Symmetry breakings in supersymmetric quantum chromodynamics with $N_{\text{color}} < N_{\text{flavor}}$

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In supersymmetric quantum chromodynamics (SQCD) with $N_{\text{color}}(=N) < N_{\text{flavor}}(=M)$, influences of explicit supersymmetry (SUSY) and chiral-symmetry breakings are examined on the basis of an effective Lagrangian for SQCD. The effective Lagrangian reproduces symmetry breakings consistent with complementarity in the massless SQCD and with instanton analysis in the massive SQCD for $M-N$ massive flavors. A chiral SU($M-N$) symmetry, which remains unbroken in massless SQCD, is broken by the SUSY breakings even in the absence of the explicit breakings of SU($M-N$). An application to composite quarks and leptons seems unsatisfactory owing to the presence of appreciable mixings of quarks and leptons with their mirrors.

I. INTRODUCTION

Dynamical properties of supersymmetric quantum chromodynamics (SQCD) have been extensively discussed in various frameworks utilizing instantons,[1–3] effective Lagrangians,[4–7] complementarity,[8] and anomalous relations for the superpotentials.[9] SQCD with N colors and M flavors exhibits as a global symmetry $G=\text{SU}(M)_L\times\text{SU}(M)_R\times\text{U}(1)_V\times\text{U}(1)_A$, in which scalar superfields $\Phi^{(a)}$ ($a=1,2$) and chiral gauge superfield W transform according to Table I. In massless SQCD, the results so far obtained indicate dynamical breakdown of the chiral symmetries of $\text{SU}(M)_{L,R}$ and $\text{U}(1)_A$: (1) for $N>M$ the chiral symmetries are spontaneously broken and perhaps supersymmetry (SUSY) is spontaneously broken, (2) for $N=M$ the chiral symmetries except for $\text{U}(1)_A$ are spontaneously broken, and (3) for $N<M$ a chiral SU($M-N$) and modified $\text{U}(1)_A$ symmetries remain unbroken. Also realized is the difference between the massless and massive SQCD. Massless SQCD cannot be defined in the massless limit of massive SQCD that posses a definite SUSY vacuum.

An application of SQCD lies in the field of composite models of quarks and leptons.[10] Quarks and leptons are considered as being massless due to a certain symmetry principle.[11] SQCD provides the Nambu-Goldstone mechanism of generating massless fermions as supersymmetric partners of the Nambu-Goldstone bosons, which are associated with the spontaneous symmetry breakdown of G to its subgroup H (Ref. 12). Since this mechanism calls for unbroken SUSY, SQCD with $N>M$ seems to be excluded. For $N\le M$, chiral symmetries appear that protect massless fermions. To give masses for these massless particles, chiral symmetries must be explicitly broken. Furthermore, no scalar quarks and scalar leptons have yet been discovered; these scalars must be much heavier than quarks and leptons. This fact requires explicit breaking of SUSY.

Toward a dynamical calculation of these effects including those on mass generations, an effective Lagrangian approach has been utilized for SQCD with $N=M$ (Ref. 13). The model considered is a SUSY extension of the Pati-Salam model[14] with $M=6$, specifying two degrees of freedom for weak isospin ($N_W=2$) and four degrees of freedom for colors and lepton number ($N_C=4$). Quarks and leptons are included in Nambu-Goldstone superfields belonging to the coset space $G/\text{SU}(6)_{L+R}\times\text{U}(1)_A$. However, this model of one generation of quarks and leptons suffers from the existence of "light" leptoquark fermions mainly due to the global $\text{SU}(6)_{L+R}$ symmetry.

In this paper, we examine SQCD with the chiral SU($M-N$) symmetry, i.e., $(2<)N<M(<3N)$. The light leptoquark fermions are absent because of the unbroken chiral SU(4) symmetry if $M-N$ is identified with N_C (=4). By the use of complementarity, one finds that the massless SQCD leads to $G\to H=\text{SU}(N)_{L+R}\times\text{SU}(M-N)_L\times\text{SU}(M-N)_R\times\text{U}(1)'_V\times\text{U}(1)'_A$ and contains the Nambu-Goldstone superfields (NGS's) transforming as $(\mathbf{N}^*,\mathbf{M}-\mathbf{N},\mathbf{1})$ and $(\mathbf{N},\mathbf{1},\mathbf{M}-\mathbf{N}^*)$ under $(\text{SU}(N)_{L+R},\text{SU}(M-N)_L,\text{SU}(M-N)_R)$. Thus, if $\text{SU}(N)_{L+R}$ takes care of the weak isospin $\text{SU}(2)_W$ and $\text{SU}(M-N)_{L,R}$ takes care of the chiral version of the Pati-Salam $\text{SU}(4)_C$, NGS's include quarks and leptons. However, in association with explicit breaking of SUSY, the chiral symmetries may undergo spontaneous breakdown, which will accompany SUSY-breaking scale. To

TABLE I. The quantum numbers of superfields under G and the anomalous $\text{U}(1)_a$.

