (18), (20), and (21) are the direct generalization of the classical Kaluza-Klein result to the theory with terms quadratic in the curvature.

We shall not dwell here on stability properties or implications of the Maxwell or Yang-Mills theory given by (18). But a relevant remark may be of some interest. Let us consider the generalized Maxwell theory with the Lagrangian

$$L=-\tfrac{1}{4}F_{\alpha\beta}F^{\alpha\beta}+p(F_{\alpha\beta}F^{\alpha\beta})^2+q(F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\delta F_\delta{}^\alpha)\ ,$$

which is equivalent to (18) for constant $F_{\alpha\beta}$ [setting in (18) $D=1$, $n=4$, $g_{\alpha\beta}=(+1,-1,-1,-1)$]. The energy density of this theory is

$$\rho=\tfrac{1}{2}(\mathbf{E}^2+\mathbf{H}^2)+2(2p+q)[3(\mathbf{E}^2)^2-2\mathbf{E}^2\mathbf{H}^2-(\mathbf{H}^2)^2]+4q(\mathbf{EH})^2\ ;$$

E,**H** are the electric and magnetic field strengths. This can be positive definite only if $2p+q=0$. For the action of (18) this condition reads $11a_1+3a_2+a_3=0$, which is valid in particular for the Gauss-Bonnet form in (1) (namely, for $a_2=-4a_1$, $a_3=a_1$).

IV. SOME MODELS OF SPONTANEOUS COMPACTIFICATION

(1) $C^D=S^D$. Extremum of the potential (19) gives Wetterich's[2] vacuum solution with

$$\bar r=l\{2[2a_1+(D-1)a_2+D(D-1)a_3]/a_4\}^{1/2}\ .$$

Setting in (18) $n=4$ and comparing (18) with the standard form of the four-dimensional action

$$S^{(4)}=\int\left[-\frac{1}{16\pi G}R^{(4)}-\frac{1}{4g^2}F^a_{\alpha\beta}F^{a\alpha\beta}+\cdots\right]\times(\,|g^{(4)}|\,)^{1/2}d^4x\ ,$$

expressions for the Newtonian and gauge coupling constants are derived:

$$1/16\pi G=-(2\bar V/\bar r^2)[2a_1+(D-1)a_2]\ , \tag{22}$$

$$1/g^2=-8\bar V[(3D-2)a_1+(D-1)a_2]/(D+1)\ , \tag{23}$$

where $\bar V=(\bar r/l)^D\Omega^{(D)}$. Equality (23) is the result of the present paper. Conditions $U>0$ for variations of $(r-\bar r)$, and $\bar r^2>0$, $G>0$ are satisfied if[2]

$$\begin{aligned}&2a_1+(D-1)a_2<0\ ,\\&2a_1+(D-1)a_2+D(D-1)a_3<0\ ,\\&a_4<0\ .\end{aligned} \tag{24}$$

Besides, we will add an equality

$$a_2=-4a_1 \tag{25}$$

which guarantees the absence of spin-2 ghosts and tachyons in the four-space.[10] Equality (25) is accepted in the superstring theory,[11] but in contrast to Ref. 11 we shall not demand $a_3=a_1$ and leave a_3 as a free parameter (cf. Ref. 12). In Ref. 2 restrictions $a_1<0$, $4a_1+(D-1)a_2<0$ were also adopted [besides (24)], which are necessary for stability of the vacuum $M^N=M^4\times S^D$ against nonconformal variations of the "inner" metric g_{ij}. These latter inequalities are, however, in the evident contradiction with (24) and (25), so we shall discard them. This discrepancy means, probably, that (1) should be supplied with terms of higher order in the curvature.

If $a_2=-4a_1$ then (22) and (23) give

$$\begin{aligned}&1/16\pi G=4\bar V a_1(2D-3)/\bar r^2\ ,\\&g^2=\frac{8\pi G}{\bar r^2}\frac{(D+1)(2D-3)}{D-2}\ .\end{aligned} \tag{26}$$

It can be shown that in the more general case of compactification into the product of several spheres, the gauge coupling constants (for $a_2=-4a_1$) are also real and the "confinement" situation ($1/g^2=0$) is realized for two-dimensional spheres independently of a_3.

(2) $C^D=S^D$, $a_4=a_5=0$ [the case of pure quadratic-curvature action (1)]. The absence of the original Einstein-Hilbert term means that the Newtonian constant is completely induced by the compactification and stems from the $a_3R^{(N)2}$ term in (1); positivity of G implies $a_3>0$. (It would be interesting to find out, if there is a conceptual relation between this "induced" gravity and, pioneered by Sakharov,[13] pregeometry, where dynamics of gravitational field is generated by quantum fluctuations).

For $a_4=a_5=0$ compactification in S^D is possible if

$$2a_1+(D-1)a_2+D(D-1)a_3=0\ ,$$

so $U\equiv 0$ [see (19)], and the radius r is not determined by the vacuum equations. For $a_2=-4a_1$ the existence of the solution requires a fine tuning:

$$\frac{a_1}{a_3}=\frac{D(D-1)}{2(2D-3)},\quad \frac{a_2}{a_3}=-\frac{2D(D-1)}{2D-3} \tag{27}$$

($a_1>0$, $a_2<0$, $a_3>0$). Equations (26) for G,g are still valid in this case with the change $\bar r\to r$, where r is a free quantity, the scalar zero mode.

(3) $N=7$, $M^7=M^4\times S^2\times S^1$ with the Abelian magnetic monopole on S^2. This example is given to illustrate the operation of Eqs. (18), (20), and (21) to the compactification. In Ref. 14 the six-dimensional Einstein-Maxwell theory was compactified into $M^6=M^4\times S^2$ with the Abelian field having the monopole configuration on S^2. But the introduction of an external gauge field is alien to the Kaluza-Klein mentality. It would be natural to take $M^7=M^4\times S^2\times S^1$ and to consider the Abelian field A_α as the Kaluza-Klein field of S^1. In this case, however, the dynamical equations of the conventional (linear in R) theory have no solution like that in Ref. 14. The reason is the appearance of a new dynamical quantity (radius r_1 of

S^1) and an additional equation for it. We shall show that the theory (1) does have such a solution with a pure geometrical monopole.

