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Mass splitting of heavy baryon isospin multiplets

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We calculate the mass splittings of isospin multiplets of baryons containing one charmed or bottom quark, using a version of the chiral bag model which has proved successful in similar calculations of mass splittings of baryons containing only light quarks. In particular, we obtain the result $\Sigma_c^{++}-\Sigma_c^0=3.0$ MeV, in agreement with a recent measurement which gives 2.5 ± 1.0 MeV.

Macfarlane[1] recently reported a new preliminary measurement by ARGUS of the mass difference between the doubly charged and neutral charmed Σ baryons. The result of the group is that $\Sigma_c^{++}-\Sigma_c^0=2.5\pm1.0$ MeV, where our notation is that the symbol for a hadron denotes its mass. This result is at first very surprising, because the quark content of Σ_c^{++} is *uuc*, while that of Σ_c^0 is *ddc*. In all other previously measured hadrons, the replacement of a *u* quark by a *d* quark in an isospin multiplet increases the mass, as shown in Table I, taken from the tables of the Particle Data Group.[2] We also include in Table I the $\pi^+-\pi^0$ and $\rho^+-\rho^0$ mass differences for completeness, although the $d-u$ mass difference is not a relevant issue in these cases.

Despite the fact that the Σ_c^0 contains two more *d* quarks than the Σ_c^{++}, we contend that it is to be expected that the latter is heavier than the former, primarily because of the repulsive Coulomb forces between the quarks in the Σ_c^{++} baryon. In fact, some years ago one of us predicted[3] that the Σ_c^{++} would have a larger mass than the Σ_c^0 by 3.4 MeV.

In Ref. 3 we refer to a number of other calculations[4–9] of the mass difference $\Sigma_c^{++}-\Sigma_c^0$ with results that show quite a variation in magnitude and even differ in sign. A later paper by Richard and Taxil[10] makes predictions for two models which give opposite signs for the mass difference. A recent calculation by Chan[11] gives 0.32 MeV for the mass difference. In Table II we give a selection of the results of calculations of $\Sigma_c^{++}-\Sigma_c^0$ to show how sensitive the results are to the model used.

In view of this sensitivity, it is appropriate to consider the problem once again, this time using a bag model which has achieved excellent agreement with the experimental values of ordinary and strange-baryon isospin mass splittings.[12] This model makes use of ideas underlying chiral bag models[13] but also incorporates features of the original MIT bag.[14] Our calculation includes effects, such as the mass dependence of the binding energy and of the color-magnetic energy, which were omitted in the most successful of the earlier calculations.[3] We obtain agreement with the measured value of $\Sigma_c^{++}-\Sigma_c^0$ within the experimental error, and we make predictions for a number of other baryon mass splittings not yet observed.

The isospin mass splittings are often called electromagnetic mass splittings. However, we do not know the origin of the mass difference between the *u* and *d* quark, which is an essential feature of any explanation of hadron-mass differences. Once the *u* and *d* have different masses, then the strong interaction will lead to additional mass-dependent effects such as those we referred to in the previous paragraph.

The main features of the version[12] of the chiral bag model used here are first, that the bag has a small radius, second, that the one-gluon-exchange term can be treated in perturbation theory, and third, that the effect of the pion cloud can be treated in perturbation theory. Here we shall not repeat the details of the model, which are fully described in Ref. 12. Rather, we shall describe how we have adapted the model to the calculation of masses of baryons containing charmed or bottom quarks.

The model contains as input parameters the quark masses, the bag radius (which can vary depending on which quarks the bag contains), the running quark-gluon coupling strength α_s, and a phenomenological effective pion-quark coupling constant α_π. In the previous work[12]

TABLE I. Experimental values of isospin mass splittings in MeV from the Particle Data Group (Ref. 2) except for the $\Sigma_c^0-\Sigma_c^{++}$ mass difference, which is from Macfarlane (Ref. 1). In all cases except for the π and ρ, we subtract the mass of the member of the isospin multiplet containing more *u* quarks from the mass of the member containing more *d* quarks.