Superfields	$\text{SU}(N)^{\text{loc}}$	$\text{SU}(M)_L$	$\text{SU}(M)_R$	$\text{U}(1)_V$	$\text{U}(1)_A$	$\text{U}(1)_a$
$\Phi^{(1)}:(\phi^{(1)},\psi^{(1)})$	$\mathbf{N}^*$	$\mathbf{M}$	1	1	$(N-M,N)$	1
$\Phi^{(2)}:(\phi^{(2)},\psi^{(2)})$	$\mathbf{N}$	1	$\mathbf{M}^*$	-1	$(N-M,N)$	1
$W:(\lambda,G_{\mu\nu})$	ADJ	1	1	0	$(-M,0)$	0

examine dynamical properties of SQCD with $N<M$ in the presence of explicit SUSY and chiral-symmetry breakings, we employ an effective Lagrangian for SQCD (Refs. 5 and 13). The massive SQCD is also examined with the emphasis on the explicit breaking of the chiral $SU(M-N)$ symmetry by the SUSY mass terms.

The effective Lagrangian for SQCD with $N<M$ involves the effective superpotential given by[5]

$$W_{\rm eff}=S[\ln(S^{N-M}\det T/\Lambda^{3N-M})+f(Z)+M-N]-\sum_i m_i T_i^{\ i} , \tag{1}$$

with

$$Z=\det T/Y^{(1)}T^{M-N}Y^{(2)}=\det T/X , \tag{2a}$$

$$X=\sum_{i,j}\epsilon^{i_1\cdots i_M}\epsilon_{j_1\cdots j_M}Y^{(1)}_{i_1}\cdots Y^{(1)}_{i_N}T_{i_{N+1}}^{\ \ j_{N+1}}\cdots T_{i_M}^{\ \ j_M}Y^{(2)j_1}\cdots Y^{(2)j_N} , \tag{2b}$$

where $T_i^{\ j}=\sum_A \Phi^{(1)}{}_i{}^A\Phi^{(2)}{}_A{}^j$, S is made of two chiral gauge superfields, and $Y^{(1,2)}$ are made of N antisymmetrized matter superfields $\Phi^{(1,2)}$.

Our starting Lagrangian $L_{\rm eff}$ is given by $L_{\rm eff}=L_0+(W_{\rm eff}\,|_{\theta\theta}+\text{H.c.})-L_{\rm mass}$ with

$$L_0=\sum_i K(\Phi_i,\Phi_i^*)\,|_{\theta\theta\bar\theta\bar\theta} , \tag{3a}$$

$$L_{\rm mass}=\sum_{i,j}[\mu_{Li}{}^2\Lambda^{-2}T_i^{\ j*}T_i^{\ j}+\mu_{Rj}{}^2\Lambda^{-2}T_i^{\ j*}T_i^{\ j}+\mu_i{}^2(T_i^{\ i}+T_i^{\ i*})]\,|_{\theta=0}+m_\lambda(S+S^*)\,|_{\theta=0} , \tag{3b}$$

where L_0 is the kinetic term for all composite superfields $\Phi_i=S$, T and $Y^{(a)}$ $(a=1,2)$ taken to be $\partial^2K/\partial\Phi_i^*\partial\Phi_j=\delta_{ij}G_i{}^{-1}(\Phi_i^*\Phi_i)$ with $G_i(0)\equiv A_i=\text{const}$; $L_{\rm mass}$ corresponds to $\mu_L{}^2\phi^{(1)*}\phi^{(1)}$, $\mu_R{}^2\phi^{(2)*}\phi^{(2)}$, $\mu^2\phi^{(1)}\phi^{(2)}$ and $m_\lambda\lambda\lambda$.

II. EXACT SUSY

In the SUSY limit, from Eq. (1), one obtains

$$\partial W_{\rm eff}/\partial\pi_{wi}=[1+\langle z\rangle f'(\langle z\rangle)](\pi_\lambda/\pi_{wi})-m_{wi}=0 , \tag{4a}$$

$$\partial W_{\rm eff}/\partial\pi_{ci}=(\pi_\lambda/\pi_{ci})-m_{ci}=0 , \tag{4b}$$

$$\partial W_{\rm eff}/\partial\pi_{ya}=-\langle z\rangle f'(\langle z\rangle)(\pi_\lambda/\pi_{ya})=0 , \tag{4c}$$

$$\partial W_{\rm eff}/\partial\pi_\lambda=\ln\left[\prod_{i=1}^{M-N}(\Lambda\pi_{ci}/\pi_\lambda)\prod_{i=1}^{N}(\pi_{wi}/\Lambda^2)\right]+f(\langle z\rangle)=0 , \tag{4d}$$