The first step is the Abelian reduction from 7 to 6 dimensions which results in the action (18) with $D=1$, $n=6$, $r\to r_1$. The second step is to find a vacuum solution for this rather complicated six-dimensional theory. Following Ref. 14, we seek the solution $M^6=M^4\times S^2$ with an S^2 radius r_2, as well as r_1, being constant, M^4 the Minkowski space, $A_\alpha=0$ for $\alpha=0,1,2,3$, whereas on S^2 one has

$$A_\varphi=\frac{K}{2}(\cos\theta\pm1) \qquad (28)$$

(θ,φ are the angle coordinates on S^2; $K=\pm1,\pm2,\ldots$). In contrast to Ref. 14 no dimensional or dimensionless coupling constants are present as factors in the right-hand side (RHS) of (28), because of the adopted condition of dimensionality (Sec. II, item 5) and the normalization of the gauge field in the gauge-covariant derivative D_α in (3) [A_φ is the nondiagonal element of metric (2) coupling the subspaces $\widetilde{S}^2$ and $\widetilde{S}^1$, having the unit radii]. Substitution of this vacuum structure in (18), (20), and (21), taking account of

$$F_{\alpha\beta}F^{\alpha\beta}=K^2/2r_2^4,\quad F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\delta F_\delta{}^\alpha=K^4/8r_2^8\ ,$$

$$R^{\alpha\beta\gamma\delta}(F_{\beta\delta}F_{\gamma\alpha}-F_{\beta\gamma}F_{\delta\alpha}-2F_{\delta\gamma}F_{\beta\alpha})=3K^2/r_2^6\ ,$$

etc., gives the potential [which is defined by analogy to those introduced in (18)]

$$U=-(a_1+\tfrac{1}{2}a_2+a_3)z_1^2-(11a_1+3a_2+a_3)z_2^2 +2(3a_1+a_2+a_3)z_1z_2+\frac{a_4}{l^2}(z_1-z_2)-\frac{a_5}{l^4}\ . \qquad (29)$$

U is a quadratic form in variables $z_1=2/r_2^2$, $z_2=K^2r_1^2/8r_2^4$. The solution of equations $\partial U/\partial z_1=\partial U/\partial z_2=0$ yields vacuum values of r_1,r_2; the fine tuning of a_5 provides $U=0$ at its extremum, that is zero cosmological term in M^4.

We will not explore this solution, yet note that if $a_2=-4a_1$ [see (25)] then (29) takes the form

$$U=(a_1-a_3)(z_1-z_2)^2+(a_4/l^2)(z_1-z_2)-a_5/l^4\ , \qquad (30)$$

and the vacuum solution defines only (z_1-z_2), leaving one of the radii r_1,r_2 to be a free quantity.

If $a_2=-4a_1$, $a_3=a_1$, the quadratic part of (29) is zero, because the Gauss-Bonnet combination is identically zero, in general, on any three-dimensional space. (In three dimensions the Riemann tensor can always be expressed in terms of the Ricci tensor and the scalar curvature[15].)

(4) $M^N=M^4\times S^d\times S^2\times S^1$ is a generalization of the previous model to S^d with a radius r_d. For the special case $a_4=a_5=0$, $a_2=-4a_1$, the potential is now given by [cf. (30) for $a_4=a_5=0$]

$$U=\frac{4d(d-1)a_1}{d^2+3d-6}\left[\frac{2d-3}{r_d^2}-\frac{1}{r_2^2}+\frac{K^2r_1^2}{16r_2^4}\right]^2; \qquad (31)$$

the fine tuning,

$$a_3=2(2d-3)a_1/(d^2+3d-6)\ ,$$

and the stability condition $a_1>0$ are necessary. The extremum of this potential sets an only relation for three radii r_1,r_2,r_d. This freedom in scalar fields permits us (by taking $r_1\ll r_2\approx r_d$) to get the desired value (≈0.01) for the fine-structure constant of the Abelian field, together with a mass scale m independent of the Planck mass $G^{-1/2}(m\ll G^{-1/2})$. It is not difficult to show that in this model the contribution from the $(F)^3,(F)^4$ terms of the Abelian field in the effective four-dimensional action is characterized by the length m^{-1}, so it is not small. It appears, however, that the Kaluza-Klein fields of S^2 and S^d have superweak coupling ($g^2\approx Gm^2$), and it is hardly reasonable to have a single Abelian field with the normal coupling. In the next section another model with free-compactification radii is considered.

V. DOUBLE COMPACTIFICATION AND A MASS SCALE HIERARCHY

Two basic energy scales of nature, gravity GUT and electroweak hadronic, differ by many orders of magnitude. The reason for such a hierarchy is not known; hopes to understand it are related as a rule to quantum radiative corrections and dynamical symmetry (supersymmetry) breaking,[16–18] where the low-mass scale m may be calculated, in principle, by $m\approx G^{-1/2}\exp(-1/g^2)$, which is like the expression for the energy gap in superconductivity. Another way is Dirac's idea[19] to relate this hierarchy to the age of the Universe, implementing thus the large-numbers hypothesis (see, e.g., Ref. 20). In the higher-dimensional theory the problem of obtaining the low-mass scale is not yet solved. It is not difficult to obtain masses for the Kaluza-Klein fields but they are of the order of the reciprocal compactification radius, which in turn defines the gauge coupling [see, e.g., (26)]; an experimentally acceptable value of the latter entails the Planckian mass scale.