Hadrons	Mass difference (MeV)
$\pi^+-\pi^0$	4.61
$\rho^+-\rho^0$	$-0.3\ \pm2.2$
K^0-K^+	4.05 ± 0.07
D^+-D^0	$4.7\ \pm0.3$
B^0-B^+	$4.0\ \pm3.4$
$K^{*0}-K^{*+}$	$4.4\ \pm0.5$
$D^{*+}-D^{*0}$	$3\ \pm2$
$n-p$	1.29
$\Sigma^0-\Sigma^+$	3.09 ± 0.07
$\Sigma^--\Sigma^+$	7.97 ± 0.07
$\Xi^--\Xi^0$	$6.4\ \pm0.6$
$\Delta^0-\Delta^{++}$	$2.7\ \pm0.3$
$\Sigma^{*0}-\Sigma^{*+}$	$1\ \pm1$
$\Sigma^{*-}-\Sigma^{*+}$	$4.4\ \pm0.7$
$\Xi^{*-}-\Xi^{*0}$	$3.2\ \pm0.6$
$\Sigma_c^0-\Sigma_c^{++}$	$-2.5\ \pm1$

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TABLE II. A comparison of some calculated values of the mass difference $\Sigma_c^{++}-\Sigma_c^0$.

Reference	$\Sigma_c^{++}-\Sigma_c^0$ (MeV)
3	3.5
4	6.5
5	6.1
6	-6
7	-3 to -18
8	0.4
9	-2.7
10	-3 or $+2$
11	0.3
This calculation	3.0

some of these parameters were varied so as to obtain a best fit to selected data on the masses of baryons containing only u, d, and/or s quarks. Because of limits on computing time, certain ratios of bag radii, however, were not varied, but were taken the same as in the MIT bag model. Because these ratios differ from unity by only about 10 percent and because calculated isospin mass splittings are not very sensitive to small changes in bag radii, this approximation should not be too bad.

Specifically, it was found in Ref. 12 that a remarkably good simultaneous fit exists at a nucleon bag radius of around 0.45 fm for the baryon mass splittings including $n-p$, $\Sigma^- - \Sigma^+$, $\Xi^- - \Xi^0$, $\Sigma^0 - \Lambda$, $\Sigma^{*-} - \Sigma^{*+}$, $\Xi^{*-} - \Xi^{*0}$, $\Delta^{++} - p$, $\Xi^{*0} - p$, and $\Sigma^{*+} - p$. The value of the $d-u$ mass difference was found to be $m_d - m_u = 6.7$ MeV, with their mean mass constrained to be 10 MeV. The s-quark mass was 307 MeV. The value of α_s was 1.42, which is somewhat smaller than the original MIT value of 2.20 (for a nucleon bag radius of around 1 fm). The *effective* α_π was found to be 0.209, a value small enough to allow for a perturbative treatment.

Some comments are necessary about the bag radius R. In principle, R should be determined by minimizing the energy. In practice, it was determined in the model by fitting the mass differences of certain baryons.[12] The small value of R gives a good fit to the data partly because it leads to relatively large electromagnetic contributions. It remains to be seen whether other properties of baryons, such as magnetic moments, can be simultaneously fit with a bag radius as small as 0.45 fm. A nucleon radius of 0.45 fm can also be obtained by minimizing the total energy of the system with respect to R if either a larger value than in the MIT bag is assumed for the bag volume energy density B or a more negative "zero-point" energy $-Z_0/R$. For example, choosing $B^{1/4}=228$ MeV and $Z_0=2.33$, Carlson *et al.*[15] obtained a nucleon radius of 0.623 fm, which is considerably smaller than the MIT result ($R_N=0.987$ fm, corresponding to $B^{1/4}=145$ MeV and $Z_0=1.84$). As emphasized in Ref. 12, inclusion of isospin-violating effects results in a small difference in the neutron and proton radii, but, in view of the stability condition, this difference does not contribute to the neutron-proton mass difference in first order in δR.