where

$$\pi_{wi}=\langle T_i^{\ i}\rangle\,|_{\theta=0}\ (i=1,\ldots,N) ,$$

$$\pi_{ci}=\langle T_{i+N}^{\ \ i+N}\rangle\,|_{\theta=0}\ (i=1,\ldots,M-N) ,$$

$$\pi_{y1}=\langle Y^{(1)}_{1\cdots N}\rangle\,|_{\theta=0},\quad \pi_{y2}=\langle Y^{(2)1\cdots N}\rangle\,|_{\theta=0} ,$$

$$\pi_\lambda=\langle S\rangle\,|_{\theta=0} ,$$

and $\langle z\rangle$ stands for the vacuum expectation value (VEV) of $\langle Z\rangle\,|_{\theta=0}$, $m_{wi}=m_i$ $(i=1,\ldots,N)$ and $m_{ci}=m_{i+N}$ $(i=1,\ldots,M-N)$. Other components such as $\pi_i^{\ j}=\langle T_i^{\ j}\rangle\,|_{\theta=0}$ $(i\neq j)$ are required to vanish.

In the massless SUSY limit $(m_{wi}=m_{ci}=0)$, complementarity[8] leads to the SUSY vacuum specified by $\pi_{wi}\neq0$ $(i=1,\ldots,N)$ and $\pi_{ci}=\pi_\lambda=0$ $(i=1,\ldots,M-N)$ and by either (i) $\pi_{y1}\neq0$ and $\pi_{y2}=0$ (or vice versa) or (ii) $\pi_{ya}\neq0$ $(a=1,2)$. This vacuum is obtained by

$$\pi_{wi}\sim\Lambda^2 , \tag{5a}$$

$$\pi_\lambda=0 , \tag{5b}$$

$$\pi_\lambda/\pi_{ci}=0 , \tag{5c}$$

$$\langle z\rangle f'(\langle z\rangle)(\pi_\lambda/\pi_{ya})=0\quad(a=1,2) , \tag{5d}$$

$$f(\langle z\rangle)=\ln\left[\prod_{i=1}^{M-N}(\pi_\lambda/\Lambda\pi_{ci})\prod_{i=1}^{N}(\Lambda^2/\pi_{wi})\right] , \tag{5e}$$

which determine the behavior of $f(Z)$ at $Z=\langle z\rangle$. The function $f(Z)$ can then be chosen to be, for $\rho>0$,

$$f(Z)=-\rho\ln Z\quad\text{with }\langle z\rangle^{-1}=0 , \tag{6a}$$

$$f(Z)=\rho\ln(Z-\langle z\rangle)\quad\text{with }\langle z\rangle\neq0 , \tag{6b}$$

where $\langle z\rangle^{-1}=0$ for $\pi_{y1}\neq0$ and $\pi_{y2}=0$ (or vice versa) and $\langle z\rangle\neq0$ for $\pi_{ya}\neq0$ $(a=1,2)$.

In the massive SUSY limit, the breaking of the chiral $SU(M-N)$ symmetry by $m_{ci}\neq0$ is included in the massless case. We obtain

$$\pi_{wi}\sim\Lambda^2 , \tag{7a}$$

$$\pi_\lambda=m_{ci}\pi_{ci}=0 , \tag{7b}$$

$$\langle z\rangle f'(\langle z\rangle)(\pi_\lambda/\pi_{ya})=0\quad(a=1,2) , \tag{7c}$$

$$f(\langle z\rangle)=\ln\left[\prod_{i=1}^{M-N}(m_{ci}/\Lambda)\prod_{i=1}^{N}(\Lambda^2/\pi_{wi})\right] . \tag{7d}$$

The absence of m_{wi} is essential for $\pi_{wi}\sim\Lambda^2$ and consistent with the anomaly relation of the Konishi type.[9] The m_{ci} dependence is given by the instanton calculation for the gaugino and N massless flavors $(m_{wi}=0)$ (Ref. 15):

$$\prod_{i=1}^{M-N}m_{ci}\propto\langle\det(\phi_i^{(1)}\phi^{(2)j})\rangle\quad(i,j=1,\ldots,N) ,$$

which is satisfied by

$$\langle\det(\phi_i^{(1)}\phi^{(2)j})\rangle\sim\prod_{i=1}^{N}\pi_{wi}=\Lambda^{2N}\prod_{i=1}^{M-N}(m_{ci}/\Lambda) ,$$

fixing $f(\langle z\rangle)=0$. $f(Z)$ can be chosen to be

$$f(Z)=-\rho\ln(Z/\langle z\rangle) \tag{8}$$

with $\langle z \rangle$ = const and

$$\pi_{y1}\pi_{y2}\langle z \rangle = \Lambda^{2N} \prod_{i=1}^{M-N} (m_{ci}/\Lambda) .$$