However, a direct relation between the coupling constants and masses may be violated if there are two (or more) scalar zero modes in the vacuum solution. If the ten-dimensional field limit of the superstring theory is compactified into the six-dimensional Ricci-flat Calabi-Yau manifold, these two scalar zero modes are a dilaton field $\varphi(x^\alpha)$ and the scale $b(x^\alpha)$ of the Calabi-Yau space.[6] In the present case, the same situation arises if a double compactification is assumed in pure curvature-squared theory. The first step may be the compactification into S^D (of radius r) considered in Sec. IV, item 2; the second step is the same as in the theory of superstrings. Ricci flatness at the second step (with a scale b) is necessary to make two scales r and b independent. The physical gauge coupling constant is a product of certain powers of b and r, and its observable value of order unity might be compatible with the strong inequality $b\gg G^{1/2}$.

But this inequality is apparently impossible if (as it is assumed usually) the Ricci-flat compactification is related to a breakdown of the grand unification symmetry: the X-meson mass is then too small and the proton lifetime too short.[21,22] We adopt a quite different interpretation: the gauge symmetry breakings which happen at the second step of compactification are all related to the low-energy scale (electroweak, flavor, etc.). In this case the GUT symmetry breaking should take place at the first step of compactification. The result would be sub-Planckian mass of the X meson. The toy model of compactification in S^D does not simulate these effects. Our purpose is rather modest, to elucidate by the simple example the appearance of the scalar-tensor gravity with two scalar fields [low-mass scale $m(x^\alpha)$ and vacuum permittivity $\epsilon(x^\alpha)\equiv 1/g^2$] in four dimensions, and to calculate the interaction constants of this Jordan-Brans-Dicke[23] (JBD) type theory.

A. First step of compactification

The result is the action (18) (for $a_4=a_5=0$, $a_2=-4a_{\Gamma}$), where the kinetic energy terms of the field $r(x^{\hat\alpha})$ should be included. (Now the n-dimensional space-time indices are circumflexed, Sec. II, item 2.) These terms are calculated by formulas (B1a), (B2a), (B3a), and (B4a). Below only the leading terms, quadratic in $r_{,\hat\alpha}$, are retained. After a scale transformation of $g_{\hat\alpha\hat\beta}$, leading to an r independence of the gravitational constant in $S^{(n)}$, and taking account of (27) the effective action will become

$$S^{(n)}=\int\left\{-\frac{1}{2\hat l^{\,n-2}}R^{(n)}+\frac{\Delta}{\hat l^{\,n-2}}\varphi^{,\hat\alpha}\varphi_{,\hat\alpha}-\frac{e^{-\varphi}}{4\hat l^{\,n-4}}\left[F^a_{\hat\alpha\hat\beta}F^{a\hat\alpha\hat\beta}-\frac{D+1}{2(D-2)}\left[R^{(n)\hat\alpha\hat\beta\hat\gamma\hat\delta}R^{(n)}_{\hat\alpha\hat\beta\hat\gamma\hat\delta}-4R^{(n)\hat\alpha\hat\beta}R^{(n)}_{\hat\alpha\hat\beta}+\frac{2(2D-3)}{D(D-1)}R^{(n)2}\right]\right]+\frac{e^{-2\varphi}}{\hat l^{\,n-6}}\frac{(D+1)^2(2D-3)^2}{4D(D-1)(D-2)^2}\frac{\hat P}{a_3}+\frac{e^{-3\varphi}}{\hat l^{\,n-8}}\frac{(D+1)^3(2D-3)^3}{4D(D-1)(D-2)^3}\frac{\hat Q}{a_3}+\cdots\right\}(|g^{(n)}|)^{1/2}d^nx\ , \tag{32}$$

where

$$e^{-\varphi}=\text{const}\times r^{2(n+D-4)/(n-2)}\ , \tag{33}$$

$$\Delta=\frac{(n-2)(D-2)(2D^2-Dn-5D+6n-6)}{8(n+D-4)^2(2D-3)}\ ; \tag{34}$$

$\hat P,\hat Q$—see (20) and (21) with $\alpha,\beta,\ldots\to\hat\alpha,\hat\beta,\ldots$; an arbitrary length l in (1) transmutated to the arbitrary length $\hat l$ in (32). Parametrization (33) of the scalar field is selected for better comparison of (32) with the field action in superstring theory, where $n=10$, $\Delta=1$, and instead of the $\hat P,\hat Q$ terms in (32) the Lagrangian $e^{-2\varphi}H^{\hat\alpha\hat\beta\hat\gamma}H_{\hat\alpha\hat\beta\hat\gamma}$ of the third-rank totally antisymmetric tensor is present. The constant in (33) is a function of $D,r,a_3,\Omega^{(D)},l$ but its value is irrelevant.

The action in (32) describes dynamics of zero modes at the first step of compactification; one can neglect higher modes (which are technically obtained by averaging over the S^D coordinates) if all scales of M^r are much larger than the radius of S^D, which will be supposed below.

B. Second step of compactification

$M^n=M^4\times K^{n-4}$, where K^{n-4} is a Ricci-flat compact manifold; its coordinates are y^p, i.e., $x^{\hat\alpha}=(x^\alpha,y^p)$. We follow Ref. 6 and assume that its scale b and the scalar field φ are independent of y^p, but $A^a_{\hat\alpha}$ has nonzero components on K^{n-4} for certain a. Now the vacuum expectation value of $(F)^2$ (and $R\cdot R$), $\hat P,\hat Q$ terms in (32) are proportional to b^{-4},b^{-6},b^{-8}, respectively. (In superstring theory it may be nonzero $H^{pqr}H_{pqr}\sim b^{-6}$.)