The value of α_π was allowed to vary[12] in the fitting procedure partly because its value is somewhat model dependent and partly because a proper pion propagator should reflect the fact that pions should be excluded from inside the bag, while (limited by the amount of computing time required to search for best fits) the free propagator for pions was used. The value of α_π required for the best fit appears to be rather small, and it is yet to be seen whether the asymptotic πNN coupling can be reproduced. To this end, a minor modification of the model with an r-dependent pion-quark coupling[16] may be helpful. In this paper, we use the model as it is stated in Ref. 12.

In the present calculation, the values of m_u, m_d, m_s, and α_π are kept the same as in the earlier work. Although α_s should perhaps be allowed to decrease in accordance with asymptotic freedom, its value too is kept fixed in order to avoid the introduction of too many free parameters. The new parameters introduced beyond those of Ref. 12 are the masses of the c and b quarks and the bag radii of baryons containing c and b quarks. As we make clear in the following discussion, these parameters are fixed by considerations which do not involve $\Sigma_c^{++}-\Sigma_c^0$.

The bag radii should not change very much for the baryons we consider because these baryons all contain only one heavy quark and two light quarks. In the limit that the mass of the heavy quark becomes infinite, the bag radius should be determined from just the two light quarks. In the original MIT bag model,[14] it was found that the bag radius of the ρ and ω mesons was 6% smaller than the nucleon bag radius. Guided by this result, we let the bag radius of baryons containing a charmed quark by 5% smaller than the nucleon bag, and the radius of baryons containing a bottom quark be 7.5% smaller than the nucleon. In fact, our results for mass differences term out to be quite insensitive to small changes in the bag radii.

Once we fix the bag radii we choose the masses of the c and b so as to reproduce the experimental values of the Λ_c(2282 MeV) and Λ_b(5500 MeV) baryons, respectively. We find that although the values of the heavy-quark masses depend fairly sensitively on the bag radii, fortunately, the baryon mass differences do not. In fact, we have reduced the bag radii by varying amounts ranging from 0 to 10%, and found that the predicted isospin mass splittings are very insensitive to such variations, changing typically by only around 0.4 MeV. Therefore, within a few tenths of a MeV, the calculated isospin mass splittings can be regarded as definite predictions of the model.

For the sake of simplicity, we use a universal radius for baryons containing one charmed quark and another universal radius for baryons containing a single bottom quark. With a radius of 0.431 fm (i.e., smaller than the nucleons radius by 5%) for charmed baryons and 0.420 fm (i.e., smaller than the nucleon radius by 7.5%) for bottom baryons, we can reproduce the experimental values of the Λ_c and Λ_b baryons with heavy-quark masses given by

$$m_c = 1.87 \text{ GeV}, \quad m_b = 5.33 \text{ GeV} .$$

Our predictions for baryon isospin mass splittings are shown in Table III. It can be seen from this table that

TABLE III. Predicted values of isospin mass splittings of baryons containing one c or b quark. The experimental value is from Macfarlane (Ref. 1).

Baryons	Mass difference (MeV) Calculated	Mass difference (MeV) Experiment
$\Sigma_c^0-\Sigma_c^{++}$	-3.0	-2.5 ± 1
$\Sigma_c^+-\Sigma_c^{++}$	-3.5	
$\Sigma_c^{*0}-\Sigma_c^{*++}$	-2.7	
$\Sigma_c^{*+}-\Sigma_c^{*++}$	-3.3	
$\Xi_c^0(\Sigma)-\Xi_c^+(\Sigma)$	$+1.0$	
$\Xi_c^0(\Lambda)-\Xi_c^+(\Lambda)$	$+2.1$	
$\Xi_c^{*0}-\Xi_c^{*+}$	$+1.3$	
$\Sigma_b^--\Sigma_b^+$	$+7.1$	
$\Sigma_b^0-\Sigma_b^+$	$+1.5$	
$\Sigma_b^{*-}-\Sigma_b^{*+}$	$+6.5$	
$\Sigma_b^{*0}-\Sigma_b^{*+}$	$+1.2$	

TABLE IV. Predicted values of some other baryon mass splittings. The experimental value is from Macfarlane (Ref. 1). The masses m_c and m_b were adjusted to give $\Lambda_c=2282$ MeV, $\Lambda_b=5500$ MeV.

