Soliton stars and the critical masses of black holes

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New possibilities of cold stable stellar configurations, based on nontopological soliton solutions in general relativity, are examined. They represent coherent quantum states with very large masses M; depending on the theory, M can be $\sim 10^{15}$ times the solar mass, or less.

It has been commonly accepted[1,2] that for any star consisting of normal matter, after its nuclear burning source is exhausted, no stable solution exists if its mass M is greater than a critical value M_c, which at zero angular momentum is $\lesssim$ five solar masses $M_\odot$. The star would undergo violent processes, either by expelling some of its mass and becoming a neutron star or a white dwarf with $M < M_c$, or by collapsing into a black hole. This relatively low critical mass M_c has been used as a criterion for the observation of black holes. The purpose of this paper is to point out that by using the nontopological soliton solution[3-5] a new type of cold stable stellar configuration, called a soliton star, can be formed (at least theoretically). Depending on the theory, cold soliton stars may have a mass as large[6] as $10^{15} M_\odot$ without becoming black holes.

To illustrate the basic mechanism, consider the following example of a nontopological soliton, first without gravity. The theory contains an additive quantum number N (like the baryon number) carried by either a spin-$\frac{1}{2}$ field ψ, or a spin-0 complex field ϕ, with its elementary field quantum having $N = \pm 1$. In addition, there is a scalar field σ. Take the self-interaction of σ to be the typical degenerate vacuum form (in units $\hbar = c = 1$):

$$U(\sigma) = \tfrac{1}{2} m^2 \sigma^2 \left[1 - \frac{\sigma}{\sigma_0} \right]^2 . \tag{1}$$

We may assign $\sigma = 0$ to the normal vacuum state, and $\sigma = \sigma_0$ to the (abnormal) degenerate vacuum state. (Theories of this type have been studied in the literature, e.g., in connection with the spontaneous T violation,[7,8] the abnormal nuclear model,[3] the bag model,[9,10] and the Higgs mechanism.[11]) The soliton contains an interior in which $\sigma \simeq \sigma_0$, a shell of width $\sim m^{-1}$, over which σ changes from σ_0 to 0, and an exterior that is essentially the vacuum. The N-carrying field ψ, or ϕ, is confined to the interior; this produces a kinetic energy E_k (assuming for simplicity that the mass of ψ, or ϕ, is zero when $\sigma = \sigma_0$, but nonzero when $\sigma = 0$):

$$E_k \simeq \begin{cases} (3\pi)^{1/3} (\frac{3}{4} N)^{4/3} / R & \text{for } \psi , \\ \pi N / R & \text{for } \phi . \end{cases} \tag{2}$$

The shell contains a surface energy

$$E_s = 4\pi s R^2 ,$$

where s is the surface tension related to σ_0 and m by

$$s \simeq \tfrac{1}{6} m \sigma_0^2 . \tag{3}$$

The radius R can be calculated by minimizing the total energy $E = E_k + E_s$. Setting $\partial E / \partial R = 0$, we have the equipartition

$$E_k = 2 E_s . \tag{4}$$

Hence, the soliton mass M (which is the minimum of E) can be written as

$$M = 3 E_s = 12 \pi s R^2 , \tag{5}$$

the total conserved quantum number is

$$N = \begin{cases} \dfrac{16}{3} \left[\dfrac{2}{3\pi} \right]^{1/4} s^{3/4} R^{9/4} & \text{for } \psi , \\ 8 s R^3 & \text{for } \phi , \end{cases} \tag{6}$$

and therefore, for large N,

$$M \propto \begin{cases} N^{8/9} & \text{for } \psi , \\ N^{2/3} & \text{for } \phi . \end{cases} \tag{7}$$

Because the exponent of N is < 1, when N is large the soliton mass is always less than that of the free particle solution, and that ensures its stability.

Next, we include the gravitational field. For configurations with R much greater than the Schwarzschild radius $2GM$, the effects of gravity can be treated as a perturbation. Gravity becomes important when R becomes of the same order as $2GM$. Thus, the critical mass M_c may be estimated by simply equating R with the Schwarzschild radius

$$R \sim 2 G M_c ,$$

which leads to, because of (5),

$$M_c \sim (48 \pi G^2 s)^{-1} . \tag{8}$$

The correctness of such an estimate together with the detailed solutions for soliton stars are given in a series of papers.[12-14] Since Newton's constant G is the square of the Planck length $l_P \simeq 10^{-33}$ cm, whereas a typical Higgs-type field σ may have $\sigma_0 \sim m$ about, or higher than 30 GeV (but much less than the Planck mass), we estimate

$$M_c \sim (l_P m)^{-4} m . \tag{9}$$

For example, if m is ~ 30 GeV, we have $M_c \sim 10^{15} M_\odot$

and $R \sim l_P{}^{-2} m^{-3} \sim 10^2$ light years; if m is ~ 300 GeV, then $M_c \sim 10^{12} M_\odot$ and $R \sim 10^{-1}$ light year.