In the physical action the mass terms of the physical gauge fields A^a_α arise from the mixed components of $F^a_{\hat\alpha\hat\beta}$: $F^a_{\alpha p}=f^a_{bc}A^b_\alpha A^c_p$. [Here we took into consideration that $A^a_{\alpha,p}=A^a_{p,\alpha}=0$, A^a_p with subscript are dimensionless calculable numbers, cf. Sec. II, item 5 and Eq. (28).] In fact, averaging $(F)^2$ over K^{n-4} one has

$$\langle F^a_{\hat\alpha\hat\beta}F^{a\hat\alpha\hat\beta}\rangle_K=F^a_{\alpha\beta}F^{a\alpha\beta}+\frac{2}{b^2}\mu_{ac}A^a_\alpha A^{c\alpha}+\frac{1}{b^4}\langle\tilde g^{pp'}\tilde g^{qq'}F^a_{pq}F^a_{p'q'}\rangle_K\ , \tag{35}$$

where $\mu_{ac}=f^d_{ab}f^d_{ce}\langle A^b_pA^e_q\tilde g^{pq}\rangle_K$ are dimensionless mass matrix elements. There are also corrections to the mass matrix from the $\hat P,\hat Q$ terms of the action in (32), yet as those corrections are proportional to b^{-4},b^{-6}, respectively, they are negligible as compared with the b^{-2} term in (35), since $b\gg r$.

In the following we discard the Kaluza-Klein fields of K^{r-4}: it is not difficult to show that their gauge coupling is superweak [proportional to $(Gm^2)^{1/2}$]. Thus to get the physical action in four dimensions from (32) one has to use only Eqs. (B1a)–(B4a). Certain cosmological consequences of the Calabi-Yau compactification have been studied in Ref. 24 taking account of all the $b(x^\alpha)$ gradient terms. We shall retain only the canonical kinetic energy term, quadratic in $b_{,\alpha}$. One can discard higher degrees of $r_{,\alpha}$ in (32) and $b_{,\alpha}$ in (37) if the scale of M^4 is larger than r,b. Thus the results of this section are valid provided that one has

$$\frac{r_{,\alpha}}{r},\frac{b_{,\alpha}}{b}\ll\frac{1}{b}\ll\frac{1}{r}\ . \tag{36}$$

Collecting all said above one gets the desired four-dimensional action

$$S^{(4)}=\int\left[\frac{b}{\hat{l}}\right]^{n-4}\Omega^{(\tilde{K})}\left[-\frac{1}{2\hat{l}^2}R^{(4)}-\frac{(n-4)(n-5)}{2\hat{l}^2}\frac{b^{,\alpha}b_{,\alpha}}{b^2}+\frac{\Delta}{\hat{l}^2}\varphi^{,\alpha}\varphi_{,\alpha}-\frac{e^{-\varphi}}{4}\left[F^a_{\alpha\beta}F^{a\alpha\beta}+\frac{2}{b^2}\mu_{ac}A^a_\alpha A^{c\alpha}\right]-e^{-\varphi}\frac{\lambda_1}{b^4}-\hat{l}^2e^{-2\varphi}\frac{\lambda_2}{b^6}-\hat{l}^4e^{-3\varphi}\frac{\lambda_3}{b^8}+\cdots\right](|g^{(4)}|)^{1/2}d^4x\ . \tag{37}$$

Only the canonical Lagrangian of gravitational, Yang-Mills, and scalar fields are written down in (37); $\Omega^{(\tilde{K})}$ is the volume of the Ricci-flat space $\tilde{K}^{n-4}$ of the unit scale; $\lambda_{1,2,3}$ are calculable dimensionless constants. In the method of compactification of Ref. 6, λ_1 is equal to zero, since the vacuum values of terms in (32) quadratic in $R^{(n)}_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}$ and $F^a_{\hat{\alpha}\hat{\beta}}$ cancel.

Formally the action in (37) (if $\varphi=\text{const}$ and $A^a_\alpha=0$) leads to the scalar-tensor gravitation with the Brans-Dicke field proportional to b^{r-4}, and the standard constant of the JBD theory $\omega=-(n-5)/(n-4)$. For superstrings $(n=10)$ one gets $\omega=-\frac{5}{6}$ (Ref. 24). But this result of Ref. 24 is wrong for two reasons. First, the gauge coupling in (37) is proportional to $b^{n-4}e^{-\varphi}$, not $e^{-\varphi}$, and it is constant if $e^{\varphi}\sim b^{n-4}$; hence $\varphi^{,\alpha}\varphi_{,\alpha}$ in (37) contributes to the kinetic energy of the Brans-Dicke field. Second, it is rather senseless to deal with value of ω before masses of elementary particles (constituents of observable heavy bodies) are defined. In the action (37) the coefficient at $R^{(4)}$ and the gauge field mass (which is responsible for the low-energy scale) are both depending on the scalar field and to compare (37) with JBD theory the scale transformation to scale gauges $m=\text{const}$ or $G=\text{const}$ must be performed. Fixing the latter, i.e., performing in (37) the scale transformation $g_{\alpha\beta}\to\eta^2 g_{\alpha\beta}$, with

$$\eta^2=2\hat{l}^{n-2}(16\pi G\Omega^{(\tilde{K})}b^{n-4})^{-1}\ , \tag{38}$$

and setting

$$\frac{1}{g^2}=\Omega^{(\tilde{K})}e^{-\varphi}\left[\frac{b}{\hat{l}}\right]^{n-4}\equiv\epsilon,\quad \frac{\eta}{b}\equiv m\ , \tag{39}$$

one obtains finally

$$S^{(4)}=\int\left\{-\frac{1}{16\pi G}R^{(4)}+\frac{1}{16\pi G}\left[\frac{2(n-4)}{n-2}\left[1+\frac{4(n-4)\Delta}{n-2}\right]\frac{m^{,\alpha}m_{,\alpha}}{m^2}+\frac{8(n-4)\Delta}{n-2}\frac{m^{,\alpha}\epsilon_{,\alpha}}{m\epsilon}+2\Delta\frac{\epsilon^{,\alpha}\epsilon_{,\alpha}}{\epsilon^2}\right]+\epsilon(-\tfrac{1}{4}F^a_{\alpha\beta}F^{a\alpha\beta}-\tfrac{1}{2}m^2\mu_{ac}A^a_\alpha A^{c\alpha})-\Lambda(m,\epsilon)\right\}(|g^{(4)}|)^{1/2}d^4x\ , \tag{40}$$

where

$$\Lambda(m,\epsilon)=\lambda_1\epsilon m^4+\lambda_2\epsilon^2 8\pi G m^6+\lambda_3\epsilon^3(8\pi G)^2m^8 \tag{41}$$