Baryons	Mass difference (MeV) Calculated	Mass difference (MeV) Experiment
$\Sigma_c^{++}-\Lambda_c^+$	164	168.3 ± 0.8
$\Xi_c^+(\Sigma)-\Xi_c^+(\Lambda)$	87	
$\Xi_c^0(\Sigma)-\Xi_c^0(\Lambda)$	86	
$\Xi_c^+(\Sigma)-\Sigma_c^{++}$	134	
$\Xi_c^{*+}-\Sigma_c^{*++}$	122	
$\Omega_c^0-\Sigma_c^{++}$	264	
$\Omega_c^{*0}-\Sigma_c^{*++}$	238	
$\Sigma_c^{*++}-\Sigma_c^{++}$	168	
$\Sigma_b^0-\Lambda_b$	227	

baryons containing a bottom quark agree with the usual rule that replacing a u quark by a d quark in a baryon increases the mass. However, baryons containing a charmed quark lead to several exceptions to this rule. It is gratifying that we obtain this sign reversal for the $\Sigma_c^{++}-\Sigma_c^0$ mass difference and also obtain a magnitude for the splitting which agrees with the preliminary measurement within experimental error.

It is a very good approximation that a single radius can be used for the baryons appearing in a given entry of Table III (and belonging to the same isospin multiplet). This makes our predictions very reliable, since the predicted isospin mass splittings are not very sensitive to reasonable adjustment of the chosen bag radius. To obtain 3.0 MeV for $\Sigma_c^{++}-\Sigma_c^0$, we find that the contributions from the electromagnetic energy, the color-electric energy, the color-magnetic energy, the pion cloud effect, and the quark eigenenergy difference are, respectively, $+8.21$, $+1.22$, $+0.02$, -0.03, and -6.44 MeV. The charmed quark with charge $+\frac{2}{3}$ makes the electromagnetic contribution large enough to overcome the effects of the $d-u$ mass difference and results in a positive prediction for $\Sigma_c^{++}-\Sigma_c^0$.

In addition to calculating isospin mass splittings, we have also used the same model to predict several other mass splittings as shown in Table IV. It may not be a good approximation to adopt a single radius for both members of each baryon pair which enter Table IV, especially for the $\Sigma_c^{*++}-\Sigma_c^{++}$ pair. Thus, the predictions listed in Table IV need to be modified when we have a better way of determining the radius of each baryon. Nevertheless, such modification should be small if both members belonging to the pair have approximately the same radius. In the case of $\Sigma_c^{++}-\Lambda_c^+$, for which recent measurements have been made,[1,2] the calculated value agrees surprisingly well with experiment. Here we note that $\Xi_c^+(\Sigma)$ and $\Xi_c^+(\Lambda)$, which differ in mass by 87 MeV according to our calculation, are, respectively, the *usc* baryons with octet wave functions in the forms of Σ and Λ. Likewise, the baryons denoted by $\Xi_c^0(\Sigma)$ and $\Xi_c^0(\Lambda)$ are the corresponding *dsc* octet baryons.[17] For completeness, we have also included in Table IV a predicted value (168 MeV) for $\Sigma_c^{*++}-\Sigma_c^{++}$. This specific value depends very much on whether one adopts the same radius for both Σ_c^{*++} and Σ_c^{++} (as we do).

