Asymmetries in e^+e^- collisions at the Z^0 pole from E_6 grand unified theories

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The effects of an extra neutral gauge boson predicted by E_6 grand unified theories are studied in e^+e^- collisions at $\sqrt{s}=M_{Z^0}$. We find that measurements of the left-right and forward-backward asymmetries are a sensitive test of the mixing between the Z^0 and Z' bosons and can be used to constrain the mass of the Z'.

Recently, there has been renewed interest in E_6 grand unified theories due to the possibility that they are the low-energy limit of $E_8 \times E_8$ superstring theories.[1] Apart from the prediction of exotic fermions,[2] the main testable feature of E_6 theories is the presence of at least one extra neutral gauge boson, which we denote the Z' (Ref. 3). Several groups have studied the properties of the Z' and obtained limits on its mass from the analysis of low-energy neutral-current data and from the measured masses of the $W^\pm$ and Z^0 bosons.[4] Using these constraints on the Z' properties we found that substantial deviations from the standard model are allowed for the left-right and forward-backward asymmetries in $e^+e^- \to \mu^+\mu^-$ and in Bhabha scattering.[5] In this short note we expand on our previous work to utilize the high statistics available at $\sqrt{s}=M_{Z^0}$ to study the properties of the Z' boson including its mass and the mixing between the standard-model Z^0 and the Z' boson. We begin by reviewing some of the formalism and notation and in particular the couplings of the neutral gauge bosons to the conventional fermions. We will then present the deviations for asymmetries between this extended model and the standard electroweak model for measurements at the Z^0 pole and discuss how to extract information about the underlying theory from these measurements.

A general property of E_6 theories is the prediction of at least one extra neutral gauge boson at low energies[6] (below the Planck mass).[7] How this neutral gauge boson will couple to fermions will depend on how the E_6 symmetry is broken. Since the U(1) generator corresponding to the extra neutral boson must be orthogonal to all generators of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model a convenient parametrization is given in terms of the generators of $U(1)_\chi$ and $U(1)_\psi$ in the following subgroup chain:

$$E_6 \to SO(10)\times U(1)_\psi \to SU(5)\times U(1)_\chi \times U(1)_\psi ,$$

where the standard model is embedded in the SU(5) group. The couplings of the fermions to the standard model Z^0 are given, as usual, by

$$Q^0 = I_{3L} - Q_{EM}\sin^2\theta_W , \tag{1}$$

while the couplings to the Z' will in general be given by a linear combination of the $U(1)_\chi$ and $U(1)_\psi$ charges:

$$Q' = Q_\chi \cos\theta_{E_6} + Q_\psi \sin\theta_{E_6} \tag{2}$$

with the values of Q_χ and Q_ψ given in Table I.

With the extra Z^0 boson the neutral-current Lagrangian is generalized to contain an extra term and is now given by

$$\mathscr{L}_{NC} = eA_\mu J^\mu_{EM} + g_{Z^0} Z^0_\mu J^\mu_Z + g_{Z'} Z'_\mu J^\mu_{Z'} , \tag{3}$$

where J^μ_{EM} and $J^\mu_{Z^0}$ are the electromagnetic current and the Z^0 current of the standard model and $J^\mu_{Z'}$ is given by

$$J^\mu_{Z'} = \sum_f \bar\psi_f \gamma^\mu (C'_V - C'_A\gamma_5)\psi_f \tag{4}$$

with $C'_{V,A} = \frac{1}{2}(Q'_f \mp Q'_{f^c})$ whose values are given in Table I. In general the physical fields, Z^0_p and Z'_p, are given by a linear combination of the gauge fields Z^0 and Z' with a mixing angle ϕ. Because this mixing angle comes from the diagonalization of the Z^0-Z' mass matrix it can be expressed in terms of the standard-model prediction for the Z^0 mass, $M_{SM} = M_W/\cos\theta_W$, and the physical Z^0 and Z' masses:[8]

TABLE I. Values of Q_χ and Q_ψ for the fermions and their vector and axial-vector couplings for the two neutral gauge fields.