is the scalar fields' potential, i.e., the physical cosmological term. If $\epsilon=\text{const}$ and $A^a_\alpha=0$, then (40) is the JBD action with the potential of (41), and the standard interaction constant is

$$\omega=\frac{2(n-4)^2}{(n-2)^2}\Delta-\frac{n-1}{n-2}\ . \tag{42}$$

[In the JBD theory, written in the scale gauge $G=\text{const}$, the scalar field kinetic term is $(4\omega+6)m^{,\alpha}m_{,\alpha}/16\pi Gm^2$.] For superstrings $(\Delta=1,\ n=10)$ (42) gives $\omega=0$. If the JBD theory is applied to the modern stage of the history of the Universe it may be compatible with observations only if $|\omega|$ is very large [$|\omega|\gtrsim500$ (Ref. 25)]. The present model admits any ω. In fact, Eqs. (34) and (42) yield

$$\Delta=(n-2)/8,\quad \omega=(n-10)/4\quad \text{for } D\gg n\ ,$$

$$\Delta=-(D-2)(D-6)/8(2D-3)\ ,$$

$$\omega=-D^2/4(2D-3)\quad \text{for } n\gg D\ .$$

Clearly, arbitrary large $|\omega|$ can be obtained for sufficiently large D,n, but this is a rather artificial way of getting on with experiment. Perhaps some other model of compactification will be more natural, but perhaps a theory of the type (40) should be applied only to initial stages of cosmological scenario (see Sec. VI).

What is the mass of the external fields in the double-compactification approach? Unlike the toy $C^D=S^D$ model presented here, a realistic theory should provide chiral fermions on C^D. The Fermi field which is the zero mode of the first step of compactification, but not that for the second Ricci-flat step, respects the Dirac equation $(\gamma^\alpha D_\alpha-\mu/b)\psi=0$ [in the same scale gauge as in (37)], where D_α is given in (3) and μ is a calculable constant. After the scale transformation (38) the fermion mass is $\mu\eta/b$, i.e., has the order of the low-mass scale m [see (39)]; the gauge coupling of the fermion field is $g=\epsilon^{-1/2}$, where ϵ is the scalar field in (40).

The cost of achieving the independent energy scales in the Kaluza-Klein theory is rather heavy: two main characteristics of elementary particles, gauge coupling constant, and mass, turn out to be dynamical fields. If the fine-structure constant were a field of this kind the equivalence principle would be violated.[26] It is not yet clear how to fix the desired value of the gauge coupling constant in "no-scale" theories, and in particular in (40); in superstring theory this problem is connected with the problem of fixation of the dilaton field vacuum condensate.[27,28] In discussing the possible cosmological consequences of the theory we just set $\epsilon=\text{const}$ in the action (40).

VI. DISCUSSION

As was mentioned above, for the Ricci-flat compactification of Ref. 6, $\lambda_1=0$ in the action (37), and the vacuum energy (41) is of the order Gm^6 or less. In Ref. 29 the JBD theory with potential Gm^6 was studied and the Dirac cosmological theory[19] was shown to be one of its solutions (this is not the case for classical general relativity and for the classical JBD theory). In Ref. 29 the scale gauge $m=\text{const}, G(t)$ was employed. In the alternative scale gauge of the present paper $[G=\text{const}, m(t)]$ Dirac's cosmology with dust matter gives

$$m \approx (H/G)^{1/3} \sim t^{-1/3}, \quad a \sim t^{5/9}, \quad \rho \sim t^{-2}, \tag{43}$$

where H is Hubble's "constant," a is the scale factor of the Universe, ρ is the energy density, and t the proper cosmological time. The substitution of $H=0.5\times 10^{-10}$ yr into (43) yields $m \approx 1$ GeV, the hadronic mass. The possibility of a cosmological origin of the hierarchy was discussed also in context of supersymmetric theories with flat directions (valleys) of potential at its minimum.[17] The serious drawback of all this approach is that Dirac's theory is hardly compatible with observations.[8,30,31]

A widespread opinion should be accepted that at present the fundamental constants do not vary with time, the low-mass scale is fixed by some quantum radiative corrections, while the coincidence of large numbers in microphysics and cosmology may be explained by a "weak anthropological principle" (Ref. 20, Chap. 5). Meanwhile, the hierarchy problem is by no means reduced to the large-numbers problem. The scalar-tensor theory of the type (40) may be applied to the initial stages of the Universe. Recently theory with the action

$$S=\int \left[-\frac{1}{16\pi G}R^{(4)}+\frac{(2\omega+3)}{8\pi G}\frac{m^{,\alpha}m_{,\alpha}}{m^2} - \text{const}\times m^q\right](|g^{(4)}|)^{1/2}d^4x \tag{44}$$

was considered in quite a different context to get the power-law inflation[32] (PLI). In fact, this theory has a vacuum cosmological regime

$$a \sim t^{4(2\omega+3)/q}, \quad m \sim t^{-2/q} \tag{45}$$

(for the four-dimensional metric $ds^2=dt^2-a^2d\mathbf{x}^2$), which for sufficiently big $\omega>0$ provides the solution of flatness, horizon, and other problems. But in the papers of PLI the inflaton scalar field has nothing to do with the elementary particle mass scale. If, as it is proposed in the present paper, these two are identified, then for $q=6$, (45) leads to a time period of 10^{10} yr for m to attain the observed hadronic level $[m(t)$ is the same as in nonvacuum Dirac theory (43)], whereas for $q=4$ this time period is 10^{-3} sec.