III. APPROXIMATE SUSY

The effective potential $V_{\rm eff}$ for $\pi_{wi,ci,ya,\lambda}$ is given by

$$V_{\rm eff} = G_T \left[\sum_{i=1}^{N} |W_{;wi}|^2 + \sum_{i=1}^{M-N} |W_{;ci}|^2 \right] + \sum_{a=1}^{2} G_{Y(a)} |W_{;ya}|^2 + G_S |W_{;\lambda}|^2 + \sum_{i=1}^{M} M_i{}^2 + m_\lambda(\pi_\lambda + \pi_\lambda^*) , \quad (9)$$

where

$$M_i{}^2 = (\mu_{Li}{}^2 + \mu_{Ri}{}^2)\Lambda^{-2} |\pi_i|^2 + \mu_i{}^2(\pi_i + \pi_i^*) , \quad (10)$$

and $W_{;I} = \partial W_{\rm eff}/\partial \pi_I$ ($I = wi, ci, ya, \lambda$). The conditions on the VEV's are the following: For $\alpha = zf'(z)$, $\beta = z[zf'(z)]' = z\alpha'$,

$$G_T W^*_{;ci}\pi_\lambda/\pi_{ci} = G_S W_{;\lambda} + M_{ci}{}^2 + |\pi_{ci}|^2 G'_T \sum_j^{wi,ci} |W_{;j}|^2 , \quad (11a)$$

$$(1+\alpha)G_T W^*_{;wi}\pi_\lambda/\pi_{wi} = \beta X(\pi_w,\pi_y) + (1+\alpha)G_S W^*_{;\lambda} + M_{wi}{}^2 + |\pi_{wi}|^2 G'_T \sum_j^{wi,ci} |W_{;j}|^2 , \quad (11b)$$

$$\alpha G_{Y(a)} w^*_{;ya}\pi_\lambda/\pi_{ya} = \beta X(\pi_w,\pi_y) + \alpha G_S W^*_{;\lambda} - \alpha^2 |\pi_\lambda|^2 G'_{Y(a)} , \quad (11c)$$

$$G_T \left[\sum_{i=1}^{N} (w^*_{;wi}\pi_\lambda/\pi_{wi}) + \sum_{i=1}^{M-N} (W^*_{;ci}\pi_\lambda/\pi_{ci}) \right] + \alpha X(\pi_w,\pi_y) + (N-M)G_S W^*_{;\lambda} + m_\lambda \pi_\lambda + |\pi_\lambda|^2 G'_S |W_{;\lambda}|^2 = 0 , \quad (11d)$$

where $M_{wi} = M_i$ ($i = 1, \ldots, N$) and $M_{ci} = M_i$ ($i = M - N, \ldots, M$) with

$$M_{li}{}^2 = (\mu_{Lli}{}^2 + \mu_{Rli}{}^2) |\pi_{li}|^2 \Lambda^{-2} + \mu_{li}{}^2 \pi_{li} \quad (l = c, w) , \quad (12a)$$

and

$$X(\pi_w,\pi_y) = \left[\sum_{k=1}^{N} G_T(W^*_{;wk}\pi_\lambda/\pi_{wk}) - \sum_{a=1}^{2} G_{Y(a)}(W^*_{;ya}\pi_\lambda/\pi_{ya}) \right] . \quad (12b)$$

The chiral SU($M-N$) symmetry is spontaneously broken even in the absence of the explicit SU$(M-N)_{L,R}$ breakings because of the presence of $A_S W_{;\lambda}$ in addition to $M_{ci}{}^2$ in Eq. (11a). To examine this breaking in detail, we impose $\Lambda \gg M_{\rm SS}$ for the approximate SUSY and consider two cases: $m_{ci} \ll M_{\rm SS}$ and $m_{ci} \gg M_{\rm SS}$, where $M_{\rm SS}$ stands for the SUSY-breaking scale. The limit $m_{ci} \ll M_{\rm SS} \to 0$ gives the massless exact SUSY while the limit $m_{ci} \gg M_{\rm SS} \to 0$, the massive exact SUSY. $G'_{Y(a)} - G_{Y(a)} |\pi_{ya}|^{-2}$ = independent of a (=1,2) from Eq. (11c) requires the same behavior for π_{ya}. The exact SUSY given by $M_{\rm SS} \to 0$ thus yields the massless SQCD with $\pi_{y1} \sim \pi_{y2} \sim \Lambda^N$ and the massless SQCD with

$$\pi_{y1} \sim \pi_{y2} \sim \Lambda^N \left[\prod_{i=1}^{M-N} (m_{ci}/\Lambda) \right]^{1/2} .$$