In the usual derivation of M_c for a white dwarf, or a neutron star, one simply compares the kinetic energy E_k of a degenerate Fermi gas with the gravitational attraction; that gives a critical mass

$$\sim m_N (l_P m_N)^{-3} , \tag{10}$$

which is $\sim M_\odot$ for $m_N =$nucleon mass. The different powers in the l_P dependence of these two estimates (9) and (10) make the large disparity in M_c.

At present, very little is known concerning the nature of the Higgs-type bosons, except that they should be massive, spin 0, and that their expectation values modify the masses of other fields. Thus M_c for the soliton star could also be quite different from the above estimate, depending on the theory. For example, by removing the degeneracy of the false and normal vacua and adjusting their energy difference, M_c can vary from the order of galactic mass to that of a solar mass.

In this connection, we may mention a related, but different, stable configuration. The theory consists of simply a "free" spin-0 *complex* field ϕ of mass $\mu \neq 0$, plus gravity. There is again a conserved additive quantum number N. (While the gravitational interaction of a neutral scalar field has been studied,[15] because of the absence of a conserved quantum number, there is no nontopological soliton solution.) For large N, one can show that because of gravitational attraction, stable soliton solutions exist. Because of the Bose-Einstein statistics, the radius R is of microscopic dimension (even though N is $\gg 1$)

$$R \sim \mu^{-1} . \tag{11}$$

Setting $R \sim 2GM$, one can estimate its critical mass

$$M_c \sim (l_P \mu)^{-2} \mu . \tag{12}$$

In the example of $\mu \sim 30$ GeV, M_c is $\sim 10^{10}$ kg, the radius is $\sim 6 \times 10^{-16}$ cm, and the corresponding density is extremely high, $\sim 10^{41}$ times that of a neutron star. Because of the smallness of its size, we call such a configuration a mini-soliton star.

As we shall see, for a given number of nodes, say, n of the field, the mini-soliton star has an upper bound M_n in mass, beyond which no stable solution exists. This bound increases with n. The critical mass M_c estimated above in (12) corresponds to M_n with $n = 0$, or not too large. However, by considering configurations with very large n, it seems possible to derive equilibrium configurations for *any* M, at least classically. In this case, the wavelength is about n/R; by equating the gravitational energy GM^2/R with the kinetic energy Nn/R, we estimate the critical mass to increase linearly with n. This possibility of having cold stars with very large n then adds still another complexity to the question of black-hole formation for these exotic configurations.

A classical soliton solution is, by definition, regular everywhere, and it approaches zero (usually) exponentially at infinity. The matter field amplitude is typically proportional to g^{-1}, where g is the relevant coupling constant. Once this singularity in g is factored out, a power-series expansion in the coupling constant can be established. The quantization and the formal perturbation series may then be carried out for any soliton solution.[16] The theory is renormalizable, if it satisfies the standard Dyson criteria[17] (originally developed for the plane-wave solutions, without solitons). Hence, except for graviton-loop diagrams, which remain divergent due to fluctuations at distances of the order of the Planck length l_P, a classical soliton solution automatically implies the existence of a corresponding quantum soliton solution, with all radiative corrections, to any powers in g, finite (provided that the theory is renormalizable to begin with). Thus, these soliton-star solutions that we have described are all bona fide quantum states.

The estimate of M_c given above in (9) refers to the ground state of the soliton star. For the same N, there are also excited quantum states. The level spacings between them may be a small fraction of the total mass M. Since M can be extremely large, the amount of energy release in such a transition can be comparable to that emitted by a quasar, or other ultraviolent astronomical objects.

Nonlinear field theories have been found to be of importance in all elementary particle interactions: QCD, the electroweak theory, grand unified theory, etc. Many of their physical properties are still in the developing stage. For stellar configurations, although the nonlinearity of gravitation is fully recognized through general relativity, that of the matter field is far from adequately explored. The aim of this and the subsequent papers in the series is to indicate the physical richness in these new possibilities.

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