Let us assume that $q=4$ in (44), i.e., $\lambda_1\neq 0$ in (37) and (41). Then the following big bang scenario may be supposed. All the symmetries (GUT, super, electroweak, discrete, etc.) are broken at the very start with approximately equal strength; this breakdown is correlated with (or is provoked by) the compactification of N-dimensional space at once to four dimensions: $M^N \to M^4\times K^{n-4}\times C^D$. The initial value of the physical cosmological term is Planckian $[m\approx G^{-1/2}$ in (41)]. In the following, three scales (of our four-dimensional Universe, low-mass scale of elementary particles, and Planckian mass) smoothly diverge. This divergence may be caused by the different inner geometry of the respective subspaces. When inequalities (36) become valid cosmological evolution is described by the theory of the type (44) and includes the PLI stage. [Presence of matter surely changes the vacuum regime of expansion (45); we do not touch here the entropy production, generation of the baryon, asymmetry, and other problems.] Eventually m will reach its low-energy value m_0, predicted by some as-yet unknown mechanism, and is fixed there. To get the ordinary Friedmann expansion, this fixation of m should be correlated with the final reducing to zero (or to small magnitude permitted by observations) of the Λ term, which during the previous period gradually decreased from Planckian to hadronic density. It is not difficult to realize such a correlation "by hand," e.g., by changing potential $\Lambda=\lambda m^4$ to the Coleman-Weinberg one[33]

$$\Lambda=\lambda m^4\ln(m^2/m_0^2)-\frac{\lambda}{2}(m^4-m_0^4).$$

The most characteristic feature of this scenario is, perhaps, the absence of phase transitions and all accompanying problems. In the conventional approach strongly inhomogeneous domain structure of the Universe is inevitably formed and the only way to explain why we do not observe it is the "new inflationary model" of Ref. 34, where the observable Universe is placed inside the single bubble whose walls are far beyond the horizon. In our case the breakdown of all symmetries took place at the moment when the size of the Universe was about 10^{-33} cm and there was no place for two domains.

Thus in this paper the hypothesis is suggested that the low-mass scale has the geometrical origin and so some of the "extra" dimensions should have a size substantially larger than Planckian one and approach the experimentally achievable limit. Perhaps it provides more questions than answers, but the theoretical possibility to consider from a single point of view—dilaton, inflaton, Higgs boson, and Brans-Dicke—is interesting.

ACKNOWLEDGMENT

I am grateful to Michael S. Marinov for fruitful discussions and help.

APPENDIX A

Connection coefficients calculated in the noncoordinate basis (3) by formula (5) [in (A1)–(A7) $g_{\alpha\beta}$ depends only on x, whereas A^i_α, g_{ij} are arbitrary functions of x,y; $F^i_{\alpha\beta}$ is given by (4a)] are

$$\Gamma^\alpha_{\beta\gamma}=\tfrac{1}{2}g^{\alpha\mu}(g_{\gamma\mu,\beta}+g_{\beta\mu,\gamma}-g_{\beta\gamma,\mu}) , \quad (A1)$$

$$\Gamma^i_{\alpha\beta}=-\tfrac{1}{2}F^i_{\alpha\beta} , \quad (A2)$$

$$\Gamma^\beta_{\alpha i}=-\tfrac{1}{2}g^{\beta\gamma}g_{ij}F^j_{\alpha\gamma} , \quad (A3)$$

$$\Gamma^K_{\alpha i}=\tfrac{1}{2}g^{kj}[g_{ij,\alpha}-(A_{\alpha i;j}+A_{\alpha j;i})] , \quad (A4)$$

$$\Gamma^k_{i\alpha}=\Gamma^k_{\alpha i}+A^k_{\alpha,i} , \quad (A5)$$

$$\Gamma^\alpha_{ij}=-\tfrac{1}{2}g^{\alpha\gamma}[g_{ij,\gamma}-(A_{\gamma i;j}+A_{\gamma j;i})] , \quad (A6)$$

$$\Gamma^k_{ij}=\tfrac{1}{2}g^{kl}(g_{il,j}+g_{jl,i}-g_{ij,l}) . \quad (A7)$$

Expressions (A8)–(A10) for Riemann and Ricci tensors and the scalar curvature were calculated by the general formula (11) taking account of (4) and (A1)–(A7) in the case when C^D is the product of several compact homogeneous subspaces ($D=D_1+D_2+\cdots$). These subspaces are enumerated by indices s,t,u, their dimensions are D_s, "radii" r_s, inner metrics $g_{i(s)j(s)}$, and the respective Kaluza-Klein fields $A^{a(s)}_\alpha, F^{a(s)}_{\alpha\beta}$ are given by (8)–(A10). Here $i(s), a(s)$ mean that indices i,a belong to the compact subspace number s. The auxiliary symbol $\{\cdots\}^{(s)}$ means that all the quantities in curly brackets are related to the subspace number s. In (A8)–(A10) $F_{\alpha\beta i}=g_{ij}\xi^j_a F^a_{\alpha\beta}$. The following notations are also used in (A9), (A10), and (B1)–(B4):

$$H_\alpha\equiv\sum_s D_s\frac{r_{s,\alpha}}{r_s},\quad X_{\alpha\beta}\equiv\sum_s D_s\frac{r_{s;\alpha\beta}}{r_s} ,$$

$$Y_{\alpha\beta}\equiv\sum_s D_s\frac{r_{s,\alpha}r_{s,\beta}}{r_s^{\,2}} ,$$

$$R^{(N)}_{\alpha\beta\gamma\delta}=R^{(n)}_{\alpha\beta\gamma\delta}+\sum_s[\tfrac{1}{4}g_{ij}(F^i_{\beta\delta}F^j_{\gamma\alpha}-F^i_{\beta\gamma}F^j_{\delta\alpha}-2F^i_{\delta\gamma}F^j_{\beta\alpha})]^{(s)} , \quad (A8a)$$

$$R^{(N)}_{\alpha\beta\gamma i(s)}=\left[\tfrac{1}{2}g_{ij}\xi^j_a\left[\nabla_\gamma F^a_{\alpha\beta}+2\frac{r_{,\gamma}}{r}F^a_{\alpha\beta}+\frac{r_{,\beta}}{r}F^a_{\alpha\gamma}-\frac{r_{,\alpha}}{r}F^a_{\beta\gamma}\right]\right]^{(s)} , \quad (A8b)$$