The chiral-symmetry breakings of SU($M-N$) by M_{ci} ($i = 1, \ldots, M-N$) and of $U(1)'_A$ by $m_\lambda \pi_\lambda$ are included and $\pi_{wi} = \pi_w$ ($i = 1, \ldots, N$) is required. Since $|\pi_{ci}|/\Lambda^2 \ll 1$ for the approximate SUSY [and SU$(M-N)_L \times$SU$(M-N)_R$] M_{ci} are dominated by $\mu_{ci}{}^2\pi_{ci}$ for $\mu_{ci} \neq 0$. Our results are given for the simplest canonical kinetic terms: $G_i = A_i$ = const. We have checked that this simplification does not alter our results of the SUSY-breaking effects on π_λ and π_{ci} mainly because $|\pi_{ci}|/\Lambda^2 \ll 1$ and $G_S |\pi_\lambda|^2 |W_{;\lambda}|^2 / G_S W_{;\lambda} \ll 1$ and the extra terms of $\sum |W_{;j}|^2$ and $\alpha^2 |\pi_\lambda|^2$ only modify the coefficients in the calculated π_λ and π_{ci}.

A. $m_{ci} \ll M_{\rm SS}$

The function $f(Z)$ can be chosen in the form

$$f(Z) = \ln\{(Z - \langle z \rangle)^\rho [1 + a_1(Z - \langle z \rangle) + a_2(Z - \langle z \rangle)^2 + \cdots]\} , \quad (13)$$

which gives Eq. (6b) at $Z = \langle z \rangle$. By equating the leading terms of $z - \langle z \rangle$ with $a_n = 0$, we find that, for real VEV's,

$$A_T(\pi_\lambda/\pi_{ci})^2 = A_S W_{;\lambda} + M_{ci}{}^2 + A_T m_{ci}(\pi_\lambda/\pi_{ci}) , \quad (14a)$$

$$A_T(\pi_\lambda/\pi_w)^2 = -[(N-2)\alpha^2\beta]^{-1}(\alpha^2 + 2\beta)A_S W_{;\lambda} , \quad (14b)$$

$$A_{Y(a)}(\pi_\lambda/\pi_{ya})^2 = [(N-2)\alpha^2\beta]^{-1}(\alpha^2 + N\beta)A_S W_{;\lambda} , \quad (14c)$$

$$A_S W_{;\lambda} = \alpha^{-2}\beta \left[\sum_{i=1}^{M-N} M_{ci}^2 + m_\lambda \pi_\lambda \right] , \quad (14d)$$

where

$$\alpha = \rho z/(z - \langle z \rangle) , \quad (15a)$$

$$\beta = -\rho z \langle z \rangle/(z - \langle z \rangle)^2 , \quad (15b)$$

$$M_{ci}{}^2 \simeq \mu_{ci}{}^2 \pi_{ci} . \quad (15c)$$

These equations indicate the following properties. (1) $(\pi_{ci}/\pi_{wi})^2 \ll 1$ is ensured by the singular behavior of $f(Z)$ at $Z = \langle z \rangle$, which yields $(\pi_{ci}/\pi_{wi})^2 \sim (z - \langle z \rangle)^2 (\sim \alpha^{-2} - \beta^{-1})$. Had $f(Z)$ been given by $\sim \ln Z$ (Ref. 8) leading to $\alpha \neq 0$ and $\beta = 0$, $\pi_{wi} \sim \pi_{ci}$ would have been dynamically required as seen from Eqs. (11a) and (11b). (2) $2 < \rho < N$, $A_S W_{;\lambda} > 0$, and

$$A_T \pi_{y1}\pi_{y2} = [(\rho - 2)/(N - \rho)](A_{Y(1)}A_{Y(2)})^{1/2}\pi_w{}^2$$

are required from Eqs. (14b) and (14c). (3) The effects of

TABLE II. (a) The calculated VEV's for the chiral SU$(M-N)$ and U$(1)'_A$ breakings by (i) m_λ with $\gamma=(\rho-2)/(N-2)$ and (ii) μ_{ci} with $\gamma=n(\rho-2)/(N-2)(n-\rho)$ in the case of $m_{ci}\ll M_{SS}$. (b) The same as in (a) but in the case of $m_{ci}\gg M_{SS}$ with $\gamma=(1-\rho)[N-(N-2)\rho]$ and $M_{SS}=m_\lambda$ or $\sum\mu_{ci}^2/m_{ci}$. In the SUSY limit, $\pi_w^N\equiv\langle z\rangle\pi_{y1}\pi_{y2}=\Lambda^{2N}\pi(m_{ci}/\Lambda)$ satisfies the instanton-induced $\langle\det(\phi_i^{(1)}\phi^{(2)j})\rangle$ for $i,j=1,\ldots,N$.