Fermion	$Q_\chi^{(f)}$	$Q_\chi^{(f^c)}$	$Q_\psi^{(f)}$	$Q_\psi^{(f^c)}$	C'_V	C'_A	C^0_V	C^0_A
e	$\frac{3}{2\sqrt{10}}$	$\frac{-1}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{\sqrt{10}}\cos\theta_{E_6}$	$\frac{1}{2\sqrt{10}}\cos\theta_{E_6}+\frac{1}{2\sqrt{6}}\sin\theta_{E_6}$	$\frac{-1}{4}+\sin^2\theta_W$	$\frac{-1}{4}$
ν	$\frac{3}{2\sqrt{10}}$	$\frac{-5}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{\sqrt{24}}$	$\frac{2}{\sqrt{10}}\cos\theta_{E_6}$	$\frac{-1}{2\sqrt{10}}\cos\theta_{E_6}+\frac{1}{2\sqrt{6}}\sin\theta_{E_6}$	$\frac{1}{4}$	$\frac{1}{4}$
u	$\frac{-1}{2\sqrt{10}}$	$\frac{-1}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{\sqrt{24}}$	0	$\frac{-1}{2\sqrt{10}}\cos\theta_{E_6}+\frac{1}{2\sqrt{6}}\sin\theta_{E_6}$	$\frac{1}{4}-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{4}$
d	$\frac{-1}{2\sqrt{10}}$	$\frac{3}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{\sqrt{24}}$	$\frac{-1}{\sqrt{10}}\cos\theta_{E_6}$	$\frac{1}{2\sqrt{10}}\cos\theta_{E_6}+\frac{1}{2\sqrt{6}}\sin\theta_{E_6}$	$\frac{-1}{4}+\frac{1}{3}\sin^2\theta_W$	$\frac{-1}{4}$

$$\tan^2\phi = \frac{M_{SM}^2 - M_{Z_p^0}^2}{M_{Z_p'}^2 - M_{SM}^2} . \quad (5)$$

This mixing will alter the fermions' couplings to the Z^0 from the standard-model values for C_V^0 and C_A^0 to the following:

$$C_{V,A} = C_{V,A}^0 \cos\phi + (g_{Z'}/g_{Z^0}) C'_{V,A} \sin\phi . \quad (6)$$

Thus, in addition to the parameters of the standard model, there are now three new independent parameters to be determined in E_6 theories with one extra Z^0 boson. These are the mass of the Z', or equivalently the Z^0-Z' mixing angle, θ_{E_6} (the angle which characterizes the direction of the generator of the Z' in E_6 group space), and $g_{Z'}/g_{Z^0}$ (the ratio of the coupling constants). In general, using renormalization-group arguments one finds that $(g_{Z'}/g_{Z^0})^2 \le \frac{5}{3}\sin^2\theta_W$ with the exact value dependent on the symmetry-breaking scheme. Here we will assume the value $(g_{Z'}/g_{Z^0})^2 = \frac{5}{3}\sin^2\theta_W$ which results when all U(1) groups are broken at the same mass which for the case of current interest is the compactification scale.

Since at $\sqrt{s} = M_{Z^0}$ the cross section for $e^+e^- \to \mu^+\mu^-$ is dominated by the pole in the Z^0 propagator, asymmetry measurements at this energy will provide an accurate determination of the couplings C_V and C_A (Ref. 9). To elaborate on this we first consider the left-right asymmetry defined by

$$A \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} , \quad (7)$$

where $\sigma_{L(R)}$ are the cross sections for the scattering of a left-handed (right-handed) electron on an unpolarized positron. Since the cross section is dominated by the Z^0 pole we can neglect, for the accuracy desired here, all terms with a photon or a Z' propagator so that A_{LR} is approximated by

$$A_{LR} \simeq \frac{2C_V C_A}{C_V^2 + C_A^2} \quad (8)$$

which is independent of the mass of the Z'. With the same approximation the forward-backward asymmetry for unpolarized beams is given by

$$A_{FB} \simeq 3 \left[\frac{C_A^e C_V^e}{C_V^{e2} + C_A^{e2}} \right] \left[\frac{C_A^f C_V^f}{C_V^{f2} + C_A^{f2}} \right] , \quad (9)$$

where $C_{V,A}^f$ are the couplings of the final-state fermions. We also note that because of the dominance of the s-channel Z^0 pole, the left-right asymmetry in Bhabha scattering is almost equal to A_{LR} in $e^+e^- \to \mu^+\mu^-$ and therefore cannot provide any new information.