$$R^{(N)}_{\alpha\beta i(s)j(s)}=[-\tfrac{1}{2}(F_{\alpha\beta i;j}-F_{\alpha\beta j;i})-\tfrac{1}{4}g^{\mu\nu}(F_{\alpha\mu i}F_{\beta\nu j}-F_{\alpha\mu j}F_{\beta\nu i})]^{(s)} , \quad (A8c)$$

$$R^{(N)}_{\alpha\beta i(s)j(t)}=-\frac{g^{\mu\nu}}{4}(F_{\alpha\mu i(s)}F_{\beta\nu j(t)}-F_{\alpha\mu j(t)}F_{\beta\nu i(s)})\quad (s\neq t) , \quad (A8d)$$

$$R^{(N)}_{\alpha i(s)\beta j(s)}=\left[-\tfrac{1}{2}F_{\alpha\beta i;j}+\tfrac{1}{4}g^{\mu\nu}F_{\alpha\mu j}F_{\beta\nu i}-g_{ij}\frac{r_{;\alpha\beta}}{r}\right]^{(s)} , \quad (A8e)$$

$$R^{(N)}_{\alpha i(s)\beta j(t)}=\tfrac{1}{4}g^{\mu\nu}F_{\alpha\mu j(t)}F_{\beta\nu i(s)}\quad (s\neq t) , \quad (A8f)$$

$$R^{(N)}_{\alpha i(s)j(s)k(s)}=\left[\frac{1}{2}g^{\mu\nu}\frac{r_{,\nu}}{r}(g_{ij}F_{\alpha\mu k}-g_{ik}F_{\alpha\mu j})\right]^{(s)} , \quad (A8g)$$

$$R^{(N)}_{\alpha i(s)j(s)k(t)}=\tfrac{1}{2}g_{i(s)j(s)}g^{\mu\nu}\frac{r_{s,\mu}}{r_s}F_{\alpha\nu k(t)}\quad (s\neq t) , \quad (A8h)$$

$$R^{(N)}_{\alpha i(s)j(t)k(t)}=0,\quad R^{(N)}_{\alpha i(s)j(t)k(U)}=0\quad (s\neq t\neq u) , \quad (A8i)$$

$$(R^{(N)}_{ijkl})^{(s)}=\left[R^{(D)}_{ijkl}+\frac{r^{,\mu}r_{,\mu}}{r^2}(g_{il}g_{jk}-g_{ik}g_{jl})\right]^{(s)} , \quad (A8j)$$

$$R^{(N)}_{i(s)j(t)k(s)l(t)}=-\frac{r_s^{,\mu}r_{t,\mu}}{r_s r_t}g_{i(s)k(s)}g_{j(t)l(t)} , \quad (A8k)$$

$$R^{(N)}_{i(s)j(s)k(t)l(t)}=0 , \quad (A8l)$$

$$R^{(N)}_{\alpha\beta}=R^{(n)}_{\alpha\beta}-\sum_s(\tfrac{1}{2}g_{ij}F^i_{\alpha\mu}F^{j\mu}_\beta)^{(s)}-X_{\alpha\beta} , \quad (A9a)$$

$$R^{(N)}_{\alpha i(s)}=\left\{-\tfrac{1}{2}g_{ij}\xi^j_a\left[\nabla_\mu F^{a\mu}_\alpha+\left[2\frac{r_{,\mu}}{r}+H_\mu\right]F^{au}_\alpha\right]\right\}^{(s)} , \quad (A9b)$$

$$R^{(N)}_{i(s)j(t)}=\tfrac{1}{4}F_{\alpha\beta i(s)}F^{\alpha\beta}_{j(t)}+\delta_{st}\left[R^{(D)}_{ij}-g_{ij}\left[\frac{\Box r}{r}-\frac{r^{,\mu}r_{,\mu}}{r^2}+\frac{r^{,\mu}}{r}H_\mu\right]\right]^{(s)} , \quad (A9c)$$

$$R^{(N)}=R^{(n)}+Y^\mu_\mu-2X^\mu_\mu-H^\mu H_\mu+\sum_s(R^{(D)}-\tfrac{1}{4}g_{ij}F^i_{\mu\nu}F^{j\mu\nu})^{(s)} , \quad (A10)$$

APPENDIX B

Components of the effective Lagrangian in M^n defined by (16) and (17), taking account of (12)—(15), i.e., as

$$\frac{1}{l^{N-4}}\int R^{(N)ABCD}R^{(N)}_{ABCD}(|g^{(N)}|)^{1/2}d^Nx=\frac{1}{l^{n-4}}\int V(L_1^{(g,r)}+L_1^{(A)}+L_1^{(A,g,r)})(|g^{(n)}|)^{1/2}d^nx\ ,$$

and analogously L_2 for $R^{(N)AB}R^{(N)}_{AB}$, L_3 for $R^{(N)2}$, L_4 for $R^{(N)}$ terms of the action (1) (see notations in Sec. II and Appendix A) are

$$L_1^{(g,r)}=R^{(n)\alpha\beta\gamma\delta}R^{(n)}_{\alpha\beta\gamma\delta}+2Y^{\alpha\beta}Y_{\alpha\beta}+\sum_s\left[\langle R^{(D)ijkl}R^{(D)}_{ijkl}\rangle-4\langle R^{(D)}\rangle\frac{r^{,\alpha}r_{,\alpha}}{r^2}+4D\frac{r^{;\alpha\beta}r_{;\alpha\beta}}{r^2}-2D\left[\frac{r^{,\alpha}r_{,\alpha}}{r^2}\right]^2\right]^{(s)}, \tag{B1a}$$