(a)

	Case (i): m_λ	Case (ii): $\bar\mu_c=\mu_{ci}$ $(i=1,\ldots,n)\gg\mu_{ci}$ $(i=n+1,\ldots,M-N)$
π_{ci}	$\pm\sqrt{\gamma}\frac{\xi\pi_w}{\rho\langle z\rangle}$	$-\sqrt{\gamma}\frac{\xi\pi_w}{\rho\langle z\rangle}\left[1-\frac{\rho}{2n\sqrt{n-\rho}}\sum_{j=n+1}^{M-N}\left[\frac{\mu_{cj}}{\bar\mu_c}\right]^2\right]$ $(i=1,\ldots,n)$ $-\left[\frac{\gamma(n-\rho)}{n}\right]^{1/2}\frac{\xi\pi_w}{\rho\langle z\rangle}\left[1+\frac{\rho\sqrt{n-\rho}}{2n\sqrt{n}}\left[\frac{\mu_{ci}}{\bar\mu_c}\right]^2\right]$ $(i=n+1,\ldots,M-N)$
π_λ	$-\gamma\frac{m_\lambda\xi^2\pi_w^2}{\rho^3\langle z\rangle^2A_T}$	$\pm\left[\frac{n-\rho}{\rho}\right]^{1/2}\left[-\sum_{i=1}^{M-N}\frac{\mu_{ci}^2\pi_{ci}^3}{A_T}\right]^{1/2}$
ξ	$\xi^{[\rho-(M-N)]}=C\left[\frac{m_\lambda}{\Lambda}\right]^{M-N}$	$\xi^{[2\rho-(M-N)]}=C\left[\frac{\bar\mu_c}{\Lambda}\right]^{2(M-N)}$
ρ	$\max(2,M-N)<\rho<N$	$\max\left[2,\frac{M-N}{2}\right]<\rho<N,\ \rho<n<M-N$
C	$C=\left[\sqrt{\gamma}\left[\frac{\pi_w}{\rho^2\langle z\rangle A_T}\right]\right]^{M-N}\left[\frac{\Lambda}{\sqrt{\pi_w}}\right]^N$	$C=\left[\sqrt{\gamma}\left[\frac{(n-\rho)\pi_w}{\rho^2\langle z\rangle A_T}\right]\right]^{M-N}\left[\frac{\Lambda}{\sqrt{\pi_w}}\right]^N$

(b)

	Case (i): m_λ	Case (ii): $\mu_{ci}(i=1,\ldots,M-N)$
π_{ci}	$\frac{\pi_\lambda}{m_{ci}}(1+\Delta_{ci})$	
Δ_{ci}	$-(1-\rho)\left[\frac{\xi\pi_w}{\gamma A_T}\right]^2\left[\frac{M_{SS}}{m_{ci}}\right]^2$	$-\left[\frac{\xi\pi_w}{\gamma A_T}\right]^2\left[\frac{M_{SS}}{m_{ci}}\right]\left[(1-\rho)\left[\frac{M_{SS}}{m_{ci}}\right]-\gamma\left[\frac{\mu_{ci}}{m_{ci}}\right]^2\right]$
π_λ	$-\frac{M_{SS}\pi_w^2}{\gamma A_T}$	$-\frac{\pi_w^2}{\gamma A_T}\left[M_{SS}+\sum_{i=1}^{M-N}\frac{\pi_{ci}\mu_{ci}^2}{m_{ci}}\right]$
ρ	$0<\rho<1$	

m_{ci} can be treated as corrections of order m_{ci}/M_{SS} since $m_{ci}\ll M_{SS}$.

B. $m_{ci}\gg M_{SS}$

The function $f(Z)$ can be given by

$$f(Z)=-\ln\{(Z/\langle z\rangle)^\rho[1+b_1(Z-\langle z\rangle)+b_2(Z-\langle z\rangle)^2+\cdots]\} \quad (16)$$

with $\langle z\rangle=\text{const}$, which coincides with $f(Z)$ in Eq. (8) at $Z=\langle z\rangle$ in the SUSY limit, where $\pi_w^N=\Lambda^{2N}\prod_{i=1}^{M-N}(m_{ci}/\Lambda)=\langle z\rangle\pi_{y1}\pi_{y2}$. In leading order with $b_n=0$, giving $\alpha=-\rho$ and $\beta=0$, we find that

$$A_T(\pi_\lambda/\pi_{ci})[(\pi_\lambda/\pi_{ci})-m_{ci}]=A_SW_{;\lambda}+M_{ci}^2\ , \quad (17a)$$

$$A_T(\pi_\lambda/\pi_w)^2=(1+\alpha)^{-1}A_SW_{;\lambda}\ , \quad (17b)$$

$$A_{Y(a)}(\pi_\lambda/\pi_{ya})^2=-\alpha^{-1}A_SW_{;\lambda}\ , \quad (17c)$$

$$A_SW_{;\lambda}=-[N+(N-2)\alpha]^{-1}\left[\sum_{i=1}^{M-N}M_{ci}^2+m_\lambda\pi_\lambda\right]. \quad (17d)$$