To investigate the sensitivity of our predictions to the input parameters, we have decreased the radii for baryons containing one charmed or bottom quark by an additional 5%. Using these new radii, we can reproduce the experimental values of the Λ_c and Λ_b baryons with heavy-quark masses given by

$$m_c=1.83\text{ GeV},\ m_b=5.30\text{ GeV}\ .$$

Then we obtain 3.4 MeV for $\Sigma_c^{++}-\Sigma_c^0$, 167 MeV for $\Sigma_c^{++}-\Lambda_c^+$, and 236 MeV for $\Sigma_b^0-\Lambda_b$. These predictions differ very little from those given in Tables III and IV. On the other hand, if we use the nucleon radius for baryons containing one charmed or bottom quark, the values for m_c and m_b need to be adjusted slightly higher by about the same amount in order to reproduce the masses of Λ_c and Λ_b. We then obtain 2.6 MeV for $\Sigma_c^{++}-\Sigma_c^0$, 158 MeV for $\Sigma_c^{++}-\Lambda_c^+$, and 214 MeV for $\Sigma_b^0-\Lambda_b$.

In conclusion, although the model[12] was designed to treat baryons containing only the light quarks u,d,s, we have used the model (with only minimal changes) to predict the mass dependences of baryons containing one charmed or bottom quark. These predictions have been remarkably successful in the few instances we have been able to make contact with experiment. Experiments to measure the mass differences predicted in Tables III and IV will provide further tests of the model: i.e., a specific version of the chiral bag model in which the quark core of a baryon is small, and both the pion cloud and one-gluon exchange are treated perturbatively.

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[1] D. Macfarlane, in Proceedings of the Twenty-Third International Conference on High Energy Physics, Berkeley, California, 1986, edited by S. Loken (World Scientific, Singapore, to be published).

[2] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).

[3] D. B. Lichtenberg, Phys. Rev. D **16**, 231 (1977).

[4] C. Itoh, T. Minamikawa, K. Miura, and T. Watanabe, Prog. Theor. Phys. **54**, 908 (1975).

[5] S. Ono, Phys. Rev. D **15**, 3492 (1977).

[6] K. Lane and S. Weinberg, Phys. Rev. Lett. **37**, 717 (1976).

[7] N. Deshpande, D. Dicus, K. Johnson, and V. Teplitz, Phys. Rev. D **15**, 1885 (1977).

[8] L.-H. Chan, Phys. Rev. D **15**, 2478 (1977).

[9] C. S. Kalman and G. Jakimov, Lett. Nuovo Cimento **19**, 403 (1977).

[10] J. M. Richard and P. Taxil, Z. Phys. C **26**, 421 (1984).

[11] L.-H. Chan, Phys. Rev. D **31**, 204 (1985); Phys. Rev. Lett. **51**, 253 (1983).

[12] W-Y.P. Hwang, Phys. Rev. D **31**, 2826 (1985).

[13] G. E. Brown and M. Rho, Phys. Lett. **82B**, 177 (1979); G. E. Brown, M. Rho, and V. Vento, *ibid.* **84B**, 383 (1979); G. A. Miller, A. W. Thomas, and S. Théberge, *ibid.* **91B**, 192 (1980); R. L. Jaffe, in *Pointlike Structure Inside and Outside Hadrons,* proceedings of the Seventeenth International School of Subnuclear Physics, Erice, Italy, 1979, edited by A. Zichichi (Plenum, New York, 1982); W-Y.P. Hwang, Phys. Lett. **116B**, 37 (1982); Phys. Rev. D **29**, 1465 (1984).

[14] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974); A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, *ibid.* **10**, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, *ibid.* **12**, 2060 (1975).

[15] C. E. Carlson, T. H. Hansson, and C. Peterson, Phys. Rev. D **27**, 1556 (1983); **28**, 2895(E) (1983).

[16] W. Weise, in *Quarks, Mesons and Isobars in Nuclei,* proceedings of the 5th Topical School on Nuclear Physics, Motril, Spain, 1982, edited by R. Guardiola and A. Polls (World Scientific, Singapore, 1983); A. Faessler and F. Fernandez, Phys. Lett. **124B**, 145 (1983).

[17] J. Franklin, D. B. Lichtenberg, W. Namgung, and D. Carydas, Phys. Rev. D **24**, 2910 (1981).