$$L_1^{(A)}=\sum_s[3K_{ab}F^a_{\alpha\beta}F^{a\alpha\beta}-\tfrac{3}{2}r^2f^d_{ab}h_{cd}F^{a\beta}_{\alpha}F^{b\gamma}_{\beta}F^{c\alpha}_{\gamma}-r^2h_{ab}(\nabla^\gamma F^{a\alpha\beta})(\nabla_\gamma F^b_{\alpha\beta})]^{(s)}$$
$$+\sum_{s,t}\frac{1}{8}r_s{}^2r_t{}^2E_{a(s)b(s)|c(t)d(t)}[3(F^{a(s)}_{\alpha\beta}F^{c(t)\alpha\beta})(F^{b(s)}_{\gamma\delta}F^{d(t)\gamma\delta})+4\phi^{a(s)b(s)c(t)d(t)}+\phi^{a(s)c(t)b(s)d(t)}]\ , \tag{B1b}$$

$$L_1^{(A,g,r)}=\sum_s\left\{r^2h_{ab}\left[-\tfrac{1}{2}R^{(n)\alpha\beta\gamma\delta}(F^a_{\beta\delta}F^b_{\gamma\alpha}-F^a_{\beta\gamma}F^b_{\delta\alpha}-2F^a_{\delta\gamma}F^b_{\beta\alpha})-6\frac{r^{,\alpha}r^{,\gamma}}{r^2}F^a_{\alpha\beta}F^{b\alpha\beta}+2F^a_{\alpha\gamma}F^{b\gamma}_{\beta}\left[\frac{r^{;\alpha\beta}}{r}-2\frac{r^{,\alpha}r^{,\beta}}{r^2}-y^{\alpha\beta}\right]\right.\right.$$
$$\left.\left.-\,2(\nabla^\gamma F^{a\alpha\beta})\left[2F^b_{\alpha\beta}\frac{r_{,\alpha}}{r}+F^b_{\alpha\gamma}\frac{r_{,\beta}}{r}-F^b_{\beta\gamma}\frac{r_{,\alpha}}{r}\right]\right]\right\}^{(s)}, \tag{B1c}$$

$$L_2^{(g,r)}=(R^{(n)\alpha\beta}-X^{\alpha\beta})(R^{(n)}_{\alpha\beta}-X_{\alpha\beta})+\sum_s\left[\langle R^{(D)ij}R^{(D)}_{ij}\rangle-2\langle R^{(D)}\rangle\left[\frac{\Box r}{r}-\frac{r^{,\alpha}r_{,\alpha}}{r^2}+\frac{r^{,\alpha}}{r}H_\alpha\right]\right.$$
$$\left.+D\left[\frac{\Box r}{r}-\frac{r^{,\alpha}r_{,\alpha}}{r^2}+\frac{r^{,\alpha}}{r}H_\alpha\right]^2\right]^{(s)}, \tag{B2a}$$

$$L_2^{(A)}=\sum_s\left[\tfrac{1}{2}K_{ab}F^a_{\alpha\beta}F^{b\alpha\beta}-\frac{r^2}{2}h_{ab}(\nabla_\gamma F^{a\gamma}_{\alpha})(\nabla_\delta F^{b\alpha\delta})\right]^{(s)}$$
$$+\sum_{s,t}\tfrac{1}{16}r_s{}^2r_t{}^2E_{a(s)b(s)|c(t)d(t)}[(F^{a(s)}_{\alpha\beta}F^{c(t)\alpha\beta})(F^{b(s)}_{\gamma\delta}F^{d(t)\gamma\delta})+4\phi^{a(s)b(s)c(t)d(t)}]\ , \tag{B2b}$$

$$L_2^{(A,g,r)}=\sum_s\left[r^2h_{ab}\left\{\tfrac{1}{2}F^a_{\alpha\gamma}F^{b\gamma}_{\beta}\left[2R^{(n)\alpha\beta}-2X^{\alpha\beta}-\left[2\frac{r^{,\alpha}}{r}+H^\alpha\right]\left[2\frac{r^{,\beta}}{r}+H^\beta\right]\right]\right.\right.$$
$$\left.\left.+\,\tfrac{1}{2}F^a_{\alpha\beta}F^{b\alpha\beta}\left[\frac{\Box r}{r}-\frac{r^{,\alpha}r_{,\gamma}}{r^2}+\frac{r^{,\alpha}}{r}H_\gamma\right]-(\nabla_\gamma F^{a\alpha\gamma})F^{b\beta}_{\alpha}\left[2\frac{r_{,\beta}}{r}+H_\beta\right]\right\}\right]^{(s)}, \tag{B2c}$$

$$L_3^{(g,r)}=\left\langle\left[R^{(n)}-2X^\alpha_\alpha+Y^\alpha_\alpha-H^\alpha H_\alpha+\sum_s(R^{(D)})^{(s)}\right]^2\right\rangle, \tag{B3a}$$

$$L_3^{(A)}=\sum_s\left[\tfrac{1}{2}r^2F^a_{\alpha\beta}F^{b\alpha\beta}\langle(-\widetilde{g}_{ij}\xi^i_a\xi^j_b)\sum_t(R^{(D)})^{(t)}\rangle\right]^{(s)}+\sum_{s,t}[\tfrac{1}{16}r_s{}^2r_t{}^2E_{a(s)b(s)|c(t)d(t)}(F^{a(s)}_{\alpha\beta}F^{b(s)\alpha\beta})(F^{c(t)}_{\gamma\delta}F^{d(t)\gamma\delta})]\ , \tag{B3b}$$

$$L_3^{(A,g,r)}=(R^{(n)}-2X^\alpha_\alpha+Y^\alpha_\alpha-H^\alpha H_\alpha)\sum_s\left[\frac{r^2}{2}h_{ab}F^a_{\alpha\beta}F^{b\alpha\beta}\right]^{(s)}, \tag{B3c}$$

$$L_r^{(g,r)}=R^{(n)}-2X^\alpha_\alpha+Y^\alpha_\alpha-H^\alpha H_\alpha+\sum_s(R^{(D)})^{(s)}\ , \tag{B4a}$$

$$L_4^{(A)}=\sum_s(\tfrac{1}{4}r^2h_{ab}F^a_{\alpha\beta}a^{b\alpha\beta})^{(s)}\ , \tag{B4b}$$

$$L_4^{(A,g,r)}=0\ . \tag{B4c}$$

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