The following points can be read off from these equations: (1) π_{ci} depend on m_{ci}; (2) $\rho=\{1+[\langle z\rangle(A_{Y(1)}A_{Y(2)})^{1/2}/A_T\pi_w^{N-2}]\}^{-1}$ is obtained by using $\pi_w^N=\langle z\rangle\pi_{y1}\pi_{y2}$ in the SUSY limit and Eqs. (17b) and (17c); (3) $0<\rho\ (=-\alpha)<1$ and $M_{ci}^2+m_\lambda\pi_\lambda<0$ for $A_SW_{;\lambda}>0$ are required from Eqs. (17b)–(17d).

Now let us consider the SUSY-breaking effects from (i) m_λ and (ii) M_{ci}. The calculated VEV's of π_{ci} and π_λ are listed in Table II. The results are summarized as follows: For $m_{ci}\ll M_{SS}$, $|\pi_{ci}|/\Lambda^2\sim\xi$ and $\pi_\lambda/\Lambda^2\sim-m_\lambda\xi$ with $\xi\equiv z-\langle z\rangle\sim(m_\lambda/\Lambda)^{(M-N)/[\rho-(M-N)]}$ and $\rho>M-N$ for $\xi\to0$ as $m_\lambda\to0$ in (i) and, in (ii), $\pi_{ci}/\Lambda^2\sim-\xi$ and $|\pi_\lambda|/\Lambda^2\sim\bar\mu_c\xi^{3/2}$ with $\xi\sim(\bar\mu/\Lambda)^{2(M-N)/[2\rho-(M-N)]}$ and $2\rho>M-N$; and for $m_{ci}\gg M_{SS}$, $\pi_\lambda\sim m_{ci}\pi_{ci}\sim-M_{SS}\Lambda^2$ with $M_{SS}=m_\lambda$ or $=\sum\mu_{ci}^2/m_{ci}$. In (ii), $\mu_{ci}\equiv\bar\mu_c$ $(i=1,\ldots,n)\gg\mu_{ci}$ $(i=n+1,\ldots,M-N)$ has been assumed for $m_{ci}\ll M_{SS}$. The ξ dependence of π_{ci} and π_λ is not specific to the canonical kinetic terms but also valid in noncanonical ones. Similarly for their M_{SS} dependence. The use of the canonical kinetic terms require constraints on ρ: $2<\rho<N$ and $\rho<n<M-N$ (thus, $n\geq3$) for $m_{ci}\ll M_{SS}$ and $0<\rho<1$ for $m_{ci}\gg M_{SS}$.

IV. SUMMARY AND DISCUSSIONS

We have discussed the symmetry breakings in SQCD with $N_{\text{color}}(=N)<N_{\text{flavor}}(=M)$ on the basis of the effective Lagrangian, which describes

$$\mathrm{SU}(M)_L\times\mathrm{SU}(M)_R\times\mathrm{U}(1)_A\rightarrow\mathrm{SU}(N)_{L+R}\times\mathrm{SU}(M-N)_L\times\mathrm{SU}(M-N)_R\times\mathrm{U}(1)'_V\times\mathrm{U}(1)'_A$$

in the massless SUSY limit. The explicit mass terms have been included to give further breakdown of the chiral $\mathrm{SU}(M-N)$ and $\mathrm{U}(1)'_A$ symmetries.

The difference between the massless and massive SQCD lies in the superpotential $W_{\rm eff}$ for $Y^{(1)}$ and $Y^{(2)}$ that generates the spontaneous $\mathrm{U}(1)_V$ breaking: $W_{\rm eff}\sim\ln(Y^{(1)}Y^{(2)}-\text{const})$ in the massless SQCD and $W_{\rm eff}\sim\ln Y^{(1)}Y^{(2)}$ in the massive SQCD. This behavior ensures consistent breakings with complementarity in massless SQCD and with instanton analysis in massive SQCD. For SQCD with $m_{ci}\ll M_{\rm SS}$, the breaking of the chiral $\mathrm{SU}(M-N)$ symmetry is induced once SUSY is broken even in the absence of the explicit $\mathrm{SU}(M-N)$-symmetry breakings. In the example of the $\mathrm{SU}(M-N)$ singlet m_λ, the order parameters are given by

$$\langle\phi_i^{(1)}\phi^{(2)i}\rangle\sim\Lambda^2\quad(i=1,\ldots,N)\,,$$

$$\sim\Lambda^2(m_\lambda/\Lambda)^{(M-N)/[\rho-(M-N)]}\quad(i=N+1,\ldots,M),$$

$$\langle\lambda\lambda\rangle\sim-\Lambda^3(m_\lambda/\Lambda)^{\rho/[\rho-(M-N)]}\,.$$

This is consistent with the result of QCD, the spontaneous breaking of chiral symmetries.