We present our results as differences between the asymmetries predicted by the E_6 theories and those given by the standard model with the standard-model predictions obtained from Eqs. (8) and (9) with the substitution $C_{V,A} \to C_{V,A}^0$. We have not included detector-dependent radiative corrections in our calculations since they depend on the detector resolution ΔE and on the experimental acceptance cuts, and as such are best included by experimentalists via Monte Carlo simulations involving the details of their detector.[9] Of the remaining radiative corrections most are absorbed into the renormalization of $\sin^2\theta_W$ with only a small residual amount remaining which shifts A_{LR} by about 0.005. The deviations from the standard model are denoted by δA_{LR} and δA_{FB} and are plotted in Fig. 1 as a function of ϕ for the cases where $\theta_{E_6}=0$ and $\tan\theta_{E_6} = -\sqrt{5/3}$. The first angle corresponds to the Z' being entirely in SO(10) and the second angle corresponds to the case where E_6 is broken by a non-Abelian discrete symmetry to a rank-5 group which may occur in superstring theories. The nearly linear dependence of δA_{LR} and δA_{FB} on ϕ can easily be verified by substituting Eq. (6) in Eqs. (8) and (9) for A_{LR} and A_{FB}, respectively, and by making a series expansion around $\phi=0$. Note that the magnitude of δA_{LR} is greater than that of δA_{FB} since A_{FB} is, roughly, the square of A_{LR}. It is expected that A_{LR} can be measured to 0.005 (Ref. 9), so this will test the value of ϕ to 0.01 for the examples given. In Fig. 2 we generalize the results to all possible values of θ_{E_6} by plotting contours of constant δA_{LR} and δA_{FB} as functions of ϕ and θ_{E_6}. It is clear that for $\theta_{E_6} = \pi/2$, the asymmetries are insensitive to the extra Z^0 since $C'_V = 0$ as can be seen from Table I. Actually, C'_V vanishes for all fermions for this particular direction of symmetry breaking and even the low-energy neutral-current data offers poor constraints on the mass of the Z'.

In addition one could make further measurements of A_{FB} for $e^+e^- \to q\bar{q}$ since C_V and C_A for the quarks are also functions of ϕ and θ_{E_6}. However, the precision with which the measurements can be made depends on the flavor-tagging efficiency for heavy quarks and on theoretical uncertainties due to strong interactions in the final state. Unfortunately without knowledge of the symmetry-breaking pattern we can only constrain ϕ and θ_{E_6} to a set of curves in the ϕ-E_6 plane due to the similar

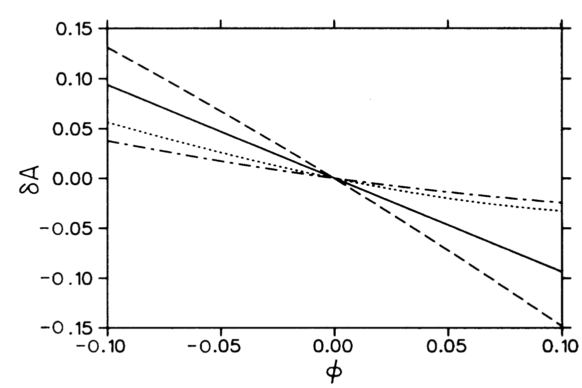

FIG. 1. δA_{LR} and δA_{FB} in $e^+e^- \to \mu^+\mu^-$ as a function of ϕ, the Z^0-Z' mixing angle, with $M_{Z'} = 200$ GeV. The solid line is for δA_{LR} with $\tan\theta_{E_6} = -\sqrt{5/3}$, the dashed line is for δA_{LR} with $\theta_{E_6}=0$, the dot-dashed line is for δA_{FB} with $\tan\theta_{E_6} = -\sqrt{5/3}$, and the dotted line is for δA_{FB} with $\theta_{E_6}=0$.