We make comments on SQCD viewed as the underlying dynamics of composite quarks and leptons. Composite quarks and leptons are contained in the Nambu-Goldstone superfields (NGS's) if $\mathrm{SU}(N)_{L+R}$ includes the weak isospin $\mathrm{SU}(2)_W$ while $\mathrm{SU}(M-N)_L\times\mathrm{SU}(M-N)_R$, the Pati-Salam $\mathrm{SU}(4)_C$ (Ref. 14) involving the color $\mathrm{SU}(3)_C^{\rm loc}$ of QCD (Ref. 16). For definiteness, we choose $N=4$ and $M=8$.

The large mass splittings between scalars and fermions are induced by the SUSY breakings from scalar masses $\gg m_{ci}$ and those between quarks and leptons in $T_i{}^m$ and $T_m{}^i$ and exotics in $T_m{}^{m'}$ $[i=1,\ldots,N\subset\mathrm{SU}(N)_{L+R};\ m,m'=N+1,\ldots,M\subset\mathrm{SU}(M-N)_{L,R}]$, leptoquarks and $\mathrm{SU}(3)_C^{\rm loc}$-octet fermions, can also be generated because these exotics are not in the Nambu-Goldstone modes. In fact, masses for these exotics are given by $\partial^2W_{\rm eff}/\partial\pi_m^{m'}\partial\pi_{m'}^m=\pi_\lambda/\pi_{ck}\pi_{ck'}\sim\bar\mu\xi^{-1/2}$ ($m=N+k;m'=N+k'$) which imply the decoupling in the SUSY limit ($\bar\mu=0$) because $\xi^{1/2}\sim(\bar\mu)^{1+\delta}$ with $\delta=2(M-N-\rho)/[2\rho-(M-N)]>0$ from $\rho<M-N$. SQCD with $N=M$ $(=6)$ is suffering from the existence of "light" leptoquarks.[13]

However, phenomenologically unfavorable mixings of the quark-lepton states with their mirror state are induced. The $\mathrm{SU}(N)_{L+R}$-preserving VEV's $\langle T_i{}^i\rangle|_{\theta=0}$, $\langle Y^{(1)}_{1\cdots N}\rangle|_{\theta=0}$ and $\langle Y^{(2)1\cdots N}\rangle|_{\theta=0}$ generate NGS's being admixtures of $T_m{}^i$ and $Y^{(1)}_{i_1\cdots i_{N-1}m}$ $(\equiv Y^i_m)$ and of $T_i{}^m$ and $Y^{(2)i_1\cdots i_{N-1}m}$ $(\equiv Y_i^m)$. It can be shown that composite fermions in (the mixed) NGS's and their orthogonal states acquire masses of $\sim\bar\mu\xi^{1/2}$ and $\sim\bar\mu\xi^{-1/2}$. The latter mass diverges in the SUSY limit, implying the decoupling of the one combination consistent with complementarity. If $T_i{}^m$ ($T_m{}^i$) are assigned to ordinary left- (right-) handed quarks and leptons, $\Phi_i^{(1)}$ ($\Phi^{(2)i}$) in $T_i{}^m$ ($T_m{}^i$) belong to doublets (singlets) of the $\mathrm{SU}(2)_L^{\rm loc}$ symmetry and then Y_i^m composed of $\Phi^{(2)}$ and Y_m^i of $\Phi^{(1)}$ become their mirrors.

SQCD provides a beautiful mechanism of generating light composite fermions. However, the dynamical consideration of the mass spectrum based on an effective Lagrangian for SQCD with $N<M$ has revealed an unpleasant feature: The appreciable mixings of quarks and leptons with their mirrors owing to the condensates that generate the "same" Nambu-Goldstone fields. Combining the previous negative result from SQCD with $N=M$, we conclude that the simplest supersymmetric gauge theory, SQCD, is not suitable to underlying dynamics for composite quarks and leptons. Therefore, other possibilities such as SQCD with extra symmetric $N(N+1)/2$ and/or antisymmetric $N(N-1)/2$ superfields should be considered[17] if SUSY has something to do with the compositeness of quarks and leptons.

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[16]Alternative choice is $M=6$ and $N=4$: $H=\mathrm{SU}(4)_{L+R}\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)'_V\times\mathrm{U}(1)'_A$, where $\mathrm{SU}(2)_L$ is the electroweak symmetry. The successive breakdown of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\to\mathrm{SU}(2)_{L+R}$ is induced by the SUSY breaking as $\pi_c\sim\xi\Lambda^2$ with ξ depending upon m_λ or $\bar{\mu}_c$ [see Table II(a)]. Since $\mathrm{SU}(4)_{L+R}$ is a vacuum symmetry, light leptoquarks and color-octet fermions are generated.

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