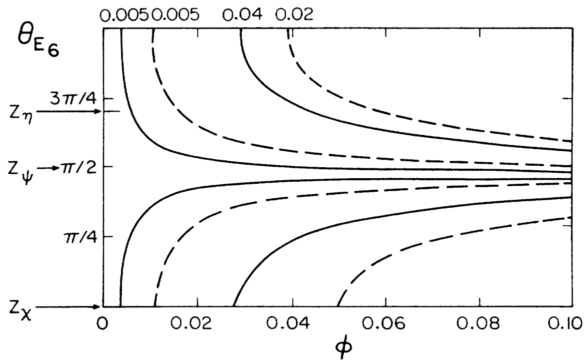

FIG. 2. Contours of constant δA for $M_{Z'}=200$ GeV as a function of θ_{E_6} and ϕ. The solid line is for δA_{LR} and the dashed line is for δA_{FB}.

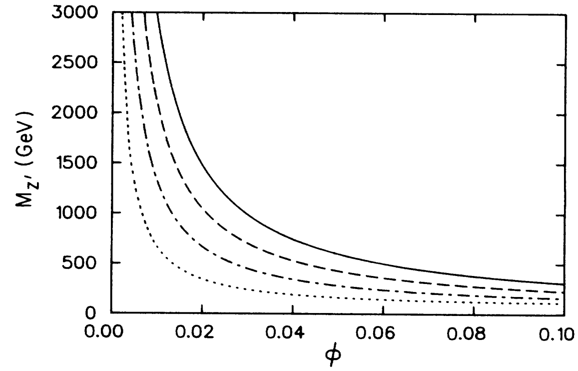

FIG. 3. Contours of constant $1-\rho$ as a function of $M_{Z'}$ and ϕ with $M_W=82$ GeV and $\sin^2\theta_W=0.222$. The solid line is for $1-\rho=0.1$, the dashed line for $1-\rho=0.05$, the dot-dashed line for $1-\rho=0.02$, and the dotted line is for $1-\rho=0.005$.

linear dependence of the various asymmetries on C_V and C_A (or ϕ and θ_{E_6}). Thus, the best one can do with measurements at the Z^0 pole is pick a specific scheme to fix θ_{E_6}, say that of Cohen *et al.* in Ref. 4, and then constrain ϕ and $M_{Z'}$. As we have shown previously,[5] to determine θ_{E_6} it will be necessary to perform asymmetry measurements at energies above the Z^0 pole.

Since $M_{Z'}$ depends on ϕ through Eq. (5) an accurate measurement of the M_{Z^0} together with the determination of $\sin^2\theta_W$ from measurements of M_W and G_μ will give $M_{Z'}$ (Ref. 8). Defining the parameter

$$\rho \equiv \frac{M_W{}^2}{M_{Z^0}{}^2\cos^2\theta_W} \tag{10}$$

we obtain

$$1-\rho \simeq \tan^2\phi \left[\frac{M_{Z'}{}^2\cos^2\theta_W}{M_W{}^2}-1\right] . \tag{11}$$

In Fig. 3 we plot contours of constant $1-\rho$ as a function of ϕ and $M_{Z'}$ where we have used the values $M_W=82$ GeV and $\sin^2\theta_W=0.222$. Although this plot seems counterintuitive recall that Eq. (5) comes from the diagonalization of a mass matrix. Therefore, for the physical Z^0 mass to be much different than the standard-model value necessitates either the off-diagonal elements of the mass matrix to be large, and hence large mixing, or for the Z^0-Z' mass difference to be small. The converse of this is that small deviations of ρ from 1 come about from either small Z^0-Z' mixing or large mass splittings. The line $1-\rho=0.05$ corresponds to the present upper limit on this parameter.

In summary, we found that measurements at $\sqrt{s}=M_{Z^0}$ in e^+e^- colliders offer a good test of new physics beyond the standard model. Measurable deviations from the standard model above the level of radiative corrections might be present in the measurements of left-right and forward-backward asymmetries and of M_{Z^0}. If such deviations are observed, the parameters of E_6 theories such as the magnitude of the Z^0-Z' mixing and $M_{Z'}$ can be measured for a given symmetry breaking. However, the direction of the Z' charge in E_6 group space can only be determined by asymmetry measurements off the Z^0 pole.

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